F.J. Ragep

Naṣīr al-Dīn al-Ṭūsī's *Memoir on Astronomy*

(al-Tadhkira fī ᶜilm al-hay'a)

Volume I

Introduction, Edition, and Translation

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to Anwar to Lina

who endured
this most recalcitrant
of siblings

كم كتاب كتبته بيدي
سوف تبلا يدي ويبقى الكتاب
كانت الدنيا لقوم غيرنا رحلوا منها وخلوها لنا
سوف نسكنها ونرحل بعدهم
ونخليها كما خليت لنا

How many books have I written with my hand!
My hand shall wither but the book will remain.
Once the world was to another people
But they have since departed
Leaving it with us.
We too shall dwell in it
But we also shall follow them
Leaving it as it was left for us.

Anon., Damascus, Ẓāhiriyya MS 4871, f. 36a

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Preface

I was introduced to Ṭūsī and his *Tadhkira* some 19 years ago. That first meeting was neither happy nor auspicious. My graduate student notes from the time indicate a certain level of confusion and frustration; I seem to have had trouble with such words as *tadwīr* (epicycle), which was not to be found in my standard dictionary, and with the concept of solid-sphere astronomy, which, when found, was pooh-poohed in the standard sources. I had another, even more decisive reaction: boredom. Only the end of the term brought relief, and I was grateful to be on to other, more exciting aspects of the history of science.

A few years later, I found myself, thanks to fellowships from Fulbright-Hays and the American Research Center in Egypt, happily immersed in the manuscript collections of Damascus, Aleppo, and Cairo. Though I had intended to work on a topic in the history of mathematics, I was drawn, perhaps inevitably, to a certain type of astronomical writing falling under the rubric of *hay'a*. At first this fascination was based on sheer numbers; that so many medieval scientists could have written on such a subject must mean something, I told myself. (I was in a sociological mode at the time.) As I began to read, or rather try to read, these manuscripts, some of daunting size, I became more and more engaged in a world of mostly forgotten scientists, many from a period that modern scholarship had deemed, with the hubris that only modernism can muster, both invisible *and* unworthy. But these late medieval astronomers of "decline" seemed to me to be saying interesting things, and significantly they themselves thought they were saying interesting things as they spoke to one another over geographical and chronological distance. And two names kept recurring with astonishing frequency in these works: Naṣīr al-Dīn al-Ṭūsī and his *Tadhkira fī ʿilm al-hay'a*. I had come full circle.

To write this book, I have incurred enormous debts to both institutions and individuals, and it gives me great pleasure to be able at long last to express my gratitude in print. They are, of course, neither responsible for the opinions expressed nor for the remaining shortcomings. The National Endowment for the Humanities generously awarded me a research grant (RL-20578-84) that allowed me to work without interruption on the edition and translation. Another grant, this one from the National Science Foundation (SES-8618656), was for research on trepidation, and I have incorporated some of the results in the commentary. The Department of the History of Science at Harvard University appointed me a postdoctoral fellow on two occasions and provided me with the necessary facilities to take full advantage of those grants. A year at the Society for the Humanities at Cornell was enormously stimulating and gave me the opportunity both to learn and to test my ideas with a very talented group of non-historians of science. Travel funds from the Department of the History of Science at the University of Oklahoma allowed me to check and recheck countless details and footnotes during preparation of the final copy.

During the many years of research and writing, I have benefited from my acquaintance and friendship with a number of extraordinary individuals, far more than I had ever expected to meet in a lifetime. In Cairo, I had the great fortune to have Edward S. Kennedy and David King as next-door neighbors *(ya^{c}nī)*, and they were, despite some scepticism concerning my project, always ready and able to provide advice and guidance on matters great and small. Also in Cairo, Aḥmad Harīdī was unceasingly patient in teaching this *khwāja* about the beauty and intricacy of Arabic and Arabic paleography. Back home, Aron Zysow, my next-carrel neighbor at Widener Library, shared his very considerable knowledge of Islam and Islamic history, Wheeler Thackston offered advice and assistance on all things Persian, and Marina Tolmacheva graciously looked over the maps and made several helpful suggestions. Over many long, sometimes difficult years, Raine Daston, Mollie Palchik, and Noel Swerdlow provided inspiration and encouragement and were always there when needed most.

A. I. Sabra, *shaykhunā al-ra'īs*, was the one who introduced me to the *Tadhkira* those many years ago and, for reasons known only to himself, thought that I could be entrusted with its study. His teachings, methodology, and inspiration are such an integral part of this work that it would be less than elegant to provide a list; let me simply say that it was he who made *hay'a* such an important part of my intellectual vocabulary.

D. E. Pingree, *ḥakīm extraordinaire*, has been unceasing in his efforts to help me broaden my horizons and use what he calls common sense, which, in his version, is quite uncommon. I very much benefited from his critique and suggestions on an earlier draft of a chapter concerning the tradition of the *Tadhkira*, and his "common sense" led me to uncover the relationship between the various versions of the text.

G. J. Toomer, φιλόπονος τε ὁμοῦ καὶ φιλαλήθης, bestowed upon this book a degree of attention that went far beyond what the designation editor normally calls for. He saved me from innumerable blunders of detail and interpretation, and his stern kindness made me feel that I should and could meet his soaring standards.

S. P. Ragep, friend and co-worker of some twenty years, drafted and photographed the figures and collated them with those in the manuscripts, prepared the concordance of manuscripts, typed the Arabic text and apparatuses (in multiple versions), offered advice on both style and substance, much of which was adopted, helped prepare the index, and somehow managed in addition to have a career and the dedicatees. But above all, she helped me find words and meaning where once there was only a cacophony of silence.

F.J.R.

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Part I

General Introduction

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§1. Naṣīr al-Dīn al-Ṭūsī

A. Life

1. Introduction

Abū Ja^{c}far Muḥammad b. Muḥammad b. al-Ḥasan Naṣīr al-Dīn al-Ṭūsī was born on Saturday (at dawn according to Riḍawī[1]), 11 Jumādā I, 597 H./17 February 1201 A.D. in Ṭūs or its environs[2] and died in Baghdad (at sunset according to Baḥrānī[3]), 18 Dhū al-Ḥijja 672 H./25 June 1274 A.D. In a remarkable century that began with the death of Chu Hsi and further witnessed the birth, death or both of such luminaries as Roger Bacon, Ibn ^{c}Arabī, Maimonides, Gregory Chioniades, Ibn Taymiyya, Thomas Aquinas and Levi ben Gerson, Ṭūsī's breadth and depth of learning stand out. Even in his lifetime he was something of a legend and acquired the honorific title of *khwāja* (distinguished scholar and teacher);[4] later he came to be referred to by the rather more impressive *ustādh al-bashar* (teacher of mankind) and *al-mu^{c}allim al-thālith* (the third teacher, *i.e.* after Aristotle and Fārābī).[5] After his death, Ṭūsī's influence, which can only be described as monumental, continued in fields as diverse as ethics, natural philosophy, mathematics, Sufism, astronomy, *kalām* (dialectical theology), *fiqh* (law), and logic.

A full-scale political and intellectual biography of Naṣīr al-Dīn, though much needed, would be out of place here; my much more limited aim is simply to provide a chronology of his life and to give a sense of why he maintained a

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[1] *Aḥwāl*, p. 3.
[2] Naṣīr al-Dīn gives his place of origin as Ṭūs in the introduction to the *Īlkhānī Zīj*; see Boyle [1963], pp. 246–247. Ṭūs may indicate either the district or city in Khurāsān; the city itself, near modern Mashhad, was destroyed in 1389 A.D. ("Ṭūs," *EI*^{1}, 4: 974).
[3] *Lu'lu'at al-Baḥrayn*, p. 246.
[4] Bar Hebraeus (1226–1286), who knew Naṣīr al-Dīn at Marāgha, calls him "*Khwāja Naṣīr*" (*Chronography*, 1: 451).
[5] Riḍawī, *Aḥwāl*, p. 3.

lifelong interest in astronomy. But even this circumscribed goal must inevitably lead us into difficult questions of the typicalness of his education, the role of Hellenism in his worldview, the importance of patronage on the quantity and content of his scientific production, and the impact of his changing religious beliefs on his science. In order to approach these questions and, in general, to organize the details of Ṭūsī's multifaceted life, I shall make a basic assumption, namely that Ṭūsī was committed throughout his life, though in varying degrees, to what we may broadly call a Hellenistic attitude. This does not mean that he abandoned Islam (though he was accused of such); it does mean that he was heir to the intellectual traditions of late Greek antiquity that had begun to be "naturalized"[6] within an Islamic context by such personages as Kindī (fl. 3rd/9th c.), Fārābī (fl. 4th/10th c.) and Ibn Sīnā (fl. 5th/11th c.). Ṭūsī's contribution to this tradition was to continue the rapprochement of Hellenism and the Islamic disciplines. But more fundamentally, Ṭūsī's Hellenism led him to pursue knowledge for its own sake; thus we would be deeply mistaken to view his pursuit of the sciences as being primarily for the purpose of serving the practical needs of the Islamic community or his patrons. Only by situating Ṭūsī within this "ancient" tradition can we hope to comprehend his lifelong love and devotion to astronomy, a commitment that should be understood as epistemological rather than utilitarian.[7]

2. Education and Early Life

How did one come to be educated in this Hellenistic tradition over and above the more customary religious one? Although Ṭūsī's education may not be typical, it does provide some insight into the factors that would lead a young student to pursue the ancient sciences and the manner in which he might go about such a task. Over the course of his life, Naṣīr al-Dīn would travel widely to satisfy his craving to be educated in virtually all fields of learning; his first instruction, however, occurred in his own home under the tutelage of his father, Wajīh al-Dīn Muḥammad b. al-Ḥasan al-Ṭūsī. Wajīh al-Dīn could trace his own education through a line of Shīᶜite scholars back to ᶜAlam al-Hudā al-Murtaḍā (355–436/966–1044), the *naqīb* (head) of the ᶜAlīds in Baghdad and defender of

[6] This term was coined by Sabra [1987a].

[7] This *should* be obvious; however the insistence by numerous modern commentators that Islamic scientists could never rise to the Greek view that knowledge was to be pursued for its own sake compels one to state the obvious.

the Imāmī (Twelver) Shīᶜites[1] against the attacks of the Muᶜtazilite[2] Judge ᶜAbd al-Jabbār (died 415/1024);[3] he continued this lineage by teaching his son *sharīᶜa* (law) through the reading of al-Murtaḍā's works. The young Naṣīr al-Dīn also studied with his father's maternal uncle, Naṣīr al-Dīn ᶜAbdallāh b. Ḥamza al-Ṭūsī, another eminent Imāmī scholar, and his own maternal uncle Nūr al-Dīn ᶜAlī b. Muḥammad al-Shīᶜī.[4] But there was evidently more to Naṣīr al-Dīn's family than a simple adherence to exoteric Shīᶜite dogma. In his so-called autobiography, Ṭūsī tells us that his father, someone who had seen the world [*jahān-dīdah*], encouraged the young boy to study different sciences and listen to the masters of various sects and opinions.[5] This seemingly broad-minded attitude may have been indirectly inspired by Tāj al-Dīn al-Shahrastānī, who had been a teacher of Naṣīr al-Dīn ᶜAbdallāh b. Ḥamza, who had in turn taught his nephew, *i.e.* Ṭūsī's father. Tāj al-Dīn had what Madelung calls "crypto-Ismāᶜīlī thought,"[6] and this may have given the otherwise rather strait-laced Twelver family a somewhat more "worldly" view; at least it may have been part of the widened horizon that would lead the young Naṣīr al-Dīn to give both philosophy and perhaps the heretical Ismāᶜīlī doctrine a hearing. At any rate, Ṭūsī took his father up on his suggestion and studied the branches of philosophy (*ḥikma*) and especially mathematics with a certain Kamāl al-Dīn Muḥammad al-Ḥāsib, a student of Afḍal al-Dīn al-Kāshī.[7]

After this initial period of his education, Naṣīr al-Dīn apparently traveled to Nīsābūr, a major city in Khurāsān located to the west of Ṭūs, to study with the noted physician Quṭb al-Dīn al-Miṣrī and with the polymath Farīd al-Dīn Dāmādh. Both provide us with evidence that he was actively pursuing further education in the ancient sciences and philosophy.

Quṭb al-Dīn, though originally from the Maghrib, had studied for some time in Egypt (whence the appellation *al-Miṣrī* [the Egyptian]), and then had traveled

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1 Shīᶜa is the designation of a large minority group in Islam (the majority being called Sunnīs) that are united in their belief in the legitimacy of the claim of ᶜAlī, the son-in-law of the prophet Muḥammad, and his family to the caliphate. Though there were (and are) several sects that can be called Shīᶜite, the two we shall be concerned with in this chapter are the Imāmīs (or Twelvers) and the Ismāᶜīlīs (or Seveners), the two disagreeing on the identity (whether twelfth or seventh) of the last, and at present hidden, Imām. Today the Imāmīs, by virtue of their majority status in Iran, are the largest Shīᶜite sect. The Ismāᶜīlīs, who are now led by the Āghā Khān, are a relatively small group that is no longer the major political force it was during the Middle Ages.

2 The Muᶜtazilites were rationalist theologians who held a predominant position during the early centuries of ᶜAbbāsid rule.

3 Brockelmann, "al-Murtaḍā," *EI*¹, 3: 736.

4 Ibn al-Fuwaṭī, vol. V (in *Oriental College Magazine*, suppl. 1939), p. 177; Riḍawī, *Aḥwāl*, pp. 158–161.

5 Ṭūsī, *Sayr*, p. 38.

6 Madelung [1985], pp. 87–89.

7 Ṭūsī, *Sayr*, p. 38.

eastward to study with the well-known *mutakallim* (dialectical theologian) Fakhr al-Dīn al-Rāzī (ca. 543–606/1149–1209). Eventually he came to reside in Nīsābūr where Ṭūsī studied medicine and *ḥikma* (presumably philosophy) under him.[8] Quṭb al-Dīn wrote a commentary on the *Kulliyyāt* (which is the part dealing with general topics) of Ibn Sīnā's medical work, the *Qānūn*, in which he is reported to have been rather critical of the author.[9]

Farīd al-Dīn Dāmādh was well-versed in philosophy, *uṣūl* (jurisprudence), logic, and medicine.[10] He also had studied with Fakhr al-Dīn al-Rāzī and, of even more interest, could trace his pedagogical lineage back to Ibn Sīnā.[11] Presumably then, Naṣīr al-Dīn studied the works of his famous Persian predecessor with Farīd al-Dīn.[12]

We can, with a fair degree of confidence, place Naṣīr al-Dīn in Nīsābūr sometime after 610/1213, when he would have been a mere lad of twelve or thirteen, and before 618/1221, when the armies of Chingiz Khān completed their devastation of Khurāsān. Indeed, Ibn abī Uṣaybiᶜa informs us that Quṭb al-Dīn al-Miṣrī was one of the victims of the final Mongol onslaught on Nīsābūr that occurred in that year.[13]

Perhaps because of the precarious situation in his native Khurāsān, or perhaps simply out of a desire to further his studies, Naṣīr al-Dīn traveled to Iraq[14] to study with the Shīᶜite legal scholar Muᶜīn al-Dīn Sālim b. Badrān al-Miṣrī[15] and with the extraordinary Kamāl al-Dīn ibn Yūnus (551–639/1156–1242), whose expertise ranged from Qur'ān exegesis to the *Almagest*.[16]

We know from several sources that Ṭūsī studied with this Muᶜīn al-Dīn; in addition, there is extant an *ijāza* (a license allowing the recipient to teach a given work) granted by Muᶜīn al-Dīn to Naṣīr al-Dīn in 619/1222.[17] Little more is

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8 Riḍawī, *Aḥwāl*, pp. 167–169.

9 Ibn abī Uṣaybiᶜa, *ᶜUyūn*, 2: 30.

10 Ibn al-Fuwaṭī, vol. IV (Damascus, 1962–65), 3: 458–459 and 466.

11 Shūshtarī gives the names of four intermediaries (*Majālis*, 2: 203 [7th *Majlis*]).

12 Apparently Zand has claimed that Ṭūsī studied Ibn Sīnā's *Al-Ishārāt wa-'l-tanbīhāt* with Farīd al-Dīn. I do not have access to Zand's article and have been unable to confirm this independently; I here depend on Siddiqi [1963], 1: 564–565.

13 Ibn abī Uṣaybiᶜa, *ᶜUyūn*, 2: 30; Juwaynī, *History*, 1: 176–178.

14 On this early trip to Iraq (Baghdad and/or Mosul), see Siddiqi [1963], p. 565 (quoting Zand) and Riḍawī, *Aḥwāl*, p. 179. In the *Sayr* (p. 41), Ṭūsī refers to a trip from Iraq to Khurāsān.

15 I am basing Muᶜīn al-Dīn's residence in Baghdad on the citation by Siddiqi of the Zand article.

16 Ibn Khallikān, *Wafayāt*, 5: 311–313 (English trans. 3: 467–468); see also Ibn abī Uṣaybiᶜa, *ᶜUyūn*, 1: 306–308.

17 This was for Ṭūsī's successful completion of the study of the third part and most of the second of a work on Shīᶜite law by ᶜIzz al-Dīn Ḥamza b. Zuhra, who had been Muᶜīn al-Dīn's teacher. Ibn Zuhra himself was apparently a mainstream Twelver Shīᶜite. (I am indebted to A. Zysow for this information.) We also know that Naṣīr al-Dīn collated the third part of this work, completing it in 624/1227 (Riḍawī, *Aḥwāl*, pp. 163–166).

known about Mu^cīn al-Dīn, however. That he or his ancestors came from Egypt (or perhaps simply passed through as we saw above with Quṭb al-Dīn al-Miṣrī) is clear from his name. Ṣafadī and Kutubī both inform us[18] that he was also *al-Mu*ᶜ*tazilī*[19] and *al-Rāfiḍī*.[20] Despite the little we do know about him, he does seem to have played an important part in Naṣīr al-Dīn's education. Ibn Kathīr (ca. 700–74/1300–73), for example, states that his basic education came from Muᶜīn al-Dīn who, he continues, influenced Ṭūsī to such an extent that he undermined his religious belief (*afsada i*ᶜ*tiqādahu*).[21] Ibn Kathīr, who held strong Ḥanbalī biases,[22] may have somewhat exaggerated the influence of Muᶜīn al-Dīn who was, as far as he was concerned, twice cursed—once with Shīᶜism and a second time with Muᶜtazilism. We should keep in mind, however, that his influence is also implied in the account of Ṣafadī/Kutubī by the fact that he is referred to twice. Furthermore he is one of only two of Naṣīr al-Dīn's teachers who there receive specific mention.

The other is Kamāl al-Dīn ibn Yūnus.[23] In all probability, Naṣīr al-Dīn studied astronomy and mathematics with Kamāl al-Dīn who was noted for his

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[18] Ṣafadī, *Wāfī*, 1: 179; Kutubī, *Fawāt*, 3: 246. The biography of Ṭūsī given in these two works is virtually identical; Iḥsān ᶜAbbās in the introduction to his edition of Kutubī would seem to explain this not uncommon occurrence by asserting that Kutubī copied much of his work from Ṣafadī (1: 4–5). Though both men were contemporaries and indeed died in the same year (764/1363), Ṣafadī seems to have completed his work first.

[19] There is a possibility that *al-Mu*ᶜ*tazilī* may be a misreading of *al-Māzinī*, which is given by Muᶜīn al-Dīn in the *ijāza* referred to above. However, we should note that *al-Mu*ᶜ*tazilī* is also recorded by Ibn Kathīr.

[20] During this period, the Imāmī Shīᶜites had accepted some of the doctrines of the Muᶜtazilites; thus it is not surprising that one person could be considered to be a member of both groups despite the earlier animosity between them. (This is a rather complicated issue; for some indication of the relationship between Muᶜtazilism and Shīᶜism, see Madelung [1970] and Momen [1985], pp. 76–82.) *Al-Rāfiḍī* may identify him as belonging to a certain Shīᶜite sect that formed in opposition to Zayd b. ᶜAlī's (d. 122/740) refusal to allow the denunciation of the Companions of the Prophet, or, more probably, it may simply identify him as a Shīᶜite. (For this latter use of *Rāfiḍa* by the opponents of Shīᶜism, see Madelung [1970], p. 3; for the former usage, see Aḥmad Amīn, *Ḍuḥā*, 3: 136 and Lane, *Lexicon*, 3: 1121.)

[21] Ibn Kathīr, *Bidāya*, 13: 267–268.

[22] The Ḥanbalīs were hostile to speculative theology as well as to legal approaches that did not adhere to a strict reading of the Qur'ān and the traditions of the Prophet.

[23] Because of his great fame—at least among his contemporaries—one would have expected that many of Naṣīr al-Dīn's biographers would have recorded that he studied with the great scholar of Mosul. Surprisingly, we find this information only in Ṣafadī/Kutubī, who rely on the authority of al-Shams, the son of Mu'ayyad al-Dīn al-ᶜUrḍī (Ṣafadī, *Wāfī*, 1: 181; Kutubī, *Fawāt*, 3: 249.) Since ᶜUrḍī was one of Naṣīr al-Dīn's associates at the Marāgha observatory, one can probably accept this report as reliable, especially in that al-Shams correctly cites Muᶜīn al-Dīn b. Badrān as another of Ṭūsī's teachers.

expertise in these subjects;[24] it is less probable that Ṭūsī pursued religious studies with him since Kamāl al-Dīn was a Sunnī of the Shāfiᶜite school of law.[25] Ibn al-Fuwaṭī tells us that students came to study with him from east and west (*i.e.* of Islamdom); we may assume that Ṭūsī was one of them, most likely traveling to Mosul sometime in the 620's/1223–32 after completing his legal studies with Muᶜīn al-Dīn b. Badrān.[26]

Although there can be little doubt that both Naṣīr al-Dīn and his education were in many ways unique, we may yet draw some preliminary conclusions that can help us understand the initiation of a 13th c. individual into the "Ancient Sciences." An obvious point is that during this period the rift between those who studied the *awā'il* sciences and those who dealt mainly with the Islamic sciences was not as great as it had been in earlier centuries.[27] Farīd al-Dīn Dāmādh, Quṭb al-Dīn al-Miṣrī, Kamāl al-Dīn b. Yūnus, and Naṣīr al-Dīn himself are clear examples of this trend to do work in both the *manqūlāt* (the transmitted, *i.e.* traditional Islamic, sciences) and the *maᶜqūlāt* (the rational sciences).[28] It is also evident from the diverse religious affiliations of these individuals that we are here dealing with a tendency current among both Shīᶜites and Sunnīs.[29]

An important consequence of the closing of the gap between religious and nonreligious studies was the opportunity this afforded to bring the *awā'il*

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[24] Ibn Khallikān, *Wafayāt*, 5: 311–312 (English trans., 3: 467–468); Ibn abī Uṣaybiᶜa, *ᶜUyūn*, 1: 306.

[25] The complexity of medieval Islam should always lead one to be tentative when making such assertions, however. It may not be irrelevant to note here that Kamāl al-Dīn is reported by Ibn Khallikān (*Wafayāt*, 5: 312; English trans., 3: 468) to have taught both the Torah and New Testament to Christian and Jewish students (*ahl al-dhimma*).

[26] Kamāl al-Dīn was associated with a number of schools in Mosul [Ibn al-Fuwaṭī, vol. V (in *Oriental College Magazine*, suppl. 1940), p. 293; Ibn Khallikān, *Wafayāt*, 5: 311, 313, 316 (English trans., 3: 467, 469, 472); Ibn abī Uṣaybiᶜa, *ᶜUyūn*, 1: 306] and would seem to have spent most of his time there except for several years of study at the Niẓāmiyya College in Baghdad in the early 570's/ca.1175 [Ibn Khallikān, *Wafayāt*, 5: 311 (English trans., 3: 467)].

[27] Cf. Sabra [1987a], pp. 236–238.

[28] For another example, see Makdisi's [1981] account of ᶜAbd al-Laṭīf al-Baghdādī (d. 629/1231), pp. 84–91.

[29] We should not conclude, however, that all the educated felt compelled to study philosophy or mathematics. Ibn Khallikān relates the story of a certain religious personage, Ibn al-Ṣalāḥ, who wished to study logic—in secret—with Kamāl al-Dīn b. Yūnus. After some time and little progress, Kamāl al-Dīn advised him to give up the study which was not going to do him any good on the one hand and might besmirch his reputation on the other. Kamāl al-Dīn himself was accused of suspect religious beliefs because of his assiduous devotion to the rational sciences. But it is important to note that Ibn Khallikān, himself a Shāfiᶜī jurist and sometime *Qāḍī al-Quḍāt* (Chief Justice) in Damascus, is apologetic for bringing up such matters; in general, he cannot restrain his admiration for Kamāl al-Dīn's erudition in *all* of the sciences.

sciences into the religious schools (*madrasa*; pl. *madāris*).[30] Though this probably did not mean that mathematics, the sciences, and philosophy became a part of the curriculum of the *madrasa*, whose purpose, above all, was the teaching of law, it did mean that someone like Kamāl al-Dīn b. Yūnus, who directed or taught at various schools in Mosul, had the opportunity to teach nonreligious subjects if he so chose.[31] Naṣīr al-Dīn's appropriation of *waqf* (religious endowment) funds at the Marāgha observatory, one of the few known instances of the use of such funds for a nonreligious purpose, may perhaps be better understood in the context of this changing relation between religious and nonreligious disciplines.[32]

A second point that should be stressed concerning Ṭūsī's education is that a student was willing to travel a considerable distance to study with the teacher of his choice. The trek of Naṣīr al-Dīn from Khurāsān to Mosul is an example of this; we have also seen that Quṭb al-Dīn al-Miṣrī traveled from the Maghrib to Egypt and from there on to Nīsābūr. Furthermore, one may conclude that, based on the example of Ṭūsī, a Shīᶜī would not hesitate to study with persons of other sects. Kamāl al-Dīn b. Yūnus and Quṭb al-Dīn al-Miṣrī, for example, were both Sunnīs of the Shāfiᶜī school of law. As we shall see in the concluding section below, this broad-minded attitude is an important part of Ṭūsī's mature philosophy.

3. The Ismāᶜīlī Stage

His studies completed, the young Naṣīr al-Dīn faced the unenviable task of finding an appropriate position. We know that sometime around 632/1235 Naṣīr al-Dīn found a patron in ᶜAbd al-Raḥīm b. abī Manṣūr Nāṣir al-Dīn Muḥtasham (died 655/1257), the last Ismāᶜīlī governor of Qūhistān, who ruled from the town of Qā'in during the reigns of the seventh and eighth Grand Masters of Alamūt, ᶜAlā' al-Dīn Muḥammad (ruled 618–53/1221–55) and his son Rukn

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30 Makdisi in various places (*e.g.* [1981]), pp. 77–80) has emphasized that the foreign sciences were formally excluded from the *madrasa*; as he puts it, "neither the madrasa nor its cognate institutions harbored any but the religious sciences and their ancillary subjects." But even a superficial perusal of the situation, including Makdisi's own examples of *actual* curricula (pp. 81–91), is enough to convince one that the individualistic nature of the teaching in the *madrasa* allowed the "forbidden" subjects to be brought in, especially in the later Middle Ages. Makdisi's discussion takes this into account but without acknowledging the puzzle that he has foisted on his reader. For a rather more balanced approach and a critique of Makdisi, see Sabra [1987a], pp. 232–235.

31 For example, ᶜAlam al-Dīn Qayṣir b. abī al-Qāsim ᶜAbd al-Ghaniy Taᶜāsīf traveled specifically to study music with Kamāl al-Dīn, who was happy to be able to teach this rarely requested subject.

32 Sayili [1960], pp. 207–211.

al-Dīn Khurshāh (ruled 653–54/1255–56).[1] The Nizārī Ismāʿīlīs, whose allegiance was to Nizār, the defeated son of the Fāṭimid[2] Imām al-Mustanṣir (died 487/1094), had in the late eleventh and early twelfth centuries established themselves in Syria and Iran under their redoubtable leader Ḥasan-i Ṣabbāḥ.[3] Their headquarters were located at the fortress of Alamūt in the Alburz mountains of Daylamān from which they ruled, in theory, the other Ismāʿīlī strongholds in Iran and Syria, some of which were located in Qūhistān in eastern Iran.

What led Ṭūsī to enter the service of the Ismāʿīlīs, a group greatly despised by the Sunnīs and more often than not shunned by the other Shīʿa? Naṣīr al-Dīn gives us *one* answer in his *Risālah-i sayr wa-sulūk* (Epistle on the Journey and Conduct). Naṣīr al-Dīn in no uncertain terms tells us that he had become disillusioned with the exoteric Islam of his childhood and had "converted" to esoteric Ismāʿīlism because he felt that one needed the divinely inspired Ismāʿīlī Imām to make the Truth known.[4] But after the fall of the Ismāʿīlī strongholds to the Mongols in 1256, Ṭūsī had a rather different story to tell. In the introduction to the *Īlkhānī Zīj*, for example, he states:

> At the time that [Hülegü] seized the dominions of the heretics [*i.e.* the Ismāʿīlīs], I Naṣīr al-Dīn who am of Ṭūs and had fallen into the power of the heretics—me he brought forth from that place and ordered to observe the stars.[5]

Furthermore in the *Akhlāq-i Nāṣirī*, he tried to justify his earlier praise of the "heretics":

> ...to save both himself and his honour, he [*i.e.* Naṣīr al-Dīn speaking in the third person] completed the composition of an exordium in a style appropriate to the custom of that community for the eulogy and adulation of their lords and great ones. This is in accordance with the sense of the verse:
>
> And humour them while you remain in their house;
> And placate them while you are in their land.

[1] 632/1235 is the date for the completion of the *Risālah-i Muʿīniyya*, which Ṭūsī dedicated to Muʿīn al-Dīn, the son of Nāṣir al-Dīn Muḥtasham. The *Akhlāq-i Nāṣirī*, which he dedicated to Nāṣir al-Dīn himself, was probably completed about 633/1235–36 (see *Nasirean Ethics*, pp. 9, 178 and Riḍawī, *Aḥwāl*, p. 454).
[2] The Fāṭimids were an Ismāʿīlī dynasty that ruled in parts of North Africa (and eventually Egypt) from 297/909 until 567/1171.
[3] The fundamental work on the Nizārī Ismāʿīlīs is Marshall Hodgson [1955], *The Order of Assassins: The Struggle of the Early Nizārī Ismāʿīlīs against the Islamic World.*
[4] See pp. 15–17 for a discussion of this "conversion."
[5] Translated by Arberry [1958], pp. 259–260.

> And also the well-attested tradition: "With whatsoever a man protects himself and his honour, it shall be recorded to him as a favour." While such a course is contrary to the belief, and divergent from the path, of the People of the Sharīᶜa and the Sunna, there was nothing else I could do.[6]

Lending some credence to Ṭūsī's obviously self-serving recantation is the historical reality that he, as many Persians, had to face during the period of the Mongol invasions. Naṣīr al-Dīn, after his sojourn in Iraq, would have needed a secure refuge if he wished to return to his homeland; but that refuge could hardly be Ṭūs or any other town in Khurāsān.[7] In Rabīᶜ I 617/May 1220 the initial contingent of the Mongol forces arrived at Nīsābūr; by the Spring of 618/1221, Ṭūs and its environs had been subjected to wholesale slaughter and destruction while in Nīsābūr itself virtually all the inhabitants who had remained in the city were killed.[8] The historian Juwaynī, who lived through the ensuing chaos, tells us that even by 658/1259–60, Khurāsān was still in ruin from years of turmoil.[9] A graphic example of this was Ṭūs where in 637/1239 only fifty houses remained in the once thriving city.[10] Naṣīr al-Dīn himself was, no doubt, away from Khurāsān during the years 617–18/1220–21; we have seen that he was studying in 619/1222 with Muᶜīn al-Dīn b. Badrān, probably in Baghdad.[11] But wherever he was at the time, returning to or remaining in Khurāsān was out of the question. One of his few options was to seek refuge with the Ismāᶜīlīs, who seemed to be the only group in Iran that could offer any security.[12] Ṭūsī himself confirms this in the revised introduction to his *Akhlāq-i Nāṣirī*:

> ...he [*i.e.* Naṣīr al-Dīn] had been compelled to leave his native land on account of the turmoil of the age, the hand of destiny having shackled him to residence in the territory of Qūhistān.[13]

But whether he felt shackled or not, Ṭūsī did spend many secure and productive years at the court of Nāṣir al-Dīn Muḥtasham. But sometime before 654/1256, Ṭūsī moved, or was relocated, from Qūhistān and ended up in Alamūt, which was the Ismāᶜīlī capital. The reason for this move is not entirely

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[6] Ṭūsī, *Nāṣirean Ethics*, p. 24.
[7] This is obliquely, but clearly, referred to in Ṭūsī's *Sayr*, p. 41.
[8] Juwaynī, *History*, 1: 145–146, 171–172, 176–178.
[9] Ibid., 1: 96–97.
[10] Ibid., 1: 500–501.
[11] See pp. 6–7 and footnotes 14–15. The date 619/1222 is coincidentally that given by Riḍawī for the beginning of Nāṣir al-Dīn Muḥtasham's rule in Qūhistān (*Aḥwāl*, p. 9).
[12] Shūshtarī, *Majālis*, 2: 203; cf. Riḍawī, *Aḥwāl*, pp. 9–10. Shūshtarī seems to imply that Ṭūsī may not have been overly enthusiastic about going to Qūhistān since he had to be induced by an enticing offer (*laṭā'if al-ḥiyal*).
[13] Ṭūsī, *Nāṣirean Ethics*, p. 124.

clear and to a certain extent its interpretation depends on how one is to understand Ṭūsī's relationship with his hosts. There is evidence that during this time Ṭūsī was not entirely content to remain with the Ismā[c]īlīs; according to one account he communicated with Mu'ayyad al-Dīn Muḥammad ibn al-[c]Alqamī, the Shī[c]ite Imāmī (*i.e.* non-Ismā[c]īlī) wazīr to al-Musta[c]ṣim, the last [c]Abbāsid caliph (ruled 640–56/1242–58), apparently in hopes that his fellow Twelver would rescue him from Qūhistān with an invitation to the court in Baghdad.[14] But according to this version, Nāṣir al-Dīn Muḥtasham discovered his plans and placed him under arrest.[15] During his next trip to Alamūt, he deposited Ṭūsī with the Grand Master [c]Alā' al-Dīn Muḥammad for safekeeping.

Whether these latter events actually occurred in the way described or whether this account was a precautionary fiction invented by Ṭūsī or his friends, we do know that Naṣīr al-Dīn was in Alamūt at its fall to the Mongols in 654/1256. We do not know, however, when he arrived there from Qūhistān. A reasonable possibility is between 642/1244–45, the year of the assumption of the wazīrship by Ibn al-[c]Alqamī and the date of composition of the *Asās al-iqtibās*, Ṭūsī's Persian work on logic written while still in the service of Nāṣir al-Dīn, and Ṣafar 644/June–July 1246, the date on which the *Ḥall mushkilāt "al-Ishārāt,"* a commentary on Ibn Sīnā's well-known *Al-Ishārāt wa-'l-tanbīhāt*, was completed. In this latter work, dedicated to Shihāb al-Dīn Muḥtasham who was probably in Alamūt at the time (thus providing us with some indication of Naṣīr al-Dīn's whereabouts), Ṭūsī complains bitterly about "the difficult circumstance" (*ḥāl ṣa[c]b*) and the "disturbed state of mind" (*kudūrat bāl*) under which the work was written, which seems oddly out of character with the convert's fulsome prose in the *Sayr*. Since the latter can be dated between 644 and 654,[16] and it was almost certainly written while he was still in Qūhistān, 644/1246 becomes a likely candidate for a rather significant crisis that would eventually lead to his relocation to Alamūt.

However one takes his Ismā[c]īlī stage, one must admit that some of his most creative work dates from this period. In addition to the influential work on ethics, the *Akhlāq-i Nāṣirī*, Ṭūsī presented the first of his new lunar and planetary models in his *Ḥall-i Mushkilāt-i Mu[c]īniyya*, a work dedicated to Mu[c]īn al-Dīn Shams, the son of Nāṣir al-Dīn Muḥtasham. These models were to appear many years later in the *Tadhkira*.[17] And if one accepts our assumption

[14] Thus the account in Shūshtarī, *Majālis*, 2: 203 and Khwāndmīr, *Ḥabīb al-siyar*, 3: 105–106.

[15] In the accounts of Khwāndmīr and of Waṣṣāf al-Ḥaḍra in *Tazjiyat al-amṣār*, Nāṣir al-Dīn Muḥtasham discovers this by way of Ibn al-[c]Alqamī himself; see Riḍawī, *Aḥwāl*, pp. 11–12. But as Riḍawī points out, this is unlikely in view of the good relations between Ṭūsī and Ibn al-[c]Alqamī in the aftermath of the fall of Baghdad in 656/1258 (p. 143).

[16] See Riḍawī, introduction to the *Sayr*, page *w*.

[17] See pp. 65–70 of this volume for details of the relationship between the *Tadhkira* and its earlier Persian version.

that Ṭūsī was relocated, for whatever reason, to Alamūt in or about 644/1246, then one may conclude that his scholarly activity continued unabated—indeed flourished—during this time when one may assume that he had access to the famous Alamūt library. One would then be able to date many of his recensions of Greek and early Islamic works—and perhaps even his decision to undertake such a formidable task—to the period of his Alamūt residence. Interestingly enough, he began with the most difficult first, completing his edition of the *Almagest* on 5 Shawwāl 644/13 February 1247 and of the *Elements* on 22 Shaᶜbān 646/10 December 1248. He continued to work on this project until it was concluded in Marāgha under the Mongols in Shaᶜbān 663/June 1265 with his edition of the *Spherics* of Menelaus.[18]

Naṣīr al-Dīn's relationship with the Ismāᶜīlīs, whatever it was, ended on the first of Dhū al-Qaᶜda 654/19 November 1256 with the fall of Alamūt to the Mongols. It is significant that he himself played some role in the final negotiations and in fact accompanied Rukn al-Dīn Khurshāh, who had replaced his assassinated father a year earlier, down the steep path from the mountaintop to Hülegü.[19] He may have even been instrumental in persuading Khurshāh to give up what by then must have appeared a hopeless cause.[20] This, of course, tends to put a somewhat different light on Ṭūsī's true position at Alamūt. On the other hand, it would not be unreasonable to assume that Ṭūsī's fidelity to Ismāᶜīlī dogma must have seemed far less important than the pressing need to use his skills during the sensitive negotiations.

4. The Mongol Period

Whatever the true explanation, Naṣīr al-Dīn was immediately enlisted into the service of Hülegü; he even accompanied him during the campaign against Baghdad and provided an eyewitness account of the end of the ᶜAbbāsid Caliphate.[1] When asked by Hülegü whether the attack on Baghdad would lead to a series of cataclysms as predicted by one of the Muslim astrologers in the Mongol entourage, Ṭūsī, "frightened and thinking he was being tested," answered in the negative.[2]

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[18] For a convenient listing of Ṭūsī's recensions, see Krause [1936], pp. 499–505. Most of these, the so-called "Middle Books," which were to be studied between the *Elements* and the *Almagest*, have been published in Ṭūsī, *Majmūᶜ al-rasā'il* (2 vols.). Ṭūsī's *Elements* and *Almagest*, however, have yet to be edited, though the former has been printed several times. A work purporting to be his *Taḥrīr* of the *Elements* was published in 1594 in Rome, but Sabra [1969] has noted that this is a false attribution (p. 18).

[19] Rashīd al-Dīn, *Jāmiᶜ al-tawārīkh*, pp. 210–213.

[20] Hodgson [1955], pp. 267–268.

[1] See Boyle [1961], pp. 145–161; this contains a translation of the appendix of Naṣīr al-Dīn to the *Ta'rīkh-i jahān-gushā* of Juwaynī. See also Wickens [1962], pp. 23–35.

[2] Rashīd al-Dīn, *Jāmiᶜ al-tawārīkh*, pp. 260–263.

Naṣīr al-Dīn thereafter became a trusted advisor to Hülegü. Ṣafadī/Kutubī report that Hülegü was dependent on him to such an extent that he would not ride or travel without his sanction. Naṣīr al-Dīn was also made a *wazīr* and put in charge of *waqf* (religious endowments).[3]

But for our purposes, the major event that resulted from the relationship between Naṣīr al-Dīn and Hülegü was the building of the Marāgha observatory.[4] Apparently both Hülegü and his brother Möngke, who ruled the vast Mongol domains from China, were patrons of the sciences. The latter wished to construct an observatory in Beijing, but this was not accomplished in his lifetime. However an observatory was constructed—whether at the instigation of Hülegü or Naṣīr al-Dīn is not clear—in Ādharbayjān at Marāgha under the direction of Ṭūsī. It was begun shortly after the fall of Baghdad in Jumāda I 657/April–May 1259.

The significance of the Marāgha observatory has been well documented by Sayili. Naṣīr al-Dīn invited the most renowned scientists of the time, among whom we should mention Mu'ayyad al-Dīn al-ᶜUrḍī and Muḥyī al-Dīn al-Maghribī, to participate in the construction of the instruments as well as the actual observations. A fairly substantial number of students was also in attendance who benefited from an impressive library.[5] The observatory also had an international character due to the several Chinese astronomers on its staff. To pay for such a massive undertaking, Naṣīr al-Dīn, in his capacity as administrator of *waqf* funds, used the proceeds from these religious endowments to finance the observatory. This was a striking and significant departure from past practices; it would seem to indicate, as we have mentioned previously, a breaking down of the barrier between the Islamic and the pre-Islamic sciences. The use of *waqf* funds for the Marāgha observatory would also explain its relatively long life of some fifty years.

It is unfortunate that we know so little about the astronomical significance of the Marāgha observatory. The *Īlkhānī Zīj* itself contains surprisingly little that is new, but this seems to be more a function of the intended audience than of what actually occurred in Marāgha. It hardly seems likely that this work, in which Ṭūsī felt the necessity to inform his Mongol patrons that Muḥammad was from Mecca, was intended for the Muslim intellectual community.[6] Fortunately, other

[3] Ṣafadī, *Wāfī*, 1: 182; Kutubī, *Fawāt*, 3: 250.

[4] In what follows I merely summarize the account in Sayili [1960], pp. 187–223, whose full and excellent treatment need not be repeated here.

[5] The report by Ṣafadī/Kutubī of a library of 400,000 volumes seems to be an exaggeration. Recent excavations at the sight reveal a library structure that could not hold anything near that number [Vardjavand [1366 H. Sh.], p. 12 (French section), pp. 233–234 (Persian section)].

[6] Boyle [1963], pp. 251–252.

zījes that were based on the Marāgha observations exist and await scholarly examination.[7]

In 672/1274, Naṣīr al-Dīn left Marāgha with a group of his students for Baghdad. The reasons for this trip are unknown but it would seem that he intended to remain for awhile since his students had decided to accompany him. In the same year, Naṣīr al-Dīn died and was given a memorable funeral attended by a number of leading dignitaries.[8] It was during this brief final sojourn in Baghdad that Ṭūsī made his final revision of the *Tadhkira*, the version that is the basis for the edition contained herein.

5. Ṭūsī's Astronomy and Its Relation to His Hellenism and Religious Beliefs

Because Ṭūsī's religious beliefs ostensibly changed several times during his lifetime, it is well worth asking whether these changes had any effect upon his scientific, and in particular his astronomical, work. In other words, were Ṭūsī's Hellenistic cosmology and astronomy independent of his apparently distinct religious commitments that entailed, by his own account, different epistemological positions? An answer to this sort of question depends upon a much more thorough analysis of Ṭūsī's writings than is possible here; I would, though, like to offer a preliminary response since I believe that there are larger lessons to be learned concerning the attempt by Islamic astronomers to immunize the mathematical core of their discipline from the charge that it was inextricably tied to a suspect Aristotelian metaphysics.

Before discussing the implications of these "conversions" for his astronomy, we should briefly review the nature and justifications of his religious beliefs during the different stages of his life. For Ṭūsī's conversion to Ismāᶜīlism, we are in the fortunate position of having his spiritual autobiography written when he was in the service of Nāṣir al-Dīn Muḥtasham, the Ismāᶜīlī governor of Qūhistān. This work, which we have had occasion to mention previously, his *Risālah-i sayr wa-sulūk* (Epistle on the Journey and Conduct),[1] is a kind of intellectual and spiritual autobiography with a liberal dose of Ismāᶜīlī apologetics thrown in. In the introduction, the author states his name to be Muḥammad al-Ṭūsī, and several crucial details agree with what we know independently of his life; there

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[7] One obvious example is Shams al-Munajjim al-Wābiknawī's *Al-Zīj al-Muḥaqqaq al-Sulṭānī ᶜalā uṣūl al-raṣad al-Īlkhānī* (the verified Sulṭānī *zīj* based upon the Īlkhānī observations); see Kennedy, *Survey*, p. 130 (no. 35). Cf. Sayili [1960], pp. 214–215. For recent discussions of this problem with particular reference to the work of al-Maghribī, one of the principals at Marāgha, see Saliba [1983], [1985], and [1986].

[8] Ṣafadī, *Wāfī*, 1: 183; Kutubī, *Fawāt*, 3: 252.

[1] Even the title of this controversial work is in dispute; Riḍawī states in his introduction to it (p. *h*) that it was not the title given it by the author but was made up at the time it was first printed.

can be little question of attributing the book to our Ṭūsī.[2] In the main body of the work itself, Ṭūsī tells of his growing disenchantment with exoteric theology (*kalām*), which he came increasingly to see as little more than apologetics; this transformation he seems to attribute in part to the subtle influences of his father, his uncle and his mathematics teacher Kamāl al-Dīn Muḥammad al-Ḥāsib.[3] He then became attracted to philosophy (*ḥikma*), which he felt gave an opportunity to the intellect for finding truth; but Ṭūsī came to believe that intellect was deficient in apprehending the "Giver of intellect" and thus philosophy could not give him answers to the ultimate questions. But philosophy, he tells us, did teach him the important lesson about potentiality and actuality. Taking the example of a body that is potentially in motion, he argues that it could never become actually in motion unless there is something else influencing it. Likewise a potential knower needed someone else, *i.e.* a teacher, to actualize that knowledge.[4] Eventually he chanced upon the *Fuṣūl-i muqaddas* (Sacred Articles) of the Ismāᶜīlī Lord of Alamūt Ḥasan ᶜalā dhikrihi al-salām (reigned 1162–66), and he at last found the religious teacher he was seeking, namely the divinely chosen Ismāᶜīlī Imām. He thus decided to join the court of Nāṣir al-Dīn Muḥtasham.[5]

After the fall of Alamūt, Ṭūsī, as we have seen, felt it necessary to repudiate his previous service to the "heretics." But Ṭūsī did more than protest his "innocence"; he also sought to disestablish his Ismāᶜīlism by returning to an essentially philosophical worldview. During this post-Ismāᶜīlī period he wrote a devastating attack upon Tāj al-Dīn al-Shahrastānī's refutation of Ibn Sīnā's philosophical position.[6] We should recall that it was this Tāj al-Dīn who had Ismāᶜīlī sympathies and had taught Naṣīr al-Dīn's great-uncle (also a Naṣīr al-Dīn). Ṭūsī in this work came down strongly in favor of (Hellenistic) philosophy, especially the Ibn Sīnā variety, and returned to his earlier notion that special pleading for a particular religious cause (and in Tāj al-Dīn's case this was Ismāᶜīlism) was incompatible with the search for truth. As he would state in his new introduction to the *Ethics*, philosophy (*ḥikmat*) "bears no relation to the agreement or disagreement of school or sect or denomination."[7]

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[2] Although I at one time ([1982], 1: 12–17) agreed with Riḍawī that Ṭūsī was never an Ismāᶜīlī and thus was somewhat sceptical of the attribution of this work to Ṭūsī, and in any event very dubious about Ṭūsī's sincerity if he were indeed the author (cf. *Majmūᶜa*, pages *h-w* and Riḍawī, p. 591), I have now come to accept at least his authorship in large part because of the large number of statements about his life in the work that can be independently verified. Note that Poonawala [1977], p. 261 and Madelung [1985], pp. 88–89 accept the authenticity of the work and both see it as representing a sincerely held stage in his intellectual development.

[3] *Sayr*, p. 38.

[4] *Sayr*, p. 40.

[5] *Sayr*, p. 41.

[6] Tāj al-Dīn's book was called *Kitāb al-Muṣāraᶜa* (The Wrestling Match), while Ṭūsī's work was called *Maṣāriᶜ al-muṣāriᶜ* (The Downfalls of the Wrestler). For a cogent discussion of these works see Madelung [1985], pp. 87–88 and Madelung [1976].

[7] Ṭūsī, *Nāṣirean Ethics*, p. 24; cf. Madelung [1985], pp. 87, 100.

One could engage in a long, but inevitably fruitless, debate about which Naṣīr al-Dīn was the true one. But leaving aside the question of the sincerity—or expediency—of his conversions, we should ask whether his association with the Ismāᶜīlīs and their doctrines had any significance for his scientific work. Was there any discernible change in it due to his new found faith? And more to the point, did the submission to a religious authority parallel any similar submission to scientific authority?[8] The answer must be negative for both; although Ṭūsī was not a scientific revolutionary—he remained committed throughout his career to a modified Ptolemaic worldview, for example—he was not hesitant to criticize and modify both his Greek and Islamic predecessors. And this attitude is clearly detectable during his Ismāᶜīlī period. As we shall see,[9] several of the original versions of the non-Ptolemaic models of the *Tadhkira* were first presented in his *Ḥall-i mushkilāt-i Muᶜīniyya*, which was written for Muᶜīn al-Dīn, the son of his patron Nāṣir al-Dīn Muḥtasham. And despite the ups and downs of a tumultuous life, Ṭūsī's interest in astronomy and the content of his work in it remained remarkably stable. But then we are faced with another question. Can we separate Ṭūsī's scientific work from his philosophical and religious views? Here I would go beyond a mere positive response to anticipate my argument in the following chapter on *hay'a*, namely that Islamic astronomers *themselves* were seeking—sometimes consciously, sometimes not—to free their physical theories from a metaphysical taint.[10] In short, I would argue that whatever the effects of Ṭūsī's conversion on his philosophical worldview, and thus on his "Hellenism" (taken here to mean a commitment to rational discourse in all matters), I believe that we may safely conclude that the effects on his astronomy were nonexistent.[11]

How influential was Ṭūsī's attitude? Was his attempt to place philosophy, and by implication the mathematical sciences, above religious disputes successful? As we shall see later, his position was quite influential in the East and we

[8] Cf. Sabra's [1984] attempt to draw a connection between Averroes's glorification of Aristotle and the theological literalism of Ibn Ḥazm (d. 1064). As he puts it, "it is difficult not to regard this attitude of Averroes's and the commentatorial style he adopted in most of his philosophical writings as a literalism in philosophy that paralleled the theological literalism of Ibn Ḥazm" (p. 144). Though Sabra does not dwell on the problem, he opens up the possibility that Averroes's change in attitude toward Ptolemaic astronomy—from embracing it in his earlier epitome (*talkhīṣ*) of Aristotle's *Metaphysics* to rejecting its fundamental basis in his large commentary (*tafsīr*) of the same work—had something to do with a parallel change in religious philosophy. Averroes's exaltation of Aristotle represents a rather extreme position in Islamic intellectual history. A much more sceptical approach toward authority is manifested in Ibn al-Haytham's *Shukūk*, pp. 3–4. (For a translation of this latter passage, see Sabra [1989], 2: 3.)

[9] See pp. 66, 69–70 of this volume.

[10] See pp. 38–41, 45–46.

[11] On the other hand there could be possible consequences for such things as his ethics; cf. Madelung [1985].

even have a very spirited defense of astronomy by the astronomer/theologian ᶜAlī al-Qūshjī (d. 879/1474) that explicitly attempts to raise it above religious objections.[12] But this philosophical position should be seen within the context of a broader intellectual trend in Eastern Islam, especially Iran, one that was not always well received further west in Syria and Egypt. To understand this we must examine how Ṭūsī's intellectual positions came to be played out on the political stage.

First one must acknowledge Ṭūsī's political genius,[13] which is nowhere more in evidence than in his ability not only to convince the Mongols, certainly not a gullible lot, that he had been an unwilling resident of Alamūt but more importantly in his skill in protecting the interests of the Shīᶜites in particular, and the Muslim community in general, from the caprice of the Mongol rulers. As we have seen, part of this involved writing new introductions to a number of works in which he had lavished praise on his former patrons,[14] and one could well conclude that this was the worst type of opportunism. One might even go so far as to claim that this is just an example of Ismāᶜīlī *taqiyya* (dissimulation); indeed, the author of *Rawḍat al-Taslīm*, which has been attributed to Ṭūsī, tells his audience that he must write "in a hurry, secretly, in a dark corner" during a period of *satr* (concealment) ordered by the Imām.[15] But this seems hardly in character with the reports in Ṣafadī/Kutubī that during his years of service under Hülegü, Muslims in general and, as one might expect, the Shīᶜa in particular benefited from him.[16] One of the Sunnīs he aided was none other than the historian Juwaynī, whom Naṣīr al-Dīn saved through elaborate astrological trickery from a death sentence imposed by Hülegü.[17] It is not beside the point to note here that it was this Juwaynī who, as he himself tells us, asked permission from Hülegü after the taking of Alamūt

[12] See Ragep [1982], 1: 141–145 for a translation of this passage as quoted by Tahānawī, *Dictionary*, 1: 48–49.

[13] Some would use a different characterization; see, for example, Reuben Levy [1923], p. 64: "The verdict of history on Naṣíru 'l Dín is a most unfavourable one. It might have been expected that the conduct of a man of his undoubted mental qualities would have been regulated by some standard higher than that of personal advantage. Yet he appears not only to have betrayed his Ismáᶜílí master to Húlágú, but to have been instrumental in bringing the last Caliph treacherously to his death at the hands of the Mongols."

[14] This included his ethical work, the *Akhlāq-i Nāṣirī*, and two astronomical works, the *Risālah-i Muᶜīniyya* and the *Ḥall-i mushkilāt-i Muᶜīniyya*.

[15] *Taṣawwurāt*, pp. 134, 126. Though Madelung [1985] and Poonawala [1977], p. 262 seem to accept that Ṭūsī is the author, there is, as far as I know, little substantial evidence. Even Ivanow, who more or less convinced himself that Ṭūsī was a lifelong Ismāᶜīlī, grudgingly acknowledged the insufficiency of the claims on Ṭūsī's authorship; nevertheless, he wished to "conventionally accept Ṭūsī's authorship...till...an incidental find may answer our quest decisively" (*Taṣawwurāt*, p. xxvi).

[16] Ṣafadī, *Wāfī*, 1: 182; Kutubī, *Fawāt*, 3: 250.

[17] See Ṣafadī, *Wāfī*, 1: 179–180. Ṣafadī is poking fun at the gullibility of the foreign conquerors, but it should be remembered that he is living safely in the following century.

> to examine the library, from which I extracted whatever I found in the way of copies of the Koran and [other] choice books...As for the remaining books, which related to their heresy and error and were neither founded on tradition nor supported by reason, *I burnt them all* [emphasis added].[18]

It is difficult to believe that Ṭūsī's *taqiyya* would extend to saving the life of someone who had almost single-handedly destroyed the literary heritage of his religious faction.

The reason this is an important point to keep in mind is that Ṭūsī's subsequent reputation in the East was very high among both Sunnīs and Shīᶜīs, a state of affairs that would be very hard to imagine if he had not acted to protect all Muslim interests during the Mongol hegemony. Indeed one need only refer to his theological work, the *Tajrīd al-ᶜaqā'id*, to note that it was commented on and used by both Sunnīs and Shīᶜīs. As with astronomy so also with *kalām* (dialectical theology) itself, Ṭūsī would seem to have been at least partially successful in transcending the religious disputes that had disturbed him so much as a youth.

But Ṭūsī did not fare so well further West in the Mamlūk areas of Syria and Egypt. There he was mercilessly denounced by such Ḥanbalī luminaries as Ibn Kathīr (ca. 700–74/1300–73)[19] and Ibn Qayyim al-Jawziyya (691–751/1292–1350). One may quote the latter to get the flavor of these invectives:

> We now reach the time of the defender (*naṣīr*) of idolatry, unbelief and heresy, the minister to the heretics, al-Naṣīr al-Ṭūsī, the minister to Hülegü. He relieved himself from following the Prophet and the members of his religion, putting them to the sword so as to satisfy his fellow heretics and save himself. He killed the Caliph, judges, jurists, and traditionists; he protected the philosophers, astrologers, natural philosophers, and magicians. He transferred the endowments of the religious schools, the mosques, and the hospices attached to them, making them his personal property...He also established schools for the heretics. He hoped to substitute the *Ishārāt* of the leader of the heretics Ibn Sīnā in place of the *Qur'ān*, but he could not accomplish that! He said: "The former is the *'Qur'ān'* of the elect while the latter is for the masses."...Finally he taught magic, for he was a sorcerer who worshipped idols.[20]

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[18] Juwaynī, *History*, 2: 719.
[19] See his *Al-Bidāya wa-'l-nihāya*, 13: 267–268.
[20] Ibn Qayyim, *Ighāthat al-lahfān min maṣāyid al-shayṭān*, 2: 267; this is quoted by Ziriklī, *Aᶜlām*, 7: 258. Later authors of the same persuasion as Ibn Qayyim have also used this passage in their works; see, *e.g.* Ibn al-ᶜImād al-Ḥanbalī, *Shadharāt al-dhahab*, 5: 339–340.

In wading through the hyperbole of this passage, one should keep in mind the real issue involved. Ṭūsī, insofar as he was a follower of Ibn Sīnā, did place philosophy beyond the pale of religious disputation or, as we have seen him put it in the *Ethics*, it "bears no relation to the agreement or disagreement of school or sect or denomination."[21] This was the crux of the battle between the philosophers and the traditionists of Islam, and Ṭūsī's power and influence were rightly seen by the latter as a direct threat.[22] They also rightly saw that Ṭūsī was seeking to carry on the work, if not the details, of Fakhr al-Dīn al-Rāzī in "naturalizing" the Hellenistic heritage in Islam.[23]

In this regard one might put into perspective the work on religious cosmology by the Egyptian al-Suyūṭī (849–911/1445–1505). As A. Heinen has noted, he hoped it would be *the* Islamic cosmology and he even went so far as to appropriate the word *hay'a*, the term for mathematical cosmology, into his title.[24] But in the East, Ṭūsī's legacy would have made the possibility of a religious cosmology *in competition* with (and potentially supplanting) a mathematical one far less likely. For Ṭūsī and his successors, science and religion could coexist within the intellectual scheme he was so instrumental in establishing.[25]

B. Works

A comprehensive evaluation of Ṭūsī's enormous corpus would be premature. Sheer size is only the beginning of the problem;[1] Naṣīr al-Dīn saw fit to try his hand at virtually all fields of both ancient and Islamic learning. Since neither my training, taste nor ability can match Ṭūsī's, my rather modest aim is simply to

[21] See p. 16, footnote 7.

[22] Cf. Madelung [1985], pp. 100–101.

[23] Cf. Sabra [1987a] and Peters [1968], pp. 193–202.

[24] Heinen [1982], p. vii.

[25] Here I would disagree strongly with Sabra's [1987a] implication that this coexistence in the later centuries of medieval Islam meant that science, and astronomy in particular, was afflicted with a kind of instrumentalism in the service of religion that "confines scientific research to very narrow, and essentially unprogressive areas" that did not generally include observational and theoretical astronomy (pp. 241–242). These "narrow, unprogressive" areas did include spherical astronomy, which was useful for such religious purposes as finding the *qibla* (direction of Mecca). While this might have been true in Mamlūk Egypt and Syria (cf. King [1983], pp. 537–539), where Ṭūsī was reviled, it was most emphatically not true in Persia and Central Asia where theoretical and observational astronomy flourished for many centuries after the Marāgha episode. My hope is that the following pages will be a modest contribution toward proving this contention.

[1] Riḍawī lists, apart from his poetry, some 190 works. But as a number of these are clearly misattributions and duplicates, Ivanow's estimate of something over 150 seems closer to the mark (*Taṣawwurāt*, p. xxvi).

give some sense of his writings and to provide a context within which to place his writings on *hay'a*.

Ṭūsī's work has often been characterized as being that of "revival" rather than of "origination."[2] But this seriously misrepresents his contribution to astronomy and, perhaps, to other fields as well. There is a kernel of truth to this "revivalist" tag, however. His role in post-13th century A.D. Islamic intellectual history cannot easily be underestimated when we consider that many of his works became the standard in a variety of disciplines up until modern times. His recensions of Euclid's *Elements*, Ptolemy's *Almagest* and the "Middle Books" of mathematics and astronomy virtually replaced earlier editions. His works in logic and *kalām* were also widely read and commented on, and it is hard to imagine Ibn Sīnā's *Al-Ishārāt wa-'l-tanbīhāt* being studied without Naṣīr al-Dīn's valuable commentary. His work on ethics, the *Akhlāq-i Nāṣirī*, was also of considerable importance, especially in the eastern domain of Islam. We should in addition mention here that among his works are books on *fiqh* and *ṣūfism*. In all these examples we have, for the most part, a consolidation of the endeavors of earlier generations of Islamic scholars. This occurred at an important point in Islamic history as there had been an unmistakable, but not easily understood, lull in intellectual activity in the century or so previous to that of Ṭūsī. Historically, then, Naṣīr al-Dīn easily falls into the role of consolidator rather than innovator.

But I would hesitate to leave the matter here. As is clear from astronomy, most of Ṭūsī's work can be viewed as simply a well-integrated account of what others had previously accomplished. Nevertheless his own contributions are not insubstantial and, when seen in the perspective of future developments, take on added importance.

I would like to venture a number of preliminary, working hypotheses about Ṭūsī's writings that, needless to say, may need to be revised as more of them are examined and edited. First, many of Ṭūsī's most creative and original work was composed while in the service of Nāṣir al-Dīn Muḥtasham in Qūhistān (ca. 630/1232 or 1233–ca. 643/1245 or 1246). Among these are the *Akhlāq-i Nāṣirī*, the *Risālah-i Muᶜīniyya* and its sequel the *Ḥall-i mushkilāt-i Muᶜīniyya* in which one finds Naṣīr al-Dīn's new models for the moon and planets, and the logical work *Asās al-iqtibās*. Furthermore, he worked on the commentary to Ibn Sīnā's *Al-Ishārāt wa-'l-tanbīhāt* during these years. As one might expect, all these works, with the notable exception of the latter, were written in Persian. Second, after going to Alamūt (ca. 643 or 644/1245, 1246 or 1247), Ṭūsī seems to have devoted himself in large measure to working on his recensions of Greek and early Islamic scientific work. Many of the editions of the so-called "Middle Books," in addition to his editions of the *Almagest* and *Elements*, date from this period. The next, and final, period of his life (654–72/1256–74), which coincides with his service under the Mongols, seems to have been a time of consolidating earlier gains rather than of making new departures. An important

[2] See, for example, Siddiqi [1963], pp. 566, 580.

preoccupation during this period was the Arabizing of earlier Persian works. The *Tadhkira*, completed in 659/1261, is basically an Arabic synthesis of the *Muʿīniyya* and the *Ḥall-i mushkilāt-i Muʿīniyya*, which were composed some 25 years previously. It is likely that we are dealing with a similar situation in the case of the Arabic treatise on logic, *Tajrīd al-manṭiq* (written 656/1258) and the earlier Persian work, the *Asās al-iqtibās* (written 642/1244 or 1245), though we should mention that the Arabic version is considerably abridged. It was also during this latter part of his life that Ṭūsī attempted to diminish the significance of his former relationship with the Ismāʿīlīs by claiming that he had only served them out of dire necessity. As we have noted, he even went so far as to rewrite the introductions of several works that had been composed for his Ismāʿīlī patrons.[3] A final point we should make is that longer and shorter versions of works on the same subject matter occur among Ṭūsī's writings. Whether the purpose of these different versions is pedagogical or otherwise is not immediately clear. One example we shall note later (pp. 66–67) is the *Zubdat al-idrāk fī al-hay'a*, which is an abridged version of the *Tadhkira*.

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Inventories of Ṭūsī's works are given by Brockelmann (*GAL*, 1: 508–512 [= 670–676] and S1: 924–933) and Riḍawī (*Aḥwāl*, pp. 333–628), but each should be used with the standard caution. His Persian astronomical works are listed in Storey (*Persian Literature*, II.1, pp. 52–60); Matvievskaya/Rozenfeld catalogue his works in the exact sciences (*Mat. i astr.*, 2: 392–408).

[3] See p. 18.

C. Chronology of Ṭūsī's Life*

	597/1201	Naṣīr al-Dīn born in Ṭūs, N.E. Iran
ca.	603–10/1206–13	Early education with father, uncles, and Kamāl al-Dīn al-Ḥāsib
ca.	613/1216	Studies philosophy (*ḥikma*) and medicine in Nīsābūr
	618/1221	Chingiz Khān ravages Khurāsān
	619/1222	Ṭūsī studying with Shī^{c}ite jurist Mu^{c}īn al-Dīn in Iraq
ca.	625/1228	Studies astronomy and mathematics with Kamāl al-Dīn ibn Yūnus in Mosul
ca.	630/1233	Finds patron in Nāṣir al-Dīn Muḥtasham, Ismā^{c}īlī governor of Qūhistān
	632/1235	Dedicates *Risālah-i Mu^{c}īniyya* on astronomy to Mu^{c}īn al-Dīn, son of Nāṣir al-Dīn Muḥtasham
ca.	632–33/1235–36	Writes *Ḥall-i mushkilāt-i Mu^{c}īniyya*; first appearance of rectilinear Ṭūsī couple
ca.	633/1235–36	Dedicates *Akhlāq-i Nāṣirī* (*Nasirean Ethics*) to Nāṣir al-Dīn Muḥtasham
	642/1244–45	Completes *Asās al-iqtibās* on logic while still in Qūhistān
	644/1246	Probable date of completion of autobiography (*Sayr wa-sulūk*) as well as relocation to Ismā^{c}īlī fortress of Alamūt; completes *Ḥall mushkilāt "al-Ishārāt,"* a commentary on Ibn Sīnā's *Al-Ishārāt wa-'l-tanbīhāt*
	644/1247	Beginning of project to edit "Middle Books" of mathematics and astronomy as well as *Almagest* and Euclid's *Elements*; recension of *Almagest* completed
	646/1248	Recension of *Elements*
	654/1256	Alamūt falls to Mongols; Ṭūsī begins his service to Hülegü
	656/1258	Baghdad falls to Mongols; end of ^{c}Abbāsid caliphate
	657/1259	Construction of Marāgha observatory begins under Ṭūsī's direction
	659/1261	Marāgha version of *Tadhkira* appears
	663/1265	Recension of Menelaus's *Spherics*; "Middle Books" project completed; Hülegü dies
ca.	670/1271	*Zīj-i Īlkhānī* completed
	672/1274	Baghdad version of *Tadhkira* completed; Ṭūsī dies in Baghdad

* The first number is the *hijra* date; the second is the corresponding Christian date.

§2. The *Tadhkira*

The full title of the work is *Al-Tadhkira fī ᶜilm al-hay'a,* meaning "Memoir on the science of *hay'a*." Although *hay'a* is used to mean astronomy in a general sense, we shall find that it has a more specialized meaning as well that we shall explore to help us situate the *Tadhkira* within both an astronomical tradition and a literary genre. In addition to tradition, we shall also need to discuss its innovations, the rather remarkable history of the text, and its enormous influence.

A. Purpose of the *Tadhkira*

To understand a work, we would be well-advised to begin by asking what is its purpose. For the *Tadhkira,* we are fortunate that we have Ṭūsī's own words to guide us:

> The scientific exposition that we wish to undertake will be a summary account of [astronomy] presented in narrative form. The details are expounded and proofs of the validity of most of them are furnished in the *Almagest.* Indeed, ours would not be a complete science if taken in isolation from the *Almagest* for it is a report of what is established therein.

The idea of a summary seems innocent enough and, up to a point, it is. Anyone who has ventured beyond the introductory sections of the *Almagest* knows that a summary of the contents or a running commentary can be of great help (not to say indispensable). But the *Tadhkira* is neither a commentary nor a straightforward summary of the contents of the *Almagest.* It rather sets forth those contents within a textual structure meant to give a physical accounting of the Universe—in short, a cosmography. Why was it that someone in the 13th c. came to feel that the *Almagest* should be summarized from the point of view of physical bodies, bodies that made a cameo appearance there but can hardly be said to have been a main feature? And furthermore, why did this someone, namely our Naṣīr al-Dīn, adopt the textual structure that he did? To answer these questions, it will be convenient to distinguish the tradition within which the *Tadhkira* falls from the genre of astronomical work that came to be characterized by the *Tadhkira* itself. In what follows, I shall attempt to sketch an answer to these questions using a rather broad stroke indeed.[1]

B. The *Tadhkira*'s Ancient Forebears

To understand the tradition of the *Tadhkira*, we need to isolate three aspects of his work that Ṭūsī feels are crucial. First the notion that astronomy has a strong physical component is put forth explicitly in I.Intr. [2] when he states that "the subject of astronomy is the simple bodies." Second he makes clear how he views the relationship between his work and the *Almagest* in II.5 [10] with the following, revealing passage:

> These then are models and rules that should be known. We have only stated them here; their geometric proofs are given in the *Almagest*. Restricting oneself to circles is sufficient in the entirety of this science for whoever studies the proofs. However, one who attempts to understand the principles of the motions (*mabādi'*) must know the configuration (*hay'a*) of the bodies.

Here we can gain some understanding of what Ṭūsī meant in the introductory passage quoted above in which he indicated that the *Tadhkira* stood in a kind of dependent relationship to the *Almagest* inasmuch as the proofs of the statements being "narrated" in the former could presumably be found in the latter. But again, because Ptolemy in the *Almagest* did not go out of his way to discuss the bodies, Ṭūsī would surely be aware that he was doing something that, though arguably implicit there, was certainly not explicit.[2] The *Almagest* then, despite what Ṭūsī at times seems to be saying, is clearly not the only source for his work. Third Ṭūsī, though he never comes right out and says so, is clearly attempting to give a coherent and unified account of astronomy. Again we can contrast this with the *Almagest*, where Ptolemy treats each planetary system separately and is seemingly unconcerned there with such problems as how all

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[1] Unfortunately the book in which "the details are expounded and proofs of the validity of most of them are furnished" has yet to be written.

[2] Several medieval Islamic commentators did not even think it was implicit. Ibn al-Akfānī (d. 1348), for example, claims that "the ancients were always constricting themselves to abstract circles for the configuration of the orbs, until Ibn al-Haytham openly declared their [the orbs'] corporeality...the later [scholars] have followed him with regard to this" [cited and translated by Langermann [1990], p. 13]. Ṭāshkubrīzāde (1495–1561), *Miftāḥ*, 1: 373 repeats this: "The ancients confined themselves (*iqtaṣarū*) in the configuration of the orbs to abstract circles."

the individual parts fit into a complete whole, how one system might affect another, and the actual (as opposed to relative) size and distance of the orbs. On the other hand, Ṭūsī provides just such an accounting.[3]

Can we find in antiquity works that could be considered earlier instances of a tradition of which the *Tadhkira* is a later member? I would venture to answer yes though I am thinking of an earlier tradition that should be considered in terms of providing inspiration rather than of establishing a specific genre. The first example I wish to consider occurs in Book λ, Chapter 8 of Aristotle's *Metaphysics*. In seeking to determine the actual number of eternal movers, Aristotle finds that he needs help from the "mathematicians" (τῶν μαθηματικῶν) so that "our thought may have some definite number to grasp." But though he is dependent on the astronomers, he is clearly not afraid to propose modifications in order to "explain the phenomena." In practice, this means that Aristotle accepts that the Eudoxan-Callippic mathematical theory of the celestial motions accounts for the phenomena of each individual planet. What he is seeking to rectify is the problem that results from Eudoxus having treated the orbs of each planet separately. When one fits the individual sets of homocentric spheres inside one another, the lower spheres will necessarily partake of the motion of the higher orbs thus upsetting the mathematical modeling for each planet. Hence Aristotle proposes "unrolling" spheres whose purpose is to undo the motions (with the exception of the daily motion) for a given planet so that one can in effect start over for the next one.

How should we understand this relationship between Aristotle and the astronomers? The traditional view would have it that Aristotle is acting as a physicist whereas Eudoxus and Callippus are astronomers whose purpose is to use geometrical (rather than physical) hypotheses to "save the phenomena."[4] Recent studies, however, have pointed out that the use of spheres and uniform rotation indicate physical commitments, and it would be a mistake to view Eudoxus and Callippus as exponents of some ancient version of instrumentalism.[5] How then should we view the relationship between Aristotle and Eudoxus? L. Wright has argued we should not regard Aristotle as someone who physicalized purely mathematical models, but rather as someone who took preexisting *physical* models and "showed how this model could be fitted into a

[3] For the individual parts, see II.2 (overview) and II.6–II.10 (details for each planet). For the dynamics of celestial motion, see II.4 [6–7]. And for the problem of planetary sizes and distances, Ṭūsī devotes the entirety of Book IV.

[4] The classic statement of this is by Duhem [1908], pp. 3–5 (Engl. trans. pp. 5–7). Heath [1913] states that "we have seen that that system [of Eudoxus and Callippus] was purely geometrical and theoretical; there was nothing mechanical about it...Aristotle, as we shall see, transformed the purely abstract and geometrical theory into a mechanical system of spheres" (p. 217). For other examples, see Wright [1973], p. 165. For a review of this viewpoint, see Lloyd [1978], pp. 202–204.

[5] This point was made forcefully by Wright [1973]; cf. Lloyd [1978], p. 219, who recants an earlier commitment to the conventional view.

unified planetary system which had consequences that the model as used by Eudoxus did not."[6] If this is the case, then we can point to the same three fundamental aspects that we have identified with Ṭūsī's *Tadhkira*: (1) Aristotle is providing a physical account of the All; (2) he is accepting as given the models of the astronomers, which he assumes to be at base physical; and (3) he is attempting to provide a coherent and unified account, which compels him to provide additional spheres. It is on this basis that I would claim kinship between the *Tadhkira* and this chapter of the *Metaphysics*.

A second example of an "inspirational" ancient predecessor of the *Tadhkira* is undoubtedly Ptolemy's *Planetary Hypotheses*, which is composed of two books.[7] Writing sometime after completing the *Almagest*, Ptolemy begins the first book by proclaiming his desire to set forth the results of the *Almagest* in a summary fashion (κεφαλαιωδῶς; Arabic trans. *jumal*)[8] so that they might be more easily conceived and also as an aid to those who wish to construct a mechanical model.[9] He then proceeds to catalogue the celestial circles, giving their positions, motions, and relative sizes (with the largest circle for each system of circles being 60). This part of Book I is followed by a discussion of the order of the planets, which, once determined, allows one to use the relative sizes of the circles to establish the absolute distances (in either Earth radii or stades) of the planets from the Earth. Computations of the sizes of the planets and stars followed by a passage on planetary visibility round out Book I. This part of the *Planetary Hypotheses* can then be understood as a summary of the main results of the *Almagest* presented without proofs for those who wish to gain a general picture of the heavens and their motions or who wish to construct actual physical models.

[6] Wright [1973], pp. 171–172. We need not follow him down the slippery slope of analogy with Kepler and Newton to accept his fundamental point.

[7] Only the first part of Book I is extant in Greek. Both books are available in an anonymous Arabic translation "corrected" by Thābit ibn Qurra. There is also a Hebrew translation from the Arabic. The Greek text of Book I (lacking the part on sizes and distances) with a French translation is in Halma [1820], pp. 41–56; the Greek text is also in Heiberg [1907], which in addition contains a German translation begun by L. Nix and completed by F. Buhl and P. Heegaard of the Arabic. Somehow the three translators missed the second part of Book I; this has been rectified by an English translation due to B. Goldstein [1967], who also included a commentary, a facsimile reproduction of one of the two extant Arabic manuscripts, namely British Museum MS Arab. 426 (hereafter referred to as BM 426) and variant readings of the other manuscript.

[8] Heiberg [1907], p. 70, line 12; BM 426, f. 81b, line 10 (Goldstein [1967], p. 13).

[9] Swerdlow/Neugebauer [1984] see this as basically an equatorium (p. 40).

Book II is a summary of a rather different sort. Ptolemy tells us that

> having pictured (*ja^c^alnā al-mithālāt*) their motions and the arrangements of their positions in a simple manner on great circles (*aflāk*), which they describe by their motions, it remains for us to describe the forms (*ashkāl*) of the bodies by which we may understand those circles (*aflāk*).[10]

Ptolemy then plunges into a rather involved discussion of the nature of the celestial bodies.[11] This is followed by a description of how the bodies are arranged for each planetary system.[12]

It is not difficult to cite numerous differences between the *Planetary Hypotheses* and the *Tadhkira* as well as other Arabic and Persian works of its type.[13] The two-book structure of the *Hypotheses*—the first a summary of the *Almagest* and the second an attempt to represent physically the orbs—is not followed in any Islamic astronomical text known to me. More importantly, the attempt to make the orbs into *manshūrāt* [πρίσματα or "sawed-off sections"] is rejected by Arabic astronomers out of hand.[14] Nevertheless, I would maintain that the *Planetary Hypotheses* is a prototype for the *Tadhkira*. The dual treatment of the motions and orbs in the two books of the *Hypotheses* have in the *Tadhkira* been combined to form a unified account for each body. The section on sizes and distances of the celestial bodies at the end of Book I of the *Hypotheses* has been placed in the final book (*bāb*) of the *Tadhkira*, which is meant to provide a unified view of the system taken as a whole. And most importantly, the purpose of each work is basically the same: (1) both seek to provide a summary of the *Almagest* and both assume the results and proofs of the *Almagest* as given; (2) both seek to provide a physical account of the celestial

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[10] BM 426, f. 93a, lines 3–6 (Goldstein [1967], p. 36).

[11] Ptolemy's basic purpose in this part of Book II (BM 426, ff. 93a–95b; Goldstein [1967], pp. 36–41) would seem to be the defense of his proposal to do away with "complete spheres." He suggests using only those parts of the spheres where the planet is seen to move, leaving the so-called tambourines (*adfāf*) or sawed-off sections (*manshūrāt*) (BM 426, ff. 94b–95a). Ptolemy at no time, however, accepts a vacuum as some modern commentators would have us believe (*e.g.* Schramm [1963], pp. 19–20 and Neugebauer, *HAMA*, 2: 923). He instead suggests that an aether fills the gaps left after detaching the truncated orbs (BM 426, f. 95b, lines 8–13). Although the exact details of how the motion of the orb might occur in the aether remain obscure, Ptolemy's reason for adopting this somewhat radical position is explicit, namely that there is nothing useless in nature. As justification for this, he cites the fact that the orbs of Venus and Mercury fill almost exactly the space between the moon and sun (BM 426, f. 95a, lines 1–8).

[12] BM 426, ff. 96a–102b (Goldstein [1967], pp. 42–55).

[13] For an extensive discussion of this with regard to Ibn al-Haytham's *Hay'at al-^c^ālam*, see Langermann [1990], pp. 11–25.

[14] See, for example, the *Tadhkira*, II.11 [16].

bodies and their motions based on the mathematical models of the *Almagest*; and (3) both seek to give a unified account of the entire system, perhaps best exemplified by the sizes and distances section of each (I.1 of the *Planetary Hypotheses* and Book IV of the *Tadhkira*). It is these three fundamental aspects, which we have also identified in a somewhat different form in Aristotle's account of the Eudoxan-Callippan system, that I would maintain link the *Hypotheses* to the *Tadhkira*.

C. The Islamic Tradition of the *Tadhkira*

The existence of classical texts that correspond in some fashion with the Islamic tradition of which the *Tadhkira* is a member does not imply, of course, that those texts are in a direct historical line with, or are responsible for, that tradition. In particular, there is little evidence that any Islamic writer would have connected his work with Aristotle and Ptolemy in quite the way this 20th century writer has done above. We also are in the unfortunate position of not understanding very clearly how late Hellenistic astronomy was assimilated into the early Islamic science of the 8th and early 9th centuries. Though theoretical concerns seem to have been at a minimum during this period, there are at least hints that an interest in cosmography and the physics underlying astronomy may have been present at the earliest stages of Islamic science.[1] Yaᶜqūb ibn Ṭāriq gave one of his works the suggestive title *Tarkīb al-aflāk* ("the arrangement of the orbs"), which was probably written in 161/777–78. Though only fragments have reached us, one of these, significantly for our purposes, is a table of planetary sizes and distances; although Bīrūnī thinks these are Indian, Pingree feels they are more in the spirit of *Planetary Hypotheses* I,2 though he is quick to point out that it is difficult to be definitive about a Greek influence.[2] Indeed, Ibn Hibintā (ca. 950) in his *Astrology* points to an Indian source for Ibn Ṭāriq's system of orbs.[3] Another work that is possibly from this period based on its attribution to the astrologer Māshā'allāh (762–ca. 815) is *De scientia motus orbis* (also known as *De elementis et orbibus coelestibus*).[4] This astronomical treatise in 27 chapters, which is extant only in Latin translation, has sections on the physical basis of astronomy, though the motivation here may be astrological.[5] Pingree has postulated a Syriac source that may be connected with the Hellenized pagan community of Ḥarrān; though this is far from proved, it does provide us with a

[1] Pingree [1973] provides a penetrating account of this period. For a taste of the muddled sources that must be used to reconstruct this period, see Hāshimī, *ᶜIlal*, who may simply be reflecting the eclectic nature of astronomy during the 9th century.
[2] Pingree [1968], pp. 105–109.
[3] Langermann [1990], p. 29.
[4] Pingree [1975] has summarized the contents, pp. 9–12. See also Sezgin, *GAS*, 6: 129.
[5] As Pingree [1975], p. 10 states: "Clearly this is Peripateticism for the astrologer."

possible conduit for the early interest in physical cosmography within Islam as well as a possible motivation having to do with the desire to put forth a unified system that would provide the rational basis for astrology.[6]

But were these early Islamic forays into physical cosmography sufficient to define a genre or to establish a *tradition* that can account for the *Tadhkira*? Based on the testimony of Ibn al-Haytham, a prominent, not to say crucial, figure in developing that tradition, the answer must be no. In the introduction to his *Al-Maqāla fī hay'at al-ᶜālam*, no doubt written at a fairly early stage in his career,[7] he identifies a group of mathematicians (*aṣḥāb al-taᶜlīm*) who have published works dealing with astronomical problems "by speaking generally" [*lit.*, "grossly"] (*bi-'l-jalīl min al-qawl*), but at the same time have "followed a procedure in apparent conformity with the detailed investigation of the science of astronomy" (*li-l-daqīq min al-naẓar fī ᶜilm al-hay'a*).[8]

> ...their goal is to collect that which has been stated in a detailed way and to present it as a summary (*jumal*) free of proofs...for the sole [purpose] of simplifying for those who would benefit from an acquaintance (*maᶜrifa*) with the configurations of those motions by acceptance rather than inquiry and by following the practitioners of the science rather than by reflection...[9]

One can see that Ibn al-Haytham is characterizing a relationship not unlike the one Ṭūsī describes between his *Tadhkira* and the *Almagest*, *i.e.* one of a general account that takes for granted an existing detailed study replete with mathematical proof. But the two other aspects we have identified as fundamental to the *Tadhkira*, a summary that takes the bodies as explicit starting points rather than simply implied and a presentation that is holistic rather than particular, are lacking in these earlier accounts according to Ibn al-Haytham. For he states that they are

> based upon the motions of imaginary points on the circumferences of intellected circles...resulting in what they [*i.e.* earlier writers] have expounded being limited to those circles and points alone since they have not made it their intention to clarify the way in which those various motions may be consummated while being assumed on the surface of solid spheres.[10]

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[6] Pingree [1975], pp. 10–12.
[7] Cf. Sabra [1972], p. 197.
[8] Langermann [1990], p. 5 (Arabic numeration), lines 7 and 10–11. I have changed Langermann's translation (no. [1], p. 53) somewhat in order to emphasize what I take to be Ibn al-Haytham's rhetorical use of *jalīl* (general or gross) and *daqīq* (precise or detailed) to contrast two types of astronomical work.
[9] Langermann [1990], p. 5 (Arabic numeration), lines 19–23; cf. Langermann's translation and commentary, no. [3], pp. 53–54 and pp. 2–4.
[10] Langermann [1990], p. 5 (Arabic numeration), lines 12–18; cf. Langermann's translation, nos. [2–3], pp. 53–54.

Furthermore these summarizers have not provided a coherent account since neither have they clarified "the manner, given the various centers involved, that spheres can carry [other spheres]."[11]

It is important to recognize that Ibn al-Haytham is not saying that previous astronomical work has been "instrumentalist" in some Duhemian sense; in fact, he seems to go out of his way to indicate that previous work has assumed the existence of solid spheres. He will refrain, he tells us, from "criticism and rebuke" since the earlier summarizers "despite ignoring the things they neglected," a clear reference to the solid spheres, nevertheless "probably conceived of them and appreciated their importance" (*mutaṣawwirīn lahā wa-muqayyimīn* [text: *qayyimīn*] *bihā*).[12] Ibn al-Haytham even gives an explanation for this neglect. Since they were dependent upon works that used figures and proofs based on circles, their summaries likewise were "limited to those circles and points alone."[13] But what of those who made these detailed studies with proofs that ignored the solid bodies, Ptolemy being for Ibn al-Haytham the most eminent? Ibn al-Haytham excuses him by remarking that his "aim was to construct proofs for which circles and points sufficed";[14] there is no hint that he thinks that his great predecessor did not understand the importance of the solid orbs for astronomy.

Rather than seeing himself as a realist who must combat the instrumentalism of his predecessors, Ibn al-Haytham instead wishes to make explicit what is implicit in previous work,[15] namely to assume

> the imaginary circles and points that [Ptolemy] took to be detached (*akhadhahā mujarrada*)[16] to be on the surfaces of spheres moving on their own...since that is more true in terms of reality (*min ṣifat al-ḥāl*) and more clear for [providing] an understandable explanation.[17]

Here then is the first aspect that we have identified as crucial for the tradition we are tracing, namely providing a physical account that would explain the

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[11] Langermann [1990], p. 5 (Arabic numeration), lines 18–19; cf. Langermann's translation, no. [3], p. 54.

[12] Langermann [1990], p. 6 (Arabic numeration), lines 1–3; cf. Langermann's translation, no. [5], p. 54.

[13] Langermann [1990], p. 5 (Arabic numeration), lines 13–17; cf. Langermann's translation, no. [2], p. 53.

[14] Langermann [1990], p. 6 (Arabic numeration), lines 14–15; cf. Langermann's translation, no. [8], p. 55.

[15] That the solid orbs were implicit in the work of ancient astronomers was not a unanimously held opinion; see footnote 2, p. 25.

[16] Here the context makes it clear that Ptolemy took these circles and points to be detached for the purposes of his proofs.

[17] Langermann [1990], p. 6 (Arabic numeration), lines 10–12; cf. Langermann's translation, no. [6], p. 54.

celestial motions. Ibn al-Haytham obliges us with clear statements of the other two aspects as well. He explicitly acknowledges the dependence of his work on the *Almagest* when he states that

> our statements concerning all the motions are according to the opinion and belief of Ptolemy; but all that we shall present is a summary by which one can understand the general picture of the form, the position and the motion but not the particulars of these.[18]

As for the problem of fitting all the parts together into a coherent whole, he addresses this as follows:

> ...it is possible to combine together all those bodies assumed for each one of the motions without there occurring any hindrance, resistance, or impediment; rather their motions in being combined are [still] continuous and permanent.[19]

Ibn al-Haytham's importance in establishing the *Tadhkira's* tradition should not be underestimated. Not only has he defined explicitly the crucial elements of that tradition, he has also placed himself in the historical position of being its originator. And insofar as his successors believed his story, and there is no reason to doubt that Ṭūsī, for one, did, this would more or less solve our problem of situating the *Tadhkira* within an Arabic tradition.

There remain, however, numerous problems with *our* privileging of Ibn al-Haytham's position. First there is the inescapable problem of the *Planetary Hypotheses* and the relation of the *Hay'at al-ᶜālam* to it. Since Ibn al-Haytham does not mention it in the *Hay'at al-ᶜālam*, we are in the rather uncomfortable position of claiming a work, *i.e.* the *Hypotheses*, as an important source for a tradition that is either unknown or ignored by a prime member of that tradition. I have little in the way of a solution of this puzzle except to offer the following facts. Ibn al-Haytham does know it by the time of writing his later *Al-Shukūk ᶜalā Baṭlamyūs* (Doubts about Ptolemy) and in fact presents a blistering attack on it.[20] Besides the fact that he considered the *Hypotheses* defective in not providing a physical body for each motion, a criticism applicable to his own ear-

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[18] Langermann [1990], pp. 6–7 (Arabic numeration), lines 27–1; cf. Langermann's translation, no. [12], p. 55.

[19] Langermann [1990], p. 6 (Arabic numeration), lines 19–22; cf. Langermann's translation, no. [10], p. 55. Ṭūsī, as well as Ibn al-Haytham himself later in his life, would come to see this as an overly optimistic view. The point I wish to emphasize here, though, is simply that Ibn al-Haytham is explicitly acknowledging this as part of the work he is undertaking.

[20] *Shukūk*, pp. 42–64.

lier work, he also is disenchanted with the use of truncated orbs, the *manshūrāt* or πρίσματα.[21] Without attempting to be definitive, I would suppose that Ibn al-Haytham, if we assume that he knew the *Hypotheses* at the time of writing the *Hay'at al-ᶜālam*, may have regarded the use of anything other than spheres as so idiosyncratic as to disqualify it from the tradition he saw himself as establishing.[22] There is also the problem of how much we should accept Ibn al-Haytham's assertion that his predecessors had been derelict in attending to the physical side of astronomy.[23] Another problem I have not attempted to deal with here is that of his contemporaries, such as Ibn Sīnā and his student al-Jūzjānī, who may have been engaged in similar enterprises.[24]

Despite these difficulties in establishing Ibn al-Haytham as having single-handedly established physical cosmography in Islam, a position I would not at all wish to maintain, there remains the fact that he and several of his successors saw him in that light. The role of al-Khiraqī (d. 533/1138–39) seems to me particularly germane. He explicitly cites Ibn al-Haytham in both his *Muntahā al-idrāk* and *al-Tabṣira* as inspiration for his work. In particular, he states that he wishes to continue the work of Ibn al-Haytham in considering solid spheres as opposed to imaginary circles in astronomy.[25] And Ṭūsī specifically cites the *Muntahā* in his *Ḥall-i mushkilāt-i Muᶜīniyya*.[26] But of even more importance is the fact that it is this Khiraqī, working explicitly under the influence of Ibn al-Haytham, who establishes the basic structure, the genre, that Ṭūsī follows in both his *Muᶜīniyya* and *Tadhkira*.

D. The *Tadhkira* as Genre

Ibn al-Haytham's advocacy of an astronomical summary in which the celestial *bodies* played a fundamental role was quite influential. But his *Hay'at al-ᶜālam* acted more as a pioneering inspiration rather than as a prototype to be emulated. This latter role was assumed by Ṭūsī's Persian *Risālah-i Muᶜīniyya*

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[21] *Shukūk*, pp. 47–58, especially p. 50.

[22] This is a widespread disenchantment; see Bīrūnī, *Qānūn*, 2: 635, lines 3–17 and Ṭūsī, *Tadhkira*, II.11 [16].

[23] For a discussion of this with particular reference to Battānī and Farghānī, see Langermann [1990], pp. 25–30. Whether Farghānī's *Elements of Astronomy* should be considered to be in line with Ibn al-Haytham's *Hay'a* is an interesting question deserving of serious study; I would merely point out here that Ibn al-Haytham certainly did not think so.

[24] See Saliba [1980].

[25] Shams al-Dīn abū Bakr Muḥammad b. Aḥmad al-Khiraqī, *Muntahā*, Tehran, Majlis-i Shūrā-i Millī MS (uncatalogued), ff. 2b–3a; idem, *Tabṣira*, Istanbul, Ayasofya(?) MS 3398, f. 2a. A German translation of the introductions to both works is due to Wiedemann and Kohl [1926–27]; reprinted in Wiedemann [1970], 2: 628–643.

[26] *Ḥall*, p. 16, line 1.

and by its later Arabic incarnation the *Tadhkira*; they defined by their structure and their approach to the problems of astronomy a certain category of work that would be the model for a distinct genre of astronomical writing.[1]

The development of this genre reflected a certain conceptualization of astronomy in Islam that seems to have evolved during the early centuries of Islamic science. Though a detailed examination would neither be appropriate nor possible here, even a brief discussion of this problem will be useful for situating the *Tadhkira* within the discipline of Islamic astronomy and for dispelling the notion that Ṭūsī's understanding of astronomy represents a new departure.

Encyclopedias and popular accounts of the sciences provide some of our primary sources for how astronomy was conceived. In the 10th century, *ᶜilm al-nujūm* (the science of the stars) was used by Fārābī, Abū ᶜAbd Allāh Muḥammad al-Khwārazmī, and the Ikhwān al-ṣafā' (Brethren of Purity) to designate astronomy in its widest sense, and Khwārazmī indicates that the term is equivalent to the Greek ἀστρονομία.[2] For all three, *ᶜilm al-nujūm* includes both mathematical astronomy[3] and astrology.[4] Khwārazmī and the Ikhwān al-ṣafā' use the term *ᶜilm al-hay'a* (the science of *hay'a*) to designate a certain branch of astronomy that Khwārazmī specifically identifies with "knowledge (*maᶜrifa*) of the arrangement (*tarkīb*) of the orbs, their configuration (*hay'a*), and the configuration of the Earth."[5]

With Avicenna (980–1037), however, we find something new. In his *Aqsām al-ᶜulūm al-ᶜaqliyya* (Classification of the Rational Sciences), *ᶜilm al-hay'a* has replaced *ᶜilm al-nujūm* as the general term for the discipline,[6] and astrology is no longer considered part of this reformulated astronomy.[7] This move is of more than passing significance. For what had once been considered a subdivision of astronomy has now been made to encompass the entire field. Thus whether one is composing an astronomical handbook (*zīj*), making instruments, finding the direction of Mecca, or doing anything astronomical, one would be engaged in this *ᶜilm al-hay'a*, which literally means "the science of configuration"; thus astronomy has implicitly come to designate, at least in this classification, a discipline having to do with bodies inasmuch as *ᶜilm al-hay'a* originally was used to indicate the study of the configuration of the Earth and orbs, and the arrangement of the latter. Without offering an explanation of why this might have occurred at this time, we can point to the related work of Avicenna's contemporary Ibn al-Haytham, who, as we have discussed at length above, was also intent on making the physical bodies central to astronomy.

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[1] This distinct category was first pointed to by Livingston [1973].

[2] Fārābī, *Iḥṣā'*, pp. 84–86; Khwārazmī, *Mafātīḥ*, Bk. II, Ch. 6, esp. p. 215; Ikhwān al-ṣafā', *Rasā'il*, 1: 73.

[3] Fārābī calls this *ᶜilm al-nujūm al-taᶜlīmī*.

[4] For Khwārazmī, I base this on the fact that Ch. 6 of Bk. II, entitled "*Fī ᶜilm al-nujūm*," has a section devoted to astrology.

[5] Khwārazmī, *Mafātīḥ*, p. 215.

[6] Ibn Sīnā, *Aqsām*, p. 112.

[7] Astrology is here a subdivision of physics (*al-ḥikma al-ṭabīᶜiyya*); ibid., p. 110.

While Avicenna is making *hay'a* synonymous with astronomy, he also is, as we have seen, removing astrology from its purview. This may seem odd at first sight since one would assume that an emphasis on the physical aspects of astronomy would make astrology even more central to it. I believe the reason this did not happen was that *ᶜilm al-hay'a* was held to deal with the external aspect of the physical bodies and not their internal character, a point we shall return to below.

This new understanding of astronomy was not confined to Avicenna; one need merely look at late medieval (*i.e.* post-12th c.) classifiers of the sciences such as Ṭāshkubrīzāde (901–68/1495–1561) and Tahānawī (12th/ 18th c.) to see that this view of *ᶜilm al-hay'a* and all that it entailed had become commonplace.[8] Thus under this classification one finds, among other things, theoretical works, astronomical handbooks (*zījes*), books on the making and use of instruments, treatises on observational astronomy, and tables of prayer times. In addition, certain topics in geography are accepted as being part of this astronomy.[9] But just as with Avicenna, *ᶜilm al-hay'a* among these later categorizers does not include works on astrology or the body of literature related to Aristotle's *De caelo* (*al-samā' wa-'l-ᶜālam*), both of which were usually considered to fall under the Peripatetic rubric of physics.

But while *ᶜilm al-hay'a* came to mean astronomy in its most general sense, the original meaning of the term was not completely lost; those works that were intended to give a general view of astronomy from the perspective of the configuration of the orbs would, of course, be *hay'a* works *par excellence*. But in order to distinguish this usage from the all-inclusive one, works of this genre were categorized under *hay'a basīṭa* (plain *hay'a*) since, as Ṭāshkubrīzāde tells us, they "lack (*yujarrada ᶜan*) proofs; they are limited to conceptualizing (*taṣawwur*) and imagining (*takhayyul*) without certainty (*yaqīn*)."[10] It is this type of work that is exemplified both by *Al-Tadhkira fī ᶜilm al-hay'a* (Memoir on the science of *hay'a*) and by one of its more elementary, and consequently very widespread, offshoots, Jaghmīnī's *Al-Mulakhkhaṣ fī al-hay'a al-basīṭa* (Epitome of plain *hay'a*).[11]

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[8] Ṭāshkubrīzāde, *Miftāḥ*, 1: 371; Tahānawī, *Dictionary*, 1: 41.

[9] The reason for this will be explored on p. 38.

[10] Ṭāshkubrīzāde, *Miftāḥ*, 1: 372.

[11] For Jaghmīnī's dependence on the *Tadhkira*, see commentary to III.1 [8]. There is a great deal of conflicting evidence concerning the date of this author, whose full name is Maḥmūd b. Muḥammad b. ᶜUmar al-Jaghmīnī. Suter [1900], p. 164 held that this Jaghmīnī was the same person who wrote a work on medicine called *Qānūnča* and who died in 745/1344–45; however a copy of the *Mulakhkhaṣ*, Istanbul MS Lâleli 2141, is purportedly dated 644/1246–47 (Krause [1936], p. 510), and King states that Ḥājjī Khalīfa, in a manuscript copy of the *Kashf al-ẓunūn*, gives the date of composition as 618/1221–22 (*EI*², 5: xvii). Cf. *GAL*, 1: 473 and S1: 865, King [1986], p. 150 and Suter and Vernet, *EI*², 2: 378. If Krause and King are correct, then Jaghmīnī would obviously not be dependent on the *Tadhkira*; however, at this point I find the evidence from III.1 [8] compelling. Clearly, though, further investigation of the manuscript evidence is needed to resolve the matter.

The three general aspects we have identified above as fundamental to the tradition of physical cosmography underlie, of course, *hay'a basīṭa*; it is at this level of generality that we can connect the *Tadhkira* with the *Planetary Hypotheses* and the *Hay'at al-ālam*. But the *Tadhkira* has certain other features—both stylistic and substantive—that distinguish it in varying degrees from these predecessors. Many, if not most, of these became codified in the large number of works modeled after the *Tadhkira* and it is in this sense that I believe we can refer to it as exemplifying a certain genre over and above its role of continuing the tradition of physical cosmography. In what follows, we shall list and discuss these features.

1. The Structure of the *Tadhkira*. As I have indicated above, we need to look at the *Tadhkira* not only as part of a tradition of cosmographical summaries but also as structured in a way that Ṭūsī felt would most effectively allow him to give a summary of astronomy from the point of view of the physical bodies. The structure comes from the four-part division of the work itself: (a) an introduction that gives the mathematical and physical principles used in the discipline; (b) a section divided into several chapters dealing with the configuration ("cosmography") of the celestial region (*hay'at al-samā'*); (c) another section also divided into several chapters dealing with topics related to the configuration ("geo-graphy") of the Earth (*hay'at al-arḍ*) in which Ṭūsī deals with the astronomically determined divisions of the Earth as well as with other astronomically determined phenomena that in general vary depending on the latitude of the observer; and (d) a section dealing with the size of the Earth and the celestial bodies, and the distances of those bodies from the Earth.

A four-part structure, as far as I have been able to determine, first appears in Khiraqī's *Muntahā al-idrāk*, which has an introduction and three books dealing with the arrangement (*tarkīb*) of the celestial bodies and their motions, the configuration (*hay'a*) of the Earth, and chronology. Ṭūsī himself has basically taken this structure as a model for both his *Risālāh-i Muᶜīniyya* and his *Tadhkira*, but he has made the sizes and distances chapter of the cosmographical section of the *Muntahā* into a separate part (Book IV) and has more or less dispensed with chronology.[12] It is this structure that came to define the genre of writing referred to as *hay'a basīṭa* (plain *hay'a*).

2. A *hay'a basīṭa* work should contain no geometrical proofs. Since a *hay'a basīṭa* work was an account meant to give a general overview of astronomy, it was generally held that it should be devoid of mathematical proof.

[12] He deals with it somewhat in III.10, but, perhaps with the *Muntahā* in mind, he insists that the details of chronology do not have a place in a work such as the *Tadhkira*; see III.10 [3]10–11.

Ṭūsī emphasizes this in I.Intr. [3], as we have seen earlier (p. 24), where he refers the reader to the *Almagest* for proofs of what is being presented "in narrative form." This is stated again in II.5 [10]: "These then are models and rules that should be known. We have only stated them here; their geometric proofs are given in the *Almagest*." And in IV.1 [4], he forgoes presenting one of Bīrūnī's methods for determining the size of the Earth since "it contains geometrical proofs." The one major exception occurs in II.11, where he gives several proofs related to his new models for planetary motion. But this exception is acknowledged by Ṭūsī in such a way that it proves the more general rule; for he states that he is presenting proofs "even though it was not our intention to provide geometrical proofs in this compendium" (II.11 [3]).

3. The *Tadhkira* as a summary for nonspecialists. In addition to not giving proofs, Ṭūsī also indicates his desire to avoid too much detail; for this he refers his reader to specialized accounts. That he could depend on these other works is a clear indication that Islamic astronomy was a highly developed field of study. I also think that this is an indication that Ṭūsī saw the *Tadhkira* as a work useful both for students of astronomy and for nonspecialists.[13]

As examples of this, we find in II.4 [12] that he cuts his discussion of the fixed stars short with the following remarks:

> Knowledge of the fixed stars and that which concerns them being a separate discipline, it is best that we confine ourselves to just what has been presented.

For those interested in numbers, he sends his readers to the "practical handbooks" (*kutub al-ᶜamal*) in II.14 [1]14–15 (for sector measurements) and in III.8 [8]4 (for the equation of time). When an example is needed, he usually will give the simplest one possible as in the case of the *qibla* determination in III.12 [4]; as we saw earlier, he avoided Bīrūnī's determination of the size of the Earth since it involved geometrical proofs.

[13] One difficulty with this is the presentation of non-Ptolemaic models in II.11, which would certainly seem to be written with the specialist in mind; evidently Ṭūsī himself recognizes this when he apologizes for going into detail "even though it was not our intention to provide geometrical proofs in this compendium" (II.11 [3]). Twenty-five years earlier this consideration had led him to delay publishing his newly discovered models in the *Risālah-i Muᶜīniyya*, the Persian prototype of the *Tadhkira*, since their "presentation in this compendium would not be appropriate"; instead he put them in his appendix to that work, the *Ḥall-i mushkilāt-i Muᶜīniyya*. (There are other cases in the *Muᶜīniyya* where Ṭūsī speaks of the inappropriateness of going into the details of complicated matters; see Section J, p. 68 of this volume.) Perhaps by the time of writing the *Tadhkira*, Ṭūsī felt that these "new" developments in the field had come to be an integral part of *hay'a* and were something the student or even general reader should be aware of despite their difficulty.

Finally we should note that Ṭūsī seems quite cognizant of disciplinary boundaries and will not hesitate curtailing his discussion when he feels a boundary is about to be traversed. As we have noted earlier with reference to Khiraqī's *Muntahā*, he dispenses with the historical aspects of chronology since he feels that it has nothing to do with astronomy.

> Every people has an epoch to which they refer the years of their history; understanding the details of that does not pertain to this science. (III.10 [3])

4. All simple bodies as the subject matter of astronomy. Ṭūsī is quite explicit on this point in that he states in I.Intr. [2]:

> The subject of astronomy is the simple bodies, both superior and inferior, with respect to their quantities, qualities, positions, and intrinsic motions.

Several points need to be noted. First it is the *simple* bodies that are the subject matter of astronomy, *i.e.* the four elements plus the celestial aether. Second the fact that the four elements are included means that a *hay'a* work would need to pay some attention to the sublunar world. In the *Tadhkira* we find that not only is the shape and situation of the Earth considered,[14] the cosmographical wrap-up given in II.2 includes a paragraph listing the various levels of the sublunar region. In addition, the entirety of Book III is devoted to "the configuration of the Earth" (*hay'at al-arḍ*), and the measurement of the Earth is taken up in IV.1.

The inclusion in a *hay'a* work of a distinct section on the configuration of the Earth was often taken to distinguish the Islamic and Greek astronomical tradition.[15] This seems something of an exaggeration since Ptolemy devoted Book II of the *Almagest* to many of the problems referred to in Book III of the *Tadhkira*. But the fact that some Islamic astronomers saw themselves as doing something new is significant and again highlights the importance they attached to the notion that astronomy should deal with *all* the simple bodies.

5. Hay'a should only deal with the external aspect of the bodies. Although Ṭūsī makes bodies the subject of astronomy, he is not thereby making astronomy a branch of physics. For one thing, it is clear that he follows Aristotle in taking it to be a "mixed science" inasmuch as he states that its principles come from metaphysics, geometry, and natural philosophy. Furthermore, he tells us that astronomy is interested in bodies in a particular way, namely "with respect to their quantities, qualities, positions, and intrinsic motions" (I.Intr. [2]).

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[14] See II.1.

[15] See, for example, Qāḍīzāde al-Rūmī's (ca. 1364–ca. 1436) commentary to Jaghmīnī's *Mulakhkhaṣ*, f. 3a, lines 9–10.

The significance of such a restriction is underscored by Tahānawī who addresses this question in the introduction to his *Dictionary of Scientific Terms*. He notes that *ᶜilm al-hay'a* should be distinguished from works on *al-samā' wa-'l-ᶜālam*, *i.e.* the *De caelo* tradition of Aristotle, which was taken to be part of the physical or natural philosophy corpus. The reason was not because of subject matter but because of the different aspects of the subject matter studied by each discipline. Though both investigate the simple bodies, *ᶜilm al-hay'a* examines them with regard to their "quantities, qualities, positions, and motions." On the other hand, *al-samā' wa-'l-ᶜālam* studies them from the point of view of "their natures" (*ṭabā'iᶜihā*).[16] Thus it was for *ᶜilm al-hay'a* to examine the outward manifestations of simple bodies, whereas it was for *al-samā' wa-'l-ᶜālam* to investigate their essential nature.[17]

What this means in practice is illustrated in II.1, which deals with proofs for such things as the sphericity of the Earth and sky. In the final paragraph of the chapter, Ṭūsī tells us that the proofs presented

> are *inniyya*, which convey existence; those which convey the necessity of that existence are *limmiyyāt* proofs and are given in natural philosophy in the book *De caelo*.

These *innī* "proofs" are based upon observations; for example, the observations listed in II.1 [2] are the evidence Ṭūsī feels is sufficient to prove that [*inna*] the Earth is, generally speaking, a sphere. This sort of proof does not, however, indicate the physical or metaphysical reasons that would show us why [*limā*] the Earth must be spherical and no other shape. This would require that one investigate the *nature* of the bodies, but that, as we have seen, is the province of natural philosophy, not astronomy.

Although the notion of *innī* and *limmī* proofs ultimately derive from the fact/reasoned fact distinction made by Aristotle in the *Posterior Analytics* (Bk. II, Ch. 13), the manner being used here to differentiate two sciences that study the same subject matter has no real precedent in the Aristotelian corpus.[18] There are, however, other Greek works that one might point to as sources for the dichotomy made in Islamic astronomy. Simplicius (6th c. A.D.), in his commentary on Aristotle's *Physics*, quotes Geminus (1st c. A.D.)[19] to the effect that

> in many cases the astronomer and the physicist will propose to prove the same point, *e.g.* that the sun is of great size or that the Earth is spherical, but they will not proceed by the same road. The

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[16] Tahānawī, *Dictionary*, 1: 47–48; actually Tahānawī is here quoting Bīrjandī's supercommentary on Qāḍīzāde's *Sharḥ al-Mulakhkhaṣ*. See also Nallino [1911], p. 32.

[17] This would also be the reason to exclude astrology from the purview of *hay'a*; cf. pp. 34–35.

[18] For an elaboration, see the commentary to II.1 [8].

[19] On dating Geminus, see *HAMA*, 2: 579–581.

> physicist will prove each fact by considerations of essence or substance, of force, of its being better that things should be as they are, or of coming into being and change; the astronomer will prove them by the properties of figures or magnitudes, or by the amount of movement and the time that is appropriate to it.[20]

Ptolemy in Book I of the *Almagest* would seem to exemplify this attitude since his proofs of the basic cosmological features (the sphericity of the Earth and universe, the Earth's centrality, and so forth) *generally* rely upon mathematics and observations. This, of course, would be perfectly in keeping with his stated preference for the potential certainty of mathematics over the "guesswork" of physics and metaphysics.[21]

But some Islamic astronomers felt he had not gone far enough. Bīrūnī, for example, chides Ptolemy for using arguments based on physics in *Almagest* I.3 to prove the sphericity of the heavens. Going outside one's disciplinary boundaries, he asserts, does not strengthen one's arguments but makes them merely persuasive rather than necessary.[22] Bīrūnī may be mildly upset with Ptolemy for some of his antics in the *Almagest*, but he is truly exasperated with what he sees as a blatant disregard for the confines of astronomy in the *Planetary Hypotheses*.

> Ptolemy in his book *al-Manshūrāt* [*i.e.* the *Planetary Hypotheses*] deviated from the path which he had followed in the *Almagest* [and took up] that which is related to opinions outside of this science, that is in the belief of people (*al-qawm*) that the celestial bodies have life (*ḥayāh*), perception (*shuᶜūr*), sensation (*iḥsās*), and the choice (*ikhtiyār*) of the noblest (*al-afḍal*) motions...so that he even said that the paths of the planets traverse spheres that resemble anklets (*khalākhīl*) or bracelets (*aswira*) and are called *manshūrāt*. He then eliminated (*asqaṭa*) the rest of the sphere...he thus repudiated (*nabadha*) his physical and persuasive proofs (*istidlālātihi*) in the *Almagest* concerning the sphericity of the sky...and he did not explain what is on either side of the *manshūrāt*, whether it is of the genus of the aether thereby returning to what he rejected...or whether it is of a genus of what is below the aether...or whether it is a sixth genus...These are separate subjects of investigation that are [dealt with] in their own particular places.[23]

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[20] Heath [1913], p. 276; cf. Lloyd [1978], pp. 212–214.
[21] *Alm.*, p. 36 (H6–H7).
[22] Bīrūnī, *Qānūn*, 1: 27; see also 1: 49. Cf. Pines [1963], p. 199.
[23] Bīrūnī, *Qānūn*, 2: 634–635.

Bīrūnī's attitude represents an extreme, but by no means unique, position among Islamic astronomers. Ṭūsī would certainly have agreed that Ptolemy had broken a cardinal rule of *hay'a* by discussing the essential nature of the heavenly bodies in an astronomical work. But he would not have agreed that astronomy could do without natural philosophy; in II.1 [6] he acknowledges that he must depend on the *physical* principle of the Earth's rectilinear inclination, rather than on observational evidence, to prove that the Earth does not rotate. This position did not go unchallenged, however; Ṭūsī's own student Quṭb al-Dīn al-Shīrāzī opposed him on this point and the debate continued for several centuries after Ṭūsī.[24]

That Islamic astronomers were insistent that *hay'a* should only deal with the external aspect of bodies, with some even holding that this meant that astronomy could be independent of physics and metaphysics, leads us into fascinating questions concerning the reasons this development took the form it did in Islam and the possible relationship of this with the emergence of non-Ptolemaic astronomy. Even to begin answering these questions, however, would take us very far afield; as Ṭūsī might have said, "it is best that we confine ourselves to just what has been presented."[25]

E. The Physical Principles Underlying the *Tadhkira*

Ṭūsī tells us in I.Intr. [2] that the principles (*mabādi'*) of astronomy come from metaphysics, geometry, and natural philosophy. Geometry is mainly dealt with in I.1,[1] while metaphysical principles of astronomy are generally ignored.[2] On the other hand, many of the principles of natural philosophy, or physics, are listed in I.2, while others are implied. Starting with Duhem, modern historians have tended to view these physical principles as somehow "metaphysical" in the modern sense; this is often reflected in the rather dismissive way in which they are referred to as "philosophical."[3] But this seriously misrepresents the historical situation of these principles and the manner in which they were understood by the practitioners of *hay'a*. For one thing, the physical principles of astronomy were conceived in a way that conformed to the notion that *hay'a* was about the external aspect of bodies. As we shall see later (p. 46), motion, within astronomy, was said to be "investigated [in terms of] its quantity and direction"; the ultimate origin of the motion and like questions was left to natural philoso-

[24] See the commentary to II.1 [6].

[25] For a preliminary viewpoint on these matters, see Ragep [1982], 1: 129–189.

[1] Other geometrical propositions are stated in IV.1 [1].

[2] I am not certain whether this is intentional.

[3] See, for example, Kennedy [1966], pp. 366–367; Neugebauer, *HAMA*, 1: 1 and 2: 572, 942; Hartner [1975], p. 9; and Goldstein [1980], p. 142. For different viewpoints, see Sabra [1984], especially note 3, pp. 145–146 and Ragep [1987], pp. 330–331. On Duhem's role in fostering this attitude, see Ragep [1990], especially p. 210.

phy. This is clearly analogous to the *innī/limmī* dichotomy discussed above. Thus Ṭūsī formulates his principles, in particular Principle 5, in a way that circumvents the "why" (*limā*) and concentrates on the "that" (*inna*).

But despite this disciplinary separation of astronomy from natural philosophy, Ṭūsī, as we have already noted, is not willing to dispense with natural philosophy.[4] As if to emphasize this, Ṭūsī gives his physical principles a certain prominence by placing them in a separate chapter at the beginning of his work (I.2). This is reinforced by a liberal sprinkling of references to these principles throughout the text in order to emphasize the inadequacies of certain of Ptolemy's models. Such a formalistic methodology is not nearly so manifest in earlier *hay'a* works, whose aims had not included the proposal of new models. Ibn al-Haytham's physical principles occur in an appendix to his *Hay'at al-ᶜālam* and this only in one manuscript.[5] In the *Tabṣira*, Khiraqī does provide a separate introduction on the geometrical principles but his physical principles are in a chapter on the ordering of the four elements.[6]

In the following, I shall present a listing of these principles, indicate their origins, which are mainly but not exclusively Aristotelian, and discuss their implications for Ṭūsī's astronomy.

1. A void is impossible (I.1 [1]4). This most fundamental of the principles requires that the universe be completely filled with bodies and thus provides the "raison d'etre" for *hay'a* itself. This, of course, is central to Aristotelian physics.[7]

2. The universe is finite. Although Ṭūsī does not specifically state this principle, it is obviously implied in II.2, which deals with the arrangement and order of the orbs; the ninth orb, responsible for the daily motion, is the "highest orb"

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[4] As we have noted earlier (p. 41), this reluctance in the case of the basic cosmography was due to Ṭūsī's contention that the Earth's state of rest could not be proven from observations; he maintained that he needed to "borrow" a principle from natural philosophy. As for those physical principles themselves, he may have also felt that in order to be established they needed arguments that were neither mathematical nor observational. Needless to say, we can hardly be definitive without investigating Ṭūsī's other writings on the relationship between the various sciences. It is worth noting here that there were some Islamic astronomers who held that their science could be established without recourse to metaphysics or natural philosophy; for an example, see Ragep [1982], pp. 141–145.

[5] Langermann [1990], p. 7.

[6] This chapter, which is Bk. I, Ch. 1, is entitled *Fī bayān aqsām al-ajsām ᶜalā al-ijmāl* (A General Exposition on the Parts of the Bodies). In his *Muntahā*, Khiraqī places this material in his Bk. I, Ch. 2, which he calls *Fī sharḥ maᶜnā ism al-ᶜālam wa-taqsīm ajzā'ihi al-awwal* (An Explanation of the Meaning of the Term "World" and a Preliminary Division of Its Parts). In general, these chapters correspond to the last part of II.2 of the *Tadhkira*.

[7] Aristotle develops his views against the void in Book IV, Chapters 6–9 of the *Physics*.

(II.2 [4]) and all the other orbs and elemental levels are embedded within it. Aristotle sets forth his proofs for the impossibility of an infinite body in *De caelo*, Book I, Chapters 5–7. In particular, an infinite heavenly body could not complete a rotation in a finite time (*i.e.* every 24 hours).[8]

3. A body is either simple...or compound (I.2 [1]20–21). From (1) and (2), we have established that the universe is a finite plenum; therefore, it must be filled in some manner. Since the universe is finite in size and Aristotle precludes infinitely small bodies,[9] there must be a finite number of distinct bodies. Of these, some will be simple while others will be composed of simple bodies. One could probably take this to follow more or less automatically from Aristotle's often repeated position that the simple is prior to the compound.[10] The identity of these simple bodies is more problematic; in *De caelo*, Aristotle uses the reality of simple motion, which he takes to be straight-line and circular, to establish that there must be simple bodies that perform these simple motions.[11] It is fairly straightforward then to establish that the celestial region is made up of one of these simple bodies, the so-called *aether*, since the heavens as a whole perform a perfectly regular daily rotation. The identity and uniqueness of the four sublunar elements cannot be established from an appeal to simple motion alone, however, since there are only two simple, natural motions in the sublunar region (*i.e.* up and down). Ultimately Aristotle must depend on the different approach of *De Generatione et Corruptione* (Bk. II, Chs. 1–8) where he identifies four elementary tactile qualities—hot and cold, moist and dry—whose combination in the substratum accounts for the four elements; earth and water, the heavy elements, can then be characterized by simple downward motion while air and fire, the light elements, can be characterized by simple upward motion.[12]

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With these three principles, Ṭūsī can begin the task, undertaken in Book II, of filling his universe with spherical celestial bodies and elements with the Earth at the center of the whole but not, as required by Aristotle, the center for each individual part.[13] He still needs a means for accounting for the motions of the simple bodies, which we should recall are the subject matter of astronomy,[14] and this is provided by two final principles.

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[8] *De caelo*, Bk. I, Ch. 5, 272b30–273a4.

[9] Although a body may be potentially divided *ad infinitum*, an actual infinitely small body cannot exist (Aristotle, *Physics*, Bk. III, Chs. 5–7; Bk. VI, Ch. 2).

[10] Cf. Elders [1965], p. 86, where one may find references to the relevant passages.

[11] Aristotle, *De caelo*, Bk. I, Ch. 2, esp. 268b17–269a2.

[12] See I.2 [3].

[13] We shall return to this problem; see pp. 46–47 of this volume.

[14] See p. 38.

4. Every motion has a principle (I.2 [2]5). A moving body will, according to him, fall into one of the following four groups (I.2 [2]):

A. *Self-Moved*	B. *Moved by an External Agent*
1. monoform	3. accidental
2. nonuniform	4. by compulsion

In I.2 [2]5, Ṭūsī states that "every motion has a *mabda'*," which literally means "beginning," but since it translates Aristotle's ἀρχαί it is more appropriately rendered "principle [of motion]." In the case of a body moved by an external agent, the principle of motion is in the external agent, while in the case of a self-moved body the principle is internal or, in Ṭūsī's terms, "does not become positionally separate" from the moved body (I.2 [2]5–6).

Astronomy is primarily concerned with the monoform self-moved motion of simple bodies, which will be dealt with in Principle 5 below, and in the accidental motion resulting from one orb moving another. An example of the latter would be an epicycle embedded within a deferent that as a result is moved accidently as the deferent rotates.[15]

5. A simple body has a single nature and what issues forth from that nature does so monoformly (I.2 [1]20–21). Since this is the basis for holding that the celestial bodies move with uniform, circular motion, and that the elements move naturally with straight-line motion, we will need to examine with some care how Naṣīr al-Dīn conceives this principle in the name of which he will proffer models to replace those of Ptolemy.

Ṭūsī states the principle in a very general way, but he really only uses it to establish the principle of motion for the simple bodies. For the elements and the celestial bodies, Ṭūsī, as a corollary to Principle 5, states that each has a nature (*ṭab*^{c}) as their particular "principle of motion."[16] This is meant to account for their self-moved motion that is "monoform" (^{c}*alā nahj wāḥid*)[17]. In I.2 [3], he then identifies the three categories of "motion due to a nature" with the five elementary bodies: (1) earth and water, the heavy elements, move "by nature" toward the center; (2) air and fire, the light elements, move "by nature" away from the center; and (3) the celestial bodies move "by nature" with a circular motion about the center. The first two categories of natural motion are

[15] Ṭūsī considers another type of accidental motion, namely that in which "the mover is the place for [the mobile] naturally to be" (I.2 [2]11). The commentators tell us that an example of this is the "enclosing orb" of II.11 [4]. See the commentary to I.2 [2]10–11, II.4 [6–7] and II.11 [4].

[16] I.2 [2]7–9.

[17] Literally this means "in a single way." Because it includes the accelerated motion of falling bodies, "uniform" would not be an appropriate translation, and I have therefore resorted to the neologism "monoform." Cf. commentary to I.2 [1]21.

monoform in direction but not uniform in speed; celestial motion, however, is both monoform in its movement and uniform in speed. This uniformity has a precise definition: "every point on [a single, simple celestial body] produces at the center equal angles in equal times or cuts equal arcs from the circumference." This, of course, is the basic rule of celestial motion; because it arises as an immediate corollary from one of the physical principles, its violation by Ptolemy, as we shall see, is completely unacceptable to Ṭūsī.

In addition to establishing the simple motions and their character, this principle has other consequences. Because it results in an absolute dichotomy between straight-line and circular motion, Ṭūsī feels compelled in II.1 [6] to assert that the Earth's state of rest at the center of the Universe is the result of a physical principle, namely its "having a principle of rectilinear inclination" which causes it to be "precluded from moving naturally with a circular motion" (II.1 [6]13). His argument against the ability of observation to establish geostasis means that a geocentric Universe ultimately rests for him upon Principle 5.[18] Another consequence of this principle is that the celestial bodies do not undergo "any change of state except for their circular motion"; this is because they do not experience any rectilinear motion, which is the efficient cause of generation and corruption.[19] Thus the perfection of the heavens is not a metaphysical doctrine but rather the result of this physical principle.

The idea that simple bodies move with a natural motion is undeniably Aristotelian.[20] Naṣīr al-Dīn, however, has made a significant shift in the way he has presented this idea, a shift that, I believe, tells us a great deal about the way he understands the role of physical principles in astronomy. As we have seen, Ṭūsī divides self-moved motion into that which is due to what he calls "a nature" if it is monoform, "a soul" if it is not. Clearly, motion due to a nature is characteristic of simple bodies irrespective of the ultimate source of such motion since that of the elements is described as being natural (*ṭabī^{c}iyya*) while that of the orbs is voluntary (*irādiyya*).[21] Thus even though the motion of the orbs may be due to a soul, which would make it in some sense "unnatural," this is not relevant here since the motion of the orbs is regular. On the other hand, the principle of irregular motion, as is the case of vegetative and animal motion, is said to be a soul (*nafs*). In short, the principle of internal motion is dichotomized in terms of regularity and not in terms of soul.

Let us not underestimate what Ṭūsī has done; he has in essence disentangled the physical underpinnings of astronomy from the metaphysical by stating his

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[18] This was not universally held by Ṭūsī's Islamic successors; see the commentary to II.1 [6].

[19] See the commentary to I.2 [4].

[20] Cf. *De caelo*, Bk. I, Ch. 2, 268b27–269a2 where Aristotle virtually states as an axiom that "by simple bodies I mean those which possess a principle of movement in their own nature...necessarily, then, movements also will be...simple in the case of the simple bodies."

[21] I.2 [2]8–9.

physical principles for the motion of the celestial bodies in a way that avoids any reference to the ultimate source of that motion. Tahānawī nicely summarizes the situation:

> In this science [*i.e. hay'a*], motion is investigated [in terms of] its quantity and direction. The inquiry into the origin (*aṣl*) of this motion and its attribution (*ithbāt*) to the orbs is part of Natural Philosophy (*al-ṭab*ᶜ*iyyāt* [*sic*]).[22]

Thus Ṭūsī *qua* astronomer can simply point to the observed regularity of celestial motions and, using Principle 5, attribute it to "a nature." The philosophical question of the ultimate source of the celestial motion is simply not of concern to him here. Uniform, circular motion of the celestial orbs is for him based on a physical principle that is self-evidently confirmed by the overwhelming weight of experience; the metaphysical explanation of that motion may interest him *qua* natural philosopher or metaphysician, but here in the *Tadhkira* he simply does not need it to develop his cosmography.[23]

F. Modeling

1. Ptolemaic Modeling

The physical principles enumerated by Ṭūsī provide the necessary basis for establishing a cosmography. They do not, however, correspond to the laws or assumptions of a deductive system; these principles are guidelines against which various models are judged, not laws from which they are deduced. This is most easily seen by simply noting that a Ptolemaic methodology could not lead unequivocally to a unique celestial configuration. Ptolemy himself had long before conceded the point when he remarked that the solar anomaly could be represented by either an epicyclic or eccentric model, which are mathematically, but not physically, equivalent;[1] this is echoed by Ṭūsī in II.6 [2]. A homocentric cosmology, with a single center for all the celestial bodies, could conceivably provide a unique set of orbs, and this is what would seem to be demanded by a strict adherence to Aristotelian physics; indeed, Ṭūsī himself states that celestial motion is "that which is about *the* center" (I.2 [3]14–15). Homocentrism, however, had not seriously been considered since at least—and more than likely

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[22] Tahānawī, *Dictionary*, 1: 47.

[23] I hope to expand on these remarks in a forthcoming publication; for the present, see Ragep [1982], 1: 164–174 as well as notes 67–76, pp. 183–187.

[1] *Alm.*, III.4, p. 153 (H232).

before—the time of Apollonius (ca. 240–170 B.C.),[2] and its advocacy in 12th c. Spain was an exceedingly limited affair.[3]

That the Ptolemaic system could not provide a definitive cosmology has sometimes been taken to mean that the Ptolemaic orbs were not "real";[4] a further consequence that has been advocated is that Ptolemy himself held this position.[5] But the question of the reality, or lack thereof, of the orbs should be seen as both logically and historically distinct from what I consider a far more important question, namely how various astronomers viewed the relationship between the physical principles and the mathematical models. The antiquated and quaint notion that Ptolemy was an instrumentalist in the modern sense seems miraculously to be fading rapidly.[6] An alternative view, however, one that would take into account his various writings, actual procedures, and possibly changing positions, is still awaited. My objective here is simply to provide some rather preliminary and brief remarks in order to understand Ṭūsī's reservations and objections to his predecessor.

There can be little doubt that Ptolemy was committed to certain physical principles in astronomy despite his reservations about Physics as a discipline.[7] The evidence from the *Planetary Hypotheses* is incontrovertible, and even in the *Almagest* he relies on physical notions for one of his proofs for the sphericity of the World.[8] Furthermore Ptolemy commits himself in various places to the perfection of the heavens and to their uniform, circular motion.[9] Irregular motion is

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[2] Neugebauer, *HAMA*, 1: 263. An explicit rejection of homocentrism was made by Sosigenes (2nd c. A.D.), the teacher of Alexander of Aphrodisias, as reported by Simplicius in his commentary on Aristotle's *De caelo*. He proposed that each body moved about its own center in the heavens while the heavens taken as a whole revolved about the center of the World, a position not dissimilar to what one finds in Ptolemy's *Planetary Hypotheses* as well as in a number of Arabic sources (see our commentary to II.4 [6–7]). Sosigenes gives the varying brightness of the planets and the changing size of the sun and moon as revealed by an annular solar eclipse as observational reasons for rejecting a homocentric cosmology. See Schramm [1963], pp. 56–57.

[3] See Sabra [1984], esp. p. 133. The influence of Biṭrūjī's homocentric cosmology in the Latin West should not blind us to its confined and episodic quality in the Islamic context.

[4] See, for example, Ibn Khaldūn, *The Muqaddimah*, Ch. VI, Section 21, 3: 134–135 (Quatremère's Arabic text, 3: 106); cf. Nallino [1911], p. 33, who also gives the Arabic text.

[5] See, for example, Duhem [1908], pp. 16–20 (English trans. [1969], pp. 16–18). Various Islamic writers would have agreed with Duhem; see p. 25, footnote 2 of this volume.

[6] See Lloyd [1978], pp. 215–217 and Ragep [1990]. In the latter, I pointed out that Duhem himself in his *Système du monde* partially recanted his earlier view and recognized that Ptolemy, by the time of writing the *Planetary Hypotheses*, took his models to be physical; in Duhem's inimitable style, he had become "a slave of the imagination."

[7] *Alm.*, p. 36; see also p. 40 of this volume.

[8] *Alm.*, p. 40 (H13–14).

[9] For example in the *Almagest*, p. 141 (H216–217).

to be explained by means of the "hypotheses" of eccentrics and epicycles.[10] Thus Ptolemaic "modeling" essentially comes down, in theory, to putting together an interlocked set of uniformly rotating concentric, eccentric, and epicyclic orbs. But Ptolemy's commitment to this scheme goes only so far. Whenever phenomenal reality intrudes, it is the physical principles that come to be modified. Ptolemy would seem not to have been oblivious to what he was doing; in his more chatty moments, such as in *Almagest* IX.2 and XIII.2, he evidently is attempting to justify some of his complicated models and his departures from standard procedure. My own tentative interpretation is that Ptolemy is a compromiser of a certain type; he is certainly willing to modify the physical principles when they cannot accommodate the phenomena but he is equally unwilling to abandon them. The result is a rather spectacular success as far as planetary prediction is concerned but a muddle for those interested in a coherent and consistent science. At least, this is the way many of his Islamic successors chose to view him.

2. *Ṭūsī's Criticism of Ptolemy's Models*

There are several types of objections one finds in Islamic astronomy to Ptolemy's models. First there are criticisms of deficiencies in their predictive ability. These begin at a surprisingly early period in the history of Islamic science; for example, Ptolemy's failure to discover the motion of the solar apogee led to a rather severe rebuke by the 9th c. author of *Fī sanat al-shams* (On the Solar Year).[11] Next there is the blanket objection to his multicentered cosmology that targets his use of epicycles and/or eccentrics; this type of criticism is associated with the Spanish Aristotelians of the 12th c.[12] Finally we have the criticisms that accept the modeling that uses eccentrics and epicycles but cannot abide any further compromises. Ṭūsī's criticisms fall into this last category.

It is difficult at present to date in a precise manner when this type of criticism began. Qabīṣī (d. 967) mentions a book of his called *Shukūk fī "al-Majisṭī"* (Doubts on the *Almagest*), but it is not clear to what these doubts refer.[13] We are on firmer ground by the time we reach the first half of the 11th c. Ibn Sīnā's student and biographer, Abū ᶜUbayd al-Jūzjānī, tells us that his teacher had informed him that he had spent some time trying to understand the equant as well

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[10] On the term *hypothesis*, see Toomer, *Alm.*, p. 23–24. For this Ṭūsī uses the Arabic word *aṣl* (pl. *uṣūl*), which literally means source or basis. I have rendered it into English as model.

[11] Morelon [1987], p. 61; the author is not Thābit as has usually been thought but possibly, according to Morelon, one or more of the Banū Mūsā.

[12] For an overview and further bibliography, see Sabra [1984].

[13] Qabīṣī, *Risāla fī imtiḥān al-munajjimīn*, Damascus, Ẓāhiriyya MS 4871, f. 67a, lines 21–24.

as the components of latitude (*mayl, iltiwā',* and *inḥirāf*) and eventually had reached an understanding of them. Reconstructing the details, however, has proven elusive, perhaps because, as he told his student Abū ᶜUbayd, he had informed no one of his result. Jūzjānī himself offered a solution to the equant problem that has recently been published.[14] As far as we know, Ibn al-Haytham provided the first comprehensive criticism of the *Almagest* and the *Planetary Hypotheses* (as well as Ptolemy's *Optics*) in his *Shukūk ᶜalā Baṭlamyūs* (Doubts Concerning Ptolemy).[15]

It is an open question whether Ṭūsī knew the *Shukūk* directly, but I am inclined to believe, despite what Naṣīr al-Dīn himself seems to say,[16] that the objections to Ptolemy's models were fairly well-known by his time.[17] After Ṭūsī, they were to become such a commonplace that they were referred to simply as the sixteen "difficulties" (*ishkālāt*).[18] Although Ṭūsī is not as systematic as his successors, he does manage to identify all sixteen and in the following table I list them, where he identifies them in the *Tadhkira*, and the location of his alternative solutions.

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[14] Saliba [1980]. Jūzjānī's solution was little appreciated; see our commentary to II.11 [1].

[15] An edition is due to A. I. Sabra and N. Shehaby [1971]; a discussion of the text with an English translation of the objections to the prosneusis point of the Ptolemaic lunar model and to the equant is in Sabra [1978]. An English translation of the entire *Shukūk* has been done as part of a University of Chicago doctoral dissertation by D. Voss [1985].

[16] See the commentary to II.11 [1].

[17] Bīrūnī, for example, alludes to the difficulties raised by the moon's prosneusis point as well as the equant in the *Qānūn*, 2: 838.

[18] See, for example, Khafrī's commentary to the *Tadhkira*, II.11, ff. 189b–190a.

Table 1. The "Difficulties" of the *Almagest*.

Difficulty	Identified in *Tadhkira*	Ṭūsī's Solution
1. Irregular Motion of the Moon's Deferent	II.7 [25]	II.11 [1–8]
2. Irregular Motion of Mercury's Deferent	II.8 [19]	II.11 [11] admission of no solution
3–6. Irregular Motions of the Deferents of Venus, Mars, Jupiter, and Saturn	II.9 [15]	II.11 [10]
7–11. Motions on Small Circles of Epicyclic Apices of Upper and Lower Planets (Latitudinal Deviation)	II.10 [6] II.11 [15]	II.11 [19]
12–13. Motions on Small Circles of Endpoints of Mean Epicyclic Diameters of Lower Planets (Latitudinal Slant)	II.10 [6] II.11 [15]	II.11 [19]
14–15. Oscillation of Equators of Deferent Orbs of Lower Planets	II.10 [2]	II.11 [20]
16. Oscillation of Lunar Epicycle Due to Prosneusis Point	II.7 [25] II.11 [13]	II.11 [21]

For a detailed discussion of these objections and their solutions, the reader is referred to the commentary to II.11. For now we can note that these objections fall into two main categories: those that involve irregular motion and those that concern an incomplete rotation. Difficulties 1–6 are in the former category and arise from the motion of the deferent being about a point other than its own center. In the case of the upper and lower planets, this is due to Ptolemy's introduction of the equant as the point of reference for uniform motion, a point not coincident with the deferent center. For the moon, the center of the World, which is distinct from the moon's deferent center, has the role of reference point for uniform motion. Difficulties 14–16 violate the requirement that celestial motion be continuously circular. In all three cases there is an oscillation that clearly contravenes this. Difficulties 7–13 are "mixed" violations. In each case the endpoints of diameters perform circular motions without the epicycle as a whole doing so, which goes against the requirement that the entire celestial orb must be rotating uniformly. Furthermore since these motions are coordinated with the motions of the epicycle center on the deferent, they will be subject to the objections concerning irregular motion raised in Difficulties 2–6.

3. Ṭūsī's Models

Naṣīr al-Dīn dutifully presents his version of Ptolemy's planetary models in Book II, Chapters 6–10. He has a standard procedure that he follows: first, he gives the observations that need to be explained; he then lists the orbs and their motions that are meant to account for these phenomena; and finally he enumerates the anomalies that arise from these motions for the observer. Pursuant to his assessment in II.5 [10] that "one who attempts to understand the principles of the motions must know the configuration of the bodies," his orbs (*aflāk*) are solid bodies as defined in I.1 [15]. These are indicated in Figures T5, T6, T8, and T10. Although these models are avowedly Ptolemaic, they do not precisely correspond to those of Ptolemy in either the *Almagest* or the *Planetary Hypotheses*. In the case of the former, this is due to Ptolemy's use of circles rather than bodies to present his models. To give a simple example: an eccentric deferent in the *Almagest* is a circle on which the epicycle center moves whereas Ṭūsī represents it as a solid body bounded by two parallel spherical surfaces in which one embeds the solid epicycle sphere.[19] In Book II of the *Planetary Hypotheses*, Ptolemy does transform his circles into bodies, and the result is generally, though not exactly, that of the *Tadhkira*.[20]

Ṭūsī comes up with a Ptolemaic count of 22 solid orbs; he also gives a "circle" count that comes to 32. The difference results from the 2 additional circles for the upper and lower planets, the equant and inclined, that are not needed when only solid orbs are considered.[21] In Book II, Chapter 11, he presents his alternative models, which necessitate the addition of 45 extra orbs.

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[19] See II.5 [10] and Fig. T4.

[20] In the absence of a careful study, or for that matter even an edition, of Book II of the *Hypotheses*, it is difficult at present to specify those differences in exhaustive detail. We can say, though, that Ptolemy himself has not eased our task since he at various places gives 41, 34, 29, and 22 as the count for his orbs. In part this results from his ambivalence about the need for 7 spheres to account for the daily motions of the planets; in part it comes about due to his initial presentation of the models in terms of spherical orbs and then his subsequent advocacy of truncated orbs (*manshūrāt*). Cf. Neugebauer, *HAMA*, 2: 922–926.

[21] See II.8 [21] and II.9 [17].

Table 2. Orbs and Circles.

Planet	Ptolemaic Models Solid Orbs	Ptolemaic Models Circles & Motions	Ṭūsī's Models Additional Orbs
Sun	2	2	0
Moon	4	4	6 [3(long.)+3(prosneusis)]
Mercury	4	6	9 [0 (long.)+3 (deviation) +3(slant)+3(inclination)]
Venus	3	5	12 [3 (long.)+3 (deviation) +3(slant)+3(inclination)]
Mars	3	5	6 [3 (long.)+3 (deviation)]
Jupiter	3	5	6 [3 (long.)+3 (deviation)]
Saturn	3	5	6 [3 (long.)+3 (deviation)]
Totals	22	32	45

Ṭūsī therefore needs 67 orbs simply to accomplish what he admits is only a partial success.[22] This enormous loss in economy must be justifiable on some grounds, and it is clear that the benefit to Ṭūsī, as well as to his contemporaries and successors, came from the consistency of mathematical and physical principles resulting from these new models. Using the rectilinear version of his couple, Ṭūsī was able to use uniformly rotating orbs to solve the problem of the irregular motions of the planetary deferents with the exception of Mercury (Difficulties 1, 3–6) while his curvilinear version could bring about the oscillations of Difficulties 7–16, again with uniformly rotating orbs.[23]

There is another facet of Ṭūsī's achievement, namely his ability to isolate specific aspects of celestial motion, in particular rectilinear and curvilinear components. Thus with the rectilinear version of his couple he is able to bring the epicycle center linearly toward and away from the reference point for uniform motion (the equant for the upper and lower planets, the center of the World for the moon); this allows him to circumvent Ptolemy's need for a circumference, that of the deferent, to effect what is essentially a variation in rectilinear distance. For the curvilinear version, Ṭūsī can have an oscillation along an arc which can give him an inclination in either latitude or longitude; with Ptolemy, these inclinations, when he brings them about with a mechanism, are the result of motions on a small circle that result in both latitudinal and longitudinal displacements.[24] Historically it is interesting to note that Ibn al-Haytham's attempt

[22] This is because Mercury eludes him and because, as he notes, there are inadequacies associated with the curvilinear version of his couple; see II.11 [11], [19] and [21].
[23] For details, see the commentary to II.11 and Ragep [1987].
[24] See II.11 [14–15].

to give Ptolemy's small latitude circles a *hay'a*, or configuration, merely sought to reproduce those circles using a Eudoxan-style technique.[25] With Naṣīr al-Dīn, it is his additional circles, ironically, that helped break the stranglehold of circular motion.

G. Sources Named (and Unnamed) by Ṭūsī in the *Tadhkira*

Naṣīr al-Dīn was one of the best informed savants of the 13th century in both the ancient and religious sciences; there can be little doubt that he had a wide variety of sources to choose from in compiling the *Tadhkira*. But it is clear, both from his explicit statements and from his implicit choices, that he saw his primary source as the *Almagest*; even when more "modern" parameters are cited, Ṭūsī tends to use those from the *Almagest*.[1] There is little wonder then that "Ptolemy" and the "*Almagest*" are explicitly cited far more times (21) than any other author or text. (The next closest is Ibn al-Haytham at 2.)

In addition to explicitly cited sources, we can easily infer that Ṭūsī has depended on a number of classical and Islamic authors. For the mathematics introduction of I.1, Euclid is of course primary, but Ṭūsī has also used various works of Theodosius, Autolycus and possibly Hero. Archimedes is explicitly cited in IV.1 [1] for the measures of circles and spheres, and a proposition from Menelaus's *Spherics* is used in III.7 [2].[2] Euclid is referred to by name in IV.4 [2] for finding the volume of a sphere.

For physical principles, the Aristotelian corpus provides the fundamental basis, but there have been some important shifts in formulation and emphasis whose inspiration is by way of Ibn Sīnā.[3] But neither Aristotle nor any other philosopher or physicist is mentioned in this connection, and in fact the only philosophers or theologians referred to at all are Ibn Sīnā and Fakhr al-Dīn al-Rāzī, both with regard to the temperateness of the equatorial region in III.2 [2–4]. As for metaphysics, which is referred to as one of three disciplines from which *hay'a* derives its principles (the other two being natural philosophy and mathematics), it receives scant, if any, attention. There is one reference to divine providence in III.1 [6].[4]

Turning to practical astronomy, again the *Almagest* is fundamental, but my impression is that Bīrūnī's manifold and multifaceted works were an important

[25] See the commentary to II.11 [16].

[1] For example: he uses Ptolemy's eccentricity for the sun in IV.5 [1] rather than those of the moderns in calculating the nearest and farthest distances of the sun; in IV.3 [2] he gives the *Almagest* value for the moon's apparent diameter rather than the value from the *Planetary Hypotheses* or even his own stated values in II.13 [8]2–4.

[2] See the commentary for references.

[3] For these, see pp. 44–46 and the commentary to I.2.

[4] See our commentary for a brief discussion.

resource though he is explicitly referred to only once.[5] As for the early period of Islamic astronomy, Ṭūsī does mention Ma'mūn's astronomers in connection with the measurement of the Earth in IV.1 [2] and he knows of Ibrāhīm ibn Sinān's work on trepidation; in both cases, though, there is some question of how well-informed he actually is on these matters.[6]

I have discussed at some length above Ṭūsī's indebtedness to his predecessors in theoretical astronomy. Curiously he himself does not mention the major works I have identified as part of the *hay'a* tradition, namely Ptolemy's *Planetary Hypotheses*, Ibn al-Haytham's *Al-Maqāla fī hay'at al-ᶜālam* and Khiraqī's *Muntahā al-idrāk* and *al-Tabṣira*. (The *Muntahā*, though, is mentioned in Ṭūsī's *Ḥall-i mushkilāt-i Muᶜīniyya*.[7]) The lack of any explicit mention of the *Planetary Hypotheses* I find particularly surprising since a good deal of the material in Book IV on sizes and distances comes from there rather than the *Almagest*. But he uses expressions such as "they have stated" or "it seemed likely to them" rather than telling his readers that the material comes from the *Hypotheses*.[8]

As for the proposals to reform the Ptolemaic system, Ṭūsī leaves the reader with the strong impression that little, if anything, has been done by earlier astronomers. (He makes the point in both II.7 [25] and in II.11 [1] that none of his predecessors have ventured an explanation or solution of the irregular motion of the moon's deferent.) He does, though, mention an anonymous author in II.11 [12] who proposed a solution to the moon prosneusis problem. He also gives a fairly thorough explanation of Ibn al-Haytham's attempt to provide a *hay'a* (configuration) to resolve the difficulty of part of Ptolemy's latitude theory.

I do not wish to leave the impression that Naṣīr al-Dīn has willfully ignored his predecessors. Although he was quite well informed, there are real questions about what was generally available to someone in Persia during the middle (as opposed say to the end) of the 13th century. For example I am far from certain whether Ṭūsī knows the *Planetary Hypotheses* directly or even Ibn al-Haytham's *Shukūk ᶜalā Baṭlamyūs* (Doubts Concerning Ptolemy).[9] It is salutary to realize that Ṭūsī does not know the greatest of Ibn al-Haytham's works, namely his *Optics*.[10] We are still a long way from being able to write a true history of the influences on and the relationships between the principals of Islamic science.

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[5] This in connection with measuring the Earth by the "dip of the horizon" method in IV.1 [4].

[6] See the commentary to II.4 [5] and IV.1 [2].

[7] See p. 33.

[8] For references, see the introductory remarks to the commentary of IV.5.

[9] See the commentary to II.11 [1].

[10] See the commentary to II.1 [1]21–1 and II.13 [1]16.

Explicit References in the Tadhkira *to Persons and Works*

1. Ptolemy: II.5 [6]; II.6 [1], [2], [3], and [4] (2 occurrences); II.11 [14], [16] and [18]; IV.2 [2] and [4]; IV.3 [1]; IV.5 [1]; IV.6 [1], [4] and [6].

2. Ibn al-Haytham: II.11 [16] (2 occurrences).

3. Abū ᶜAlī ibn Sīnā: III.2 [2] and [4].

4. Archimedes: IV.1 [1].

5. Abū al-Rayḥān al-Bīrūnī: IV.1 [4].

6. Euclid: IV.4 [2].

7. Fakhr al-Dīn al-Rāzī: III.2 [3].

8. Ma'mūn's scientists: IV.1 [2].

9. The *Almagest*: I.Intr. [3] (2 occurrences); II.5 [10]; II.10 [6]; II.11 [14].

10. Natural Philosophy Corpus (*al-ṭabīᶜiyyāt*): I.Intr. [4]; I.2 [title]; II.1 [8].

11. Geometry Corpus (*al-handasiyyāt*): I.Intr. [4]; I.1 [title].

12. *De caelo* (*al-samā' wa-'l-ᶜālam*): II.1 [8].

H. The Influence of the *Tadhkira*

The *Tadhkira* had an enormous influence on the subsequent history of astronomy—so much so that it would be foolhardy to pretend to do justice to it in a few pages. I propose here simply to sketch a program for dealing with that influence and to summarize what we know so far.

"Influence" is one of those tricky historical concepts whose meaning is usually assumed rather than delineated. Most of the discussion of the influence of the *Tadhkira* has focused on its non-Ptolemaic models in II.11, the part these played in the so-called "Marāgha school," and the significance of this "school" in the astronomy of the European Renaissance and, in particular, that of Copernicus. But besides displaying a Eurocentric bias, such a viewpoint has serious historical limitations. "Marāgha," for all its importance in the history of Islamic observational astronomy and teaching, was simply one *episode* in a very long and complex story of Islamic theoretical astronomy. As we shall see in Section J, Ṭūsī had developed his non-Ptolemaic models long before coming to

Marāgha and compiling his *Tadhkira*. ᶜUrḍī as well seems to have developed his models prior to his Marāgha residence,[1] and Shīrāzī's *Nihāya* and *Tuḥfa* were written after leaving Marāgha. And to call an Ibn al-Shāṭir, a Qūshjī, a Bīrjandī or any other late medieval Islamic astronomer writing in various regions of the Islamic world part of this "Marāgha school" substitutes shorthand for history. Later astronomers certainly acknowledged the importance of what had occurred in the 13th c., not only at Marāgha but elsewhere, but they would have seen this as part of a long historical process that, for some, had begun with Ibn al-Haytham, for others even earlier; in short, they considered themselves not part of some "school" but ongoing members of the *hay'a* tradition.

As we have discussed at length above, the *Tadhkira* was an important part of this tradition since it provided, first and foremost, a summary of astronomy from the point of view of the solid bodies. Its influence as a *hay'a basīṭa* work can be seen in numerous ways. The large number of extant manuscript copies stand in silent testimony to this. I suspect that from the 13th until the 18th centuries, the *Tadhkira* was the text of choice for beginning students of astronomy as well as educated laypersons who wanted an introduction providing more meat than Jaghmīnī's *Mulakhkhaṣ*.[2] It is, for example, quoted extensively by the encyclopaedists Ṭāshkubrīzāde (16th c.) and Tahānawī (18th c.) and is mentioned by Ṣafadī (14th c.) in one of his literary works.[3] In addition I believe that the *Tadhkira* was an important model for those texts promising a taste of *hay'a* without tears. In addition to Ṭūsī's own *Zubdat al-idrāk fī hay'at al-aflāk* and to a lesser degree his Persian *Zubdah-i hay'a*,[4] several elementary textbooks seem to derive both their form and contents from the *Tadhkira*. An instructive example is *Kitāb al-Nuzha al-ᶜAlā'iyya*, an otherwise inconsequential school text by a certain Tāj al-Dīn al-Tabrīzī. Though incomplete, the table of contents shows that this late simplified introduction to astronomy follows the basic form of the *Tadhkira*, an indication that its approach was influential at all levels of astronomical writing. An even more important example of this may be Jaghmīnī's *Al-Mulakhkhaṣ fī al-hay'a al-basīṭa* (Epitome of plain *hay'a*), which has been mentioned previously. There is some rather strong evidence that this extremely popular and simplified introduction to astronomy, which was the subject of numerous commentaries and supercommentaries, was dependent on the *Tadhkira*.[5]

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[1] See Saliba [1979].

[2] Ṭāshkubrīzāde (901–68/1495–1561), for example, names the *Tadhkira* first as an epitome (*mukhtaṣar*) of *hay'a basīṭa* and then lists ᶜUrḍī's *Hay'a* and then Shīrāzī's *Tuḥfa* and *Nihāya* as more elaborated works of the genre. The *Mulakhkhaṣ* is then given as a "well-known" (or perhaps "much disseminated") epitome (*min al-mukhtaṣar al-mashhūr*) (*Miftāḥ*, 1: 372–373).

[3] Ṣafadī, *Al-Ghayth al-musjam fī sharḥ lāmiyyat al-ᶜajam*, 2: 257.

[4] The former seems never to have gained much popularity; the latter, in Persian and in Arabic translation, enjoyed somewhat greater success; see Section J.1, pp. 66–67.

[5] For the evidence, see p. 35, footnote 11 and the commentary to III.1 [8].

The *Tadhkira* was also influential on another level. It formed an important basis for more detailed and elaborated studies such as Shīrāzī's *Nihāya* and *Tuḥfa*, which themselves became the primary basis for future work in *hay'a*. It goes without saying that the many commentaries on the *Tadhkira*, listed in the next section, also continued, at times in very interesting and exciting ways, the development of *hay'a* in Islam. Ṭūsī's programmatic approach to the *ishkālāt* (difficulties) of astronomy in II.11, and his admission that several problems remained to be resolved such as Mercury and the latitude theory, provided a significant, indeed crucial, step—and challenge—to the further evolution of non-Ptolemaic modeling in Islam. One can hardly imagine any work after Ṭūsī dealing with the *ishkālāt* that did not mention the Ṭūsī couple, known in subsequent literature as the "model of the big and the small."

The influence of the *Tadhkira* was also felt in cultures beyond the borders of Islam. Jayasiṃha, who ruled in Rājasthāna from 1700 until 1743, clearly had an interest in the astronomy of the Yavanas (Muslims) that led to the acquisition, among other texts, of a copy of Nīsābūrī's commentary on the *Tadhkira*.[6] Of even greater interest, and somewhat surprising, is the Sanskrit translation by Nayanasukha and his assistant Muḥammad Ābida of II.11 of Bīrjandī's commentary on the *Tadhkira*, which is one of the more sophisticated and extensive discussions of non-Ptolemaic astronomy in Islam. But according to Pingree, the work had little lasting influence,[7] which is not surprising inasmuch as the *theoretical* aspects of Ptolemaic astronomy made few inroads into traditional Indian astronomy. Nevertheless this episode in Indian astronomy is of considerable historical interest, especially as it came when European influence was beginning to be felt.

Further west, the impact of the *Tadhkira* may also be detected. This was originally, but rather obliquely, suggested by Dreyer in a footnote in the course of his discussion of Ṭūsī's models that referred the reader to Book III, Chapter 4 of Copernicus's *De revolutionibus*, where the Ṭūsī couple is introduced.[8] But Dreyer suggests nothing beyond this curiosity; postulating a connection between late Islamic astronomy and Copernicus had to await another time and place. This came in the 1950s with the discovery by E. S. Kennedy of the models in Ibn al-Shāṭir's *Nihāyat al-sūl*, which were virtually identical with several of those used by Copernicus. In a series of articles culminating with Kennedy [1966], Kennedy and his collaborators laid out the circumstantial evidence linking late Islamic astronomy, including that of Ṭūsī, and Copernicus.[9] More substantive

[6] I depend here on the very informative article by Pingree [1987]. See also King [1980].

[7] Pingree [1987], p. 325. The Sanskrit translation has been studied and translated into English by T. Kusuba, who presented it as a Master's thesis to the History of Mathematics Department, Brown University.

[8] Dreyer [1906], p. 269. He knew of the *Tadhkira* from Carra de Vaux's [1893] translation .

[9] This research, as well as that of others, was exquisitely summarized by Swerdlow and Neugebauer [1984], 1: 41–48, where the reader may find extensive references to the relevant literature.

evidence came with the discovery by Neugebauer that MS Vatican Gr. 211, which was in Italy by 1475, has in it a short treatise dealing with planetary theory that contains diagrams of a Ṭūsī couple and lunar model; the treatise itself is a Greek translation by Gregory Chioniades of an Arabic original.[10] Furthermore Swerdlow [1972] has shown that the Ṭūsī couple was used by at least one other Renaissance astronomer, and Copernicus himself indicates that the device was hardly a novelty by his time.[11] As the evidence for contacts between late medieval Islamic and Renaissance astronomy has piled up, "the question," to quote Swerdlow and Neugebauer, "is not whether, but when, where, and in what form [Copernicus] learned of Marāgha theory."[12]

I would certainly concur in that judgment; but I think we should both refocus and reformulate the problem away from models and "Marāgha" and toward the *hay'a* tradition itself. Copernicus shared with his Islamic predecessors an approach to astronomy that emphasized the reintegration of physics into mathematical astronomy;[13] in that sense I would consider him as much part of the *hay'a* tradition as Ṭūsī—or for that matter Aristotle and Ptolemy. So the problem is not simply one of change in mathematical models but of the evolution of classical physics as well. In my commentary to II.1 [6], I present an interesting coincidence of views between Ṭūsī and Copernicus regarding the Earth's rotation. Both use the same arguments; each comes to a different conclusion. But as I point out, the debate involving Ṭūsī's position regarding the Earth's stasis went on in Islam until the 16th century—if not later—and involved major reformulations of Aristotelian physics. Did Copernicus know of these debates? I simply do not know. I would argue, though, that to foreclose the possibility is just as "biased" as to assert a connection without further proof. But whatever the outcome of the debate concerning Copernicus's predecessors and motivations, the role of the *Tadhkira* and its author in the history of astronomy would seem secure.

I. The Commentaries on the *Tadhkira*

The large number of commentaries written on the *Tadhkira* provide compelling evidence for its enormous influence. But they also provide a vast resource for studying the development and fate of *hay'a* in Islam. Unfortunately, our modern insistence on "creativity" and "originality" has led to a sharp downgrading of such "mere commentaries"; indeed the large number of them in astronomy and other fields has often been taken as a symptom of the declining

[10] Swerdlow and Neugebauer [1984], 1: 47–48 and Figs. 5–6, 2: 567–568.
[11] See our commentary to II.11 [2].
[12] Swerdlow and Neugebauer [1984], 1: 47.
[13] This was emphasized by Swerdlow [1976], who was responding to Rosen's [1975] ahistorical approach to the problem.

centuries of Islamic science. It is clear that those who have made these judgments have never read these works with any care or attention to detail. Even the more mediocre of them are written by competent scholars who are responding to a continuing interest in theoretical astronomy by both students and the educated public. The best of them are highly original works that provide new solutions to the *ishkālāt* (difficulties) of astronomy as well as very interesting passages concerning the status of astronomy, the relation of theory and observation, the role of physics in astronomy, and other theoretical concerns. The commentary style is simply that—a style within which the author can exhibit creativity and criticism to the extent of his abilities. In Bīrjandī's massive commentary of over 250 folios, the *Tadhkira* often becomes almost incidental as he discusses matters that Ṭūsī had either barely touched upon or ignored entirely. In short, these commentaries as a group represent an important part of the history of astronomy that need to be edited and studied if we are ever to go beyond our very fragmentary view of late medieval astronomy.

One also sees in many of these commentaries a very "modern" concern with textual criticism and historical reconstruction. This has greatly facilitated my own work, and I have not hesitated to use them as my basic "secondary" sources. My favorite quickly became that of Bīrjandī, who seems to have known almost everything there was to know about *hay'a*. My own commentary owes an enormous debt to his elaborations and insights. (I long since have forgiven him his arrogance and nit-picking.)

The following list contains those works that are self-declared commentaries or supercommentaries on one or more passages of the *Tadhkira*. They range in size from Kamāl al-Dīn al-Fārisī's 4-folio treatise, which is restricted to the *Tadhkira*'s discussion of retrograde and direct motion, all the way up to Shīrwānī's nearly 400-folio commentary on the entire work. It should be noted that virtually all the authors are Persians, but they worked and studied in places as far afield as Ḥamāh, Syria (no. 6) and Samarqand (no. 8). I have not listed those texts that are clearly derivative from the *Tadhkira* (which would include most subsequent works on *hay'a*) or that appropriate large parts of it (such as Quṭb al-Dīn al-Shīrāzī's *Nihāyat al-idrāk* and *Tuḥfa*) since they do not follow the standard format of a commentary or supercommentary. I should also mention that many copies of the *Tadhkira* and its commentaries contain extensive notes and glosses, which would be well worth studying to gain insight into the influence of the *Tadhkira* as well as the history of *hay'a*.

Unless otherwise indicated, all the works are in Arabic. (Note: As a general rule, I have throughout referred to the commentaries by the book, chapter, paragraph and line number of the *Tadhkira* established in my edition. In this way, I have avoided referring to specific copies of manuscripts that may be either unfoliated or inaccessible.)

(1) *Tibyān maqāṣid al-Tadhkira* (Exposition of the Intent of the *Tadhkira*) by Muḥammad b. ᶜAlī b. al-Ḥusayn al-Munajjim al-Ḥimādhī, composed some-

time between Jumādā I, 684/July–August 1285 and 4 Ramaḍān 710/ca. 25 January 1311; the former is the date of composition of *Al-Tuḥfa al-Shāhiyya* by Quṭb al-Dīn al-Shīrāzī, who claims in (2) that Ḥimādhī has substantially plagiarized his work, and the latter is the date of Shīrāzī's death. This commentary is only extant as an incorporated part (and therein substantially contracted) of (2).

Begins:

الحمد لله رب العالمين وصلوته على خير خلقه محمد وآله الطاهرين يقول أحوج خلق الله إليه محمد بن علي بن الحسين المنجم الحماذي لما كان كتاب التذكرة في علم الهيئة من مصنفات أفضل المتأخرين

(2) *Fa^calta fa-lā talum* (*lit.*, You have Done It, So Do Not Condemn), by Maḥmūd b. Mas^cūd Quṭb al-Dīn al-Shīrāzī, composed sometime between the boundary dates of (1). This work is a rather caustic reply to (1) that, among other things, attempts to defend the honor of the deceased Naṣīr al-Dīn, accuses Ḥimādhī of plagiarism with regard to the *Tuḥfa*, and generally directs sarcastic and at times vicious comments at that hapless astronomer.

I have used Tehran, Majlis-i Shūrā MS 3944 (= Arab League, Ba^cathat Īrān film no. 228); 232 ff.; copied 826/1422–23 from an autograph.

Cf. *GAL* I, p. 511 (= 675), SI, p. 931 (no. 40a); Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 432 (no. 387, A4).

Begins:

أما بعد حمد الله خالق الأفلاك ومديرها... فإن أحوج خلق الله إليه محمود بن مسعود بن المصلح الشيرازي ختم الله له بالحسنى يقول إني لأتفكّر في صناعة التصنيف وأجيل نظري على جمهور أصحاب التأليف

(3) *Tawḍīḥ al-Tadhkira* (Elucidation of the *Tadhkira*), by al-Ḥasan b. Muḥammad Niẓām al-Dīn al-Nīsābūrī, completed the first of Rabī^c I, 711/18 July 1311. Ḥājjī Khalīfa (under the entry for *al-Tadhkira*) remarks that this commentary is well-known and widely appreciated.

I have mainly used Najaf, Āyat Allāh al-Ḥakīm Library MS 649, 1 (= Arab League, uncatalogued *falak* film no. 315), 107 ff. I have also occasionally used London, British Library MS Add. 7472 for comparison.

Cf. *GAL* I, p. 511 (= 675), SI, p. 931 (no. 40b); Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 438–439 (no. 395, A3).

Begins:

الحمد لله الذي جعلنا من المتفكرين في خلق الأرض والسموات وشرفنا بالنظر في هيئة الأجرام المبدعات ، فهدانا التفكر في المصنوعات ، والتدبر في أمر المدبرات إلى وجود صانع قدير حكيم خبير

(4) Glosses (*Ḥāshiya*) on Nīsābūrī's *Tawḍīḥ al-Tadhkira*, by a certain Faṣīḥ al-Dīn. The only extant manuscript is Leiden or. MS 1010; 50 ff.

Cf. *GAL* I, p. 511 (= 675).

Begins:

أما على الإطلاق كالعدد للحساب قلت فيه بحث

(5) *Ḥāshiya ᶜalā dhikr aṣl al-rujūᶜ wa-'l-istiqāma fī al-Tadhkira* (Glosses on Retrograde and Direct Motion as Described in the *Tadhkira*), by Kamāl al-Dīn al-Ḥasan b. ᶜAlī b. al-Ḥusayn al-Fārisī (d. ca. 720/1320). Fārisī was a student of Shīrāzī and is particularly noted for his *Tanqīḥ al-Manāẓir*, an incisive commentary on Ibn al-Haytham's *Optics*.

The only copy of this short work known to me occurs at the end of a manuscript containing Nīsābūrī's commentary, namely Najaf, Āyat Allāh al-Ḥakīm Library MS 649, 2 (= Arab League, uncatalogued *falak* film no. 315), 4 ff.

Begins:

قال مولانا الأعظم الحبر الأعلم رئيس الحكماء سلطان المهندسين كمال الملة والدين الحسن بن علي بن الحسين الفارسي سقاه الله شآبيب رضوانه وكساه جلابيب غفرانه الحمد لوليه حاشية على ذكر أصل الرجوع والاستقامة في التذكرة الخط المستقيم المواجه في الغاية هو الذي ينصفه سهم شعاع البصر ويكون عمودا عليه

(6) *Takmīl al-Tadhkira* (Complement to the *Tadhkira*), by ᶜUmar b. Da'ūd b. Sulaymān al-Fārisī, completed during the last part of Ramaḍān 711/ January–February 1312 for Abū al-Fidā', the governor of Ḥamāh and author of a famous work on geography.

The unique manuscript is Cairo, Taymūr riyāḍa MS 128, 99 ff.; cf. King [1986], p. 152.

Begins:

قال المفتقر إلى رحمة ربه عمر بن داود بن الشيخ سليمان الفارسي... الحمد لله الذي فطر السموات بغير عمد إرشاداً للخلق الصمد

(7) *Bayān al-Tadhkira wa-tibyān al-tabṣira* (Explanation of the *Tadhkira* and Exposition of the Enlightenment [a pun on Khiraqī's *Tabṣira*]), by Jalāl al-Dīn Faḍl Allāh al-ᶜUbaydī (d. 751/1350), completed 24 Rabīᶜ I, 728/ca. 7 February 1328.

Two copies of this work are Ahmet III MS 3325, 2 (= Topkapı Saray MS 7058, 2), ff. 34b–131b and Ahmet III MS 3313 (= Topkapı Saray MS 7083). I have used the former.

Begins:

الحمد لله الذي خلق السماء متحركة على القطب والمحور والأرض ساكنة على شكل الكرة

(8) *Sharḥ al-Tadhkira* (Commentary on the *Tadhkira*), by al-Sayyid al-Sharīf ᶜAlī b. Muḥammad al-Jurjānī (740–816/1339–1413), completed Tuesday, in the middle of Dhū al-ḥijja, 811/(probably) 30 April 1409, in Shīrāz. Jurjānī is mainly noted today as a philosopher/theologian whose commentary on Ījī's *Mawāqif* became a standard work. He was for a time (1387–1405) part of Timur's entourage at Samarqand and has been described as having been a kind of court theologian there. If the evidence of extant MSS is any indication, this was an extremely popular work.

I have used Damascus, Ẓāhiriyya MS 3117, 160 ff.

Cf. *GAL* I, p. 511 (= 675), SI, p. 931 (no. 40c); Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 476 (no. 424, A1).

Begins:

تبارك الذى جعل في السماء بروجا متخالفة المراتب والآثار وزينها بثواقب الكواكب متفاوتة الأنوار والأنظار

(8_1) A work listed in *GAL* I, p. 511 (no. 40e) as a commentary by Mūsā Qāḍīzāde (Biblioteca Medicea Laurenziana or. MS 271 [= Palat. MS 311, not 313 as in *GAL*]) is actually a copy of Jurjānī's *Sharḥ* lacking the introduction and part of Book I, Chapter One.

(9) *Sharḥ al-Tadhkira* (Commentary on the *Tadhkira*), by Fatḥ Allāh b. ᶜAbd Allāh al-Shīrwānī (al-Rūmī al-Ḥanafī) (d. 891/1486), completed (according to the colophon of Ahmet III MS 3314) Wednesday evening, 3 Ramaḍān 879/Tuesday–Wednesday, 10–11 January 1475. Shīrwānī tells us that he wished to write a more detailed commentary than those of Nīsābūrī and Jurjānī; this mammoth work is certainly that. Among other things, it contains a long *tadhnīb* (appendix) on optics at the end of Book I of about 28 folios. The text is in Arabic.

Baghdādī (*Hadiyya*, 1: 815) claims that Shīrwānī was a resident of Qasṭamūnī (in Anatolia), studied with al-Sharīf al-Jurjānī, and died in 857/1453; the last two claims are untenable based on the date of composition given above as are Sarton's statements that this commentary was written in Turkish and completed in 1414 A.D. (*Introduction*, 2: 1007). I have taken Shīrwānī's death date from *GAL* SII, p. 290; repeated by Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 513. I do not know the basis of their information.

There do not seem to be many extant copies; I have used Ahmet III MS 3314 (= Topkapı Saray MS 7093), 370 ff.

Cf. Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 513 (no. 444v, A1).

Begins:

الحمد لله الذي هيّأ هيئة العالم بحكمته وشيّأ زمرتنا(؟) على التفكر فيها بشيئته

Cf. the anonymous commentary listed by Riḍawī, *Aḥwāl*, pp. 405–406 (no. 9).

(10) *Sharḥ al-Tadhkira* (Commentary on the *Tadhkira*), by ᶜAbd al-ᶜAlī b. Muḥammad b. Ḥusayn al-Bīrjandī (d. 932/1525–26), completed Rabīᶜ I, 913/July–August 1507. He was a student of Manṣūr al-Kāshī, the son of Muᶜīn al-Dīn al-Kāshī; both father and son may well have been on the staff of the Samarqand observatory (823–53/1420–49). Bīrjandī himself is noted for having written a commentary on Ulugh-beg's *Astronomical Tables* as well as a super-commentary on Qāḍīzāde's commentary on Jaghmīnī's *Mulakhkhaṣ* (Sayili [1960], p. 267).

The commentary on the *Tadhkira* makes it clear that he is well-informed not only about what took place at Samarqand, but also about earlier astronomical work. In addition to the commentaries on the *Tadhkira*, Bīrjandī refers to numerous other works in a variety of fields, all of which he is careful to cite. Curiously it contains both sophisticated criticism of previous work and rather extensive (some might say long-winded) explanations of elementary points; clearly Bīrjandī had colleagues as well as students in mind as an audience. The text is in Arabic and should not be confused with a Persian work by the same author entitled *Risālah-i hay'a* (also called *Sharḥ Mukhtaṣar al-hay'a*?); for the latter, see Storey, *Persian Literature*, II.1, p. 82 (no. 121, 3).

I have used Cambridge, Harvard College Library, Houghton MS Arabic 4285, 258 ff.

Cf. *GAL* SI, p. 931 (no. 40g) and Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 542 (no. 456, A9). (Note: Brockelmann apparently is confusing it with the Persian work noted above.) Book II, Chapter 11 was translated into Sanskrit; see p. 57 in this volume.

Begins:

الحمد لله الذي خلق السموات والأرض وجعل الظلمات والنور وبسط على بساط الساهرة بميامن[؟] قدرته الباهرة الظل والحرور

(11) *Al-Takmila fī sharḥ al-Tadhkira* (The Complement to the Commentary on the *Tadhkira*), by Shams al-Dīn Muḥammad b. Aḥmad al-Khafrī, completed

Monday, 4 Muḥarram 932/23 October 1525. The commentary to which this is a complement is Jurjānī's. *Khafr* is the name of a small village near Shīrāz (just east of Fīrūzābād) that I take to be the basis for the *nisba* of this writer. This reading is attested by Damascus, Ẓāhiriyya MS 6727 as well as by Riḍawī (*Aḥwāl*, p. 404). Other possibilities, which seem less plausible, are "Khiḍrī" and "Khafarī," which are given in *GAL*. This work appears to have been quite popular despite its late date. In addition to Jurjānī, Khafrī depends on Nīsābūrī's commentary as well as on Shīrāzī's *Tuḥfa*. Though written during the assumed precipitous decline of Islamic science, Khafrī shows real insight into and understanding of the major problems of *hay'a*.

I have mainly used Damascus, Ẓāhiriyya MS 6727, 323 ff.; Damascus, Ẓāhiriyya MS 6782, 297 ff. was used for comparison.

Cf. *GAL* SI, p. 931 (no. 40d) and Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 471 (no. 422a, A1).

Begins:

وتعاليت يا ذا العرش الأعلى وما أعظم شأنك وتباركت يا مبدع السموات العلي...فيقول الفقير إلى الله الغني محمد بن أحمد الخفري لما كان أجل العلوم بيانا وأوثقها تبيانا هو علم الهيئة

(12) *Ta^c^līqāt* (annotations) on the *Tadhkira*, by Ghiyāth al-Dīn Manṣūr. This is presumably Ibn Amīr Ṣadr al-Dīn Muḥammad al-Shīrāzī (d. ca. 950/1543–44), a prince who had been charged by the Ṣafavid King Shāh Ismā^c^īl I (ruled 1501–24) with restoring the Marāgha observatory, a task that was never brought to fruition (Sayili [1960], p. 288). Riḍawī (*Aḥwāl*, p. 406 [no. 10]) refers to a single manuscript of this work in the Madrasah-i ^c^ālī-i Sipahsālār Library; it is not mentioned by any other source.

Begins (according to Riḍawī):

ان هذه تذكرة فمن شاء اتخذ إلى ربه سبيلا

(13) A commentary by Kamāl al-Dīn Ḥusayn b. Sharaf al-Dīn ^c^Abd al-Ḥaqq al-Ardabīlī (d. 950/1543–44). As far as I know, this work is not extant. See Riḍawī, *Aḥwāl*, p. 405 (no. 8), who cites Baghdādī, *Hadiyyat al-^c^ārifīn*, 1: 318.

(14) Another anonymous commentary is contained in Paris, Bibliothèque nationale MS ar. 6085. This incomplete manuscript lacks all of Book I and Book II, Chapters One through Seven. The illustrations are missing but otherwise the text seems complete. It was copied on Thursday, 18 Rajab 1091/15 August 1691.

Cf. *GAL* SI, p. 931 (no. 40f).

J. The Evolution of the Text of the *Tadhkira*

1. Ṭūsī's Other hay'a basīṭa *Works and Their Relationship to the* Tadhkira

The *Tadhkira* was in large measure modeled after one of Ṭūsī's Persian works, the *Risālah-i Mu^cīniyya*, written during the early period of his residence at the Ismā^cīlī stronghold in Qūhistān. Ṭūsī subsequently published an appendix (*dhayl*) to it, usually called *Ḥall-i mushkilāt-i Mu^cīniyya*, whose contents, which included the first appearance of the rectilinear Ṭūsī couple, were also incorporated into the *Tadhkira*. The *Mu^cīniyya* thus forms an essential part of the background to the *Tadhkira*. Of the many astronomical and astrological writings attributed to Naṣīr al-Dīn, the only other work that I would classify within the genre of *hay'a basīṭa* (plain or simplified *hay'a*) is his *Zubdat al-idrāk fī hay'at al-aflāk*. This small treatise seems to be an abridgment of his larger *hay'a* productions, but further study will be needed to establish a more precise relationship.

Brief accounts of these works follow:

(a) *Risālah-i Mu^cīniyya* [or, mistakenly, *al-mufīd* and *al-mughniya*] *dar hay'a* (the *Mu^cīniyya* treatise on *hay'a*), a treatise written in Persian and completed Thursday, 2 Rajab 632/22 March 1235.[1] This work was written in Qūhistān for Mu^cīn al-Dīn Abū al-Shams, the son of Nāṣir al-Dīn Muḥtasham, the Ismā^cīlī governor of Qūhistān and patron of Naṣīr al-Dīn. There are two different versions of the introduction, one in which Ṭūsī lavishes praise upon his Ismā^cīlī patrons and the other in which he leaves out any direct mention of them altogether. Clearly, the second was written after the fall of Alamūt in 654/1256.

The text itself has the characteristic four-part division of a *hay'a basīṭa* work and in fact quite closely, but not exactly, anticipates the *Tadhkira* in both structure and content: Both have an introductory section consisting of two chapters dealing with geometry and natural philosophy; both have a section on the configuration of the celestial bodies (*hay'at-i ajrām-i ^calawī*) divided into fourteen chapters; both have a section on the configuration of the Earth (*hay'at-i zamīn*) divided into twelve chapters; and both have a section on distances and sizes (*ab^cād wa-ajrām*), the *Mu^cīniyya* having six chapters whereas the *Tadhkira* has seven.

A facsimile reproduction of this work is due to Muḥammad Taqī Dānish-Pizhūh, who provides both versions of the introduction in his intro-

[1] This is the date given in the colophon of Tehran, Dānishgāh (Kitābkhānah-i markazī), Āqā Mishkāt MS 1014 (1), which is transcribed by Dānish-Pizhūh on p. *z* of his introductory remarks to the facsimile reproduction of another manuscript of the *Mu^cīniyya*. (There is an indication that the former copy may be an autograph.) The date of composition is also confirmed by Riḍawī, *Aḥwāl*, p. 388, based on another manuscript source.

ductory remarks.[2] Kennedy [1984] gives a table of contents and reports that the work has been described by Usmanov [1978].

For further bibliographical details, see *GAL* I, p. 511 (no. 40), SI, p. 931; Krause [1936], pp. 494–495 (no. 2); Storey, *Persian Literature*, II.1, p. 56 (no. 7); Riḍawī, *Aḥwāl*, pp. 384–388 (no. 30)); King [1986], p. 152; and Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 404 (A10).

(b) *Ḥall-i mushkilāt-i Muᶜīniyya = Sharḥ-i Muᶜīniyya = Dhayl-i Muᶜīniyya* ("A Solution to the Problems," "A Commentary," or "An Appendix" to the *Risālah-i Muᶜīniyya*), also in Persian, and meant to be an appendix or completion of the *Muᶜīniyya*. This short work was also written for Muᶜīn al-Dīn Abū al-Shams, who asked that certain obscure points from the *Muᶜīniyya* be clarified. Once again we have the curious situation of two prefaces, one extolling the virtues of Ṭūsī's Ismāᶜīlī patrons and the other neglecting any mention of them altogether.

There are nine chapters with a rather wide range of content.[3] Dānish-Pizhūh has also brought forth a facsimile reproduction of a manuscript of this work that begins with the second preface;[4] in his own introduction, he supplies the other preface. Kennedy [1984] gives a table of contents. An unpublished edition has been completed by Wheeler M. Thackston; a translation, also unpublished, is due to him and the present writer.

For further bibliographical details, see Krause [1936], p. 495 (no. 3); Storey, *Persian Literature*, pp. 56–57 (no.7); Riḍawī, *Aḥwāl*, pp. 388–390 (no. 31); and Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 403–404 (A18 & A18a).

(c) *Zubdat al-idrāk fī hay'at al-aflāk* (The Essential Understanding of the Configuration of the Orbs), a short work in Arabic of unknown date.[5] It consists of an introduction and two chapters (one on the *hay'a* of the celestial region, the other on the *hay'a* of the Earth), and a concluding section on sizes and distances. Ṭūsī states in the introduction that his intention is to epitomize the available

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[2] Tehran: Intishārāt Dānishgāh Tahrān (no. 300 in the series), 1335 H. Sh.

[3] For example, Chapter Nine deals with the rather mundane topic of using the Indian circle for finding directions whereas Chapter Five deals with the difficult subject of Ibn al-Haytham's latitude theory. In Chapter Three, he presents the Ṭūsī couple for the first time and his alternative models for the deferents of the moon and planets.

[4] Tehran: Intishārāt Dānishgāh Tahrān (no. 304 in the series), 1335 H. Sh.

[5] It should not be confused with the Persian work *Zubdah-i hay'a* (Essentials of *hay'a*), which was Arabized on several occasions; the Princeton translation has the title *Al-Zubda fī al-hay'a*. Though this work overlaps the *Zubdat al-idrāk* in numerous ways, it does not have the characteristic 4-part division of a *hay'a basīṭa* work but is instead divided into 30 chapters. It is thus comparable to Farghānī's *Elements of Astronomy*. For further information on this work, see *GAL* SI, p. 931 (no. 44a); Krause [1936], p. 497 (no. 13); Storey, *Persian Literature*, p. 60 (no. 15); Riḍawī, *Aḥwāl*, pp. 390–391 (no. 32)); and Mach, *Catalogue* (Princeton), p. 421.

works on *hay'a*. Ḥājjī Khalīfa compares it, sizewise, to *Al-Mulakhkhaṣ* by Jaghmīnī.

A copy of the work, which I have examined, is Istanbul, Topkapı Saray, Ahmet III MS 3430 (5), ff. 59b–92b (=Arab League *falak*, no. 123). Another copy is Paris, B. N. MS ar. 2511 (1). Further bibliographical details may be found in *GAL* I, p. 511 (no. 44) and SI, p. 931; Krause [1936], p. 497 (no. 14); Riḍawī, *Aḥwāl*, p. 391 (no. 33)); and Matvievskaya/Rozenfeld, *Mat. i astr.*, 2: 403–404 (A11).

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An obvious question that needs to be addressed is the relationship of these works to the *Tadhkira*. The *Zubdat al-idrāk* is clearly meant to be an even more abridged and simplified work than the *Tadhkira*. For the most part Ṭūsī here eschews criticisms of Ptolemy; the one exception I have been able to find is a very brief reference to Ibn al-Haytham's alternative models for latitude.[6] But without a date of composition, I am unable to say whether it was made with the *Tadhkira* in hand. The audience is also not entirely self-evident; unlike what one finds in the introduction to the *Zubdah-i hay'a*, Ṭūsī does not say here that the work was meant for his students. Perhaps it was intended, as implied by Ḥājjī Khalīfa, for the lay audience that was captured by Jaghmīnī's *Mulakhkhaṣ*; if so, it was a dismal failure, to judge from the small number of extant manuscripts and the apparent lack of any commentaries on it. The Persian *Zubdah-i hay'a* and its Arabic translations were clearly more successful in that regard.

We are on firmer ground when we come to the question of the relationship of the *Tadhkira* to the *Muᶜīniyya* and its *Ḥall*. As stated above, the *Muᶜīniyya* itself is a straightforward exposition of Ptolemaic astronomy in the *hay'a* mode, such as one finds in Khiraqī's *Muntahā* or *Tabṣira*. However the young Naṣīr al-Dīn, who was 34 at the time, had clearly been thinking about the "difficulties" of Ptolemaic astronomy and their solutions when he was writing the *Muᶜīniyya*, which was completed in 632/1235. For in the midst of his exposition of the moon (Section II, Chapter 5), he remarks that there is a doubt (*shakk*) concerning the motion of the moon's epicycle center on the deferent. In a passage rather reminiscent, but in important ways different, from II.7 [25] of the *Tadhkira*, he concludes:

> Then one of two things must follow: either the invariability of distance and closeness of the epicycle center from the center or the variability in speed and slowness in the motion of the center. And

[6] *Zubdat al-idrāk*, Istanbul, Topkapı Saray, Ahmet III MS 3430 (5), f. 76a–b. It is interesting that Ṭūsī states that neither Ptolemy's nor Ibn al-Haytham's proposals are "free from defect (*khalal*)," which is reminiscent of what we find in the *Muᶜīniyya* (see p. 68 of this volume).

> these two are prohibited. This is a serious doubt (*shakk-i* ^{c}*aẓīm*) concerning this account, which no one among the practitioners of the science has ventured anything or, if they have put it forth, it has not reached us. There is an elegant way (*wajh-i laṭīf*) of solving (*ḥall*) this doubt whose presentation in this compendium would not be appropriate. If in the future the blessed disposition of the prince of Iran, may God multiply his eminence, commands the pleasure of investigating this question, in that chapter (*bāb*) an account will be given, God willing.[7]

In the next chapter (Section II, Chapter 6), which deals with the upper planets and Venus,[8] Ṭūsī again points to the problem of the irregular motion of the deferent, this time due to the equant, and claims a solution:

> The doubt which occurs with regard to the moon occurs in exactly the same way for the motion of the epicycle center on the deferent equator with the lack of uniformity about its center and the uniformity about another center different from it...the solution of this doubt, which no one from the practitioners of the science has ventured anything, is among the puzzles (*asrār*) of the science of *hay'a*. God willing, in the future an explanation (*bayān*) of that will be made.[9]

Naṣīr al-Dīn also has doubts about Ptolemy's latitude theory, which he expresses in Section II, Chapter 8. Here, though, the work of one of his predecessors, namely Ibn al-Haytham, has reached him, and he gives a sketch of his theory.[10] But Ṭūsī is not content with what he finds:

> Yet even with this establishing [of orbs?], this variation (*ikhtilāf*) has not become ordered (*manẓūm*). Concerning that, several other defects come to the fore; but this [work] is not the place to expound on that.[11]

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[7] *Mu*^{c}*īniyya*, p. 31, lines 7–12.

[8] This corresponds to II.9 of the *Tadhkira*.

[9] *Mu*^{c}*īniyya*, p. 37, lines 6–11. Ṭūsī points to the same problem for Mercury on p. 42, lines 7–9.

[10] *Mu*^{c}*īniyya*, p. 44, line 20 – p. 45, line 2. For a discussion of Ibn al-Haytham's model, see our commentary to II.11 [16].

[11] *Mu*^{c}*īniyya*, p. 45, lines 2–3. Other references to Ibn al-Haytham's treatise occur on p. 46, lines 8–9, where Ṭūsī repeats that this is not the place to go into details, and on p. 47, lines 11–14, where he gives the Ibn al-Haytham's additional orbs needed for the latitude of Venus and Mercury.

To summarize, we have the following situation in March 1235: (a) Ṭūsī has published his *Muᶜīniyya* as a relatively short summary of astronomy in the *hay'a* mode and dedicated it to the son of his patron; (b) he claims to have a solution to the irregular motion of the deferents of the moon and planets; (c) he has been studying Ibn al-Haytham's treatise on latitude and though he presents its main results he believes there remain problems that need to be resolved; (d) he promises to present his results in a future work.

From this, it is reasonable to draw the following conclusions. Naṣīr al-Dīn had been inspired at a fairly early period of his career with the rectilinear version of his couple, which he can use to resolve the irregular deferent motions in longitude (Difficulties 1–6; see p. 50).[12] He has also studied Ibn al-Haytham's treatise on latitude and accepts that it can give a *hay'a* (configuration) to the small circles of Ptolemy's latitude theory. But he still feels that Ibn al-Haytham has not resolved the problem completely and refers to unspecified "defects" that remain. These presumably would be Objections 2 and 3 that he refers to in II.11 [15].[13] Unlike the case with the irregular motion on the deferents, he does not claim a solution for these problems. As far as I can tell, he also does not mention the difficulty of the moon's prosneusis nor does he make the critical connection between that problem and the difficulties of Ptolemy's latitude theory.[14] In other words, though he points to Difficulties 7–15 (see pp. 50–51), he does not yet have the curvilinear version of his couple with which to resolve them. He still depends on Ibn al-Haytham despite his reservations.

The confirmation of this reconstruction comes in the *Ḥall*. There can be little if any question that it is the work that Naṣīr al-Dīn tantalizingly dangled before his patron in the *Muᶜīniyya*. And since there is nothing in the *Ḥall* that is not forecast in the *Muᶜīniyya*, there is little to recommend putting its date of composition more than a few months, perhaps a year at most, after that of the *Muᶜīniyya*.[15] It could then be viewed, as it was, simply as an appendage to the

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[12] At this stage in his investigations, Ṭūsī seems to have assumed that Mercury could be dealt with in the same manner as the moon and other planets; see *Muᶜīniyya*, p. 42, lines 7–9. In the *Ḥall* Ṭūsī does not mention Mercury explicitly but it seems to be included when he states that "the difficulty encountered in the other planets may also be resolved [with the couple] if the equant takes the place of the inclined orb and the deferent that of the eccentric" (*Ḥall*, p. 12, lines 3–4). In the *Tadhkira*, he realizes that Mercury is a special case defying an easy solution; see II.11 [11].

[13] These have to do with the irregular motion of the epicycle apex on the small circle and the disruption of the position in longitude. See also pp. 50–51.

[14] See the commentary to II.11 [13].

[15] The recent attempt by Saliba [1987a] to whisk the *Ḥall* "closer to the date of the *Tadhkira* (1260/1261) [*sic*], which it resembles much more closely" does not take into account crucial historical and intellectual developments of the 13th c. For Saliba's dating would force us to assume that Naṣīr al-Dīn was still dedicating books to the Ismāᶜīlīs after they had been completely decimated by the Mongols and after he had joined the Mongol entourage. For other reasons to reject Saliba's dating, see the commentary to II.11 [18].

earlier work.[16] Of the nine chapters of the *Ḥall*, seven are of a fairly pedestrian nature. The two that are of greatest interest are Chapter Three, rather cumbrously entitled "Concerning the Solution of the Doubt Occurring with Regard to the Motion of the Center of the Lunar Epicycle on the Circumference of the Deferent, and the Uniformity of That Motion About the Center of the World," and Chapter Five, entitled "On the Configuration of the Epicycles of the Wandering Planets According to the Doctrine of Abū ᶜAlī ibn al-Haytham." Chapter Three presents the rectilinear version of the couple, while Chapter Five simply presents Ibn al-Haytham's theory. There is no hint of the curvilinear version of the couple that he would use in the *Tadhkira* for resolving Difficulties 7–16 (on latitude and the moon's prosneusis); he is basically at the same place he was when he completed the *Muᶜīniyya*.

Between 632/1235 and 644/1247, the date of the completion of his recension of the *Almagest*, Naṣīr al-Dīn has developed the idea, if not the final form, of the curvilinear version of the couple. He has also made the crucial connection between the prosneusis problem and the latitude problem.[17] The stage for the final synthesis in the *Tadhkira* is now set.

2. The Marāgha (α) Version of the Tadhkira

The process of Arabizing Ṭūsī's Persian works went on both in his lifetime and afterwards; in the former category we may note his Arabic treatise on logic, *Tajrīd al-manṭiq* (written 656/1258), which seems to be an abridged version of his earlier Persian work, the *Asās al-iqtibās* (written 642/1244–45). In the latter category, there are several Arabic translations of the *Zubdah-i hay'a*. One translator of the treatise, a certain ᶜImād al-Dīn ᶜAlī al-Qāshī (?), tells us in his introduction that the work, though "of great usefulness," was not "of general benefit" since it was in the "Persian language whose understanding is denied to the intelligent Arab."[18]

One of the motivations for composing the *Tadhkira* was, no doubt, likewise to have his Persian *Muᶜīniyya* reach a wider audience. He would furthermore have wished in the latter part of his life to synthesize the *Ḥall* as well as his more recent discoveries mentioned in passing in the *Taḥrīr al-Majisṭī* (Recen-

[16] This is clear in the way the *Muᶜīniyya* is referred to in the *Ḥall*, namely simply by chapter and section. For example, Chapter Three begins: "In the 5th Chapter of Section II dealing with the arrangement of the orbs of the moon, this problem occurs." Sometimes Ṭūsī speaks as if the reader has the *Muᶜīniyya* in hand as when in the sentence immediately following the above he states: "as already mentioned [*i.e.* in the *Muᶜīniyya*], the same problem also occurs in the orbs of the other planets..." The intimate connection between the *Muᶜīniyya* and the *Ḥall* may be the reason that it apparently lacks a date of composition in the extant manuscripts.

[17] See the commentary to II.11 [18] for details.

[18] Princeton, Mach MS 4884 (4066), f. 2b.

sion of the *Almagest*) into a more coherent whole. The immediate occasion for the *Tadhkira* was provided by his friend ᶜIzz al-Dīn al-Zinjānī (d. 660/1261–62), who requested a work in Arabic on *hay'a*.[19] It is this Zinjānī who is evidently being referred to in the preface as "one of our dear friends" and to whom the work is being presented as a "memento" (*tadhkira*).[20]

This first version of the *Tadhkira*, which was completed in Marāgha, may be confidently dated as the beginning part (*awā'il*) of Dhū al-qaᶜda 659/September or October 1261, based on the following note that occurs in the margin of the last page of a copy of Faḍl Allāh al-ᶜUbaydī's commentary on the *Tadhkira* (Istanbul, Topkapı Saray, Ahmet III MS 3325, f. 131b):

> وفرغ المصنف من تسويد المتن أوائل ذي القعدة سنة تسع
> وخمسين وستمائة ببلدة مراغه
>
> The author completed drafting (*taswīd*) the text (*matn*) during the beginning part (*awā'il*) of Dhū al-qaᶜda in the year 659 in the town of Marāgha.[21]

It is rather curious that none of the twenty or so manuscripts of the *Tadhkira* that I have examined gives the date of composition, a point to which we shall return.

3. The Baghdad (β) Version of the Tadhkira

The completion of a *muswadda* ("draft," *i.e.* the original manuscript) did not imply an unalterable text; corrections could be made and revisions carried out either by the author alone or in the course of having it read to him by a student or copyist. There is ample evidence to indicate that this is precisely what happened to the *Tadhkira* between the time of its original completion in 659/1261 and Ṭūsī's death in 672/1274. In the following, I shall first present the evidence, both textual and secondary, for the existence of the final revision of the *Tadhkira*, the Baghdad (β) version, which was probably compiled sometime during the latter part of 672 H. (January–June 1274). I shall then discuss the nature of these revisions and the evidence that some of them were made before 672 H.

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[19] This information reliably comes from Ibn al-Fuwaṭī, (*Talkhīṣ*, Vol. IV, 1: 234–235), who was the librarian at the Marāgha observatory.

[20] Cf. commentary to I.Pref. [2]5.

[21] Virtually the same statement occurs in a copy of Jurjānī's commentary (Istanbul, Köprülü MS 927 (2), f. 218a). Riḍawī, *Aḥwāl*, p. 400 gives this same information but without indicating his source.

The most clear-cut evidence for a later, revised text comes from a number of the commentators in their discussion of II.4 [2]23–[3]16, which has to do with the possible motion of the ecliptic equator with respect to the equinoctial. One version of the text has four possibilities for this motion, another has eight.[22] Jurjānī, whose commentary is dated 811/1409, states that the newer version with eight possibilities was made an extended period (*mudda madīda*) after the first. Shīrwānī, writing some 65 years later in 879/1474, adds that the new version was completed more than 20 years after the first and that this was done in Baghdad. Bīrjandī in 913/1507 also states that the change was made in Baghdad. Khafrī, writing in 932/1525, gives the 20-year period of separation.

Certain problems immediately arise. The first, which is not serious, concerns the 20-year period between the two versions. Obviously the greatest length of time possible between the Marāgha version and this Baghdad revision would be 13 years. Could some version earlier than Marāgha, or perhaps the *Muᶜīniyya*, be intended by the earlier one? There is absolutely no evidence for the first possibility and I think that it can be safely dismissed. The second is also untenable since the *Muᶜīniyya* has neither the four nor eight possibilities.[23] Clearly the 20 years is simply a minor exaggeration.

Another more serious problem is whether we are justified in attributing the revisions to Ṭūsī himself since they were seemingly done in the last year of his life. For we know from the historical sources that he left Marāgha for Baghdad in 672 H. (July 1273–June 1274), just a few months before his death.[24] We also know that he was accompanied by a number of his students, and it might be supposed that one of them revised the text after his death. However, there is considerable evidence that Ṭūsī himself was responsible for several of the revisions and some of these can be dated prior to the relocation to Baghdad. The best evidence for this comes from MS M, which is based upon Shīrāzī's working copy of the *Tadhkira*.[25] Now we know that Shīrāzī left Marāgha, whether or not estranged from Ṭūsī being unclear, in the period[26] between 667/1268 and 672/1274; it is then reasonable to conclude that the revisions recorded in the text of MS M were effected before Shīrāzī's departure. And since Shīrāzī, according

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[22] Suter [1902], p. 175, depending on Nallino, who in turn depended on Jurjānī, originally pointed out that there were two versions of the *Tadhkira*. Note that Nallino mistakenly thought that the incomplete copy of Jurjānī's *Sharḥ* contained in Biblioteca Medicea Laurenziana or. MS 271 was by Mūsā Qāḍīzāde.

[23] *Muᶜīniyya*, Section II, Ch. 2, pp. 15–16. There is a brief mention of the possibility of the motion of the ecliptic but little in the way of the extensive treatment in the *Tadhkira*.

[24] See p. 15. Ṣafadī indicates that he was only in Baghdad a few months before his death, which occurred on 18 Dhū al-Ḥijja 672 H./25 June 1274 A.D. I have inferred from this that the trip to Baghdad, and hence the Baghdad version of the *Tadhkira*, should be dated sometime between January and June 1274.

[25] Istanbul, Süleymaniye Kütüphanesi, Lâleli MS 2116. For the colophon that establishes this, see p. 78.

[26] Walbridge [1983], pp. 17–18. A date closer to 667/1268 seems the most likely.

to the note at the end of MS M, read the entire text to Ṭūsī (*qara'ahā ilā ākhirihā ᶜalā muṣannifihā*), all the revisions to the Marāgha version were authorized ones.

An important example of a revision done before Shīrāzī's departure occurs in I.1 [2]19–20, where Ṭūsī gives his definition of a plane surface.[27] MSS DGT[28] contain what Jurjānī refers to as the older and incorrect version. MS M contains a definition that Jurjānī tells us was read (and presumably approved) by Ṭūsī; from what we have established above this would have been done while Shīrāzī was in Marāgha. Now there is a second definition contained in MSS FL and in the margin of MS T, which Jurjānī gives as an alternative to the definition of MS M. For reasons that I shall come to presently, I take this latter definition to be of a later date than MS M and more than likely to be after Shīrāzī's departure from Marāgha.

Very strong evidence that revisions continued to be made after Shīrāzī left Marāgha come from Shīrāzī himself. In his *Faᶜalta fa-lā talum*, which is a supercommentary on an early commentary of the *Tadhkira*, Shīrāzī informs us that Ṭūsī's addition to II.2 [3]22–23 of the *Tadhkira* occurred *after* he left the service (*khidma*) of Naṣīr al-Dīn.[29] MS M gives the addition but a note, perhaps due to Shīrāzī, advises the reader to ignore it.[30] MSS GFL, however, incorporate the revision into their texts without further comment whereas MSS DT do not have it at all.

We have now established that certain revisions were made at various times between 659/1261 and 672/1274, but we still need some means for identifying the original Marāgha text and the various stages in the revision process. The manner in which certain critical revisions are handled in MSS DT provide the key we need for establishing the original text. In the major revision to II.4 [2]23–[3]16 mentioned above, both manuscripts give the original version in the body of the text and provide the revision in the margin.[31] In virtually all the cases in which we can identify an older and newer version, this is the pattern that emerges. For example, in the text of MS D we find what Jurjānī, Khafrī, and Bīrjandī identify as the original version of I.1 [16]8–9, while in the margin of MS D there occurs what they identify as the revision; the copyist has also added the following remark: "The author, may God have mercy on him, changed to this text." Khafrī also specifically indicates that the change is due to the author. For II.5 [8]6–8, MS D specifically attributes the variant given in the margin to "the new, emended version" (*al-iṣlāḥ al-jadīd*); the variant occurs in the text of

[27] For this and all subsequent examples, the reader is referred to the commentary where a detailed discussion occurs.

[28] For a listing of manuscripts, see pp. 76–81.

[29] See our commentary to II.2 [3]21–23 for additional details.

[30] It is interesting that Shīrāzī made sure to be informed of continuing changes to the *Tadhkira* even after leaving Marāgha.

[31] MS T places "*kh aṣaḥḥ*" after the variant indicating that it is from a corrected manuscript.

MSS FL and in the margin of MS T, where the copyist indicates it comes from another version or manuscript (*kh*).

Now MS D has another interesting feature; in the colophon it claims that it is from an autograph. The conclusion then seems inescapable that the *texts* of MSS DT represent to a fairly high degree the original Marāgha version. In the margins, the copyists have placed the revisions.[32] My very strong suspicion is that this is precisely the way revisions were handled during much of the period between 659/1261 and 672/1274. Ṭūsī would approve an original version and then have the copyist place the revisions he had made up to that point in the margins;[33] or he might do the job himself as is indicated by a note at the end of a copy of the *Tadhkira* preserved in Tehran, Sipahsālār MS 4727:

> I voweled and collated [this copy] with a copy that was read to its author, may God continue to protect him, and I transcribed its marginal notes that were in his noble handwriting and what he added to it; I indicated some of them in haste according to [my] effort and ability.[34]

At various points an approved text with the revisions incorporated into a fair copy of the text might be produced; this is what Shīrāzī's copy, exemplified by MS M, represents.[35] (This could also help us understand why no copies of the *Tadhkira* I have examined give the date of composition; it would not have made sense to date these later fair copies with the original 659/1261 year of composition.) After Ṭūsī went to Baghdad, there seems to have been a final version produced and this is represented by MSS FL, which incorporate virtually all the revisions previously made into a new fair copy of the text; it cannot be ascertained whether this occurred before or after his death but because the major revisions can be verified as being due to Ṭūsī himself (either from the evidence of MSS DT or from the commentators), there is no reason to doubt that the other minor changes one finds in MSS FL are also due to him.

What are the nature of these changes? Compared with the changes from the *Muʿīniyya* and the *Ḥall* to the *Tadhkira*, they are fairly insignificant. A large number simply correct grammatical mistakes or stylistic infelicities, such as one finds in I.1 [14]1. In the following list of revisions, I generally ignore these but

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32 The copyist of MS T often indicates that the revisions are corrections with the abbreviation *kh aṣaḥḥ*.

33 There is also evidence that Ṭūsī provided glosses for certain words and phrases, which he did not intend to be incorporated into the text. The one example I know is a *ḥāshiya* (gloss) in the margin of MS M, f. 48b (II.13 [3]2) for the word *makth*: "What is intended by *makth* is the amount (*masāfa*) occurring on the moon from its being cut; this has been copied from the handwriting of the author, may God be pleased with him."

34 Mudarrisī, *Sar-gudhasht*, p. 114.

35 But even in such a case, additional corrections might be put in the margins by a diligent copyist as we have seen.

list the others in somewhat arbitrary categories. Starred citations (*) indicate corrections that have been discussed in the commentary.

(a) Simplifications: I.Intr. [1]10; II.11 [6]2.

(b) Corrections of Mistakes: *I.1 [2]19–20; *I.1 [16]8–9; *I.1 [17]12–13; II.7 [1]1; *II.9 [1]9; II.10 [2]14–15; II.10 [2]17; *II.11 [5]22; II.12 [5]12; II.12 [6]18; II.14 [2]21; III.2 [1]9; III.8 [7]1; *III.10 [1]13–14; *IV.1 [2]7; *IV.1 [3]18–19; IV.3 [1]24; *IV.4 [2]17–18; IV.6 [5]11; *IV.6 [5]17.

(c) Amplifications: I.1 [13]19; II.1 [7]14; *II.2 [3]22–23; *II.4 [2]2–[3]16; II.8 [14]10–11; *II.9 [6]12; II.10 [3]10; II.10 [3]11–12; II.11 [2]5; *II.11 [5]14–15; II.11 [5]19–20.

(d) Clarifications: I.1 [14]21; I.1 [14]22–23; II.3 [10]20; II.4 [2]23; *II.5 [8]6–8; *II.5 [9]3_2; II.7 [9]8; II.7 [14]10; II.7 [16]2–4; II.8 [1]15; II.8 [1]20; *II.8 [15]5; II.8 [16]8; II.9 [1]14; *II.9 [8]21; II.9 [14]11; *II.11 [4]2–4; *II.11 [4]9–10; II.11 [10]9–10; II.12 [1]16; II.12 [3]8; II.13 [9]15; III.3 [1]15; III.3 [2]5; III.3 [3]1; III.9 [1]11; IV.1 [3]12; IV.1 [3]17; IV.5 [3]19.

In addition the original of Figure T25 in III.9 [1] was revised in the Baghdad version.

In general, the revisions are more numerous in the first two books than the last two. Few, if any, can be called sensational; in particular, the revisions made in the chapter that presents Ṭūsī's new models (II.11) are fairly minor. Some of the changes are surprisingly for the worse. This is especially true of the one in II.5 [8]6–8, which in the Baghdad version gives an incorrect proportion for finding retrograde motion. Other examples are I.1 [13]18 and II.11 [5]19–20. In this regard, it is interesting that a revision might be marked for deletion; this seems to be the case for II.9 [6]12, where a phrase that was added in the margins of MSS MT, and into the text of MS G, is marked for deletion in MS F and is missing entirely in MS L, these latter two manuscripts being my best witnesses to the Baghdad version. Finally we should note that though the vast majority of the revisions are due to Ṭūsī, there are a few suspicious ones that might be due to Shīrāzī or perhaps another student or colleague. Three that I strongly suspect are due to Shīrāzī, and that occur in MS M, are variants to II.9 [1]9, II.11 [9]1–2_2 and IV.3 [5]19; both Jurjānī and Bīrjandī suggest that this is so for II.9 [1]9.

K. List of Manuscripts

In the following list, nos. 1–6 are the principal manuscripts that have been used to establish the edition (see Section M.2). The other five manuscripts that have sigla were used at an earlier stage of editing, but their variants added little to the final edition; consequently I decided to leave them out of the final apparatus. On occasion, though, they are referred to in the apparatus and commentary. In addition to the eleven manuscripts with sigla, I have also examined, with varying degrees of thoroughness, nos. 7, 10, 11, 12, 13, 16, 29, 31, 32, 33, 35.

A convenient listing of manuscript collections and their catalogues is in Sezgin, *GAS* 6.

Sigla	Description of Manuscript
D ض	1. Istanbul, Feyzullah MS 1330, 1; ff. 1b–83a; ca. 14–16 lines/page; numerous annotations, especially in the beginning; copy completed on 17 Ṣafar 757/ca. 20 February 1356 from an autograph by Aḥmad b. Maḥmūd b. Muḥammad al-Qazwīnī.

Colophon:

نقلت هذا الكتاب من خط مصنفه محمد بن محمد بن الحسن الطوسي رحمه الله في السابع عشر من الصفر ختم بالخير والظفر[؟] لسنة سبع وخمسين وسبعمائة وأنا أحوج خلق الله تعالى إلى غفرانه أحمد بن محمود بن محمد القزويني

I copied this book from the handwriting of its author Muḥammad b. Muḥammad b. al-Ḥasan al-Ṭūsī, may God have mercy on him, successfully completing it on the 17th of Ṣafar of the year 757. And I am Aḥmad b. Maḥmūd b. Muḥammad al-Qazwīnī, of God's creation the most in need of His forgiveness.

Sigla	Description of Manuscript
F ف	2. Vatican, ar. MS 319, 1; ff. 1b–64a; 17 lines/page; has occasional Latin annotations [15th–16th c. ?]; copied Friday, 5 Muḥarram 683/24 March 1284 by Maḥmūd b. Muḥammad b. al-Qāḍī Taqī al-Dīn in Baghdad at the Niẓāmiyya College.

Sigla | Description of Manuscript

Colophon:

فرغ من تحريره العبد المذنب المفتقر إلى رحمة ربه الغفور
محمود بن محمد بن القاضي تقي الدين يوم الجمعة خامس
محرم سنة ثلاث وثمانين وستمائة في مدرسة نظام الملك بمدينة
السلام حماها الله تعالى عن نوائب الدهر وحدثانه حامداً لله
تعالى ومصلياً على نبيه والحمد لله رب العالمين

The writing of this was completed by the sinner in need of the mercy of his forgiving Lord Maḥmūd b. Muḥammad son of the Judge Taqī al-Dīn on Friday, the 5th of Muḥarram of the year 683 at the College of Niẓām al-Mulk in the City of Peace, may God Almighty protect it from the misfortunes and afflictions of time, praising God Almighty and praying for His Prophet; Praise be to God, Lord of the Worlds.

G غ 3. St. Petersburg (a.k.a. Leningrad), Oriental Institute MS A 437; ff. 1b–43b; 17 lines/page; incomplete, folios disordered; copy completed 3 Ramaḍān 673/ca. 2 March 1275.

Colophon:

تمت بعون الله وحسن توفيقه في الثالث من الرمضان سنة
ثلاث وسبعين وستمئة

Completed with the aid of God and the goodness of His granting success on the 3rd of Ramaḍān of the year 673.

L ل 4. Leiden, University Library MS or. 905 (= 1093); ff. 1b–95a; 15 lines/page; very readable pointed script with numerous vowel markings; no colophon but a marginal note on f. 95a indicates that it was compared with a "reliable copy."

M م 5. Istanbul, Lâleli MS 2116; ff. 1b–83b; 17 lines/page; copy completed on 8 Ramaḍān 681/ca. 10 December 1282.

Sigla | Description of Manuscript

Colophon (in margin of f. 83b):

قوبل بالنسخة التي بخط مولانا المعظم علامة العالم أفضل
المتقدمين والمتأخرين زبدة خلق الله في العالمين قطب الملة
والدين أسبغ الله ظلال جلاله وكان يحدّث عنها وقراها إلى
آخرها على مصنفها محمد بن محمد الطوسي رضي الله عنه
في ثامن[؟] رمضان المبارك[؟] في[؟] إحدى[؟] ‹و›ثمانين[؟؟]
‹و›ستمايه

This has been collated with the copy in the hand of our great master, the most erudite in the World, the most excellent of the ancients and the moderns, the cream of God's worldly creation, the pole (*quṭb*) of the community and creed, may God widen the shadows of his grandeur, and he used to narrate from it and he read it in its entirety to its author Muḥammad ibn Muḥammad al-Ṭūsī, may God be pleased with him, on the 8th of blessed Ramaḍān in 681 [Krause [1936] reads 684].

T ط

6. Istanbul, Ahmet III MS 3453, 19 (= Topkapı Saray MS 7005, 19); ff. 261b–281a; 27 lines/page; an important manuscript containing, in addition to the *Tadhkira*, Ṭūsī's recensions of the "Middle Books"; copied 12 Rabīᶜ I, 677/ca. 3 August 1278 by ᶜAbd al-Kāfī b. ᶜAbd al-Majīd b. ᶜAbd Allāh al-Tabrīzī in Baghdad.

Colophon:

وأكتب المصنف رحمة الله إليه هذا سوّده أحوج خلق الله
محمد بن محمد الحسن الطوسي فرغ نسخ هذا الكتاب عبد
الله الفقير إليه عبد الكافي بن عبد المجيد بن عبد الله
التبريزي في الثاني عشر من شهر ربيع الأول سنة سبع
وسبعين وستمائة في مدينة بغداد دامت محميه عن الآفات
حامدا لله تعالى ومصليا على أشرف خلقه محمد وآله الطيبين
الطاهرين

Sigla	Description of Manuscript

The author, God's mercy upon him, dictated this: The most needy of God's creation, Muḥammad b. Muḥammad b. al-Ḥasan al-Ṭūsī, drafted this. The copying of this book was completed by the needful servant of God ᶜAbd al-Kāfī b. ᶜAbd al-Majīd b. ᶜAbd Allāh al-Tabrīzī on the 12th of the month of Rabīᶜ I in the year 677 in the city of Baghdad, may its protection from harm continue, praising God Almighty and praying for Muḥammad, the most noble of His creation, and for his most excellent and virtuous family.

7. Aleppo, Aḥmadiyya (*waqf*) Library MS 1284.

8. Aligarh, Subḥānullāh Oriental Library MS 121, 3; 44 ff.; Shaᶜbān 1300/June–July 1883.

9. Baghdad, MS 2958 [? see Matvievskaya/Rozenfeld, *Mat. i Astr.*, 2: 403 (no. 368, A9)].

10. Cairo, al-Azhar MS 18079, 1; ff. 1–94; copied 1290/1873–74 by Muḥammad Amīn Riḍā.

11. Cairo, National Library MS K 3957, 3; ff. 155b–253a; n. d. (ca. 900/1500) [see King [1986], p. 151].

12. Cairo, National Library, Ṭalᶜat hay'a MS 38, 1; ff. 1a–62a; copied 1076/1665–66 [see King [1986], p. 151].

13. Cairo, National Library, Taymūr majāmīᶜ MS 181, 1; pp. (*sic*) 1–120; copied ca. 1114/1702–03 [see King [1986], p. 151].

14. Diyarbakır (Turkey), MS 2213 A, 8; ff. 82a–113a; copied in 727/1326–27 (see Şeşen, 3: 33).

15. Edirne (Turkey), Selimiye MS 1244, 3; ff. 197b–250b; copied in 879/1474–75 (see Şeşen, 3: 33).

16. Heidelberg, University Library MS or. A 144; 88 ff.; n. d. (ca. 1650 A. D.).

Sigla	Description of Manuscript
S س	17. Istanbul, Ahmet III MS 3317 (= Topkapı Saray MS 7081); ff. 1b–60a; copy owned by Sultan Bāyazīd II.
H ح	18. Istanbul, Ahmet III MS 3333, 1 (= Topkapı Saray MS 7082, 1); ff. 1b–32a; copied in 728/1327–28 by Muḥammad b. Muḥammad al-Samarqandī.
	19. Istanbul, Ali Emiri Arabi MS 2735; 150 ff.; n. d. (ca. 9th–10th c./ 15th–16th c.) [see Krause [1936], p. 494].
	20. Istanbul, Aşir Hafid MS 203, 2; 90 ff.; copied 797/1394–95 [see Krause [1936], p. 494].
	21. Istanbul, Carullah MS 1457, 2; 26 ff.; copied ca. 712/1312–13 in Tabrīz; incomplete [see Krause [1936], p. 494].
	22. Istanbul, Fatih MS 3388; 77 ff.; copied 955/1548–49 in Istanbul [see Krause [1936], p. 494].
	23. Istanbul, Fatih MS 3389; 56 ff.; n. d. (ca. 7th–8th c./13th–14th c.) [see Krause [1936], p. 494].
	24. Istanbul, Feyzullah MS 1331; 66 ff.; copied 749/1348–49 in Aleppo [see Krause [1936], p. 494].
	25. Istanbul, Lâleli MS 2115; 88 ff.; n. d. (ca. 9th–10th/15th–16th c.) [see Krause [1936], p. 494].
	26. Istanbul, Ragıp Paşa MS 919, 2; ff. 71b–134a; copied 877/ 1472–73 [see Krause [1936], p. 494].
N ن	27. Leiden, University Library MS 188, 4 (= 1092); ff. 38b–111a; copied in 785/1383–84.
C ص	28. Leipzig, University Library MS 261 (= K. 203); ff. 1b–40a; copied 14 Shawwāl 790/ca. 16 October 1388.

Sigla	Description of Manuscript
	29. London, British Library MS 1339, 1 (= Add. 23,394); copied in 1107/1695–96 by Muḥammad Riḍā b. ᶜAzīz Allāh al-Tūnī.
	30. Los Angeles, UCLA Library Arabic MS 1117 [courtesy of Aḥmad Harīdī, Cairo].
	31. Mashhad, Āstān Quds Riḍawī MS 8568.
	32. Najaf, Āyat Allāh al-Ḥakīm MS 1099 (= Arab League uncatalogued *falak* film no. 321); 91 ff.; copied 1324/1906–07.
	33. Paris, Bibliothèque nationale MS ar. 2330, 8; ff. 83b–86b (contains only part of Book I).
B ب	34. Paris, Bibliothèque nationale MS ar. 2509 (= Suppl. ar. 962); ff. 2b–82a; used by Carra de Vaux [1893] for his translation of II.11, but the text is a jumble of the two versions and the figures are rather inaccurate; copied Monday (end of day) 2 Shaᶜbān 791/26 July 1389.
	35. Princeton, MS 4881 (Mach); 56 ff.; copied Muḥarram 771/August–September 1369.
	36. Tashkent, Oriental Institute MS 8990, 1 [see Matvievskaya/Rozenfeld, *Mat. i Astr.*, 2: 403 (no. 368, A9)].
	37. Tehran, Dānishkadah-i Ilāhiyyāt Library MS 275 G; 45 ff.; n. d.
	38. Tehran, Kitābkhāna Najm Ābādī; copied in 671/1272–73 [see Riḍawī, *Aḥwāl*, p. 400].
	39. Tehran, Sipahsālār MS 4727; 19 ff.; collated with a copy that had been read to Ṭūsī and that had marginal notes in his hand, which were incorporated into this manuscript; copy completed 10 Rabīᶜ II, 761/ca. 29 Feb. 1360 [see Mudarrisī, *Sar-gudhasht*, p. 114].

L. Concordance of Manuscripts

page of edition	D	F	G	L	M	T
91.	1b,1	1b,1	1b,1	1b,1	1b,1	261b,1
93.	1b,14	1b,13	1b,11	2a,1	2a,1	261b,7
95.	2b,1	2a,8	2a,3	2b,3	2b,1	261b,14
97.	3a,6	2b,8	2a,16	3a,10	3a,7	261b,23
99.	3b,9	3a,6	2b,11	4a,1	3b,10	262a,4
101.	4a,10	3b,2	3a,3	4b,4	4a,9	262a,10
103.	5a,6	4a,5	3b,2	5b,1	5a,2	262a,21
105.	5b,11	4b,3	3b,14	6a,7	5b,6	262b,2
107.	6b,9	5a,6	4a,12	7a,6	6b,1	262b,12
109.	7b,3	5b,7	4b,9	8a,1	7a,11	262b,22
111.	8a,9	6a,7	5a,5	8b,10	7b,16	263a,3
113.	9a,2	6b,8	5b,3	9b,7	8b,9	263a,14
115.	9b,6	7a,6	5b,15	10a,13	9a,15	263a,22
117.	10a,14	7b,9	6a,13	11a,11	10a,9	263b,6
119.	11a,4	8a,9	6b,10	12a,5	10b,17	263b,15
121.	11b,9	8b,9	7a,6	12b,14	11b,8	263b,25
123.	12a,12	9a,7	7b,2	13b,6	12a,14	264a,6
125.	12b,9	9b,4	7b,10	14a,12	12b,11	264a,11
127.	13b,2	10a,7	8a,7	15a,9	13b,4	264a,20
129.	14a,10	10b,10	8b,5	16a,6	14a,15	264b,2
131.	15a,3	11a,13	9a,3	17a,4	15a,9	264b,13
133.	15b,12	11b,14	9a,15	18a,2	16a,3	264b,23
135.	16a,8	12a,9	9b,7	18b,8	16b,7	265a,5
137.	16b,7	12b,7	33a,3	19a,14	17a,9	265a,13
139.	17b,6	13b,3	33a,17	20a,13	18a,3	265a,23
141.	18a,5	13b,15	33b,7	20b,14	18b,2	265b,2
143.	19a,10	14b,7	34a,9	22a,2	19b,10	265b,17
145.	19b,6	15a,1	34b,3	22b,1	20a,7	265b,23
147.	20b,7	15b,9	34b,16	23b,3	21a,7	266a,7
149.	21a,9	16a,5	35a,8	24a,6	21b,8	266a,16
151.	22a,4	16b,16	35b,5	25a,9	22b,5	266a,27
153.	22b,10	17a,16	35b,16	26a,2	23a,13	266b,9
155.	23b,4	17b,17	36a,12	26b,15	24a,6	266b,20
157.	24a,8	18a,16	36b,6	27b,7	24b,12	266b,27
159.	24b,13	18b,16	36b,17	28b,1	25b,4[m]	267a,10
161.	25b,7	19b,2	37a,13	29a,14	26a,13	267a,21
163.	26a,3	20a,1	37b,2	30a,1	27a,1	267a,25
165.	26b,6	20a,16	37b,12	30b,8	27b,3	267b,6
167.	27a,12	20b,16	38a,8	31a,15	28a,9	267b,15

page of edition	D	F	G	L	M	T
169.	28a,6	21a,17	38b,3	32a,12	29a,2	268a,5
171.	28b,13	22a,1	38b,14	33a,10	29b,9	268a,15
173.	29b,9	22b,3	39a,9	34a,7	30b,4	268a,25
175.	30b,2	23a,5	39b,4	35a,3	31a,15	268b,9
177.	30b,5	23b,1	39b,6	35a,6	31b,2	268b,11
179.	31a,1	23b,7	39b,12	35b,9	32a,1	268b,16
181.	31b,5	24a,7	40a,9	36a,15	32b,9	268b,24
183.	32a,10	24b,7	40b,3	37a,7	33a,16	269a,9
185.	33a,5	25a,9	40b,16	38a,3	34a,8	269a,20
187.	33b,7	25b,6	41a,8	38b,7	34b,11	269a,27
189.	34a,2	26a,4	41a,15	39a,5	35a,1	269b,6
191.	34b,9	26b,5	41b,10	39b,15	36a,2	269b,18
193.	35b,4	27a,7	42a,6	40b,12	36b,17	270a,1
195.	36b,2	27b,12	42b,4	41b,13	37b,7	270a,13
197.	37a,8	28a,12	42b,16	42b,6	38a,11	270a,22
199.	37b,1	28b,1	12a,1	43a,1	38b,1	270b,1
201.	38a,1	29a,4	12a,15	43b,13	39a,7	270b,9
203.	38b,4	29b,6	12b,9	44b,9	40a,3	270b,17
205.	39a,7	30a,5	13a,3	45b,4	40b,9	270b,26
207.	39b,2	30b,2	13b,1	46a,3	41a,2	271a,6
209.	39b,7	30b,8	13b,5	46a,11	41a,10	271a,18
211.	40a,14	31a,11	14a,1	47a,12	42a,3	271b,3
213.	41a,8	31b,14	14a,16	48a,11	42b,12	271b,13
215.	42a,2	32b,1	14b,13	49a,10	43b,6	271b,23
217.	42b,13	33a,6	15a,11	50a,8	44a,17	272a,6
219.	43a,8	33a,15	15a,16	50b,6	44b,11	272a,10
221.	43b,3	33b,5	15b,5	51a,7	45a,4	272a,14
223.	44b,4	34a,15	16a,5	52a,9	46a,3	272a,27
225.	45a,8	34b,14	16a,17	53a,3	46b,8	272b,9
227.	45b,2	35a,7	16b,8	53b,1	47a,2	272b,12
229.	46a,5	35b,10	17a,4	54a,11	47b,9	272b,21
231.	46b,10	36a,13	17a,17	55a,3	48a,15	273a,2
233.	46b,14	36b,2	17b,2	55a,7	48b,2	273a,5
235.	48a,1	37b,4	18a,4	56a,11	49b,9	273a,21
237.	48b,9	38a,7	18b,1	57a,7	50b,1	273b,4
239.	49a,3	38a,13	18b,4	57b,1	50b,10	273b,6
241.	49b,7	38b,16	19a,3	58a,10	51a,16	273b,18
243.	50a,11	39a,15	19a,15	59a,3	52a,6	273b,26
245.	50b,15	39b,13	19b,8	59b,8	52b,12	274a,6
247.	51b,1	40a,9	20a,1	60a,11	53a,15	274a,13
249.	52a,11	40b,15	20a,16	61a,9	54a,11	274a,23
251.	53a,6	41b,2	20b,14	62a,6	55a,7	274b,6

page of edition	D	F	G	L	M	T
253.	53b,16	42a,7	21a,11	63a,4	56a,2	274b,16
255.	54b,9	42b,10	21b,7	64a,2	56b,10	274b,24
257.	55a,12	43a,11	23a,2	64b,11	57b,1	275a,6
259.	56a,5	43b,13	23a,15	65b,6	58a,10	275a,15
261.	56b,12	44a,13	23b,11	66a,14	58b,17	275a,24
263.	57a,8	44b,5	23b,17	66b,12	59a,11	275b,1
265.	57b,12	45a,4	24a,12	67b,6	59b,16	275b,9
267.	58b,3	45b,5	24b,7	68b,1	60b,5	275b,18
269.	59a,11	46a,7	25a,3	69a,15	61a,13	275b,27
271.	60a,6	46b,10	25a,16	70a,13	62a,7	276a,10
273.	60b,3	47a,2	25b,5	70b,10	62b,3	276a,16
275.	61b,2	47b,4	25b,13	71a,14	63a,4	276a,22
277.	62a,1	48a,6	26a,5	72a,1	63b,7[m]	276b,3
279.	62b,4	48b,6	26a,12	72a,10	63b,8	276b,8
281.	63b,5	49a,15	26b,9	73a,7	64b,1	276b,20
283.	64a,12	49b,16	27a,4	74a,4	65a,7	277a,1
285.	65a,6	50a,16	27a,16	74b,13	65b,13	277a,9
287.	66a,4	51a,4	27b,14	75b,12	66b,8	277a,19
289.	66b,11	51b,5	28a,10	76b,5	67a,15	277a,27
291.	67b,7	52a,10	28b,7	77b,3	68a,12	277b,11
293.	68a,6	52b,4	28b,16	78a,3	68b,12	277b,19
295.	68b,5	53a,3	29a,4	78b,2	69a,6	277b,23
297.	69a,1	53a,14	29a,16	79a,4	69b,5	278a,4
299.	69a,5	53b,1	29b,2	79a,12	69b,13	278a,7
301.	69b,13	54a,10	29b,13	80a,3	70a,16	278a,15
303.	70b,3	54b,11	30a,8	80b,13	71a,6	278a,24
305.	71a,8	55a,12	30b,4	81b,10	71b,14	278b,6
307.	71b,15	55b,14	30b,16	82b,6	72b,6	278b,16
309.	72b,3	56a,11	31a,9	83a,11	73a,9	278b,24
311.	73a,4	56b,7	31b,1	83b,15	73b,10	279a,3
313.	73b,7	57a,5	31b,11	84b,7	74a,14	279a,11
315.	74a,15	57b,6	32a,7	85b,1	75a,5	279a,20
317.	75a,5	58a,6	32b,2	86a,9	75b,12	279b,1
319.	75b,4	58b,1	32b,10	86b,8	76a,9	279b,7
321.	76a,13	59a,4	22a,8	87b,7	76b,18	279b,19
323.	76b,3	59a,8	22a,11	88a,1	77a,1	279b,21
325.	77b,3	59b,15	22b,10	89a,1	78a,2	280a,6
327.	78a,7	60a,13	–	89b,6	78b,3	280a,13
329.	78b,6	60b,7	–	90a,9	79a,2	280a,19
331.	79a,9	61a,5	–	90b,14	79b,6	280a,27
333.	80a,2	61b,6	–	91b,8	80a,15	280b,9
335.	80b,11	62a,8	–	92b,3	81a,6	280b,19

page of edition	D	F	G	L	M	T
337.	81a,14	62b,7	43a	93a,10	81b,11	280b,27
339.	82a,5	63a,9	43a,4	94a,6	82b,4	281a,10
341.	82b,10	63b,7	43a,13	94b,12	83a,9	281a,17

Notes: (1) In MS G, pp. 327–335 are missing. The text ends on p. 325, line 14 and resumes on p. 337, line 20. (2) ^{m} = margin.

M. Editorial Procedures

1. Previous Work on the Tadhkira

The French orientalist Carra de Vaux seems to have been the first modern European to have become interested in the *Tadhkira*. He made a translation into French of Book II, Chapter 11 (*i.e.* the part in which Naṣīr al-Dīn presents his non-Ptolemaic models), which Paul Tannery saw fit to include as Appendix VI of his *Recherches sur l'histoire de l'astronomie ancienne* published in 1893. Except for a brief discussion in Dreyer [1906], which followed Carra de Vaux's translation, interest in the *Tadhkira* waned until Roberts [1957], in which the similarity between the models of the 14th c. Damascene astronomer Ibn al-Shāṭir and Copernicus was first noted. (This was based on a discovery by E. S. Kennedy and the recognition of the connection to Copernicus by O. Neugebauer.) Kennedy [1966] established the historical continuity of the work of Ibn al-Shāṭir with that of Ṭūsī and his immediate successors, dubbing the whole lot, rather ahistorically, the "Marāgha School." Hartner as well dealt with various aspects of Ṭūsī's work, in particular the lunar theory [1969] and the connection to Copernicus ([1973] and [1975]). Except for Livingston [1973], who discussed the *Tadhkira* as a genre of astronomical writing, the main interest in it has centered on the non-Ptolemaic modeling.

I have seen little need of providing a systematic criticism of my predecessors; obviously I owe them an enormous debt. But this has not meant that I have avoided indicating those places where my own understanding and interpretation have differed from theirs.

2. Establishment of the Edition

The very large numbers of extant manuscripts of the *Tadhkira*, either standing as independent texts or incorporated into a commentary, considerably complicates the problem of establishing *the* or even *a* text. I had originally conceived of the task as one of collating as many manuscripts as possible and producing a

"Platonic text" that assumed Naṣīr al-Dīn was flawless to a fault. But after producing shoebox after shoebox of variants, and an endless assortment of stemmata that bore an increasing resemblance to a New York subway map, I concluded that I, and I hope my readers, would be better off if I followed the classicists a bit less and the evidence of the manuscripts and the commentators a bit more. That evidence, which I have discussed in detail in Section J, leads inexorably to the conclusion that there was an original version of the *Tadhkira* produced in Marāgha in 659/1261 and that there was an emended version (which the copyist of MS D refers to as *al-iṣlāḥ al-jadīd*) finalized in Baghdad in the last few months of Ṭūsī's life—or perhaps shortly after his death—in 672/1274. I therefore decided to present as the main edited text this final revision (or more precisely as close an approximation to it as I could manage). I refer to this throughout as the Baghdad (β) version, which would have been Ṭūsī's final word on the subject. Revisions are marked off by slashes and the readings from the original Marāgha (α) version are presented at the foot of the page as are those from an intermediate stage (MS M).

No manuscript can be said to establish alone the Marāgha or Baghdad version; of the manuscripts I have examined, the ones that had most completely incorporated into their texts the revisions that had been made from the time of the completion of the Marāgha version in 659/1261 until Ṭūsī's death in 672/1274 were MS F (Vatican ar. MS 319, 1) and MS L (Leiden, University Library MS or. 905). I do not know the date or provenance of MS L, but MS F was completed just 10 years after Ṭūsī's death in 683/1284 and, by a happy coincidence, at the Niẓāmiyya College in Baghdad itself. MS F has thus served as my main source, but MS L has been used to corroborate it. Furthermore the marginal variants in MSS DMT have also been used to confirm that what one finds in MSS FL were indeed Ṭūsī's revisions. The main text of MS D (Istanbul, Feyzullah 1330, 1), which was copied from an autograph, is in my estimation very close to the original Marāgha version; revisions have usually been scrupulously put into the margins. Another manuscript that is very close to MS D, and in which the revisions are marginalized, is MS T (Istanbul, Ahmet III MS 3453, 19). As I discuss in Section J.3, MS M (Istanbul, Lâleli MS 2116) is a special case; it has been used to establish the revisions to the Marāgha version that were made before Shīrāzī left Marāgha (and Ṭūsī), probably in 667/1268.

Although I have examined some twenty manuscripts of the *Tadhkira* (excluding commentaries) and compared at least eleven of them in detail, I decided to restrict the final apparatus for the most part to the variants from only six manuscripts.[1] In addition to the five mentioned above (MSS DFLMT), I also included MS G (St. Petersburg (a.k.a. Leningrad), Oriental Institute, MS A 437) mainly because of its early date (673/1275). But in addition to being incomplete and hopelessly corrupt due to an incompetent copyist, it incorporates material

[1] Occasionally there are references in the apparatus or the commentary to one of the other manuscripts.

from the new version without indication and sometimes puts it in the wrong place, which would seem to show that the copyist found some of Ṭūsī's marginal revisions but did not know quite where to put them (see, for example, II.10 [3]). What is interesting is that it has very late revisions, such as II.2 [3]22–23, which was added after Shīrāzī's departure from Marāgha. On the other hand, it has a variant on II.4 [2]23–[3]16 that I have not found elsewhere, which would indicate a very early revision since it retained but modified the original four-possibility version before it was changed to eight possibilities.[2] MS G is indicative of a highly contaminated text that brings together original texts and revisions in a haphazard and confusing way; in fact, many, if not most, of the manuscripts I have looked at do this. It is only because of the care taken by the commentators and by a few copyists that "purer" textual versions can be presented here.

Let me turn to some specific points. Though MS F is fairly close to my Baghdad version, there are some differences. In the main this is due to my reluctance to assume that a variant in it represents one of Ṭūsī's revisions unless there are other witnesses or corroborating evidence. An important example where MS F was not used is IV.1 [3]18–19, where I could not confirm that its recalculated value of 3,756,231½ was due to Ṭūsī. On the other hand, MS F does not contain all the revisions that can be ascertained elsewhere; examples are I.1 [14]22–23 and IV.6 [5]11. But I would maintain that it is a better witness to the final revised version than MS L, which lacks the revisions of II.8 [14]10–11, II.9 [1]9, II.13 [9]15, II.13 [9]19–20, III.8 [7]1, III.10 [1]13–14, III.12 [2]15, and IV.5 [3]19. As for my best witnesses to the original (α) version, namely MSS DT, they occasionally seem to place revisions into the text rather than the margins. For MS T, this is the case for II.9 [8]21; for MS D, examples are II.10 [3]10 and II.10 [3]11–12.

I have not seen it as my job to correct Ṭūsī's mistakes; where it would seem that these were not corrected in the Baghdad revision, they have been retained. In addition to the particularly egregious case of II.5 [8]6–8, where the revision itself is nonsensical, examples are II.1 [6]11, II.11 [10]11&12, II.13 [6]12, and IV.3 [5]19.[3] In general, such "mistakes" are indicated by an exclamation mark. On the other hand, I have standardized the spelling of numbers as well as such words as *hāhunā* and have used the modern rules for *hamza*s. The spellings in the manuscripts are indicated in the text apparatus. Furthermore I have attempted to have subjects and verbs agree despite the distance between them; even though the grammarians disagree on this, as do the copyists, such a course seemed best for a scientific treatise where the minimization of ambiguity is desirable.

As for the text figures, I have generally followed MSS FL and have given variations in the figure apparatus. Labels that are absent in MSS FL but present

[2] See the commentary for more details.

[3] The last is particularly significant; see our commentary for details.

in the other manuscripts are put in parentheses. For Figure T25, where the divergence between the Marāgha and Baghdad versions was especially great, I placed the Baghdad figure in the text and the Marāgha version in the commentary (Figure C34).

3. The Translation

I have attempted to be as literal as possible, but I have also been concerned to make the translation sound like reasonable English; the *Tadhkira* is written with a precise, structured, scientific Arabic that developed over many centuries, and I have felt strongly that the English should reflect this. For the most part, I have followed the English vocabulary that has evolved during this century to translate scientific Arabic (which is fairly consistent with that of the classicists and Latin medievalists), but I have not hesitated to coin new renditions when this seemed appropriate (for example, dirigent for *mudīr* and monoform for *ᶜalā nahj wāḥid*). A complete listing of technical terms and their translations is given in the glossary.

Ṭūsī normally writes his numbers and fractions in words; an exception occurs in II.6 [4], where he gives the sun's eccentricity alphanumerically. He is not consistent in the way he gives fractions, employing sexagesimals, sums of unit fractions, and non-unit fractions. I have retained this inconsistency in the translation, but, for the sake of clarity, generally rendered his numbers numerically.

The translation itself follows the text of the Baghdad (β) version; with the exception of untranslatable grammatical variants, the variant readings from the Marāgha (α) version and from MS M have been indicated at the bottom of the page. Line numbers in the margin follow the edited text and not the actual lines of the translation.

4. The Commentary

Neugebauer's *History of Ancient Mathematical Astronomy*, Pedersen's *Survey of the Almagest*, and Toomer's translation of the *Almagest* have made it unnecessary to explain the Ptolemaic system in detail in a work such as this. I have therefore not belabored Ṭūsī's "summary" of the *Almagest* in the commentary but rather concentrated on those points that are rather more obscure. In general these involve matters in which Islamic astronomers had modified or recast the science they had inherited from antiquity. But I have also felt it necessary to elaborate on points that were of interest, or that may have been unclear, to Ṭūsī's successors; on these matters I have relied heavily upon Shīrāzī and the commentators.

Each comment is generally numbered in the following sequence: Roman numeral of the book; number of the chapter; bracketed number of the paragraph; and line number. The line number invariably refers to the edited text. This is usually followed by an identifying transliterated phrase followed by the English translation in parentheses.

Part II

Edition and Translation

89 ℓ

In the Name of God
the Beneficent, the Merciful

[1] Praise be to God Who brings forth good and Who inspires truth. And may His blessings be upon Muḥammad, who was sent with the Final Message, upon his family, the most excellent of families, and upon his Companions, the most excellent of Companions.

[2] We wish to present a summary of astronomy as a memento for one of our dear friends, and we ask God to grant us success for its completion; for He is the One who grants success and to Him is the Final Return. Let us now set forth what we have in mind in chapters that are contained within four books.

BOOK I

Concerning That Which Must Be Presented by Way of Introduction

[1] / Every science has: [a] a subject, which is investigated in that discipline; / [b] principles, which are either self-evident or else obscure, in which case they are proved in another science and are taken for granted in this science; and [c] problems, which are proved in this science.

[2] The subject of astronomy is the simple bodies, both superior and inferior, with respect to their quantities, qualities, positions, and intrinsic motions. Those of its principles that need proof are demonstrated in three sciences: metaphysics, geometry, and natural philosophy. Its problems aim at gaining knowledge of these bodies in and of themselves, of their shapes, of the manner of their arrangement and motions, of the amounts of their motions and distances, and of the reasons for changes in position.

/10/ …] β, M = Every science has: [a] a subject whose essential attributes are investigated in it] α.

بسم الله الرحمن الرحيم

[١] الحمد لله مفيض الخير وملهم الصواب ، وصلواته على محمد المبعوث بفصل الخطاب وعلى آله خير آل وأصحابه خير أصحاب .

[٢] نريد أن نورد جملاً من علم الهيئة تذكرة لبعض الأحباب ونسأل الله أن يوفق لإتمامه ، إنه الموفق وإليه المآب . فلنورد ما قصدناه في فصول تشتمل عليها أربعة أبواب .

الباب الأول
فيما يجب تقديمه

[١] لكل علم موضوع يُبحث / في ذلك العلم عنه / ، ومبادئ إما بينة بنفسها وإما خفية تُبيّن في علم آخر وتُستعمل في ذلك العلم على أنها مسلمة ، ومسائل تُبيّن في ذلك العلم .

[٢] وموضوع الهيئة الأجرام البسيطة العلوية والسفلية من حيث كمياتها وكيفياتها وأوضاعها وحركاتها اللازمة لها . ومبادئها المحتاجة إلى البيان تتبين في علوم ثلاثة : ما بعد الطبيعة، والهندسة ، والطبيعيات . ومسائلها معرفة تلك الأجرام بأعيانها وأشكالها ، وكيفية نضدها وحركاتها ، ومقادير الحركات والأبعاد، وعلل اختلاف الأوضاع .

/10/ ...] M , β = فيه عن أعراضه الذاتية] α .

[3] The scientific exposition that we wish to undertake will be a summary account of this presented in narrative form. The details are expounded and proofs of the validity of most of them are furnished in the *Almagest*. Indeed, ours would not be a complete science if taken in isolation from the *Almagest* for it is a report of what is established therein.

[4] In this [study], it is necessary that there be a familiarization with certain definitions and rules, which will be presented by way of introduction. For their elucidation, one is referred to the above-mentioned sciences. Despite the fact that the explanations of these [definitions and rules] occur in various places, they will be grouped into two divisions: one of them pertaining to the geometry [corpus] and the other pertaining to the natural philosophy [corpus]. Let us then present an account of them in two chapters.

CHAPTER ONE

An Account of What Needs to Be Known That Pertains to the Geometry [Corpus]

[1] Among those things having position, *i.e.* that can be indicated by the senses, are: [a] a **point**, which is that without part; [b] a **line**, which is that with length only and which ends in a point; [c] a **surface**, which is that with length and width only and which ends in a line; and [d] a **solid**, which is that with length, width, and depth and which ends in a surface. These "ends" are called **boundaries**.

[2] A **straight line** is the one on which all the given points are facing one another. / A **plane surface** is the one on which the assumption of straight lines in all directions is possible. /

/19–20/ …] β = a plane surface is the one of which it is possible to produce straight lines in all directions] M = a plane surface is the one whereby the lines assumed on it in all directions are straight] α.

[٣] والفن الذي نريد أن نشرع فيه تقرير جمل من ذلك تورد على سبيل الحكاية . وتُبيّن تفاصيلها وتُقام البراهين على صحة أكثرها في المجسطي ، فهو ليس بعلم تام إذا أفرز عن المجسطي لأنه حكاية / ما / عما ثبت فيه .

[٤] ولا بد / فيه / من تعرّف حدود وأحكام تورد على سبيل التصدير ، ويحال بيانها على العلوم المذكورة . وهي ، على اختلاف مواضع بياناتها ، تنقسم إلى قسمين : أحدهما ما يتعلق بالهندسيات والآخر ما يتعلق بالطبيعيات . فلنقدم ذكرها في فصلين .

الفصل الأول
في ذكر ما يحتاج إلى معرفته مما يتعلق بالهندسيات

[١] من الأشياء التي لها وضع أي التي يمكن أن يشار إليها بالحس : النقطة وهي ما لا جزء له ، والخط وهو ما له طول فقط وينتهي بالنقطة ، والسطح وهو ما له طول وعرض لا غير وينتهي بالخط ، والجسم وهو ما له طول وعرض وعمق وينتهي بالسطح . وتسمى النهايات حدوداً .

[٢] والمستقيم من الخطوط هو الذي يتحاذى جميع النقط التي تفرض عليه . / والمستوي من السطوح هو الذي يكون فرض الخطوط المستقيمة عليه في جميع الجهات ممكناً / .

/4/ ما] β , M = α− . /5/ فيه] β , M = α− . /19–20/ ...] β = والمستوي من السطوح هو الذي يمكن أن تخرج منه الخطوط المستقيمة في جميع الجهات؛ M = والمستوي من السطوح هو الذي تكون الخطوط المفروضة عليه في جميع الجهات مستقيمة] α .

[3] An **angle** is either: [a] a surface bounded by two lines meeting at a point in such a way that they do not form a single line; or [b] a solid bounded by surfaces meeting at a point in such a way that any two of its surfaces join at a line without forming a single surface.

[4] The point at which two lines join or intersect is a **common part** for them; and, similarly, a line for surfaces and a surface for solids.

[5] If a straight line stands erect on another straight line and there result two equal angles on its two sides, then these are **right angles**, and each one of the two lines is **perpendicular** to the other. An angle which is smaller than a right angle is **acute**, and that which is larger is **obtuse**.

[6] A straight line that stands erect on a plane surface in such a way that it along with any given line lying in the [plane] and meeting it bound a right angle is **perpendicular** to that surface. When one plane surface stands erect on [another] plane surface in such a way that any two perpendiculars that are produced in the [planes] from any given point on their common part bound a right angle, then the two [planes] **intersect at right angles**.

[7] Straight lines occurring in a plane that do not meet even if extended without limit in both directions are **parallel**. Similarly planes that do not meet even if extended without limit in all directions are also [parallel]. Non-straight [lines] and non-plane [surfaces] may also be called parallel if distances between them are completely invariable.

[8] A **circle** is a plane surface bounded by a curved line in whose interior is a point such that all the straight lines extending from it to the [curved line] are equal. That line is its **circumference**, and the point is its **center**. The extended lines are **radii**. A line extending in both directions from the [point] to the circumference is a **diameter** of the circle, which it bisects.

[٢] والزاوية سطح أحاط به خطان ملتقيان عند نقطة من غير أن يتحدا خطاً واحداً أو جسم أحاط به سطوح ملتقية عند نقطة يتصل كل سطحين منها عند خط من غير أن يتحدا سطحاً واحداً .

[٤] والنقطة التي يتصل أو يتقاطع عليها خطان فصل مشترك لهما ، وكذلك الخط للسطوح والسطح للأجسام .

[٥] وإذا قام خط مستقيم على خط مستقيم وحدثت عن جنبتيه زاويتان متساويتان فهما قائمتان وكل من الخطين عمود على صاحبه . والزاوية التي هي أصغر من قائمة حادة والتي هي أعظم منفرجة .

[٦] والخط المستقيم القائم على سطح مستوٍ بحيث يحيط مع كل خط يفرض فيه ملاقياً له بقائمة عمودٌ على السطح . وإذا قام سطح مستوٍ على سطح مستوٍ بحيث يحيط كل عمودين يخرجان فيهما من أي نقطة تفرض على فصلهما المشترك بقائمة فهما يتقاطعان على قوائم .

[٧] والخطوط المستقيمة الكائنة في سطح مستوٍ التي لا تتلاقى وإن أخرجت في الجهتين إلى غير / نهاية / هي المتوازية ، وكذلك السطوح المستوية التي لا تتلاقى وإن أخرجت في جميع الجهات إلى غير / نهاية / . وقد يقال في غير المستقيمة والمستوية منهما متوازية إذا لم تختلف الأبعاد بينهما أصلاً .

[٨] الدائرة سطح مستوٍ يحيط به خط مستدير في داخله نقطة يكون جميع الخطوط المستقيمة الخارجة منها إليه متساوية ، وذلك الخط محيطها وتلك النقطة مركزها والخطوط الخارجة أنصاف أقطارها . والخارج منها إلى المحيط في الجهتين قطر لها وهو يُنصّف الدائرة .

/17/ ...] M , β = النهاية] α . /19/ ...] M , β = النهاية] α .

[9] Every straight line that divides the circle into any two parts whatever is a **chord**, and what is separated off from the circumference is an **arc**. Half a chord is the **sine** of half the arc. The perpendicular extending from the midpoint of an arc to the midpoint of the chord is the **versed sine** of half that arc.

[10] A **sphere** is a solid bounded by a curved surface in whose interior is a point such that all straight lines extending from it to the [curved surface] are equal. That surface is its **circumference**, and the point is its **center**. The extended lines are **radii**. A line extending in both directions from the [point] to the circumference is a **diameter** of [the sphere].

[11] Every plane that divides a sphere into two segments produces a circle in it, which is the common part between them. If it bisects it, then this is the largest circle that can occur in that sphere and it will pass through its center; thus their two centers coincide.

[12] If a sphere rotates about itself, every point marked on it describes by its motion in / each / complete rotation a circle, which is the point's **circuit**. The exceptions to this are two points, which are the **poles** of the sphere. The diameter that connects them does not move either; it is the **axis**.

[13] The great circle that is equidistant from the two poles is the sphere's **equator**. All the circuits are parallel to one another as well as being parallel to the equator. The axis is perpendicular to each. Any two circuits on the two sides of the equator and equidistant from it are equal. Every great circle / or small one in the sphere / has an axis and two poles as is the case with the equator.

[14] Two given great / circles / on a sphere bisect one another at two points, and their [common] part is a straight line / that passes through the center. / The greatest distance between the two circles is the same as the distance between

/13/ each] β = –α, –M. /19/ or small one in the sphere] β = –α, –M. /21/ circles] β = –α, –M. /22–23/ that passes through the center] β = –α, –M.

[٩] وكل خط مستقيم يقطعها بقطعتين كيف اتفق فهو وتر وما يُفرَز من المحيط قوس . ونصف الوتر لنصف القوس جيب . والعمود الخارج من منتصف القوس إلى منتصف الوتر سهم لنصف القوس .

[١٠] الكرة جسم يحيط به سطح مستدير في داخله نقطة يكون كل الخطوط المستقيمة الخارجة منها إليه متساوية ، وذلك السطح محيطها وتلك النقطة مركزها والخطوط أنصاف أقطارها . والخارج منها إلى المحيط في الجهتين قطر لها .

[١١] وكل سطح مستوٍ يقطع الكرة إلى قطعتين يُحدِث دائرة فيها هي الفصل المشترك بينهما . فإن نصّفها فهي أعظم دائرة تقع في تلك الكرة وتمرّ بمركزها فيتّحد مركزاهما .

[١٢] وإذا دارت الكرة على نفسها / فعلت / كل نقطة ترسم عليها بحركتها في / كل / دورة تامة دائرة هي مدارها ، إلا نقطتين هما قطبا الكرة . والقطر الواصل بينهما أيضاً لا يتحرك وهو المحور .

[١٣] والدائرة العظيمة المتساوية البعد عن القطبين منطقتها . وتكون المدارات جميعاً متوازية وموازية للمنطقة ، والمحور عموداً على الكل . وكل مدارين عن جنبتي المنطقة / متساويتي / البعد عنها متساويان . ولكل دائرة عظمى / أو صغرى في الكرة / محور وقطبان كما للمنطقة .

[١٤] وإذا فرضت على كرة / دائرتان / عظيمتان فهما تتناصفان على نقطتين ويكون فصلهما خطاً مستقيماً / ماراً بالمركز / ، ويكون أعظم الأبعاد بين الدائرتين كالبعد بين

/12/ …] M , β = فعل] α . /13/ …] β = −M , −α . /18/ …] M , β = متساويي] α.
/19/ …] β = −M , −α . /21/ …] β = −M , −α . /22–23/ …] β = −M , −α .

their two poles. If they intersect at right angles, each of the [circles] passes through the poles of the other; the converse also holds.

[15] An **orb** is a spherical solid bounded by two parallel surfaces having the same center. The outer [surface] is called **convex** and the other **concave**. Sometimes the concave is not taken into account, as in the case of epicycles.

[16] A **circular cylinder** is a solid bounded by two equal, parallel circles, which are its **bases**, and by a circular surface that connects their circumferences. / The line that joins the two centers is an **axis** of the [cylinder]; if it is perpendicular to the planes of the two circles, the **cylinder** is a **right** one. /

[17] A **circular cone** is a circular solid that rises from a circle, which is its **base**, to a point, which is its **apex**. The line that joins the point and the center of the base is / the [cone's] **axis**; if it is perpendicular to its base, the **cone** is a **right** one. /

[18] If a cylinder or a cone is cut by a plane passing through the axis, it produces a quadrilateral in the cylinder and a triangle in the cone. If the plane is parallel to the base, then in both cases it will produce a circle.

CHAPTER TWO

An Account of What Needs to Be Accepted from Natural Philosophy in This Science

[1] A body is either **simple**, in which case it has a single nature and what issues forth from that nature does so monoformly, or else **compound**, in which case it is composed of

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/8–9/ …] β = The line that joins the two centers is perpendicular to the planes of the two circles and it is the axis of the cylinder] α, M. /12–13/ …] β = perpendicular to its base and it is its axis] α, M.

قطبيهما . فإن / تقاطعتا / على قوائم مرّ كل منهما بقطبي الأخرى وبالعكس .

[١٥] الفلك جسم كري يحيط به سطحان متوازيان / مركزاهما / واحد . ويسمى الخارج منهما محدّباً والآخر مقعّراً ، وربما لا يعتبر المقعّر كما في التداوير .

[١٦] الأسطوانة المستديرة جسم تحيط به دائرتان متساويتان ومتوازيتان هما قاعدتاها وسطح مستدير واصل بين محيطيهما . / ويكون الخط الواصل بين المركزين سهماً لها ، فإن كان عموداً على سطحي الدائرتين كانت الأسطوانة قائمة / .

[١٧] والمخروط المستدير جسم مستدير يرتفع من دائرة هي قاعدته إلى نقطة هي رأسه . والخط الواصل بين النقطة ومركز القاعدة / يكون سهمه ، فإن كان عموداً على قاعدته كان المخروط قائماً / .

[١٨] وإذا فُصل الأسطوانة والمخروط بسطح يمرّ بالسهم أحدث في الأسطوانة ذا أربعة أضلاع وفي المخروط مثلّثاً . فإن كان السطح موازياً للقاعدة أحدث فيهما دائرة .

الفصل الثاني
في ذكر ما يحتاج في هذا العلم
إلى تسليمه من الطبيعيات

[١] الجسم إما بسيط وهو الذي له طبيعة واحدة يصدر عنها ما يصدر على نهج واحد ، وإما مركّب وهو الذي يتركب من

/1/ ...] M , β = تقاطعا] α . /3/ ...] β = مركزهما] M , α . /8-9/ ...] β = ويكون الخط الواصل بين المركزين عموداً على سطحي الدائرتين وهو سهم الأسطوانة] M , α . /12-13/ ...] β = يكون عموداً على قاعدته وهو سهمه] M , α.

simple bodies and may turn out to be a species distinct from them. A simple body is either celestial or elemental. The **celestial bodies** are the orbs and the luminous bodies whose proper place is the orbs. The **elemental bodies** are the four well-known elements. The compound bodies are those things composed of them, namely minerals, plants, and animals. Their **proper places** are those of the elements. A void is impossible.

[2] Every motion has a **principle**. The mobile is said to be **self-moved** if its principle of motion does not become positionally separated from it. If it does separate from it, the movement is ascribed to the mobile, but the action of moving the mobile is attributed to that in which its principle of motion occurs. If the motion of a self-moved mobile is monoform, its principle of motion is called a **nature** whether the motion is natural and elemental or voluntary and celestial. Otherwise its [principle of motion] is called a **soul** whether the [motion] be vegetative or animal. When a mobile is moved by something other than itself, its motion is **accidental** if it is part of the mover or if the mover is the place for it naturally to be; otherwise its motion is by **compulsion**.

[3] Motion due to a nature is divided into: [a] that which is toward the center; its principle is heaviness and it is characteristic of the two heavy elements; [b] that which is away from the center; its principle is lightness and it is characteristic of the two light elements. These two [motions] are **displacing** and **rectilinear**. And [c] that which is about the center; this motion is **in place** and **circular**, and it is characteristic of the celestial bodies. The [latter motion] is divided into **simple**, which arises from a single, simple body with every given point on it producing at the center equal angles in equal times or cutting equal arcs from the circumference, and into **composed**, which arises from a combination of more than one simple body. Every motion whose angles or arcs vary in equal times is composed; however, the converse does not hold.

[4] Nothing having the principle of circular motion can undergo any rectilinear motion at all, and conversely, except by compulsion. Thus the celestial bodies neither tear nor mend, grow nor diminish, expand nor contract; neither does their motion intensify nor weaken. They do not reverse direction, turn, stop, depart from their confines, nor undergo any change of state except for their circular motion, which is uniform at all times.

بسائط /وقد يصير/ نوعاً غيرها . والبسيط إما فلكي وإما عنصري : والفلكي هو الأفلاك والأجرام النيّرة التي مكانها الأفلاك ، والعنصري هو العناصر الأربعة المشهورة . والمركّب ما يتركب منها من المعادن والنبات والحيوانات ، وأمكنتها أمكنة العناصر . والخلاء محال .

[٢] ولكل حركة مبدأ . والمتحرك إن لم يفارقه مبدؤه بالوضع قيل إنه يتحرك بنفسه ؛ وإن فارقه نُسب التحرك إليه والتحريك إلى ما فيه مبدؤه . والمتحرك بنفسه إن كانت حركته على نهج واحد سمي المبدأ طبعاً سواء كانت الحركة طبيعية عنصرية أو إرادية فلكية . وإن لم يكن كذلك سمي نفساً سواء كانت نباتية أو حيوانية . والمتحرك بغيره إن كان كجزء من المحرك أو كان المحرك مكاناً له بالطبع فالحركة عرضية ، وإلا فقسرية .

[٣] والحركة بالطبع تنقسم إلى ما إلى المركز ومبدؤه الثقل وتختص بالعنصرين الثقيلين ؛ وإلى ما من المركز ومبدؤه الخفة وتختص بالعنصرين الخفيفين ، وهما أينيّتان مستقيمتان ؛ وإلى ما على المركز وهي وضعية مستديرة وتختص بالفلكيات ، وتنقسم إلى بسيطة تصدر عن جرم واحد بسيط كل نقطة تفرض عليه تفعل عند المركز في أزمنة متساوية زوايا متساوية أو تقطع من المحيط قسياً متساوية ، وإلى مركبة تصدر عن جملة بسائط فوق واحدة . وكل حركة تختلف زواياها أو قسيها في الأزمنة المتساوية مركبة ، ولا ينعكس .

[٤] وكل ما فيه مبدأ حركة مستديرة فهو لا يقبل الحركة المستقيمة أصلاً ، وبالعكس ، إلا بالقسر . فالفلكيات لا تنخرق ولا تلتئم ، ولا تنمو ولا تذبل ، ولا تتخلخل ولا تتكاثف ، ولا تشتد في حركاتها ولا تضعف ؛ ولا يكون لها رجوع ولا انعطاف ولا وقوف ولا خروج من حيز ولا اختلاف حال غير حركتها المستديرة المتشابهة في جميع الأوقات .

/1/ ...] M , β = ويصير] α .

BOOK II

The Configuration of the Celestial Bodies

Fourteen Chapters

CHAPTER ONE

On the Sphericity of the Sky and the Earth; On the Earth Being in Relation to the Sky As the Center of a Sphere to Its Circumference; and on [the Earth] Being Completely Stationary

[1] The sphericity of the sky is evidenced by: the movement of the fixed stars along parallel circles about a stationary point; the occurrence of those [stars] nearest this [point] upon [relatively] small circuits of permanent visibility, while those further away occur on [progressively] larger circuits until those stars are reached that just touch the horizon but do not disappear and then are followed by those that disappear fleetingly, keeping one and the same rising and setting place; the proportional increase in the periods of invisibility thereafter according to the increase in distance until those [stars] are reached whose periods of visibility and invisibility are equal and then are followed by those whose period of invisibility exceeds their period of visibility; the increase as well in the period of invisibility until those [stars] are reached that are visible briefly and then are followed by those that just touch the horizon once every revolution but do not rise; the converse equality of the periods of visibility and invisibility of those [stars] equidistant and on the two sides of that circuit upon which the periods of visibility and invisibility are equal; the ascent of whatever rises occurring little by little up to a certain maximum that is at the midpoint of the visible segment of its circuit and then its descent little by little until it disappears; the [successive] rising of one part of its body after another, as also occurs during setting; the observed equality of size of a [celestial body] at all distances along its revolution except at the horizon [which can be explained by] the accumulation of vapors rising from the Earth causing objects behind them to appear larger than they should just as we find to be the case for things that we observe at one time in air and in another in water. Therefore size increases

الباب الثاني
في هيئة الأجرام العلويـة
أربعة عشر فصلاً

الفصل الأول
في استدارة السماء والأرض
وكون الأرض عند السماء كمركز الكرة عند محيطها
وكونها غير متحركة بالجملة

[١] تحرُّكُ الثوابت على دوائر متوازية حول نقطة لا تتحرك ؛ وكون ما هو أقرب منها على مدار أصغر أبدي الظهور ، وما هو أبعد على مدار أكبر إلى أن ينتهي إلى ما يماس الأفق ولا يخفى ، ثم إلى ما يخفى زماناً يسيراً حافظاً لمطلع ومغيب بعينهما ؛ وتزايُدُ أزمنة الخفاء بعد ذلك بحسب تزايد البعد على نسبة إلى أن ينتهي إلى ما يتساوى زمانا ظهوره وخفائه ، ثم إلى ما يزيد زمان خفائه على زمان ظهوره ؛ وتزايُدُ أزمنة الخفاء أيضاً إلى أن ينتهي إلى ما يظهر قليلاً ، ثم إلى ما يماس الأفق في دورة مرة ولا يطلع ؛ وتساوي زماني الظهور والخفاء للمتساوية الأبعاد عن المدار الذي يتساوى زمانا ظهوره وخفائه عن الجنبتين على التبادل ؛ وارتفاعُ ما يطلع يسيراً يسيراً إلى غاية ما عند منتصف القطعة الظاهرة من مداره ، ثم انحطاطه يسيراً يسيراً إلى أن يخفى ؛ وطلوعُه شيئاً بعد شيء من جرمه وكذلك غروبه ؛ وتساوي مقداره في النظر في جميع أبعاده في دورته إلا عند الأفق ، فإنّ تراكُم الأبخرة المرتفعة من الأرض يُري ما وراءها من الأشخاص أكبر مما يجب أن يُرى كما نشاهد فيما نرى تارة في الهواء وتارة في الماء ولذلك يزداد

when the air becomes denser and the opposite holds; the permanent visibility of half the sky—or thereabouts—to everyone on Earth whatever his location; and, in addition to the above, other characteristics that are peculiar to sphericity.

[2] The general sphericity of the Earth is evidenced by: the earlier rising and setting of the stars for those persons in the east relative to their rising and setting for those in the west, and the increase and decrease of that [priority] according to farness and nearness; the increasing altitude of the pole and the northern stars and the decreasing altitude of the southern ones for those who advance northward and the opposite for those who advance southward, this being in both cases according to the extent of their advance; and the combining of these two variations for those traveling on a course between the two [cardinal] lines [*i.e.* east-west and north-south]. The [Earth's] undulations that occur due to mountains and depressions do not cause it to depart from being basically spherical since there is no appreciable ratio between them and the [Earth] taken as a whole. For a mountain that rises to a height of half a parasang is to the [Earth] as $\frac{1}{5}$ of $\frac{1}{7}$ the width of a barleycorn is to a sphere whose diameter is approximately one cubit. This will be elucidated when investigating the area of the Earth.

[3] The sphericity of the surface of the water standing on the face of the Earth is evidenced, in addition to what has been stated above regarding the Earth, by the fact that the curvature of the sea waters conceals the lower parts of mountains rising up from them but not their higher reaches and that these [lower parts] appear little by little to someone approaching them.

[4] That the Earth is in the middle of the Universe at the center is evidenced by: the equality of the periods of ascent and descent of the stars during the time of their visibility; the permanent visibility of half the celestial sphere; the coincidence on a single straight line of the sun's shadows at the time of its rising and setting when it is on the [day-] circle on which its periods of visibility and invisibility are equal or when it is at opposite parts of the circle that it describes with its proper motion; and the eclipse of the moon at its true oppositions to the sun.

[5] The permanent visibility of half the ecliptic orb as well as the orbs below it down to the orb of the sun is evidence that the Earth does not have a perceptible size with respect to the orb of Mars and all orbs beyond; indeed, it is as a point since there is no difference

الكبر إذا صار الهواء أغلظ وبالضد ؛ وظهورُ النصف أو قريب منه دائماً لكل من على الأرض في أي موضع يكون ؛ إلى غير ذلك من الأعراض الخاصة بالاستدارة : يدل على استدارة السماء .

[٢] وتقدُمُ طلوع الكواكب وغروبها للمشرقيين على طلوعها وغروبها للمغربيين ، وزيادة ذلك ونقصانه بحسب بُعد المسافة وقربها ؛ وازديادُ ارتفاع القطب والكواكب الشمالية وانحطاط الجنوبية للواغلين في الشمال وبالعكس للواغلين في الجنوب بحسب وغولهما ؛ وتركُّبُ الاختلافين للسائرين على سمت بين السمتين : يدل على استدارة الأرض جملة . وتضاريسها التي تلزمها من جهة الجبال والأغوار لا تُخرِجها عن أصل الاستدارة إذ لا نسبة محسوسة لها إلى جملتها ، فإنّ جبلاً يرتفع نصف فرسخ يكون عندها كخُمس سُبع عرض شعيرة عند كرة قطرها ذراع بالتقريب _ يتبين ذلك عند الوقوف على مساحة الأرض .

[٣] وسترُ تقبيب مياه البحار أسافلَ الجبال الطالعة منها دون أعاليها المرتفعة وظهورها قليلاً قليلاً للمتقارب إليها ، مضافاً إلى ما مرّ في الأرض ، يدل على استدارة سطح الماء الواقف على وجه الأرض .

[٤] وتساوي زماني ارتفاع الكواكب وانحطاطها مدة ظهورها ؛ وظهورُ النصف من الفلك دائماً ؛ وتطابُقُ أظلال الشمس في وقتي طلوعها وغروبها عند كونها على المدار الذي يتساوى زمانا ظهوره وخفائه على خط واحد مستقيم أو عند كونها في جزأين متقابلين من الدائرة التي تقطعها بسيرها الخاص بها ؛ وانخسافُ القمر في مقاطراته الحقيقية للشمس : يدل على كون الأرض في وسط الكل عند المركز .

[٥] وظهور النصف من فلك البروج ومما تحته من الأفلاك إلى فلك الشمس دائماً يدل على أنّ الأرض ليست بذات قدر محسوس عند فلك المِرّيخ وما وراءه من الأفلاك ، بل هي كالنقطة إذ لا فرق

between the plane passing over the face of the Earth and separating the visible from the invisible part of these orbs, and the plane parallel to it passing through the center of the Universe. It does, however, have an appreciable size with respect to the moon's orb whose visible segment is therefore less than half; we shall make this clear in the appropriate place.

[6] The reliability of all the evidence we have adduced establishes that these bodies are according to the stated configuration. It is not possible to attribute the primary motion to the Earth. This is not, however, because of what has been maintained, namely that this would cause an object thrown up in the air not to fall to its original position but instead it would necessarily fall to the west of it, or that this would cause the motion of whatever leaves the [Earth], such as an arrow or a bird, in the direction of the [Earth's] motion to be slower, while in the direction opposite to it to be faster. For the part of the air adjacent to the [Earth] could conceivably conform to the Earth's motion along with whatever is joined to it, just as the aether conforms to the orb as evidenced by the comets, which move with its motion. Rather, it is on account of the [Earth] having a principle of rectilinear inclination that it is precluded from moving naturally with a circular motion.

[7] The sphericity of the Earth, / sky, / and water having been established, let it be known that the inclination of all heavy things is to the center of the Earth, which is the center of the Universe, and that the inclination of light things is to the circumference. Thus "up" from all sides of the Earth is what is toward the sky, while "down" is what is toward the Earth's center. Objects are situated on the Earth at the endpoints of its diameters; hence, the distance between their apices is greater than the distance between their bases. The amount of water a filled vessel contains when it is closer to the center, such as at the bottom of a well, is greater than what it contains when it is further away from it, such as at the top of a lighthouse. This is because in the former case the water is more greatly cupolaed than in the latter. This is one of the many things that those not acquainted with these matters find strange.

[8] The above proofs are *inniyya,* which convey existence; those which convey the necessity of that existence are *limmiyyāt* proofs and are given in the book *De caelo* of the Natural Philosophy.

/14/ sky] β = –α, –M.

بين السطح المارّ بوجه الأرض الفاصل بين الظاهر والخفي من تلك الأفلاك وبين السطح المارّ بمركز الكل الموازي لذلك السطح . وأما عند فلك القمر فلها قدر محسوس ، ولذلك تكون القطعة الظاهرة من فلكه أقل من النصف ، وسنبيّن ذلك في موضعه .

[٦] وثبات جميع ما ذكرنا من الدلائل يدل على ثبات تلك الأجرام على الهيئة المذكورة . ولا يمكن إسناد الحركة الأولى إلى الأرض ـ لا لِما قيل من أنّ ذلك يوجب أن لا يقع المرمي في الهواء على موضعه الأول بل يجب أن يقع في الجانب الغربي منه أو يوجب / أن الحركة / لما انفصل منها كالسهم والطائر إلى جهة حركتها أبطأ وفي خلافها أسرع ، فإنّ المتصل بها من الهواء يمكن أن يشايعها بما يتصل بها[!] كما يشايع الأثير الفلك بدلالة حركات ذوات الأذناب بحركته ـ بل لكونها ذات مبدأ ميل مستقيم ، فيمتنع أن تتحرك على الاستدارة بالطبع .

[٧] وإذا ثبت استدارة الأرض / والسماء / والماء فليُعلم أنّ ميل الأثقال جميعاً إلى مركز الأرض الذي هو مركز الكل ، وميل ما هو خفيف إلى المحيط . فالفوق من جميع جوانب الأرض ما يلي السماء ، والتحت ما يلي مركز الأرض . والأشخاص تقوم على الأرض على أطراف أقطار لها ، فيكون البعد بين رؤوسها أكثر من البعد بين قواعدها . والإناء المملوء ماءً يحوي من الماء وهو أقرب إلى المركز كقعر بئر مثلاً أكثر مما يحويه وهو أبعد منه كرأس منارة مثلاً ، وذلك لكونه هناك أشد تقبيباً من هاهنا . وهذا من جملة ما يستغربه مَن لا يعرف هذه المسائل .

[٨] وهذه الأدلة إنّية تُفيد الوقوع ، والتي تُفيد وجوب الوقوع من اللِميات ما يذكر في كتاب السماء والعالم من العلم الطبيعي .

/9/ ...] M , β = أن تكون الحركة] α . /14/ ...] β = M– , α– .

CHAPTER TWO

On the Arrangement and Order of the Bodies

[1] An observer of the two luminaries and the stars finds that they all move with the daily motion: those that rise do so in the east, travel westward and become invisible there. After a period of invisibility, they return to the east once more, rising there as they did before and continuing thus in the same way always. Those stars that do not rise have a parallel motion.

[2] With a more precise observation than the first, he will then find that they [also] move with a slower motion and in a direction opposite to the first, seemingly from west to east. Actually this latter motion may be distinguished from the former because of a difference in their equators and poles, as will be discussed more fully below, since it would be impossible to perceive two distinct motions occurring on the same sphere and sharing the same equator and poles. In such a case one would only perceive them as a single motion, either as the sum of the two motions if they are in the same direction or as the excess of the faster over the slower if they are in [opposite] directions. The same rule applies as well if more [than two motions] are involved. Both of these [primary] motions are uniform in and of themselves, and both embrace the entirety of the stars and bodies that are perceived above.

[3] [The observer] will then find that the two luminaries and the five planets have irregular motions that are neither uniform in themselves nor in relation to one another. Therefore the practitioners of this science established nine orbs upon their initial consideration—two for the above-mentioned motions and seven for the seven planets. Since the rest of the stars did not have any motion other than the two primary ones, they were satisfied with using one of these two orbs as the place for them even though their being on manifold orbs was conceivable. In addition, the attribution of one of these two primary motions to the whole rather than to a specific orb would not be precluded / if it were not for the other motion. They did not do this, however, because of its existence. /

/22–23/ if it were not…existence] β = they did not do this, however] α, M.

الفصل الثاني
في ترتيب الأجرام ونضدها

[١] الناظر في النيّرين والكواكب يجدها بأسرها متحركة بالحركة اليومية ، يطلع ما يطلع منها من المشرق ويسير إلى المغرب ويخفى فيه ، وبَعد خفائه مدة يعود إلى المشرق ثانياً ويطلع كما طلع أولاً ، وهكذا دائماً . ويتحرك ما لا يطلع منها على موازاته .

[٢] ثم يجدها بنظر أدق من الأول متحركة حركة بطيئة مخالفة للأولى كأنها من المغرب إلى المشرق . وإنما امتازت هذه الحركة من الأولى باختلاف المنطقتين والأقطاب كما سيجيء شرحه ، وذلك لأنّ الإحساس بحركتين مختلفتين في كرة واحدة على منطقة وقطبين بأعيانها ممتنع ، بل إنما يُحسّ منهما بحركة واحدة هي مركبة من مجموعهما إن كانتا إلى جهة أو حاصلة من فضل أسرعهما على أبطئهما إن كانتا إلى جهتين ، وكذلك الحكم فيما زاد على ذلك . وهاتان الحركتان متشابهتان في أنفسهما شاملتان لجميع ما يحس به علواً من الكواكب والأجرام .

[٣] ثم إنه يجد النيّرين والخمسة من الكواكب ذوي حركات مختلفة غير متشابهة لا في أنفسها ولا بقياس بعضها إلى بعض . فلذلك أثبت أهل هذا العلم تسعة أفلاك في بادئ نظرهم – اثنين منها للحركتين المذكورتين وسبعة للسيّارات السبعة . ولمّا لم تكن لباقي الكواكب حركة غير الأوليين اكتفوا بأحد فلكيهما مكاناً لها وإن كان كونها على أفلاك شتّى جائزاً . وأيضاً إسناد إحدى الأوليين إلى المجموع لا إلى فلك خاص به لم يكن ممتنعاً / لو لا الحركة الأخرى ، لكنهم لم يذهبوا إلى ذلك لوجودها / .

/22–23/ [...] = β لكنهم لم يذهبوا إلى ذلك] M , α .

[4] They then made the highest orb, in view of its being without stars, for the most conspicuous motion and called it the **orb of orbs** and the ***aṭlas* orb**. The next was for the less conspicuous motion, and they made it the place for the rest of the stars and called it the **orb of the ecliptic** [*lit.*, of the signs] and the **orb of the fixed stars.** Its stars they called "fixed" either because of the minuteness of their second motion or else because their [relative] positions are permanently fixed. The seven remaining orbs were for the seven wandering planets, arranged according to the occultation of one by another. The most remote of these [orbs] was reserved for Saturn, followed by the one for Jupiter and then Mars. The lowest was for the moon, and above it were the [orbs] of Mercury and then Venus. They placed the sun in the medial orb between the former and the latter even though it was not occulted except by the moon, deeming this the most elegant arrangement and the most excellent structure inasmuch as the six were connected to it—the upper [planets] in a certain way, the lower in another and the moon in yet another. In addition, the known distance of the [sun] from the Earth corresponded with this positioning. There are also reports that Venus has been observed at its farthest and nearest distances occulting the [sun], appearing like a mole on its surface. Each of these seven orbs must be further divided into other orbs so that its planet's compound motion results from them, consistent with what is observed. A discussion of this will be forthcoming. These nine [orbs] then were the least number they could allow; as for a maximum, there was no upper limit.

[5] The orbs terminate with the orb of the moon. Below it are the elements and they are likewise divided into various levels: [a] a level of pure fire; [b] a level composed of fire and hot air in which the smoke rising from the lower region dissolves. Comets, meteors [*nayāzik*], and the like are formed in this [level], and they are sometimes found to move in conformity with the motion of the orb; [c] a level that is predominantly air in which shooting stars [*shuhub*] occur; [d] a level of intense cold [*zamharīr*] that is the place of origin of clouds, thunder, lightning, and lightning bolts; [e] a level of dense air that is contiguous to the earth and water; [f] a level of water, part of which has been drawn aside, uncovering

[٤] فجعلوا أعلى الأفلاك للحركة الأظهر على أنه غير مكوكب ، وسموه فلك الأفلاك والفلك الأطلس ؛ وتاليه للحركة الأخفى وجعلوه مكاناً لسائر الكواكب ، وسموه فلك البروج وفلك الثوابت ، وسموا كواكبه الثوابت إما لقلة حركتها الثانية أو لثبات أوضاعها أبداً ؛ والسبعة الباقية للسيّارة السبعة على ترتيب خسف بعضها بعضاً ، أقصاها لزُحَل وما يليه للمُشْتَري ثم للمِرّيخ والأدنى للقمر والذي فوقه لعُطارِد ثم للزُهَرة ، وجعلوا الشمس في الفلك الأوسط بين هذه وتلك ، وإن / لم / تنكسف إلا بالقمر ، استحساناً لما في ذلك من حسن الترتيب وجودة النظام إذ الستة مربوطة عليها ـ العلوية بوجه والسفليان بوجه آخر والقمر بوجه غيرهما . وكان أيضاً بُعدها المعلوم من الأرض مناسباً لهذا الوضع ، وقد قيل إنّ الزهرة رُئيت في بعديها الأبعد والأقرب كاسفة إياها كخالة في صفحتها . ويجب أن ينقسم كل واحد من الأفلاك السبعة إلى أفلاك تتألف حركة كوكبه المركبة منها مطابقة لما يوجد ، وسيأتي ما قيل فيه . فهذه التسعة هي التي لم يجوّزوا أن يكون أقل منها ، وأما في جانب الكثرة فلا قطع .

[٥] وبفلك القمر تتناهى الفلكيات ويكون ما دونه العنصريات وهي أيضاً طبقات : طبقة للنار الصرف ؛ ثم طبقة لما يمتزج من النار والهواء الحار التي تتلاشى فيها الأدخنة المرتفعة من السفل ، تتكوّن فيها الكواكب ذوات الأذناب والنَيازِك وما يشبهها ، وربما توجد متحركة بحركة الفلك تشييعاً له ؛ ثم طبقة الهواء الغالب التي تحدث فيها الشُهُب ؛ ثم طبقة الزَمْهَرِير التي هي منشأ السحب والرعد والبرق والصواعق ؛ ثم طبقة الهواء الكثيف المجاور للأرض والماء ؛ ثم طبقة الماء وبعض هذه الطبقة منكشفة عن

/8/ ...] β = لم تكن] M , α .

the earth; [g] a level of earth intermixed with other [elements] in which is generated mountains, minerals, and much plant and animal life; and, finally, [h] a level of pure earth surrounding the center.

CHAPTER THREE

On the Well-Known Great Circles

[1] When mathematicians wish to assign values to circles and their diameters, it is customary for them to divide [the circle] into 360 **parts** and the diameter into 120 **parts**. These parts are then subdivided into **minutes, seconds,** and so forth. Thus one-fourth of a revolution will be 90. For every arc less than [90], its complement is what remains from one-fourth [of a revolution] after subtracting that [arc] from it.

[2] Let us now proceed with the main purpose [of this chapter]. We say: the most conspicuous of the great circles is the equator of the first motion, *i.e.* the daily universal motion. It is called the **equinoctial orb** and the **equinoctial circle**, their use of the name "orb" [*falak*] to designate its equator being permissible. It is named "equinoctial" because of the equality of night and day at all locations when the sun occurs on it. Its poles are called the **poles of the first motion**, one of them being northern and the other southern. This [circle's] parts are in units of time because time is initially measured by its motion. Every given point on the orb produces by virtue of its daily motion a circle that is parallel to the equinoctial, and all [the circles thus produced] are called the **day-circles**.

[3] The equator of the slower second motion is called the **equator of the ecliptic** and the **orb of the ecliptic**, its poles being those of the ecliptic. It intersects the equinoctial at non-right angles in all the orbs that have these two motions, and there occur between the two equators two facing intersection points called the two **equinox points**. The sun

الأرض ؛ ثم طبقة الأرض المخالطة / بغيرها التي فيها تتولد / الجبال والمعادن وكثير من النباتيات والحيوانات ؛ ثم طبقة الأرض الصرفة المحيطة بالمركز .

الفصل الثالث
في الدوائر العظمى المشهورة

[١] من عادة الحسّاب إذا أرادوا تقدير الدوائر وأقطارها تجزئتُها بثلاثمائة وستين جزءاً وتجزئة القطر بمائة وعشرين جزءاً ، ثم تجزئة الأجزاء إلى دقائقها وثوانيها وما يتلوها . فيكون الربع من الدور تسعين ، وكل قوس أقل منه فتمامها ما يبقى من الربع بعد نقصانها عنه .

[٢] ولنشرع في المقصود فنقول : أظهر الدوائر العظمى منطقة الحركة الأولى أعني حركة الكل اليومية وتسمى فلك مُعدِّل النهار ودائرة مُعدِّل النهار ، وقد يطلقون اسم الفلك على منطقته تجوّزاً . وسُميت معدل النهار لتعادل الليل والنهار في جميع البقاع عند كون الشمس عليها . ويسمى قطباها قطبي الحركة الأولى : أحدهما شمالي والآخر جنوبي ، وأجزاؤها أزماناً لأنّ الزمان يتقدر أولاً بحركتها . وكل نقطة تُفرض على الفلك فهي تفعل بحركتها اليومية دائرة موازية لمعدل النهار ويسمى جميعها المدارات اليومية .

[٣] ومنطقة الحركة الثانية البطيئة تسمى منطقة البروج وفلك البروج وقطباها قطبي البروج . وهي تُقاطِع معدِّل النهار في جميع الأفلاك التي تتحرك بالحركتين على زوايا غير قائمة . ويحدث بين المنطقتين تقاطُعان متقابلان يسميان نقطتي الاعتدال . والشمس

/1/ ...] β = بغيره التي تتولد فيها] M = لغيره التي تتولد فيها] α .

adheres to this second equator, and the intersection point which is such that the [sun], when crossing it, comes to be north of the equinoctial is the **vernal** [equinox]; the other is **autumnal**. The maximum distance between the two equators is the distance between their two poles that are in the same direction and is called the **total obliquity** [*lit.*, the complete declination].

[4] Now let there be imagined a **great circle passing through the four poles**, and it will be called by this name.* It is perpendicular to each one of the two equators, and its two poles are the equinox points. It passes through two points of the ecliptic orb at which occurs the maximum inclination from the equinoctial and which mark quarter points of the [ecliptic] equator. They are called the **solstice points**: the northern being the summer one, the southern being the winter. The arc that occurs on the **solstitial colure** between the two equators or between their two poles is the total obliquity whose value is found by observation. Its complement is that [arc] occurring on this [circle] between the pole of one of them and the other equator.

[5] The ecliptic equator is divided into 12 equal parts, each of which is called a **zodiacal sign**. Their 12 names are well-known; they were derived from forms fancied from among those fixed stars that happened to be in alignment with the [zodiacal signs] at the time they were named. Since these [forms] are no longer aligned with their [respective signs], it is up to those who name these things to change their names. A [zodiacal sign's] units are called **degrees**, and each zodiacal sign consists of 30 degrees. Every point describes by virtue of its second motion a circle parallel to the ecliptic orb, which is its circuit; all [the circles thus produced] are called **parallels of latitude**.

[6] If a circle is imagined to pass through an arbitrary part of the ecliptic orb, or through a given star, and through the poles of the equinoctial, it is a **circle of declination**. The arc occurring on it between that part [of the ecliptic] and the equinoctial is the declination of that part; this is known as a **particular declination**. The [arc] occurring between the star and the equinoctial is that star's **distance from the equinoctial**. For each [arc], its complement is the distance of [the part or the star] from the pole. The plane of this circle intersects the plane of the equinoctial at right angles.

[7] If a circle is imagined to pass through an arbitrary part of the ecliptic orb, or through a given star, and through the poles of the ecliptic orb, it is a **circle of latitude**.

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*Henceforth *al-dā'ira al-mārra bi-'l-aqṭāb al-arbaᶜa* (the circle passing through the four poles) will be translated "the solstitial colure."

تلازم هذه المنطقة فالتقاطع الذي إذا جازته صارت شمالية عن معدل النهار ربيعي والآخر خريفي . وغاية البعد بين المنطقتين هي البعد بين قطبيهما اللذين في جهة ويسمى الميل الكلي .

[٤] فتتوهم دائرة عظيمة تمرّ بالأقطاب الأربعة وتسمى بهذا الاسم . وهي تقوم على كل واحدة من المنطقتين على زوايا قائمة ، ويكون قطباها نقطتي الاعتدالين . وتمرّ بنقطتين من فلك البروج عندهما غاية الميل من معدل النهار ، تُربّع المنطقة بهما وتسميان نقطتي الانقلابين ــ الشمالية صيفية والجنوبية شتوية . والقوس الواقعة من الدائرة المارّة بالأقطاب الأربعة بين المنطقتين أو بين القطبين هي الميل الكلي ومقدارها يُعرف بالرصد ، وتمامها ما يقع منها بين قطب إحداهما والمنطقة الأخرى .

[٥] وتقسم منطقة البروج باثني عشر قسماً متساوية يسمى كل قسم بُرْجاً . وأسماؤها الاثنا عشر مشهورة وهي مأخوذة من صور تُوهِّمت من كواكب وقعت وقت التسمية بحذائها من الثوابت . وإذا انتقلت عن محاذاتها فللمُسمِّين أن يسموها بغيرها . وأجزاؤها تسمى درجاً فكل برج ثلاثون درجة . وكل نقطة تفعل بحركتها الثانية دائرة موازية لفلك البروج هي مدارها ويسمى الجميع بالمدارات العرضية .

[٦] وإذا توهمت دائرة تمرّ بجزء من فلك البروج ــ أي جزء كان ــ أو بكوكب ما وبقطبي معدّل النهار فهي دائرة الميل . والقوس الواقعة منها بين ذلك الجزء وبين معدل النهار هي ميل ذلك الجزء وهي من الميول الجزئية ، والواقعة بين الكوكب وبين معدل النهار هي بعد ذلك الكوكب من معدل النهار . وتماماهما بعداهما من القطب . وسطح هذه الدائرة يقطع سطح معدل النهار على زوايا قائمة .

[٧] وإذا توهمت دائرة تمرّ بجزء من فلك البروج ــ أي جزء كان ــ أو بكوكب ما وبقطبي فلك البروج فهي دائرة العرض .

The arc occurring on it between that part [of the ecliptic] and the equinoctial is the **latitude** of that part. An [arc] on the circle of declination is sometimes called the **first declination**, while this latter is the **second declination**. At maximum declination, they are the same since the circles of declination and latitude [thereupon] coincide becoming none other than the solstitial colure. The arc occurring on the [latitude circle] between a star and the ecliptic orb is the **star's latitude**; its **colatitude** is the [arc] between it and the ecliptic pole.

[8] The **longitude** of a star is that arc along the ecliptic orb measured sequentially from the vernal equinox point to the star if it is on the ecliptic with no value in latitude or, if it has a value in latitude, to the point at which its latitude circle intersects the ecliptic orb. The longitude is sometimes called ***taqwīm***. Taking the vernal equinox point instead of some other starting point is by convention.

[9] If six circles of latitude pass through the beginning of each of the twelve zodiacal signs—one of these circles necessarily being the solstitial colure—they will divide the orb into twelve parts that are the zodiacal signs, each of which [extends] in latitude from pole to pole and in longitude 30 degrees. Everything falling in one of these divisions is considered to be in that zodiacal sign. The ecliptic equator passes through the midpoints of the zodiacal signs and hence is also called the **orb of the middle of the zodiacal signs**.

[10] These then are five circles that can be conceived without reference to the lower regions. Three of them—the equinoctial, the ecliptic orb, and the solstitial colure—are each individual circles. The other two—the circle of declination and the circle of latitude—are classes, each consisting of an unlimited number of individual members.

[11] By referring to the lower regions we have (among others):

[12] [a] the **horizon circle**. This is the great circle that divides the visible from the invisible on the celestial sphere. One of its poles is the **zenith** and the other is directly opposite it from below.

والقوس الواقعة منها بين ذلك الجزء وبين معدل النهار هي عرض ذلك الجزء . وقد تسمى التي تكون من دائرة الميل ميلاً أوّلَ وهذه ميلاً ثانياً . وعند غاية الميل يتحدان لأنّ دائرتي الميل والعرض تتحدان فتصيران المارّة بالأقطاب الأربعة بعينها . والقوس الواقعة منها بين الكوكب وبين فلك البروج عرض الكوكب ، والتي بينه وبين قطب البروج تمام عرضه .

[٨] وطول الكوكب هو قوس من فلك البروج على التوالي تقع بين نقطة الاعتدال الربيعية وبين الكوكب إن كان على فلك البروج عديم العرض ، أو بين النقطة التي تقطع دائرةُ عرضه فلكَ البروج عليها إن كان ذا عرض . وقد يسمى الطول تقويماً . وإنما اعتبر نقطة الاعتدال الربيعية دون غيرها لأنّها جعلت مبدأً اصطلاحاً .

[٩] وإذا مرّت ستّ من دوائر العروض بأوائل البروج الاثني عشر ــ وتكون إحداها لا محالة المارة بالأقطاب الأربعة ــ قسمت الفلك باثني عشر قسماً هي البروج ، كل قسم في العرض من القطب إلى القطب وفي الطول ثلاثون درجة . وكل ما يقع في كل قسم منها يكون في ذلك البرج . ومنطقة البروج تمرّ بأوساط البروج ولذلك تسمى أيضاً فلك أوساط البروج .

[١٠] فهذه خمس دوائر تتوهم من غير ملاحظة السفليات : ثلاث منها أشخاص بأعيانها وهي معدل الهار وفلك البروج والمارّة بالأقطاب / الأربعة / ، واثنتان نوعان لهما أشخاص بلا نهاية وهما دائرة الميل ودائرة العرض .

[١١] وأما التي تكون بملاحظة السفليات فمنها :

[١٢] دائرة الأفق وهي العظيمة الفاصلة بين الظاهر والخفي من الفلك . وأحد قطبيها سمت الرأس والآخر ما يحاذيه من تحت .

.−α = β , M [... /20/

The circles parallel to it above the Earth are called **almucantars of altitude**, while those below are **almucantars of depression**;

[13] [b] the **meridian circle**. This divides the eastern from the western half of the celestial sphere as well as the ascending from the descending with respect to the primary motion. It passes through the poles of the horizon and those of the equinoctial and is perpendicular to both the horizon and the equinoctial. It bisects the visible and the invisible segments of the day-circles as well as those day-circles that are visible or invisible in their entirety. Since it passes through the poles of the equinoctial and of the horizon, these two will pass through the [meridian's] poles, which are the points of their intersection. These poles are the places of rising and setting of the equinoxes and are called the **east and west points**. The arc occurring on the [meridian] between the pole of the equinoctial and the horizon circle (or [alternatively] between the pole of the horizon and the equinoctial circle) is called the **local latitude**; its complement is the [arc] between the two poles or between the two equators;

[14] [c] the **east-west circle**. It passes through the poles of the horizon and through the poles of the meridian. Its poles are the points of intersection of the horizon and the meridian, which are called the **north and south points**. This circle is also called the **circle of the initial azimuth** [prime vertical]; the meaning of "azimuth" will be forthcoming.

[15] These three circles divide the celestial sphere into eight equal parts—triangles whose sides are quarter revolutions. Four of them are visible, while four are invisible;

[16] [d] the **ecliptic meridian circle** [*lit.*, midheaven circle of appearances]. It passes through the poles of the ecliptic orb and the poles of the horizon, bisecting the visible and invisible halves of the ecliptic orb. It is also called the **circle of the local ecliptic latitude** [*lit.*, the clime latitude circle of appearances]. The arc occurring on it between the pole of the ecliptic and the horizon circle (or [alternatively] between the pole of the horizon and the ecliptic equator) is the **local ecliptic latitude**;

وتسمى الدوائر الموازية لها فوق الأرض مُقَنْطَرات الارتفاع والتي تحتها مقنطرات الانحطاط ؛

[١٣] ودائرة نصف النهار وهي الفاصلة بين النصف الشرقي والنصف الغربي من الفلك بل الصاعد والهابط بقياس الحركة الأولى . وهي المارّة بقطبي الأفق وقطبي معدل النهار ، وتقوم على الأفق وعلى معدل النهار على زوايا قائمة ، وتنصّف القِطَع الظاهرة والخفية من المدارات اليومية والمدارات الظاهرة والخفية / بأسرها أيضاً / . ولكونها مارّة بأقطاب معدل النهار والأفق فهما تمرّان بقطبيها فيكون قطباها نقطتي تقاطعيهما وهما مطلع الاعتدالين ومغيبهما وتسميان نقطتي المشرق والمغرب . والقوس الواقعة منها بين قطب معدل النهار ودائرة الأفق أو بين قطب الأفق ودائرة معدل النهار تسمى عرض البلد ، والتي بين القطبين أو المنطقتين تمامه ؛

[١٤] ودائرة المشرق والمغرب وهي المارة بقطبي الأفق وبقطبي نصف النهار . ويكون قطباها نقطتي تقاطُع الأفق ونصف النهار وتسميان نقطتي الشمال والجنوب . وتسمى هذه الدائرة أيضاً دائرة أول السموت وسيجيء معنى السمت .

[١٥] وهذه الدوائر الثلاث تقسم الفلك بثمانية أقسام متساوية مثلّثات أضلاعها أرباع الدور – أربعة ظاهرة وأربعة خفية ؛

[١٦] ودائرة وسط سماء الرؤية وهي المارّة بقطبي فلك البروج وبقطبي الأفق . وهي تنصّف النصفين الظاهر والخفي من فلك البروج . وتسمى دائرة عرض إقليم الرؤية . والقوس الواقعة منها بين قطب فلك البروج ودائرة الأفق أو بين قطب الأفق ومنطقة البروج هي عرض إقليم الرؤية ؛

/7–8/ [...] = أيضاً بأسرها] M , α .

[17] [e] the **altitude circle**. This passes through any given point on the celestial sphere and through the poles of the horizon circle. If the point is above the Earth, then the [arc] between it and the horizon is its **altitude**; if below, the [arc] is the point's **depression**. The [arc] between this circle and the prime vertical along the horizon circle is the point's **azimuth**. An azimuth may be northeast, southeast, northwest, or southwest. The altitude circle coincides with the meridian circle if the star is at the middle of the period of its visibility or of its invisibility, with the prime vertical if the star has zero azimuth, and with the ecliptic meridian circle if the star is at quadrature to the ascendent.

[18] Each of these five circles is a class with numerous individual members.

[19] These then are the best-known great circles.

CHAPTER FOUR

On the Circumstances Occurring Due to the Two Primary Motions, and the Situation of the Fixed Stars

[1] The total obliquity as found by ancient and modern observations is not the same; rather, what the ancients found is greater than that found by the moderns. It has been supposed that the value found by a more recent observer will be less than that of one more ancient, even though the maximum that was found did not come to 24°, while the minimum was not less than $(23 + \frac{1}{2} + (\frac{1}{2} \text{ of } \frac{1}{10}))°$. Most have accepted the value of $(23 + \frac{1}{3} + \frac{1}{4})°$.

[2] Because of this difference, some have maintained that the ecliptic equator moves in latitude and approaches the equinoctial. If this were true, then another orb would need to be established / whereby the ecliptic orb would move with that motion. / Now the [ecliptic] equator, if

/23/ whereby...that motion] β = to move the ecliptic orb with that motion] α, M.

[١٧] ودائرة الارتفاع وهي التي تمرّ بأي نقطة تُفرض على الفلك وبقطبي دائرة الأفق . فإن كانت النقطة فوق الأرض فما بينها وبين الأفق ارتفاعها وإن كانت تحتها فهو انحطاطها . وما بين هذه الدائرة ودائرة أول السموت من دائرة الأفق سمتها ، فمن السمت شرقي شمالي ومنه شرقي جنوبي وكذلك غربي شمالي وغربي جنوبي . وهي تتّحد بدائرة نصف النهار إذا كان الكوكب في منتصف زمان ظهوره أو خفائه ، وبدائرة أول السموت إذا كان عديم السمت ، وبدائرة وسط سماء الرؤية إذا كان على تربيع الطالع .

[١٨] وهذه الدوائر الخمس وحدتها نوعية وتتكثر بالأشخاص.

[١٩] فهذه هي المشهورة من العظام .

الفصل الرابع
في الأوضاع التي تحدث بسبب الحركتين الأوليين وأحوال الكواكب الثابتة

[١] الميل الكلي الموجود بالأرصاد القديمة والحديثة ليس شيئاً واحداً بل كان ما وجده القدماء أكثر مما وجده المحدثون . وقد يظن أنّ ما وجده من هو أحدث زماناً كان أقل مما وجده من هو أقدم مع أنّ أكثر ما وجدوه لم يبلغ أربعة وعشرين جزءاً ، وأقله لم ينقص من ثلاثة وعشرين جزءاً ونصف جزء ونصف عُشر جزء . والجمهور على أنه ثلاثة وعشرون جزءاً وثُلث وربُع جزء .

[٢] فلهذا الاختلاف زعم بعضهم أنّ منطقة البروج تتحرك في العرض فتقرب من معدل النهار . وإن كان ذلك حقاً فيجب أن يُثبت فلكٌ آخر / يتحرّك فلكُ البروج بتلك الحركة / . ثم المنطقة إن

/23/ ...] β = يُحرِّك فلكَ البروج تلك الحركة] α , M .

it moves, may complete a revolution or it may not complete it but instead move to a certain limit then return. / This limit may be after it has coincided twice with the equinoctial equator, or at the second coincidence, or between the two coincidences. If between, then the limit may be after half a revolution, or exactly at the halfway mark, or before it. If the equator does not reach the region between the two coincidences, then it will either return upon arriving at the first coincidence or else return before this [limit is reached].

[3] These then are eight possibilities. On the first five assumptions, the northern and southern halves of the surface of the ecliptic orb, along with whatever conditions pertain to them, would interchange. In the first three cases only, each of the two halves of the equator of the ecliptic orb would coincide with each of the two halves of the equinoctial equator. For the remaining three assumptions (those after the first five), only part of the surfaces would interchange. On the basis of the first seven assumptions, one half of the ecliptic equator would coincide with that half of the equinoctial equator adjacent to it; during each coincidence, day and night would be of equal length in all localities and the seasons of the year would cease to occur. On the eighth assumption, this would not be the case; however, altitudes [of stars] and the extent of days and nights would increase and decrease in a given locality. /

[4] Furthermore there was a divergence regarding the rate of the second motion. The ancients found it to describe one degree every 100 years, while the moderns observed it to be every 66 years. One reliable group among the latter found it to be

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/2–16/ This limit...a given locality] β = This limit may be after it has coincided with the equinoctial and departed from it; it may be at its coincidence; or, it may be before its coincidence. On the first assumption, the halves of the ecliptic orb, *i.e.* the northern and the southern, could completely interchange. On the second assumption, this could partially occur. On the third assumption, this could not occur; however, day and night would become equal during the coincidence under all circumstances and the seasons of the year would cease to occur. On the fourth assumption, this would not be the case; however, altitudes [of stars] and the extent of days and nights would increase and decrease in a given locality] α, M.

تحركت فيمكن أن تتمّم الدورة ويمكن أن لا تتمّم بل تتحرك إلى غاية ما ثم تعود . / وتلك الغاية يمكن أن تكون بعد انطباقها على منطقة معدل النهار مرّتين أو حال انطباقها الثاني أو فيما بين الانطباقين ، وذلك إما بعد قطع نصف دورتها أو حال قطع النصف سواءً أو قبله ، وإن لم تصل إلى ما بين الانطباقين فإما أن تعود حال انطباقها الأول أو قبل ذلك .

[٣] وهذه ثمانية احتمالات . وعلى التقديرات الخمسة الأُوَل يتبادل نصفا سطح فلك البروج الشمالي والجنوبي مع ما يتبعهما من الأحكام . وفي الثلاثة الأُوَل منها فقط ينطبق كل واحد من نصفي منطقة فلك البروج على كل واحد من نصفي منطقة معدل النهار . وعلى التقديرات الثلاثة الباقية بعد الخمسة الأُوَل لا يتبادل غير البعض من السطح . وعلى التقديرات السبعة الأُوَل ينطبق النصف من منطقة البروج على النصف المجاور إياه من منطقة معدل النهار وعند كل انطباق يتساوى النهار والليل في جميع البقاع وتبطل فصول السنة . وعلى التقدير الثامن لا يكون ذلك إلا أنّ الارتفاعات ومقادير الأيام والليالي تزيد وتنقص في بقعة بعينها . /

[٤] وأيضاً وقع الاختلاف في مقدار الحركة الثانية . وذلك أنّ القدماء وجدوها تقطع جزءاً واحداً في كل مائة سنة والمحدثون وجدوها في كل ست وستين سنة ، وقوم من محققيهم وجدوها في

/16–2/ ...] β = وتلك الغاية يمكن أن تكون بعد انطباقها على معدل النهار ومفارقتها إياه ويمكن أن تكون حال انطباقها ويمكن أن تكون قبل انطباقها وعلى التقدير الأول يمكن تبادل نصفي فلك البروج أعني الشمالي والجنوبي بالتمام وعلى التقدير الثاني يمكن ذلك في البعض وعلى التقدير الثالث لا يمكن ذلك إلا أن النهار والليل يصيران متساويين عند الانطباق في جميع الأحوال وتبطل فصول السنة وعلى التقدير الرابع لا يمكن ذلك إلا أن الارتفاعات ومقادير الأيام والليالي تزيد وتنقص في بقعة بعينها] α , M .

every 70 years. Some of the practitioners of [the art of] talismans have claimed that the orb undergoes accession and recession, the maximum extent of each being 8 degrees, which is completed in / every / 640 years. Some of the practitioners of this discipline have entertained this idea and held that the [second] motion becomes slower due to the recession during which the vernal equinox, which is the starting point, shifts counter-sequentially from its position, and that the motion becomes faster due to the accession during which the vernal equinox shifts sequentially from its position. This would, if they are correct, also require that another mover be established other than those already mentioned.

[5] One [some?] of them came to be satisfied with one mover for both divergences. This mover would cause the ecliptic orb to move in such a way that every point on it moves about a small circle. Accession would then result from the motion in one of its halves, while recession would occur in the other half. [In addition], there would occur a decrease in the obliquity during the motion from the midpoint of one of these halves to the midpoint of the other half, while there would be an increase during the motion in the other half. This then is what has been said concerning this matter. But to establish conclusively the authenticity of a mover and its configurations depends upon ascertaining the true state of affairs; let us then put this matter aside.

[6] It should be noted that when one orb moves another, the moved retains its position with respect to the mover; its relation to it is as a part to the whole. Thus it along with its poles and the rest of its parts moves due to the motion of [the mover] just as an occupant on a ship moves by virtue of the ship's motion. At the same time, the [moved orb] has its own proper motion just as is the case with the occupant of a ship who moves back and forth on the ship—sometimes in the direction of the ship's motion, at other times opposite that motion.

[7] This having been determined, let there be imagined a similar situation with respect to the eighth orb, which is moved by the motion of the ninth orb. And it should be noted that the fixed stars—as indeed all given points on the eighth orb—do not depart from their latitude circles in any way nor do their positions relative to one another or relative to the ecliptic equator or its poles change. However, their positions relative to the equinoctial do change. Every star on the ecliptic equator crosses the equinoctial twice in

/2/ every] $\beta = -\alpha$, –M.

كل سبعين سنة . وقد زعم بعض أهل الطِلَسمات أنّ للفلك إقبالاً وإدباراً غاية كل واحد منهما ثمانية أجزاء يتم في / كل / ستمائة وأربعين سنة . فسمع ذلك بعض أهل هذا العلم فظن أنّ تلك الحركة تبطؤ بسبب الإدبار وانتقال النقطة الربيعية التي هي المبدأ من موضعها إلى خلاف التوالي ، وتسرع بسبب الإقبال وانتقالها من موضعها إلى التوالي . وذلك أيضاً إن كان كما ظنوا محوج إلى إثبات محرك آخر غير ما مرّ .

[٥] وذهب بعضهم إلى الاكتفاء بمحرّك واحد للاختلافين يحرّك فلكَ البروج فتتحرك كل نقطة منه حول دائرة صغيرة . فيكون من الحركة في أحد نصفيه[!] الإقبال ومن الحركة في النصف الآخر الإدبار . ومن الحركة من منتصف أحد النصفين إلى منتصف النصف الآخر انتقاص الميل ومن الحركة في النصف الآخر ازدياده . فهذا ما قيل فيه والقطع بإثبات محرّك وهيئاته موقوف على تحقّق الحال ، فلنُعرض عنه .

[٦] وٱعلم أنّ تحريك فلك فلكاً يكون بملازمة المتحرك لمكانه من المحرك ، وكونه منه كالجزء من الكل . فيتحرك مع قطبيه وسائر أجزائه بحركته مثل حركة ساكن السفينة بحركة السفينة . ثم إنه مع ذلك يتحرك بنفسه حركته الخاصة به كساكن السفينة إذا تردّد في السفينة تارة إلى جهة حركتها وتارة إلى خلاف تلك الجهة .

[٧] وإذا تقرر ذلك فليتصوّر في الفلك الثامن المتحرك بحركة الفلك التاسع مثل ذلك . وليعلم أنّ الثوابت بل جميع النقط المفروضة على الفلك الثامن لا تفارق مداراتها العرضية البتة ولا تختلف أوضاعها بقياس بعضها إلى بعض ولا بقياسها إلى منطقة البروج وقطبيها ، لكنّ أوضاعها بالقياس إلى معدل النهار تختلف . فكل كوكب يكون على منطقة البروج فهو يقطع معدل النهار في

. −α , −M = β [… /2/

a revolution of the second motion; on one half of its circuit it is north of it and on the other half south. Every star having a latitude less than the total obliquity will also cross the celestial equator twice; however, the north and south segments of its circuit are unequal, the larger being in the direction of the star's latitude. Every star whose latitude is equal to the total obliquity will not cross the equinoctial; rather, it will touch it once every revolution at the solstice point that is in the direction of its latitude. Every star whose latitude exceeds the total obliquity neither crosses the equinoctial nor touches it [at a point]; instead, it will approach and recede from it. If the star's latitude is equal to the complement of the total obliquity, once every revolution it will reach the equinoctial pole that is in its direction. On account of this change [relative to the equinoctial], the day-circles for each star will vary. No star remains on the same circuit but will instead shift to a larger circuit if it approaches the equinoctial or shift to a smaller one if the opposite is the case.

[8] The positions of the stars also vary with respect to the inhabitants of the climes. Those stars having a greater altitude come to have a lesser one and conversely. Some of them will transit the zenith after not having done so previously. This is so when their distance and direction from the equinoctial becomes the same as the local latitude. Some stars will become permanently visible or permanently invisible after not being so. This will occur when the complement of their distance from the equinoctial becomes equal to the local latitude in the direction of the visible pole or the invisible one after having been greater than this amount. Some stars will come to rise and set after having been permanently visible or permanently invisible. This will take place when the complement of their distance from the equinoctial becomes greater than the local latitude after having been less than or equal to it. Polaris is one of those stars that will eventually reach the northern pole. Rigil Centaurus and Canopus are two of the stars that will become permanently invisible in the Fourth Clime.

دورة من الحركة الثانية مرّتين ، ويكون في أحد نصفي مداره شمالياً عنه وفي النصف الآخر جنوبياً . وكل كوكب ذي عرض يكون عرضه أقل من الميل الكلي فهو يقطع / أيضاً معدل النهار / مرّتين ، لكن تختلف قطعتا مداره الشمالية والجنوبية ويكون أعظمهما ذات جهة العرض . وكل كوكب يساوي عرضه الميل الكلي فهو لا يقطع معدل النهار ولكن يماسه على نقطة الانقلاب التي في جهة عرضه في دورة مرة واحدة . وكل كوكب يفضل عرضه الميل الكلي فهو لا يقطع معدل النهار ولا يماسه بل يقرب منه ويبعد عنه . فإن كان عرضه مساوياً لتمام الميل الكلي فهو ينتهي في دورة إلى قطب معدل النهار الذي في جهته مرة واحدة . وبحسب هذا الاختلاف تختلف المدارات اليومية لكل كوكب ولا يبقى كوكب على مدار واحد بل ينتقل إلى مدار أكبر إن كان يقرب من معدل النهار أو إلى مدار أصغر إن كان بالضد .

[٨] وتختلف أيضاً أوضاع الكواكب بالقياس إلى سكّان الأقاليم ، فيصير ما هو أكثر ارتفاعاً أقل وبالعكس . ويحدث لبعضها مرور بسمت الرأس بعد ما لم يكن ، وذلك عند صيرورة بعده عن معدل النهار بقدر عرض البلد وفي جهته . ويصير بعضها أبدي الظهور أو أبدي الخفاء بعد أن لم يكن ، وذلك عند صيرورة تمام بعده عن معدل النهار مساوياً لعرض البلد في جهة القطب الظاهر أو الخفي بعد أن كان أكثر من ذلك . ويحدث لبعضها طلوع وغروب بعد أن كان أبدي الظهور أو الخفاء ، وذلك عند ازدياد تمام بعده عن معدل النهار على عرض البلد بعد أن كان أقل منه أو مساوياً له . والجُدَيّ مما سينتهي إلى القطب الشمالي ، ورِجْل قَنْطُورِس وسُهَيل مما يصيران أبدي الخفاء في الإقليم الرابع .

/3/ [...] M , β = معدل النهار أيضاً] α .

[9] The **fixed stars** cannot possibly be enumerated, but 1022 of them have been observed and their positions in longitude and latitude are known. They arranged their magnitudes into six grades, the first being the greatest. In order to identify them, they imagined certain constellations [*lit.*, forms] that the stars were either on or near; thus one could say "that star on the head of such and such a constellation" or "that one near the foot of such and such a constellation." The constellations are 48 in number, of which 21 are in the north, namely: Ursa Minor, Ursa Major, Draco, Cepheus, Boötes, Corona Borealis, Hercules, Lyra, Cygnus, Cassiopeia, Perseus, Auriga, Aquila, Delphinus, Sagitta, Ophiuchus, Serpens, Equuleus, Pegasus, Andromeda, and Triangulum. Twelve are on the [ecliptic] equator; they are the zodiacal signs and their names are well-known. There are 15 southern [constellations] and they are: Cetus, Orion, Eridanus, Lepus, Canis Major, Canis Minor, Argo Navis, Hydra, Crater, Corvus, Centaurus, Lupus, Ara, Corona Australis, and Piscis Austrinus. Three hundred sixty stars were observed in the northern constellations, 346 in the ecliptic constellations, and 316 in the southern constellations.

[10] The **Milky Way**, *i.e.* the galaxy, is made up of a very large number of small, tightly-clustered stars, which, on account of their concentration and smallness, seem to be cloudy patches. Because of this, it was likened to milk in color.

[11] As for the **mansions of the moon**, they are made up of the stars that are near the ecliptic equator. The Arabs used them as signs for the 28 divisions into which the [ecliptic] equator was divided so as to coincide with the number of days of the moon's revolution. Thus every night it was seen "residing" near one of these mansions whose names are well-known.

[12] Knowledge of the fixed stars and that which concerns them being a separate discipline, it is best that we confine ourselves to just what has been presented.

[٩] والكواكب الثابتة لا يمكن أن تُحصى كثرة . وقد رُصد منها ألف واثنان وعشرون كوكباً فعرف مواضعها في الطول والعرض . ورتّبوا أقدارها في ست مراتب أُولاها أعظمها ، وتوهموا لتعريفها صوراً تكون هي عليها أو بقربها فيقولون : الذي على رأس الصورة الفلانية أو بقرب رجل الصورة الفلانية . وكانت الصور ثمانية[!] وأربعين منها إحدى وعشرون في الشمال وهي : الدُبّ الأصغر ، والدُبّ الأكبر ، والتِنّين ، وقَيْقاوُس ، والعَوّاء ، والفَكّة ، والجاثي على رُكْبَتَيه ، وشِلْياق ، والدَجاجة ، وذات الكُرْسي ، وحامِل رأس الغُول ، ومُمسِك العِنان ، والعُقاب ، والدُلْفِين ، والسَهْم ، والحَوّاء ، والحَيّة ، وقِطْعة الفَرَس ، والفَرَس الأعظم ، والمرأة المُسَلْسَلة ، والمُثَلَّث ؛ واثنتا عشرة على المنطقة وهي البروج وأسماؤها مشهورة ؛ وخمس عشرة في الجنوب وهي : قِيْطُس ، والجَبّار ، والنَهْر ، والأرْنَب ، والكَلْب الأكبر ، والكَلْب الأصغر ، والسَفينة ، والشُجاع ، والباطِيَة ، والغُراب ، وقَنْطُورِس ، والسَبُع ، والمِجْمَرَة ، والإكْليل الجنوبي ، والحُوت الجنوبي . وكان من المرصودة ثلاثمائة وستون على الصور الشمالية ، وثلاثمائة وستة وأربعون على صور المنطقة ، وثلاثمائة وستة عشر على الصور الجنوبية .

[١٠] والدائرة اللَبَنِيّة أعني المَجَرّة مؤلفة من كواكب صغار متقاربة متشابكة كثيرة جداً صارت من تكاثفها وصغرها كأنها لطخات سحابية ، ولذلك شُبّهت باللبن لوناً .

[١١] وأما منازل القمر فهي من الكواكب القريبة من منطقة البروج جعلته[!] العرب علامات الأقسام الثمانية والعشرين التي قسمت المنطقة بها لتكون مطابقة لعدد أيام دور القمر ، فيرى كل ليلة نازلاً بقرب أحدها . وأسماؤها مشهورة .

[١٢] ومعرفة الثوابت وأحوالها فنّ مفرد ، فالأَولى أن نقتصر هاهنا على هذا القدر .

CHAPTER FIVE

On Basing Some of the Apparently Irregular Motions Upon Models That Bring About Their Uniformity

[1] If a celestial motion is irregular from our perspective, we must require that it have a model according to which that motion is uniform; this model should also bring about its irregularity with respect to us. For irregular [motion] does not arise from the celestial bodies.

[2] One of these models is such that the motion is uniform about a point different from the center of the World, which is in our proximity. The circumference upon which the moved thing—a planet, for example—moves about that point must either enclose the center of the World or else not enclose it. In the first case it is called an **eccentric**, while in the second it is called an **epicycle**.

[3] If an eccentric is taken by itself and a planet is assumed to move upon it about its center with a simple, uniform motion, it will cause that motion, with respect to the center of the World (as well as to points other than the [eccentric] center) to be irregular. Thus in the segment that is farther away, the motion will be slower, while in the nearer segment it will be faster. This is so because for equal arcs whose distances vary, those that are farther away appear smaller than those that are nearer. If a line is produced that passes through the [eccentric] center and through the center of the World (or through some given point other than these two), it will pass through the **farthest distance**, which is the midpoint of the far segment, and through the **nearest distance**, which is the midpoint of the near segment. Then if a line is produced perpendicular to it passing through the center of the World (or through that [other given] point) and reaching the circumference on both sides, it will pass through the two **mean distances**, which are the common part between the two segments. At these two points the motion will be at its mean between the faster and slower motions.

[4] As for the epicycle, if it is taken by itself and the planet moves on its circumference, equal arcs will likewise vary with respect to the center of the World. The line joining the two centers passes through the **farthest and nearest distances** of the epicycle.

الفصل الخامس
في إسناد بعض الحركات المختلفة في الرؤية إلى أصول تقتضي تشابهها

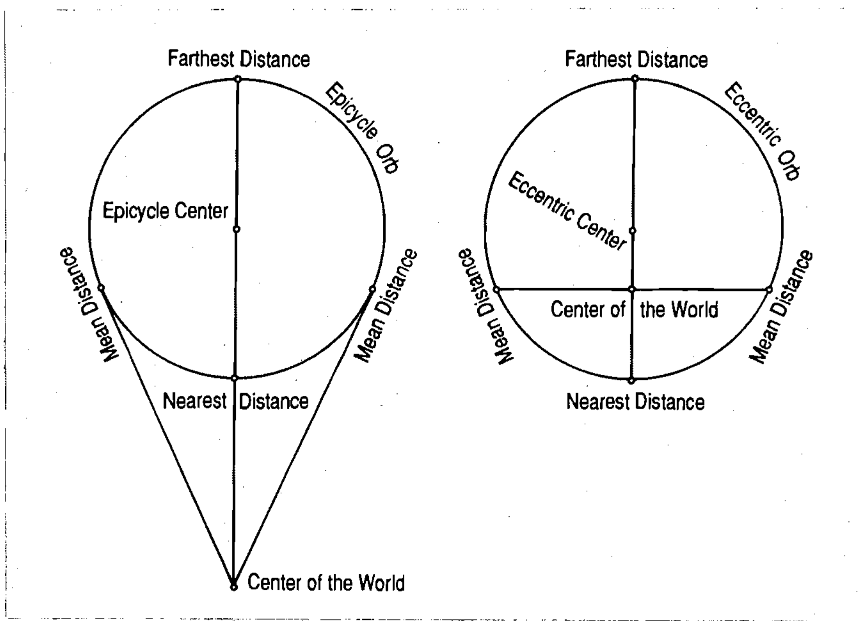
[١] إذا اختلفت حركة فلكية عندنا وجب أن نطلب لها أصلاً تتشابه تلك الحركة بحسبه ويقتضي ذلك الأصل أيضاً اختلافها بالقياس إلينا ، فإنّ المختلفة لا تصدر عن الفلكيات .

[٢] فمن الأصول كون الحركة متشابهة حول نقطة خارجة عن مركز العالم الذي نحن بقربه . ولا يخلو من أن يكون المحيط الذي يتحرك عليه ذلك المتحرك – وليكن كوكباً مثلاً – حول تلك النقطة إما محيطاً بمركز العالم وإما غير محيط به ، والأول يسمى الخارج المركز والثاني يسمى التدوير .

[٣] والخارج المركز إذا فُرض وحده وفُرض الكوكب متحركاً عليه حول مركزه حركة بسيطة متشابهة صيّر الحركةَ بالقياس إلى مركز العالم وغيره من النقط التي هي غير ذلك المركز مختلفة . فتكون في القطعة التي هي أبعد منه بطيئة وفي القطعة التي هي أقرب سريعة . وذلك لأن القسي المتساوية المختلفة بالبعد والقرب يُرى البعيد منها أصغر من القريب . وإذا أُخرج خط يمرّ بمركزه وبمركز العالم أو بالنقطة المفروضة التي هي غيرهما مرّ بالبعد الأبعد وهو منتصف القطعة البعيدة وبالبعد الأقرب وهو منتصف القطعة القريبة . ثم إذا قام عليه عمود يمرّ بمركز العالم أو بتلك النقطة ووصل إلى المحيط في الجانبين مرّ بالبعدين الأوسطين ، وهما الفصل المشترك بين القطعتين وعندهما تكون الحركة متوسطة بين السرعة والبطء .

[٤] وأما التدوير فإذا فُرض وحده وتحرك الكوكب على محيطه كانت القسي المتساوية أيضاً مختلفة بالقياس إلى مركز العالم ، وكان الخط الواصل بين المركزين ماراً بالبعدين الأبعد والأقرب منه ،

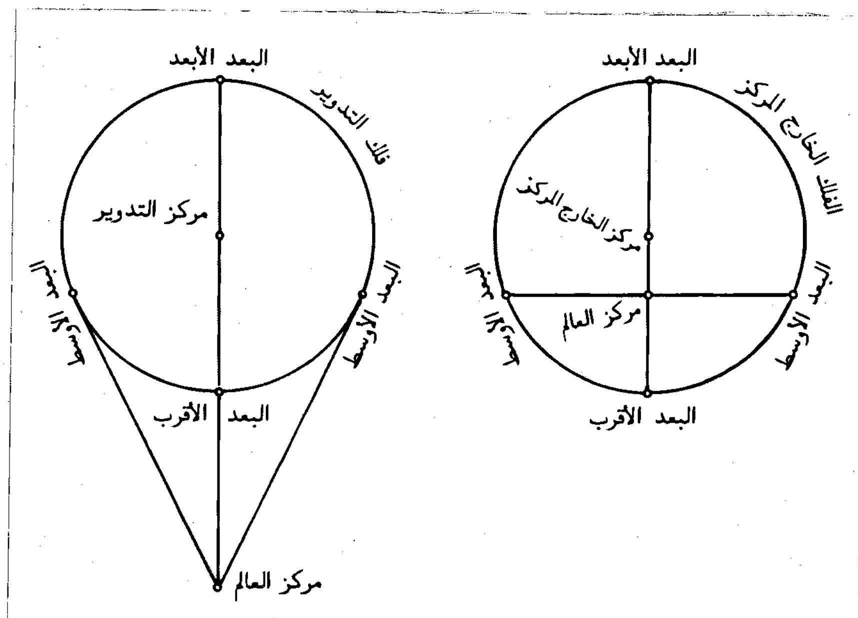
The two lines produced from the center of the World tangent to the epicycle on both sides separate the far and near segments. [As distinct from the eccentric], however, the planet is observed to retrograde in one segment from the direction in which it had been proceeding in the other until it reaches the starting point of its motion. [Also] it does not traverse by that motion all parts of the orb that surrounds the center of the World. This is their illustration:



[Fig. T1]

[5] Now if an epicycle is taken to be on a concentric orb that is its deferent such that the ratio of the deferent's radius to the radius of the epicycle is the same as the ratio of the eccentric radius to the eccentricity, and if the motion of the deferent is made the same as the motion of the eccentric and in the same direction so that they complete their revolutions simultaneously (with the epicycle center then also moving with this same motion), and if the epicycle is also made to move with a motion the same as the previous two in such a way that it is in the direction opposite the motion of the deferent in the far segment and in the same direction in the near segment, then the motion of the planet in the far segment is observed to be in the amount of the excess of the motion of the deferent over

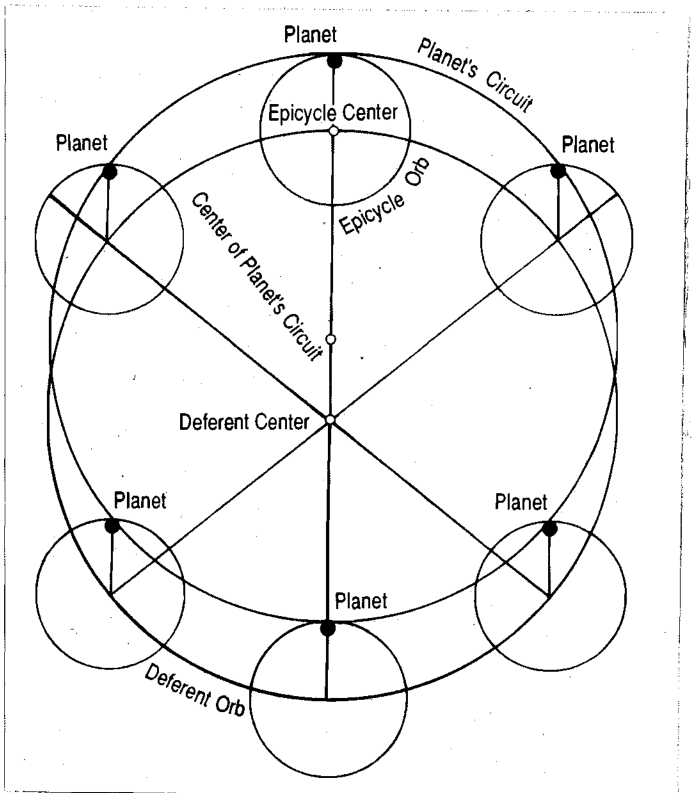
والخطّان الخارجان من مركز العالم المماسّان للتدوير من جانبيه يفصلان بين القطعتين البعيدة والقريبة ؛ إلا أنّ الكوكب يُرى في إحدى القطعتين راجعاً عن السمت الذي يقصده في القطعة الأخرى إلى أن يصل إلى المبدأ الذي تحرك منه . ولا يقطع أجزاء الفلك المحيط بمركز العالم جميعاً بتلك الحركة . وهذه صورتهما :



[**شكل ١**]

[٥] أما إن فُرض التدوير على فلك آخر حامل له موافق المركز على أنّ نسبة نصف قطر الحامل إلى نصف قطر التدوير كنسبة نصف قطر الخارج المركز إلى ما بين المركزين ، وجُعلت حركة الحامل شبيهة بحركة الخارج المركز وفي جهته بحيث يتمّمان الدورتين معاً فيتحرك مركز التدوير بتلك الحركة ، وجُعل التدوير متحركاً أيضاً بحركة شبيهة بهما على وجه تكون في القطعة البعيدة إلى خلاف جهة حركة الحامل وفي القطعة القريبة إلى جهتها ، رُئيت حركة الكوكب في القطعة البعيدة بقدر فضل حركة الحامل على

that of the epicycle and in the near segment to be their sum. Thus the apparent motion is exactly the same as that observed in the previously mentioned eccentric model, there being no difference whatsoever. The planet will describe by this compound motion an eccentric circuit that is the same as with the eccentric orb. This is its illustration:

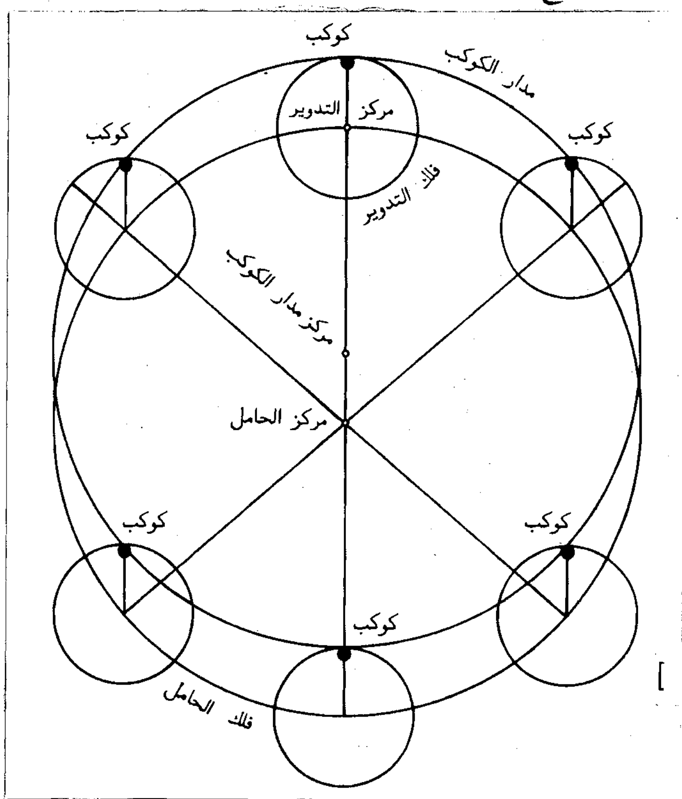


[Fig. T2]

[6] The difference between the two models in this case is twofold: first, the eccentric model is realized by means of a single motion, whereas the epicyclic model is realized with two motions; second, the epicycle entails a circuit that is eccentric, while the eccentric does not entail an epicycle. Therefore Ptolemy considered the eccentric in this case to be simpler than the epicycle.

[7] If the epicycle is assumed to move in such a way that in the far segment it is in the direction of the motion of the deferent, the faster motion will be attained in that segment and the slower in the near segment, which is the opposite of what occurred in the first case. However, the period

حركة التدوير وفي القطعة القريبة بقدر مجموعهما ، فصارت الحركة المرئية مثل ما يُرى في أصل الخارج المركز المذكور بعينه من غير تفاوت أصلاً . ويفعل الكوكب بحركته المركبة مداراً خارجَ المركز شبيهاً بالفلك الخارج المركز . وهذه صورته :



[شكل ٢]

[٦] والفرق بين الأصلين في هذا الموضع بشيئين : أحدهما أنّ أصل الخارج المركز يتم بحركة واحدة وأصل التدوير يتم بحركتين، والثاني أنّ التدوير يستلزم مداراً خارج المركز والخارج المركز لا يستلزم تدويراً . فلذلك حكم بطلميوس في هذا الموضع بأنّ الخارج المركز أبسط من التدوير .

[٧] وإن فُرض التدوير متحركاً على وجه يكون في القطعة البعيدة إلى جهة حركة الحامل حصلت السرعة في تلك القطعة والبطء في القطعة القريبة بخلاف ما كان في الأول ؛ إلا أنّ زمان

of the faster motion will be longer than the period of the slower motion in this figure, whereas previously it was shorter. This is because the far segment is larger than the near one since the dividing [line] between them cannot pass through the center; hence, it will not bisect the epicycle but rather divide it into two unequal segments, the smaller of which is the one nearest the center of the deferent.

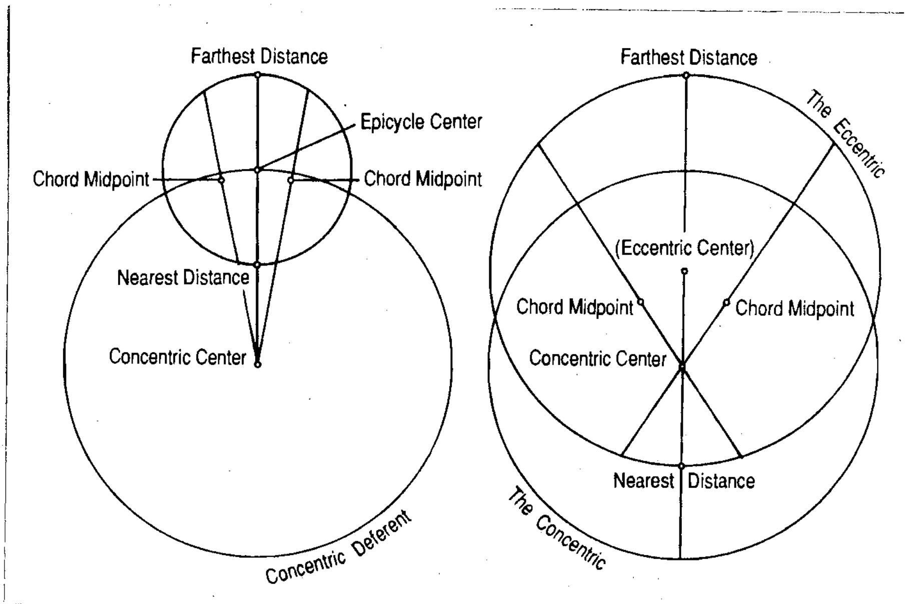
[8] Related to this discussion is the following: let us assume the eccentric also to have a concentric mover; / let us make the ratio of the line connecting the concentric center and the eccentric perigee to the eccentric radius the same as the ratio of the line connecting the concentric center and the epicyclic perigee to the radius of the epicycle; / and, let us make the two concentrics move, say, sequentially with equal motion, the eccentric counter-sequentially, and the epicycle in such a way that at its farthest distance it moves sequentially (these latter two motions likewise being assumed equal). Then the ratio of the motion of the eccentric or the epicycle to the motion of its [respective] concentric must either be: [a] less than the ratio of the line joining the center of the concentric and the nearest distance on either [the eccentric or the epicycle] to the radius of the eccentric or epicycle, respectively; [b] equal to it; or [c] greater than that [ratio]. If it is less, the planet on account of the two motions will only undergo a faster motion in the far segment and a slower motion in the near segment. For the eccentric this is because the amount reduced from the motion of the concentric in the far segment on account of the motion of the eccentric is less than the amount reduced from it in the near segment since the former arcs appear smaller. In the case of the epicycle, this is because the motion in the far segment is the sum of the two motions, while in the near one it is the excess of the motion of the concentric over the motion of the epicycle. If the ratio is equal to it, the planet will become stationary at the midpoint of the period of the slower motion, this occurring when it is at the nearest distance, which is located on the above-mentioned line. The planet, however, does not undergo retrogradation. If the ratio is greater, the planet will undergo retrograde motion in the near segment between two stations.

/6–8/ β = let us make the ratio of the eccentric radius to the [distance] between the two centers the same as the ratio of the [epicycle's] deferent radius to the radius of the epicycle] α, M.

السرعة يكون في هذه الصورة أطول من زمان البطء وهناك كان أقصر . وذلك لأنّ القطعة البعيدة تكون أكبر من القريبة ، فإنّ الفاصل بينهما لا يمكن أن يمرّ بالمركز فهو لا ينصّف التدوير بل يقطعه بمختلفين[!] أصغرهما التي تلي مركز الحامل .

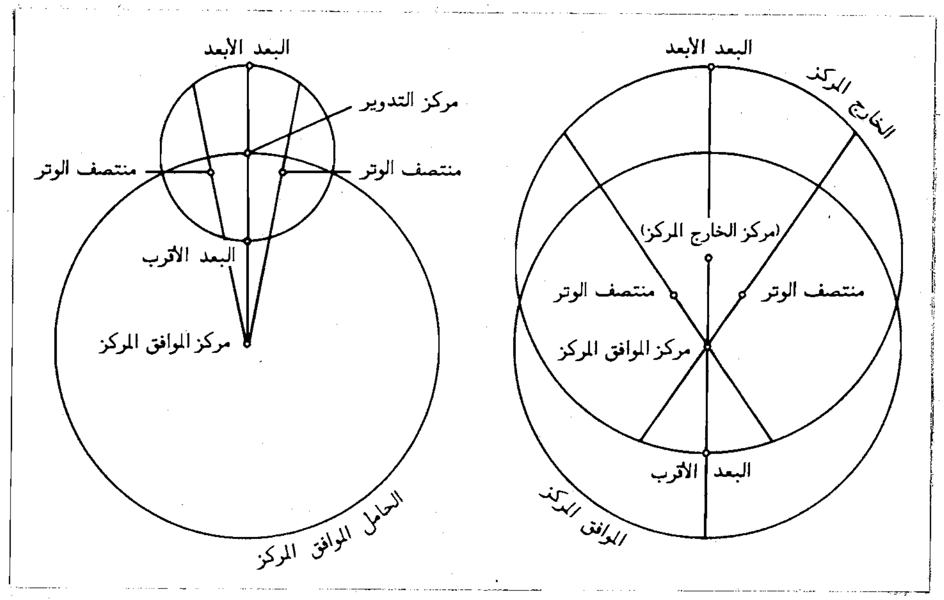
[٨] ومما يتّصل بهذا البحث أنّا إذا فرضنا لخارج المركز أيضاً محركاً موافق المركز ، / وجعلنا نسبة الخط الواصل بين مركز الموافق وحضيض الخارج إلى نصف قطر الخارج كنسبة الخط الواصل بين مركز الموافق وحضيض التدوير إلى نصف قطر التدوير / ، وجعلنا الموافقي المركز متحركين إلى التوالي مثلاً حركتين متشابهتين والخارجَ المركزِ إلى خلافه والتدويرَ على وجه يكون في بعده الأبعد إلى التوالي وحركتاهما أيضاً متشابهتان ، فنسبة حركة الخارج المركز أو التدوير إلى حركة موافقيهما لا يخلو من أن تكون إما أصغر من نسبة الخط الواصل بين مركز الموافق وبين البعد الأقرب من كل واحد منهما إلى نصف قطر الخارج المركز أو التدوير كل إلى صاحبه ، وإما مساوية لها ، وإما أكبر منها . فإن كانت أصغر فلا يحدث للكوكب بسبب الحركتين إلا السرعة في القطعة البعيدة والبطء في القطعة القريبة : أما في الخارج المركز فلأنّ ما ينقص في القطعة البعيدة بسبب حركة الخارج المركز من حركة الموافق المركز يكون أقل مما ينقص في القطعة القريبة لكون تلك القسي أصغر في الرؤية ، وأما في التدوير فلأنّ الحركة في القطعة البعيدة مجموع الحركتين وفي القريبة فضل حركة الموافق على حركة التدوير . وإن كانت مساوية حدث للكوكب في منتصف زمان البطء وقوف وهو عند كونه في البعد الأقرب على الخط المذكور ولا يكون له رجوع . وإن كانت أكبر حدث للكوكب رجوع في القطعة القريبة بين وقوفين .

/6–8/ ...] β = وجعلنا نسبة نصف قطر الخارج المركز إلى ما بين المركزين كنسبة نصف قطر الحامل إلى نصف قطر التدوير] α , M .



[Fig. T3]

[9] Let two lines be produced from the two concentric centers on both sides of the above-mentioned line, *i.e.* that joining the concentric center and the nearest distance for each of the two orbs, to the circumferences of the eccentric and the epicycle on both sides in such a way that the ratio of the motion of the eccentric or the epicycle to the motion of its respective concentric is equal to the ratio of that part of each of those two lines lying between the concentric center and the circumference of the eccentric or epicycle that is on the nearer side to half the chord, likewise from that line, that divides each of the orbs, respectively, into two segments. This is possible for this type of eccentric or epicycle [in which the ratio is greater] but not for the first two cases. The planet will thereby, after a gradual slowing down, become stationary when it reaches the first of the two lines in the near segment. From there until it reaches the second line, it will retrograde, gradually going from a slower to a faster speed whose maximum occurs at the nearest distance and then slowing until it reaches the second line whereupon it once more becomes stationary. After that, its motion becomes



[**شكل ٣**]

[٩] وليخرج خطّان عن مركزي الموافق عن جنبتي الخط المذكور ـ أعني الواصل بين مركز الموافق وبين البعد الأقرب في كل واحد من الفلكين ـ إلى محيطي الخارج المركز والتدوير في الجانبين بحيث تكون نسبة حركة الخارج المركز أو التدوير إلى حركة الموافقين ، كل إلى صاحبه ، مساوية لنسبة ما وقع من كل واحد من ذينك الخطّين بين مركز الموافق ومحيط الخارج المركز أو التدوير من الجانب الأقرب إلى نصف الوتر الفاصل لكل واحد من الفلكين إلى قطعتين أيضاً من ذلك الخط ، كل إلى صاحبه . وذلك يكون في مثل هذا الخارج المركز والتدوير ممكن[!] دون الأولين . فيكون الكوكب عند وصوله إلى أول الخطّين في القطعة القريبة واقفاً بعد بطء متدرج إلى الوقوف ، ومنه إلى وصوله إلى الخط الثاني راجعاً رجوعاً متدرجاً من بطء إلى سرعة غايته في البعد الأقرب ، ثم منها إلى بطء ينتهي عند الخط الثاني ، وعند وصوله إلى الخط الثاني واقفاً وقوفاً ثانياً ، وبعد ذلك يستقيم

direct as it gradually goes from being stationary to its fastest speed. The two mean speeds between the slower and faster speeds occur at the two mean distances. This speed / in the case of the epicycle / is that of the motion of the concentric alone. If the concentric and eccentric motions are set opposite the directions we have assumed previously, while the deferent's motion is as it was before and the motion of the epicycle is such that at the farthest distance it is counter-sequential, and all the rest of the conditions remain unchanged, then the situations of the near and far segments become reversed.

[10] These then are models and rules that should be known. We have only stated them here; their geometric proofs are given in the *Almagest*. Restricting oneself to circles is sufficient in the entirety of this science for whoever studies the proofs. However, one who attempts to understand the principles of the motions must know the configuration of the bodies, which move with these motions in such a way that these motions manifest themselves on their equators. He should conceive of the **concentric** and the **deferent** each as an orb bounded by two parallel surfaces with the same center and the eccentric as an orb within the thickness of the concentric and bounded by two parallel surfaces with the same center, which is eccentric to the concentric center by an amount dictated by the anomaly. The convex of the two surfaces is tangent to the convex surface of the concentric at a single point that is the farthest point on it from the center of the concentric. Its concave surface is tangent to the concave surface of the concentric at a single point that is opposite the first and is the nearest point on it to the [concentric center]. Its thickness is such that it is large enough to include in it an epicycle or planet so that the latter's convex surface is tangent to the [eccentric's] two surfaces at two points. Its **[inner] equator** is the circuit of the epicycle's center or the planet's center. The **[inner] equator of the concentric** is a circle whose center is the concentric center and is equal to the [inner] equator of the eccentric, intersecting it in two points. One group makes it a circle that is tangent to the eccentric's [inner] equator at a point facing the farthest distance. The epicycle [should be conceived of] as a sphere in the thickness of its deferent with its convex surface tangent to the [deferent's] two surfaces at two points that are the farthest and nearest points on it

/3/ in the case of the epicycle] $\beta = -\alpha$, –M.

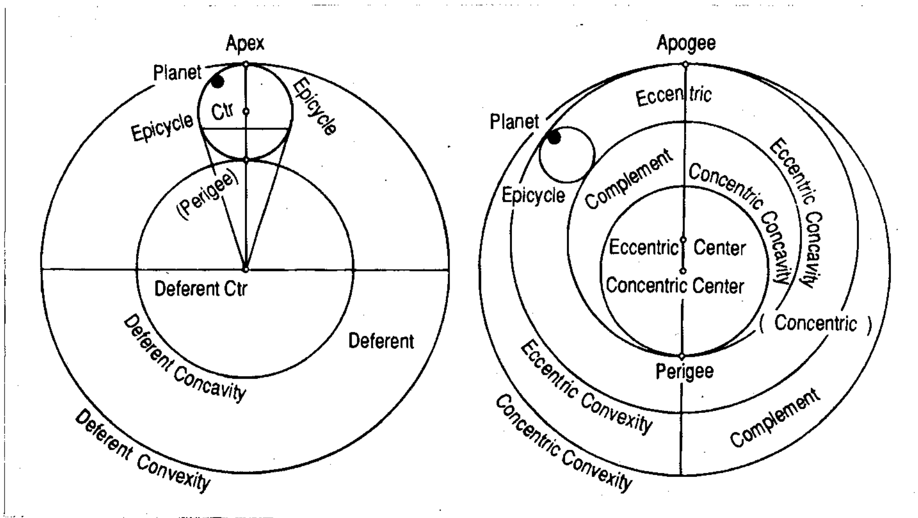
متدرجاً من وقوف إلى سرعة سير . ويكون السيران المتوسطان بين البطء والسرعة عند البعدين الأوسطين ، وذلك السير هو حركة الموافق وحدها / في التدوير / . وإن جُعل حركتا الموافق المركز والخارج المركز مخالفتين في الجهة لِما فرضنا ، وحركة الحامل كما / كان / ، لكنّ حركة التدوير على وجه تكون في البعد الأبعد إلى خلاف التوالي ، وسائر الشروط بحالها ، تبادلت حالتا القطعتين القريبتين والبعيدتين .

[١٠] فهذه أصول وقوانين لا بدّ من معرفتها أوردناها ههنا على سبيل الحكاية وبراهينها مذكورة بالخطوط في المجسطي . والاقتصار على الدوائر كافٍ للناظر في البراهين في جميع هذا العلم . أما لمن يحاول تصور مبادئ الحركات فلا بدّ من معرفة هيئة الأجسام المتحركة بتلك الحركات على وجه تظهر تلك الحركات في مناطقها . وعليه أن يتصور كلاً من الموافق المركز والحامل فلكاً يحيط به سطحان متوازيان مركزاهما واحد ؛ والخارجَ المركز فلكاً في ثخن الموافق المركز يحيط به سطحان متوازيان مركزاهما واحد خارج عن مركز الموافق بقدر ما يوجبه الاختلاف ، والمحدّب من سطحيه مماسّ لمحدّب الموافق على نقطة واحدة هي أبعد نقطة عليه من مركز الموافق ومقعّره مماسّ لمقعّر الموافق على نقطة واحدة مقابلة للأولى هي أقرب نقطة عليه منه ، وثخنه بحيث يسع ما يجب أن يكون فيه من تدوير أو كوكب بحيث يماسّ محدّبه سطحيه على نقطتين ، ومنطقته مدار مركز التدوير أو مركز الكوكب ، ومنطقة الموافق دائرة مركزها مركز الموافق مساوية لمنطقة الخارج مقاطعة إياها في نقطتين ، وقوم يجعلونها دائرة تماسّ منطقة الخارج على نقطة محاذية للبعد الأبعد ؛ وفلكَ التدوير كرة في ثخن حامله محدّبها مماسّ لسطحيه على نقطتين هما أبعد نقطة عليه وأقربها

/3/ … [β = –M , –α . /5/ … [β = كانت [M , α .

from the center of its deferent. The planet is embedded in it in such a way that its outer surface is tangent to the convex surface of the epicycle at a point. For neither [planet nor epicycle] is their concave surface taken into account. The **[inner] equator of [the epicycle]** is a circle that is the circuit of the planet's center. The **[inner] equator of the deferent** is a circle that is the circuit of the epicycle center. After removing the eccentric from the concentric, there are left two solid, curved bodies that are thick in the middle, becoming more narrow until the thickness disappears altogether at a point opposite the maximum thickness. The two bodies bound the eccentric in accordance with the alternating position of their two thicknesses. They are called the **complementary bodies**.

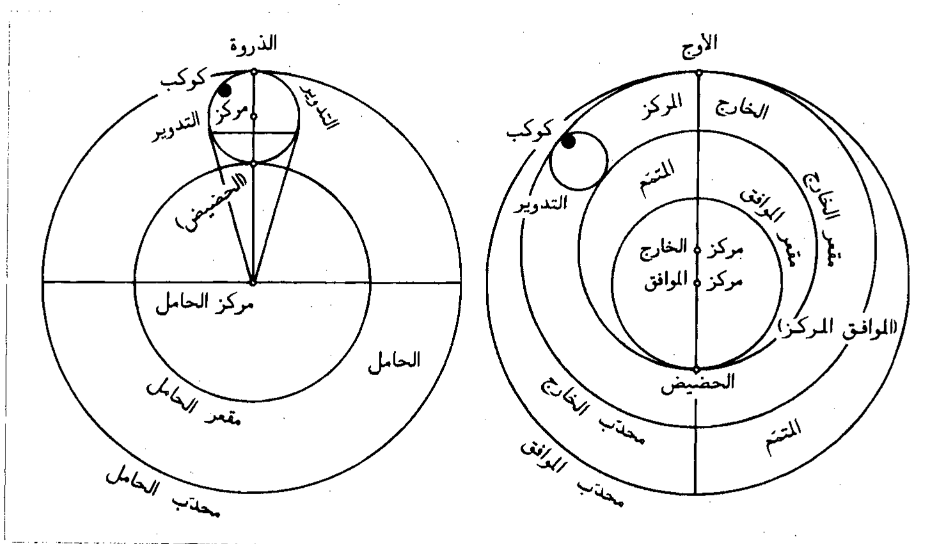
[11] The farthest distance for the eccentric is called the **apogee**; for the epicycle, it is called the **apex**. For both, the nearest [distance] is called the **perigee**. The eccentric is sometimes referred to as the **orb of the apogee**. A moving object on either orb is [called] **descending** from the farthest distance to the nearest and **ascending** from the [nearest] to the farthest. This is their illustration:



[Fig. T4]

من مركز حامله ، والكوكب مركوز فيه بحيث يماس سطحه الخارج محدّب التدوير على نقطة ولا يعتبر مقعرهما ، ومنطقته دائرة هي مدار مركز الكوكب ، ومنطقة الحامل دائرة هي مدار مركز التدوير . ويفضل من الموافق المركز بعد انفصال الخارج المركز منه جسمان مستديران ثخينان غليظا الوسط يستدق ذلك الغلظ إلى أن ينعدم عند نقطة مقابلة لغاية الغلظ يحيطان بالخارج المركز على تبادل وضع غلظيهما ، ويسميان المُتمِّمين .

[١١] والبعد الأبعد في الخارج المركز يسمى الأوج وفي التدوير يسمى الذروة ، والأقرب فيهما يسمى الحضيض . وقد يسمى الخارج المركز بفلك الأوج . والمتحرك في الفلكين من البعد الأبعد إلى الأقرب هابط ، ومنه إلى الأبعد صاعد . وهذه صورتهما :



[شكل ٤]

CHAPTER SIX

On the Orbs and Motions of the Sun

[1] When the situation of the sun was considered, its motion was found to vary in different parts of the ecliptic equator: it was slower in a certain half, faster in the other. The center of its body was found to adhere always to the ecliptic equator, deviating neither north nor south of it; for this reason, the ecliptic is sometimes known as the solar circuit. By careful examination of solar eclipses, the solar body was found during the middle of the period of slower motion to be somewhat smaller than during the middle of the period of faster motion. It was inferred from this that during its slower motion the sun was farther away from the center of the World, while during the faster motion it was nearer. The moderns have found the midpoints of its slower and its faster motions—and, indeed, every position whatever the circumstance—to have a movement through the parts of the ecliptic equator in the sequence [of the signs] that is approximately the movement of the fixed stars due to the second motion. This was something Ptolemy did not find.

[2] The above required that there be established for the sun either: [a] an eccentric whose equator would be in the plane of the ecliptic equator. The sun would be within its thickness, and the [eccentric] would move, thus moving the sun, in the sequence of the signs at the rate of the sun's mean motion (the motion of the apogee being subtracted from the mean motion among those who propound it). This is called the motion of its center; or [b] an epicycle and deferent whose equators are as in the previous case [in the plane of the ecliptic]. The sun would be on the epicycle, which moves it in the upper half countersequentially at the rate of the motion of the sun's center. The deferent moves the epicycle sequentially, likewise at the rate of that motion so that the two revolutions will be completed together. The center of the sun will undergo exactly the same motion as produced by the eccentric. This motion will be slower in the apogeal half, faster in the perigeal half. Ptolemy chose the former model—there being no necessity to do so—because it is simpler.

[3] For the eccentric model, there must be established a concentric orb in whose thickness occurs the eccentric and which exceeds the eccentric by its two complementary bodies. It is called the **pareclipic orb** since its center, equator, and two poles correspond to those of the [ecliptic orb].

الفصل السادس
في أفلاك الشمس وحركاتها

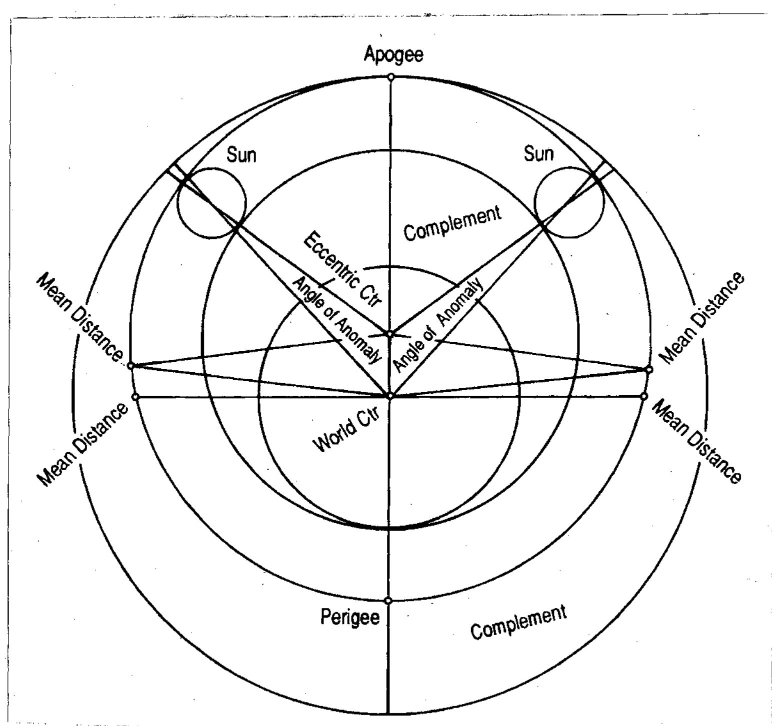
[١] لمّا تؤمّل في أحوال الشمس وُجدت حركتها مختلفة في أجزاء منطقة البروج بأن كانت بطيئة في نصف بعينه ، سريعة في النصف الآخر . ووُجد مركز جرمها دائماً ملازماً لمنطقة البروج غير مائل عنها إلى الشمال ولا إلى الجنوب ، ولذلك ربّما تعرّف بمدار الشمس . ووُجد بالنظر الدقيق في الكسوفات جرمها في أواسط زمان البطء أصغر قليلاً منه في أواسط زمان السرعة . فاستدلوا من ذلك على كونها في البطء أبعد من مركز العالم وفي السرعة أقرب . والمتأخرون وجدوا لمنتصفي بطئها وسرعتها — بل لكل موضع حال من أحوالها — انتقالاً في أجزاء منطقة البروج على التوالي قريباً من انتقالات الثوابت بالحركة الثانية . وبطلميوس لم يجد ذلك .

[٢] فاقتضى ذلك أن يثبت لها إما خارج مركز ، منطقته في سطح منطقة البروج ، تكون الشمس في ثخنه وهي[!] تتحرك وتحرك الشمس على توالي البروج بقدر حركة وسط الشمس إذا نقص منها حركة أوجها عند من يقول بها ، وتسمى حركة مركزها ؛ وإما تدوير وحامل ، منطقتاهما كذلك ، تكون الشمس على التدوير وهو يحرّكها في النصف الأعلى إلى خلاف التوالي بقدر حركة مركز الشمس ، والحامل يحرك التدوير إلى التوالي أيضاً بقدر تلك الحركة لتتم الدورتان معاً . وتحدث لمركز الشمس حركة كما أحدثها الخارج المركز بعينها ، وتكون تلك الحركة في النصف الأوجي بطيئة وفي النصف الحضيضي سريعة . وبطلميوس اختار الأول من غير ضرورة لكونه أبسط .

[٣] ويلزم على أصل الخارج المركز إثبات فلك موافق المركز يكون الخارج المركز في ثخنه ويفضل عليه بمتمّميه . ويسمى الفلك المُمَثّل بفلك البروج لكونه بالمركز والمنطقة والقطبين موافقاً له ، وهو

According to the moderns, it has the motion of the fixed stars and moves the apogee and perigee. For the epicyclic model, the eighth orb suffices for the moving of the apogee and the perigee since it moves everything below it; thus the deferent is the parecliptic. Since the sun is always in the plane of the eccentric or epicyclic equator, which are themselves in the plane of the parecliptic, [the sun] does not have any latitude. We have set forth the illustration of the sun's two orbs according to the eccentric model as was Ptolemy's preference.

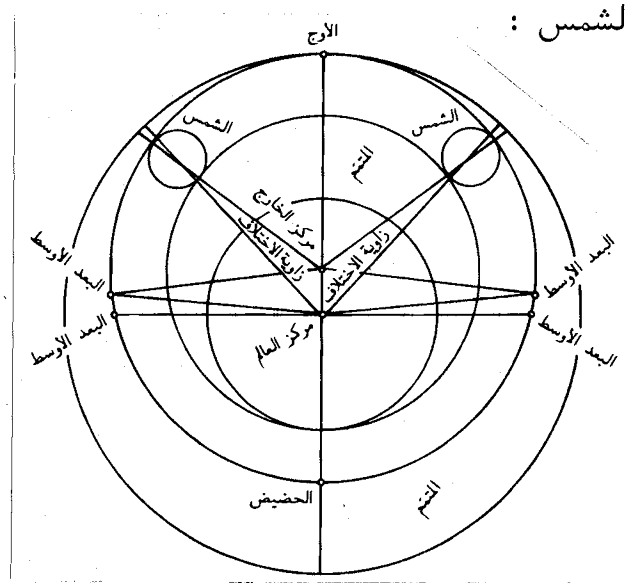
[4] The sun has a single **anomaly** that is in the amount by which the sun's observed motion differs from its mean motion, it being an angle that is formed at the center of the sun from two lines produced from the centers of its two orbs to it. This angle attains its greatest possible value at the two mean distances and disappears at the two other distances; it depends upon the amount of the eccentricity. According to Ptolemy, it is 2;30; according to more recent observational [astronomers], it is about 2;05 (the radius of the eccentric being 60). The position of the apogee, according to Ptolemy, is 24½° ahead of the summer solstice point. According to the moderns, the value varies, as they have stated in their *zījes*, depending on the date. This is the illustration of the orbs of the sun:



[Fig. T5]

يتحرك حركة الثوابت فيحرك الأوج والحضيض وذلك عند المتأخرين . وأما على أصل التدوير فالفلك الثامن كافٍ في تحريك الأوج والحضيض إذ هو محرك لجميع ما دونه فالحامل هو المثل . ولكون الشمس دائماً في سطح منطقة الخارج أو التدوير ــ وهما في سطح المثل ــ لا يكون لها عرض . ونحن أوردنا صورة / فلكيها / على أصل الخارج كما مال إليه بطلميوس .

[٤] ويلزم للشمس اختلاف واحد / بقدره / تخالف حركتها المرئية حركتها الوسطى ، وهو زاوية تحدث عند مركز الشمس من خطّين يخرجان من مركزي / فلكيها / إليه . ويصير أعظم ما يمكن في البعدين الأوسطين وينعدم عند البعدين الآخرين ، ويكون بقدر ما بين المركزين وهو عند بطلميوس بَلَ وعند أصحاب الأرصاد من المتأخرين قريب من بَهَ على أن يكون نصف قطر الخارج المركز ستين . وموضع الأوج عند بطلميوس متقدم على نقطة الانقلاب الصيفية بأربعة وعشرين جزءاً ونصف ، وعند المتأخرين مختلف فيه كما ذكروه في زيجاتها بقيد التأريخ . وهذه صورة أفلاك الشمس :



[شكل ٥]

/5/ ... [فلكيه = β [M , α . /7/ ... [بقدر ما = β , M [α . /9/ ... [فلكيه = β [M , α .

[5] One group has made the **mean distance** so that the two produced lines from the two centers to the sun are equal. This [mean] distance is based on distance, whereas what we have previously stated is based on motion. The above having been determined, it should be known that:

[a] the **solar apogee** designates the [arc] measured sequentially occurring on the parecliptic between the first of Aries and the apogee point;

[b] the **solar center** designates the [arc] measured sequentially occurring on the eccentric between the apogee and the center of the sun;

[c] the **mean** designates the sum of [a] and [b];

[d] the **true position** [*taqwīm*] designates the [arc] occurring on the parecliptic between the first of Aries and the endpoint of the line produced from the center of the World to the sun's body. This will be less than the mean by the amount of the anomaly as long as the sun is descending, greater than the mean when it is ascending.

[6] Thus the situation of the sun has been ordered by means of two orbs and two motions, and this is what we intended.

CHAPTER SEVEN

On the Orbs and Motions of the Moon

[1] The moon was found to move along a circuit different from the solar circuit that intersected the latter in two places facing one another. These [points] were not fixed, however, but shifted in the counter-sequence [of the signs]. Thus the moon is north of the ecliptic equator in half its circuit and south of it in the other half, the maximum distance in both directions being the same amount. Its motion on that circuit was found to be nonuniform, having anomalous speeds [that did not recur] in the exact same parts of the ecliptic but instead shifted, each anomaly returning not to its exact same value but rather, a short time after the completion of a lunar revolution, to what was comparable to it. The distance of the [moon] from the Earth was also found to vary; during the slower motion, it was sometimes nearer and at other times farther away, and likewise during the faster motion. It was found at mean conjunction and opposition to the sun, [this being] at a farther distance, to increase and decrease [in distance], it being slower with increasing, faster with decreasing [distance];

[٥] وقوم يجعلون البعد الأوسط حيث يتساوى الخطّان الخارجان من المركزين إليه ، وهذا بُعد بحسب المسافة وما ذكرناه أولاً هو بحسب الحركة . وإذا تقرر هذا فآعلم أنّ أوج الشمس يقال لِما يقع من الممثل بين أول الحمل ونقطة الأوج على التوالي ؛ ومركز الشمس لِما يقع من الخارج المركز بين الأوج ومركز الشمس على التوالي ؛ والوسط لمجموعهما ؛ والتقويم لِما يقع من الممثل بين أول الحمل وطرف الخط الخارج من مركز العالم إلى جرم الشمس ، وهو ناقص من الوسط بقدر الاختلاف ما دامت الشمس هابطة ، زائد عليه ما دامت صاعدة .

[٦] فإذن انتظم أمر الشمس بفلكين وحركتين ، وذلك ما أردناه .

الفصل السابع
في أفلاك القمر وحركاته

[١] وُجد القمرُ متحركاً على مدار غير مدار الشمس مقاطع إياه في موضعين متقابلين غير ثابتين بل منتقلين إلى خلاف التوالي ، فيكون القمر في نصف مداره شمالياً عن منطقة البروج وفي النصف الآخر جنوبياً عنها ، وغاية البعد في الجهتين بمقدار واحد ؛ وحركتُه على ذلك المدار غير متشابهة بل مختلفة بالبطء والسرعة في أجزاء لا بأعيانها من فلك البروج بل منتقلة ، عائدٌ كلُ اختلاف لا إلى مثله بعينه بل إلى ما يشبهه بعد تمام دور القمر بزمان قليل ؛ وبعدُه عن الأرض أيضاً مختلفاً اختلافاً يكون في البطء تارة قريباً وتارة بعيداً وكذلك في السرعة . ووُجد في مقارنة الشمس ومقابلتها الوُسطيين في بعد أبعد يزيد وينقص ، فيكون أبطأ كلما زاد وأسرع كلما نقص ، وتختلف مقادير جرمه في

/ similarly, / its body size varies during lunar and solar eclipses. At its quadrature to the sun, [this being] at a nearer distance, it increases and decreases as well. The lunar body was found to have different illuminated shapes according to its position from the sun. Its markings were found to be fixed.

[2] They therefore established four orbs and four simple motions for the [moon].

[3] The first orb is the parecliptic whose convex surface is contiguous with the concave surface of Mercury's parecliptic.

[4] The concave surface of the [moon's parecliptic] is contiguous with the convex surface of the second of its orbs, which is called the **inclined orb**. The concave surface of the inclined orb is contiguous with the sphere of fire from among the four elements. It is called inclined because its equator has a fixed inclination to the parecliptic equator whose maximum value as found by observation is five degrees. Its center is the center of the World.

[5] The third orb is an eccentric in the thickness of the inclined orb. Its equator is in the plane of the inclined equator.

[6] The fourth orb is an epicycle in the thickness of the eccentric, which is its deferent. The moon is embedded in the epicycle and adheres always to its equator that occurs in the plane of the eccentric equator.

[7] The equators of the parecliptic and inclined orbs intersect at two points facing one another called the **nodes** and the *jawzahr*. One of them, the northern crossing point or the **head**, is such that the moon when crossing it comes to be north. The other node is the southern crossing point or the **tail**.

[8] Turning to the motions, the first is the motion of the parecliptic with the motion of the nodes; it is three minutes plus a fraction daily in the countersequence [of the signs] about the center of the World. All the orbs of the moon move with this motion, and hence the head and tail shift as well; the motion, for this reason, is attributed to them. The motion of the fixed stars is indistinguishable from the other lunar motions but not because of what is [sometimes] said, namely that it is imperceptible since its ratio to these very fast motions is so small. For the small over a long period multiplies,

/1/ similarly] β = because of that] α, M.

الخسوفات والكسوفات / كذلك / ، وفي تربيعه للشمس في بعد أقرب يزيد وينقص أيضاً ؛ وجرمه مختلفَ الأشكال في النور بحسب أوضاعه من الشمس ؛ ومَحْوه ثابتاً .

[٢] فأثبتوا له أربعة أفلاك وأربع حركات بسيطة .

[٣] الفلك الأول هو المثل بفلك البروج محدّبه يماسّ مقعّر المثل لعطارد .

[٤] ومقعّره يماسّ محدّب الفلك الثاني من أفلاكه وهو المسمى بالفلك المائل ، ومقعّر المائل يماسّ كرة النار من العناصر الأربعة . وإنما سُمي مائلاً لكون منطقته مائلة عن منطقة المثل ميلاً ثابتاً ، غايته على ما وُجد بالرصد خمسة أجزاء . ومركزه مركز العالم .

[٥] والفلك الثالث فلك خارج المركز في ثخن المائل ، ومنطقته في سطح منطقة المائل .

[٦] والفلك الرابع فلك تدوير في ثخن الخارج المركز وهو حامله . والقمر مركوز في التدوير ملازم أبداً لمنطقته الكائنة في سطح منطقة الخارج المركز .

[٧] ومنطقتا المثل والمائل تتقاطعان على نقطتين متقابلتين تسميان العقدتين والجَوْزَهْر ، إحداهما التي إذا جاوزها القمر أخذ في الشمال هي المجاز الشمالي والرأس ، والأخرى هي المجاز الجنوبي والذنب .

[٨] وأما الحركات فالأولى حركة المثل بحركة الجوزهر ، وهي كل يوم ثلاث دقائق وكسر إلى خلاف التوالي حول مركز العالم . وبها تتحرك جميع أفلاك القمر ، فينتقل الرأس والذنب ولذلك تُنسَب إليهما . وأما حركة الثوابت فغير متميّزة عن غيرها في القمر ـ لا لِما قيل من أنها غير محسوسة لقلة نسبتها إلى هذه الحركات السريعة جداً ، فإنّ القليل في المدد الطويلة يتكثر

/1/ ... [β = لذلك] M , α .

and the models of the moon cannot tolerate much variation since the matter of the solar and lunar eclipses would thereby become disturbed. Rather, the motion [of the fixed stars] is indistinguishable from the motion of the nodes since their circumstances are the same in all respects. Hence the perceptible motion of the nodes is, in actuality, composed, being the excess of the motion of the nodes over the simple motion [of the fixed stars].

[9] The second motion is that of the inclined [orb] in the counter-sequence [of the signs], also about the center of the World, and it is 11°9′ daily. The eccentric is moved by this / motion, / which is called the **motion of the apogee** because it manifests itself there.

[10] The third motion is that of the eccentric in the sequence [of the signs], also about the center of the World, and it is 24°23′ daily. It is called the **motion of the center** because the epicycle center is moved by it this amount.

[11] Because the center of the epicycle is moved by the motions of the parecliptic and inclined [orbs] counter-sequentially, these motions being 11°12′, and sequentially by the above amount [*i.e.* 24°23′], its daily elongation from the apogee is this amount [24°23′] and from a fixed point on the ecliptic orb in the amount of the excess of the motion of the center over the sum of the first two motions, this being 13°11′. This is called the **mean motion of the moon**. The mean sun is always aligned with the center of the epicycle when the latter is at the apogee. Since [the mean sun] moves sequentially 0;59° daily, its elongation from the lunar apogee will be 12°11′, and its elongation from the epicycle center will by subtraction be the same. The sun then, after the epicycle center has departed from the apogee, will always be midway between the apogee and the center until the apogee is directly opposite the center at the [sun's] quadrature. The [center] will once more meet the [apogee] at the [sun's] opposition, will be opposite it at the other quadrature, and will then return to conjunction with the apogee. The motion of the center is therefore called the **double elongation**, *i.e.* the elongation of the epicycle center from the sun is doubled.

/8/ motion] $\beta = -\alpha$, –M.

وأصول القمر لا تحتمل كثير تفاوت لأنّ أمور الكسوفات والخسوفات تختل بذلك ـ بل لأنّ تلك الحركة لا تتميز عن حركة الجوزهر لاتّحاد موضوعيهما من جميع الوجوه . فإذن الحركة المحسوسة من الجوزهر مركبة في الحقيقة ، أعني أنها فضل حركة الجوزهر على تلك الحركة البسيطة .

[٩] والحركة الثانية حركة المائل إلى خلاف التوالي حول مركز العالم أيضاً ، كل يوم إحدى عشرة درجة وتسع دقائق . ويتحرك الخارج المركز بتلك / الحركة / وتسمى حركة الأوج لظهورها فيه .

[١٠] والثالثة حركة الخارج المركز إلى التوالي حول مركز العالم أيضاً ، كل يوم أربعاً وعشرين درجة وثلاثاً وعشرين دقيقة . وتسمى حركة المركز لانتقال مركز التدوير به[!] ذلك القدر .

[١١] ولكون مركز التدوير متحركاً بحركتي الممثل والمائل إلى خلاف التوالي ، وهما إحدى عشرة درجة واثنتا عشرة دقيقة ، وإلى التوالي هذا القدر ، يكون بعده عن الأوج كل يوم هذا القدر وعن النقطة الثابتة من فلك البروج بقدر فضل حركة المركز على مجموع الأوليين ، وهو ثلاث عشرة درجة وإحدى عشرة دقيقة . وتسمى هذه حركة وسط القمر . والشمس بوسطها تكون أبداً مع مركز التدوير عند كونه في الأوج ـ وهي تتحرك كل يوم تسعاً وخمسين دقيقة إلى التوالي ـ فيصير بعدها عن أوج القمر اثنتي عشرة درجة وإحدى عشرة دقيقة ، ويبقى بعدها عن مركز التدوير مثله . فتكون الشمس ، بعد مفارقة مركز التدوير الأوجَ ، متوسطة دائماً بين الأوج والمركز إلى أن يقابل الأوجُ المركزَ عند تربيعها ، ويلاقيه مرة أخرى عند استقبالها ، ويقابله في التربيع الآخر ، ويعود إلى الاجتماع مع الأوج . ولذلك تسمى حركةُ المركز البعدَ المضعَّف ، يعني بُعد مركز التدوير من الشمس مضعّفاً .

/8/ ...] β = M–, α– .

In this way, the center will be at the eccentric apogee at mean conjunction and opposition, and at its perigee at the quadratures.

[12] Because all the above motions are about the center of the World, they are all uniform with respect to us.

[13] The fourth motion is that of the epicyclic orb. The moon, due to its motion, moves 13°4′ per day such that it is counter-sequential in the upper half [of the epicycle]. This is called the [moon's] **proper motion**.

[14] Because the ratio of this last motion to the mean motion is less than the ratio of the line joining the center of the World and the epicyclic perigee to the [epicyclic] radius, the moon does not have a station or retrogradation; rather, / its motion / will be slower in the apex half [of the epicycle], faster in the perigeal half. At conjunction, opposition and the two quadratures, the moon will have a slower speed with increasing distance and a faster one with decreasing distance. And because the motion of the epicycle is less than the mean motion, the slower and faster motions will not [recur] in the exact same parts of the ecliptic orb but instead their positions will shift; the return to the same anomaly will occur after the return to the same part of the ecliptic orb. For this reason as well, an eccentric alone cannot substitute for this epicycle. Because the radius of the epicycle varies in size with respect to the center of the World due to the varying distance of [the epicycle] from it in the two orbs, the rates of the slowest and fastest speeds are not always the same but will also vary; thus the slowest speed will sometimes return to a less slow, sometimes to a more slow speed, and the same holds for the fastest speed as well as the other anomalous speeds.

[15] These then are the motions of the moon.

/10/ its motion] β, M = $-\alpha$.

فعلى هذا الوجه يكون المركز في الاجتماع والاستقبال الوَسطيّين في الأوج من خارج المركز ، وفي التربيعين في الحضيض منه .

[١٢] ولكون جميع هذه الحركات حول مركز العالم تكون الجميع عندنا متشابهة .

[١٣] والحركة الرابعة حركة فلك التدوير . ويتحرك القمر بحركته إلى غير التوالي في النصف الأعلى كل يوم ثلاث عشرة درجة وأربع دقائق . وتسمى حركته الخاصة .

[١٤] فلكون نسبة هذه الحركة إلى حركة الوسط أصغر من نسبة الخط الواصل بين مركز العالم وحضيض التدوير إلى نصف قطره لا يكون للقمر وقوف ولا رجوع ، بل تصير / حركته / بطيئة في نصف الذروة ، سريعة في نصف الحضيض . ويكون للقمر في الاجتماع والاستقبال والتربيعين بطءٌ مع زيادة بعد وسرعةٌ مع نقصانه . ولكون حركة التدوير أقل من حركة الوسط لا يكون البطء والسرعة في أجزاء بأعيانها من فلك البروج ، بل تنتقل مواضعها ، ويكون العود إلى اختلاف بعينه بعد العود إلى جزء بعينه من فلك البروج . ولا يقوم خارج مركز وحده بدل هذا التدوير لهذا السبب أيضاً . ولكون نصف قطر التدوير مختلف المقادير بالقياس إلى مركز العالم لاختلاف أبعاده منه في الفلكين تكون أقدار البطء والسرعة غير متشابهة بل مختلفة ، فيعود البطء تارة إلى بطء أقل وتارة إلى بطء أكثر ، وكذلك السرعة وغيرهما من الاختلافات .

[١٥] فهذه حركات القمر .

/10/ ...] M , β = α– .

[16] Turning now to the anomalies that inhere in the [moon] as a result of these motions, / the first is that anomaly effected by the radius of the epicycle at conjunctions and oppositions. It is an angle occurring at the center of the World from the production of two lines from it, one to the epicycle center and the other to the lunar body. / Its maximum is based upon the radius of the epicycle being at [the epicycle's] two mean distances whose size has been found by observation to be $5\frac{1}{4}$ parts, the radius of the inclined orb being 60 parts. The [anomaly] disappears at the apparent apex and perigee; it is subtractive from the mean as long as the moon is descending on the epicycle, additive when it is ascending. This is called the **independent equation**.

[17] The second anomaly is that resulting from an increase in the previous one when the epicycle is at a distance other than the farthest distance. Its maximum occurs when the epicycle is at the two quadratures, *i.e.* at the perigee, and it is $2\frac{2}{3}°$ for this [epicycle] radius if the first anomaly is at its maximum. When the [first anomaly] decreases, the [second anomaly] will decrease accordingly. It is additive when the first anomaly is additive, subtractive when it is subtractive. It is called the **anomaly of the nearest distance**.

[18] The moon has another anomaly whose maximum occurs when the center of the epicycle is at the sextiles or trines with respect to the sun. It is caused by the apex of the epicycle, which is the starting point of its proper motion, and its perigee, which is opposite the apex, not being aligned with either the eccentric center or the center of the World except when the epicycle center is at the apogee or perigee. At that time they will be aligned with them since the diameter passing through [the apex and epicyclic perigee] will coincide with the diameter passing through the apogee, the [eccentric] perigee, and the centers. At other times, they will always be aligned with a point that is in the direction of the [eccentric] perigee whose distance from the center of the World

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/2–4/ …] β, M = the first is that anomaly effected by the radius of the epicycle. It is an angle occurring at the center of the World from the production of two lines from it, one to the epicycle center and the other to the lunar body at conjunctions and oppositions] α.

[١٦] وأما الاختلافات التي تلزمه بسبب هذه الحركات / فالاختلاف الأول الذي بسبب نصف قطر التدوير في الاجتماعات والاستقبالات ، وهو زاوية تحدث على مركز العالم من خروج خطّين منه — أحدهما إلى مركز التدوير والآخر إلى جرم القمر / . وتكون غايته بحسب نصف قطر التدوير في البعدين الأوسطين منه ، وقد وُجد بالرصد مقداره خمسة أجزاء / وربعاً / على أنّ نصف قطر المائل ستون جزءاً . وينعدم في الذروة والحضيض المرئيين ، وهو ناقص من الوسط ما دام القمر هابطاً في التدوير ، زائد ما دام صاعداً . ويسمى التعديل المفرد .

[١٧] والاختلاف الثاني هو الذي يكون بسبب زيادة الاختلاف المذكور عند كون التدوير في بعد غير الأبعد . وتكون غايته عند كون التدوير في التربيعين — أعني في الحضيض — وهي لنصف القطر جزءان وثُلثا جزء ، وذلك إذا كان الاختلاف الأول في الغاية . ولما نقص منه يكون بحسب نقصانه ، ويكون زائداً مع زيادة الاختلاف الأول ، ناقصاً مع نقصانه . ويسمى اختلاف البعد الأقرب .

[١٨] وللقمر اختلاف آخر تكون غايته عند كون مركز التدوير على تسديس الشمس أو / تثليثها / . وسببه أنّ ذروة التدوير التي هي مبدأ حركته الخاصة وحضيضه المقابل لها لا يحاذيان مركز الخارج ولا مركز العالم إلا عند كون مركز التدوير في الأوج أو الحضيض ، فإنهما حينئذ يحاذيانهما لانطباق القطر المارّ بهما على القطر المارّ بالأوج والحضيض والمراكز . أما في غير ذلك الوقت فيحاذيان أبداً نقطة مما يلي الحضيض ، بُعدها عن مركز العالم

/2–4/ ...] β , M = فالاختلاف الأول الذي بسبب نصف قطر التدوير وهو زاوية تحدث على مركز العالم من خروج خطّين منه أحدهما إلى مركز التدوير والآخر إلى جرم القمر يكون في الاجتماعات والاستقبالات] α. /6/ ...] β = وربع] α , M. /17/ ...] β = تثليثه] α , M.

is the same as the distance of the eccentric center, which is in the direction of the apogee, from the [center of the World]. This point is called the **point of alignment**. Each of these distances, according to what has been found by the observational [astronomers], amounts to 10 parts and 19 minutes, the radius of the inclined orb being 60. Because of this alignment, the **mean apex**, from which is the beginning of the proper motion, will always differ from the **apparent apex** at which the first two anomalies disappear. The situation is the same for the two [epicyclic] perigees. Thus the moon will have an anomaly when it may be thought to have none, and it will not have one when it is thought to occur. The maximum of this anomaly is based upon the above-mentioned distance. It disappears when the center is at the [eccentric] apogee or perigee. It is additive when the center is descending and subtractive when it is ascending. This is called the **equation of the proper** [motion].

[19] In addition, the moon has another anomaly that is the difference between the distance from the nodes of its two positions on the parecliptic and inclined equators. This anomaly is taken into account if one wishes to convert from one position to the other.

[20] All the above are concerned with longitude.

[21] As for latitude, it should be clear from what has already been stated. The return of the moon in both directions is always to the maximum [latitude]. The moon is north from the head to the tail and south from the tail to the head. It is ascending from its maximum latitude in the south to its maximum in the north and descending in the other half.

[22] As regards the changes in the illuminated shapes of the lunar body according to its position from the sun, this will be forthcoming in a separate chapter.

[23] The variation in the parts of the lunar surface in receiving light, which is called the **lunar markings**, is due to an anomaly whose true character has yet to be ascertained. Most likely, there exist various bodies occurring with the moon in its epicycle that do not accept illumination equally, either because of a difference in type or because of a difference in position.

[24] This then is the situation of the moon.

[25] There is a difficulty that arises regarding the motion of the epicycle center on the circumference of the eccentric about the center of the World and also regarding the alignment of its diameter with a point other than the center of the deferent. To see this, [we note] that when the deferent moves the epicycle with a simple, uniform motion:

كبعد مركز الخارج مما يلي الأوج عنه ، تسمى تلك النقطة نقطة المحاذاة . ومقدار كل واحد من البعدين عشرة أجزاء وتسع عشرة دقيقة على أنّ نصف قطر المائل ستون بحسب ما وجده أهل الرصد. وبسبب هذه المحاذاة تخالف الذروة الوسطى التي منها مبدأ الحركة الخاصة أبداً الذروة المرئية التي عندها ينعدم الاختلافان الأولان وكذلك الحضيضان ، فيوجد للقمر اختلاف عندما يظن عدمه ويعدم اختلافه عندما يظن وجوده . وغاية هذا الاختلاف بحسب البعد المذكور ، وينعدم عند كون المركز في الأوج أو الحضيض .ويكون زائداً ما دام المركز هابطاً ، وناقصاً ما دام صاعداً . ويسمى تعديل الخاصة .

[١٩] وأيضاً له اختلاف آخر وهو التفاوت بين بعد موضعيه في منطقتي المثل والمائل عن العقدتين . ويعتبر ذلك إذا أريد تحويل أحدهما إلى الآخر .

[٢٠] وهذه الأمور كلها تتعلق بالطول .

[٢١] وأما العرض فقد تبيّن مما مرّ . ويكون عود القمر في الجهتين دائماً إلى غايته . فيكون شمالياً من الرأس إلى الذنب وجنوبياً من الذنب إلى الرأس ، وصاعداً من غاية عرضه في الجنوب إلى غايته في الشمال وهابطاً في النصف الآخر .

[٢٢] وأما اختلاف التشكّلات النورية في جرمه بحسب وضعه من الشمس فسيجيء في باب مفرد .

[٢٣] وأما اختلاف أجزاء سطحه في قبول النور المسمى بالمَحْو فلاختلاف فيه لم يوقف على حقيقته . والأشبه وجود أجرام مختلفة معه في تدويره غير قابلة للإنارة بالتساوي إما لاختلاف نوعي أو لاختلاف وضعي .

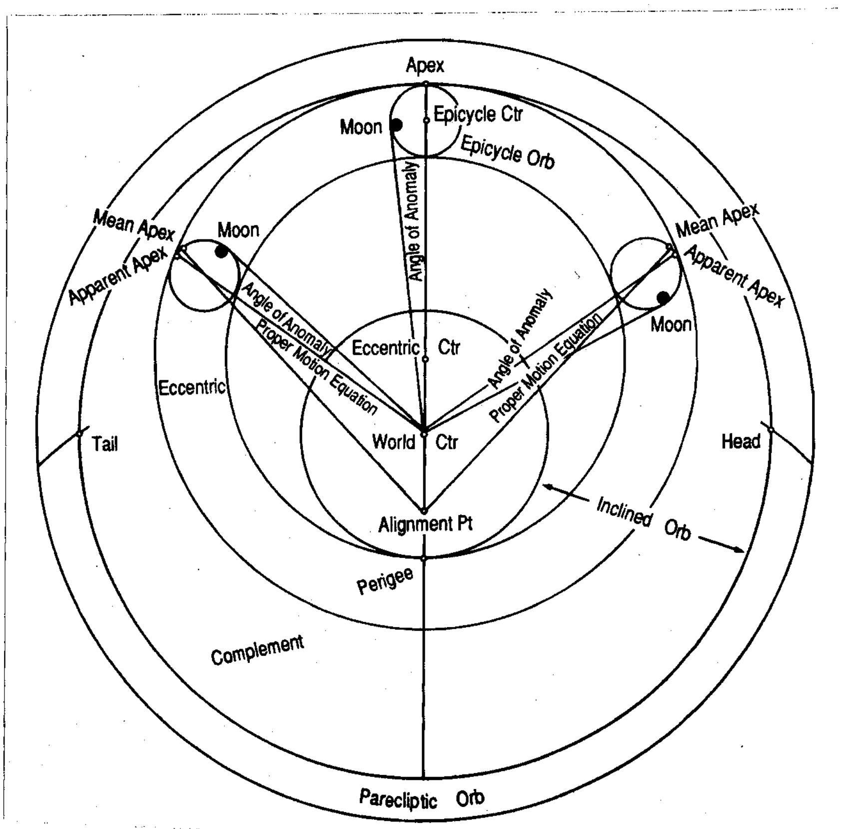
[٢٤] فهذه أحوال القمر .

[٢٥] وورد على حركة مركز التدوير في محيط الخارج المركز حول مركز العالم ومحاذاة قطره نقطة غير مركز الحامل إشكالٌ . وبيان ذلك أنّ الحامل إذا حرك التدوير حركةً بسيطة متشابهة

[a] the distances of the epicycle center from the [deferent] center must be equal under all circumstances; [b] the angles [formed] at the [deferent center] must be equal in equal times; and [c] the diameter passing through the apex and the [epicyclic] perigee must be aligned with the [deferent center] under all circumstances. If any one of these three were not to hold, this would be due to the motion being composed. In fact, we do find these things violated in the case of the moon. For while the distances of its epicycle center *are* equal with respect to the center of the eccentric, the equality of angles occurs at the center of the World and the alignment of the diameter is to the point of alignment. The practitioners of the profession have not yet explained in what way this motion is composed; in fact, they have not ventured any explanation of this at all. I shall present below what I have regarding this matter, God willing.

[26] The moon has another anomaly called parallax whose description will be forthcoming.

[27] This is the illustration of the orbs of the moon:

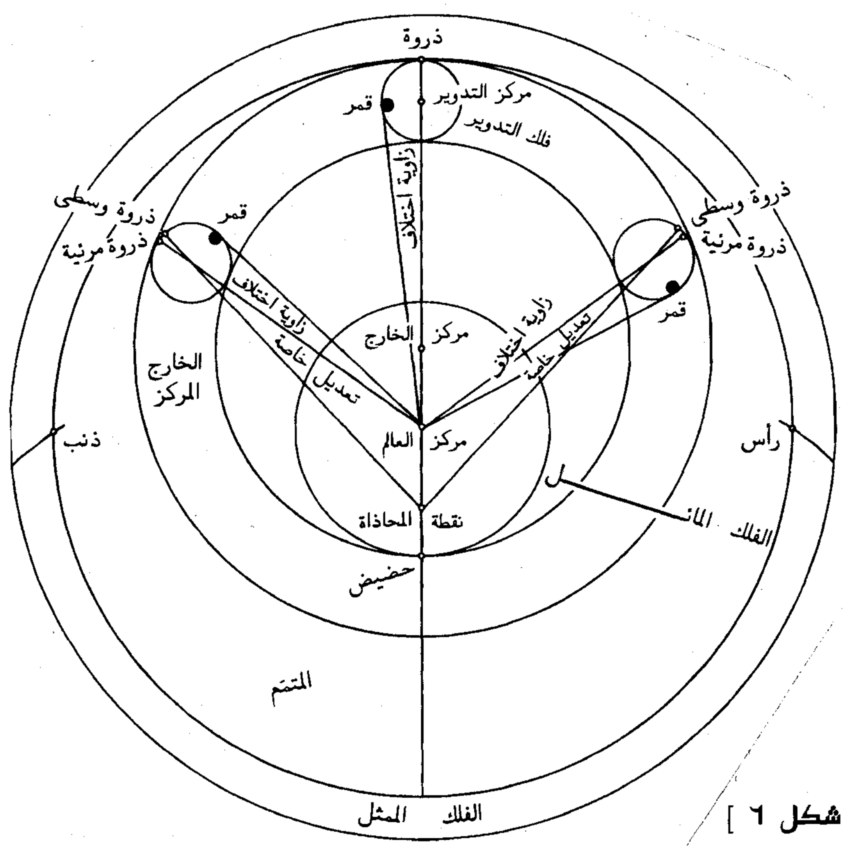


[Fig. T6]

وجب تساوي أبعاد مركز التدوير عن مركزه في جميع الأحوال ، وتساوي الزوايا حوله في الأزمنة المتساوية ، وكون القطر المارّ بالذروة والحضيض محاذياً له في جميع الأحوال . فإن اختلف بعض هذه الأمور الثلاثة فذلك يكون لتركُب في الحركة . ثم إنّا نجد هذه الأمور مختلفة في القمر ، فإنّ تساوي أبعاد مركز تدويره إنما يكون عند مركز الخارج المركز ، وتساوي الزوايا عند مركز العالم ، ومحاذاة القطر لنقطة المحاذاة . وأهل الصناعة لم يبيّنوا الوجه في كيفية هذا التركيب ، بل لم يتعرضوا لبيان شيء من ذلك . وسأورد ما عندي فيه إن شاء الله تعالى .

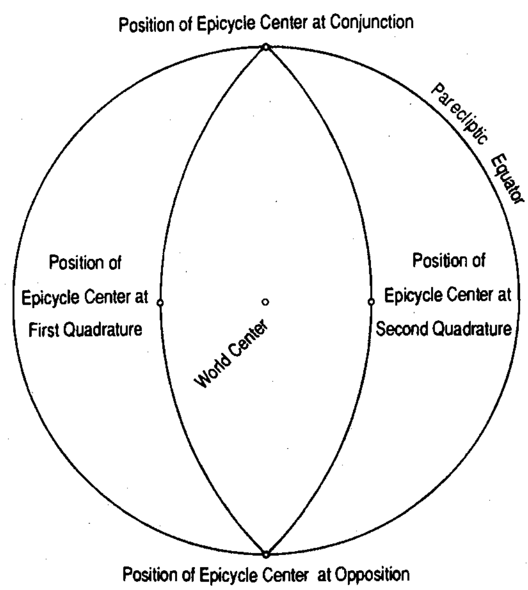
[٢٦] وللقمر اختلاف آخر يسمى اختلاف المنظر سيجيء وصفه .

[٢٧] وهذه صورة أفلاك القمر :



[شكل ٦]

[28] Those who limit themselves to circles make the parecliptic and the inclined equators intersect one another, the eccentric equator tangent to the inclined [equator] at the apogee point, and the equator of the epicycle such that its center is on the eccentric equator. If the motion of the sun is disregarded, the circuit of the center of the epicycle, which reaches each of the apogee and the perigee twice during its revolution, would be thus:

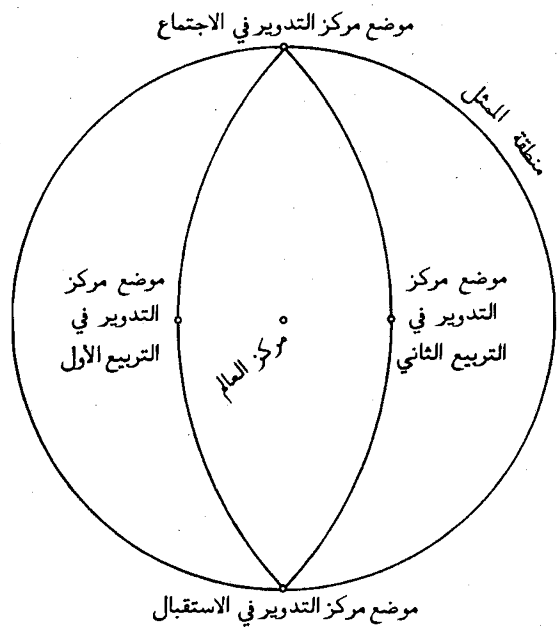


[Fig. T7]

[29] Let us end this chapter with the meanings of those expressions related to the moon:

- [a] the **nodal mean** is the [arc measured] counter-sequentially along the parecliptic between the first of Aries and the head;
- [b] its [*i.e.* the **node's**] **true position** [*taqwīm*] is the [arc measured] sequentially along it between them;
- [c] the **lunar apogee** is the [arc measured] sequentially along the inclined [equator] between the point aligned with the first of Aries, this point being assumed fixed, and the apogee point;
- [d] the **center of the [moon]** or its **double elongation** is the [arc measured] sequentially along the inclined equator between its apogee and the endpoint of the line produced from the center of the World through the epicycle center to the inclined equator;
- [e] its **mean** is the [arc measured] sequentially along the inclined equator between the point aligned with the first of Aries, this point being assumed fixed, and the endpoint of the line cited above [in [d]];

[٢٨] ومَن اقتصر على الدوائر أورد منطقتي المثل والمائل متقاطعتين ، ومنطقةَ الخارج المركز مماسّةً للمائل على نقطة الأوج ، ومنطقةَ التدوير على أنّ مركزها على منطقة الخارج المركز . ومدار مركز التدوير – لو لا حركة الشمس – في وصوله إلى الأوج في الدورة مرتين وإلى الحضيض مرتين يكون هكذا :



[شكل ٧]

[٢٩] ولنختم الفصل بمعاني ألفاظ تتعلق بالقمر فنقول : وسط الجوزهر ما بين أول الحمل ونقطة الرأس من الممثل على خلاف التوالي ؛ وتقويمه هو ما بينهما منه على التوالي ؛ وأوج القمر هو ما بين النقطة المحاذية لأول الحمل ، على أنها لا تتغير ، ونقطة الأوج من المائل على التوالي ؛ ومركزه أو بعده المضعّف هو ما بين أوجه وطرف الخط الخارج من مركز العالم إلى مركز التدوير ومنه إلى منطقة المائل من منطقة المائل على التوالي ؛ ووسطه ما بين النقطة المحاذية لأول الحمل ، على أنها لا تتغير ، وطرف الخط المذكور

[f] its **mean proper anomaly** [*khāṣṣa*] is the [arc measured] sequentially (as defined for the [epicycle]) along the equator of its epicycle between the mean apex and the center of the [lunar] body.

Among those whose motion is irregular:

[g] the [moon's] **apparent proper anomaly** is the [arc] along the equator of its epicycle between its apparent apex and the center of its body;

[h] its **true position** [*taqwīm*] is the [arc measured] sequentially along the parecliptic equator between the first of Aries and the point at which its latitude circle intersects the parecliptic;

[i] its **argument of latitude** [*ḥiṣṣat ᶜarḍihi*] is the [arc measured] sequentially along [the parecliptic equator] between the head and the intersection point cited above [in [h]].

CHAPTER EIGHT

The Orbs and Longitudinal Motions of Mercury

[1] Mercury was found not to move in longitude precisely along the ecliptic equator but rather in its vicinity, approaching it sometimes from the north and at other times from the south, the limits [north and south] not being exactly the same. It will increase in speed, thus moving ahead of the sun after having been in conjunction with it, and appear in the west. It then gradually slows until it becomes stationary. Thereupon it retrogrades—disappearing, coming into conjunction with the sun, and departing from it; the sun will then move ahead of it, [the planet now] appearing in the east. / It will thereafter become stationary and undergo a direct motion, / gradually increasing in speed until it disappears. It will then reach the sun and be in conjunction with it. Thus it is with [the sun] during the middle of its periods of direct and of retrograde motion. Its elongation ahead of or behind [the sun] does not exceed 27°. If one compares one retrogradation to another retrogradation, a direct motion to another direct motion, a slower motion to another slower motion, or a faster motion to another faster motion, one will not find them to be identical; instead, in some parts of the ecliptic they will be less in terms of both / extent / and time, while in other parts they will be greater. That part in which the slower motion is at its slowest and the time periods are at their shortest is not fixed but shifts with the movement of the fixed stars. The opposite of these circumstances does not occur

/15/ It will thereafter become stationary and undergo a direct motion] β, M = It will become stationary and thereafter undergo a direct motion] α. /20/ extent] β, M = revolution] α.

من منطقة المائل على التوالي ؛ وخاصته الوسطى ما بين ذروته الوسطى ومركز جرمه من منطقة تدويره على التوالي المفروض فيه . ومما تختلف حركته : خاصته المرئية وهي ما بين ذروته المرئية ومركز جرمه من منطقة تدويره ؛ وتقويمه وهو ما بين أول الحمل والنقطة التي تقاطع دائرة عرضه المثل من منطقة المثل على التوالي ؛ وحِصة عرضه / وهي / ما بين نقطة الرأس ونقطة التقاطع المذكورة منه على التوالي .

الفصل الثامن
في أفلاك عطارد وحركاته الطولية

[١] وُجد عطارد متحركاً في الطول لا على نفس منطقة البروج بل / حواليها ، يقرب منها تارة في شمالها وتارة في جنوبها / لا إلى حدّين بعينهما . وهو يسرع في سيره فيسبق الشمسَ بعد مقارنتها ويظهر مُغرَّباً ، ثم يأخذ في البطء متدرجاً إلى أن يقف ، ثم يرجع ويختفي ويقارن الشمسَ ويفارقها فتسبقه الشمسُ ويظهر مشرَّقاً ، / ثم يقف ويستقيم / ويتدرج إلى السرعة إلى أن يختفي ، ثم يُدرِك الشمسَ ويقارنها . فيكون معها في منتصف زماني استقامته ورجوعه ، ولا يبعد عنها من قدامها وخلفها أكثر من سبعة وعشرين جزءاً . وإذا قيس رجوعٌ إلى رجوع أو استقامة إلى استقامة أو بطء إلى بطء أو سرعة إلى سرعة لم توجد متشابهة بل كانت في بعض أجزاء البروج أقل / قدراً / وزماناً وفي بعضها أكثر . والجزء الذي يوجد البطء فيه أشد والزمان أقل لا يكون ثابتاً بل منتقلاً انتقال الثوابت . وأضداد تلك الأحوال ليست في

/6/ …] M , β = وهو] α . /11/ …] β = حواليه يقرب منه تارة في شماله وتارة في جنوبه] M , α . /15/ …] M , β = ويقف ثم يستقيم] α . /20/ …] M, β = دوراً] α .

directly facing that part [of the ecliptic] but at the two trines. Directly facing that, one finds a situation similar to what occurs at this part but not to that maximum extent.

[2] They therefore established for [Mercury] four orbs and four motions.

[3] The first orb is the parecliptic whose convex surface is contiguous with the concave surface of Venus's orb. Its concave surface is contiguous with the convex surface of the moon's parecliptic.

[4] The second orb is an eccentric and is called the **dirigent**. It is in the thickness of the parecliptic just as we have described an eccentric being in the thickness of a concentric. Its equator is not in the plane of the parecliptic equator but has an inclination to it that is not fixed; its description will be forthcoming. The apogee of the [dirigent] occurs at the place of the maximum inclination. The plane of its equator intersects the plane of the parecliptic equator at acute and obtuse angles. There thus occurs a great circle in the parecliptic orb whose center is the center of the World and which intersects the parecliptic in two places that are called the nodes—the "head" and the "tail"—of this planet. This great circle is called [Mercury's] **inclined orb**.

[5] The third orb is another eccentric called the deferent for the epicycle. It is in the thickness of the dirigent just as the dirigent is in the thickness of the parecliptic. The [deferent's] equator is in the plane of the [dirigent's] equator.

[6] On account of its two eccentric orbs, this planet has four complementary bodies: two for the dirigent from the parecliptic and two for the deferent from the dirigent.

[7] The fourth orb is the epicycle, and it is in the thickness of the deferent. Its equator is not fixed in the [deferent's] equator as will be explained below. Mercury is on the epicycle, being embedded in it, and moves along its equator.

[8] Turning now to the motions, the first is the motion of the parecliptic, [which moves] with the motion of the fixed stars sequentially about the center of the World. It manifests itself in the apogee and perigee of the dirigent and in the head and tail.

[9] The second is the motion of the dirigent and is the same as the motion of center of the mean sun, *i.e.* the excess of its mean motion over the motion of its apogee; it is counter-sequential about its own center. This motion manifests itself in the apogee and perigee of the deferent.

مقابلة ذلك الجزء بل في تثليثيه . وفي مقابلة ذلك يوجد مثل ما يوجد في ذلك الجزء ولكن لا في تلك الغاية .

[٢] فأثبتوا له أربعة أفلاك وأربع حركات .

[٣] الفلك الأول الممثل بفلك البروج محدّبه مماسّ لمقعّر فلك الزهرة ومقعّره مماسّ لمحدّب ممثل القمر .

[٤] والفلك الثاني خارجُ مركزٍ يسمى بالمُدير . ويكون في ثخن الممثل كما وصفنا في كون الخارج المركز في ثخن الموافق المركز . ومنطقته ليست في سطح منطقة الممثل بل مائلة عنها غير ثابتة الميل ، وستجيء صفتها ، وأوجُه عند موضع غاية الميل . وسطح منطقته يقاطع سطح منطقة الممثل على زوايا حادّة ومنفرجة ، فتَحدث في الفلك الممثل دائرةٌ عظيمة مركزها مركز العالم مقاطعة للممثل في موضعين يسميان عقدتي الرأس والذنب لهذا الكوكب . وتسمى تلك العظيمة فلكه المائل .

[٥] والفلك الثالث خارج مركز آخر يسمى الحامل للتدوير . ويكون في ثخن المدير مثل كون المدير في ثخن الممثل ، ومنطقته في سطح منطقته .

[٦] وتكون لهذا الكوكب بحسب فلكيه الخارجي المركز أربعةُ متمّمات : اثنان للمدير من الممثل ، واثنان للحامل من المدير .

[٧] والفلك الرابع فلك التدوير وهو في ثخن الحامل . ومنطقته ليست بثابتة في منطقته على ما سيجيء بيانه . وعطارد على التدوير مركوز فيه يتحرك على منطقته .

[٨] وأما الحركات فالأولى حركة الممثل بحركة الثوابت حول مركز العالم على التوالي . وتظهر في أوج المدير وحضيضه وفي الرأس والذنب .

[٩] والثانية حركة المدير وهي مثل حركة مركز الشمس الوسطى ، أعني فضل حركة وسطها على حركة أوجها إلى خلاف التوالي حول مركزه . وتظهر هذه الحركة في أوج الحامل وحضيضه .

On account of it, the deferent center comes to have a circuit about the dirigent center called the **deferent orb for the deferent orb's center**.

[10] The third is the motion of the deferent; it is equal to twice the sun's motion of center and is sequential. [It moves] not about its own center nor about the center of the World nor about the dirigent center; it is instead about a point we will discuss below. This motion manifests itself in the epicycle center.

[11] The center of the epicycle is always in conjunction with the position of the mean sun. When it is at the dirigent apogee, it will be at the deferent apogee as well. The [center] and the [deferent apogee] then separate from the [dirigent apogee]: the deferent apogee will move counter-sequentially away from the dirigent apogee at the rate of the sun's motion of center, and the epicycle center will move sequentially away from the dirigent apogee at the rate of the excess of its motion over the motion of the deferent apogee, which is likewise equal to the sun's motion of center. Therefore the dirigent apogee is always at the midpoint between the deferent apogee and the epicycle center as was the case for the moon with respect to the center of the sun being midway between the apogee and the epicycle center. When each [of the deferent apogee and the epicycle center] describes a quarter [revolution], the center will reach the perigee of the deferent, and they will each be at quadrature with respect to the dirigent apogee. After describing another quarter, they will meet opposite the dirigent apogee and thus the center will be at the dirigent perigee and at the deferent apogee. Then [the center and deferent apogee] will separate, and they will be in opposition at the quadratures. They will once again meet at the dirigent apogee. Thus the epicycle center's farthest distance is when it is at both apogees simultaneously. Its nearest distance, however, is not directly opposite this position on account of it thereupon being at the deferent apogee and the dirigent perigee, nor is [this distance] at the quadratures since the two opposite distances [from the center of the World] occurring at the apogee [of the dirigent] and at the point directly opposite it are not equal. Instead, it occurs at two positions whose distance from the dirigent apogee is greater than from the point directly opposite it. These two positions are at the trines with respect to the apogee, a result dictated by the composition of the two perigees.

[12] This latter motion and the motion of the apogee together yield Mercury's **mean motion**.

ويظهر بسببها لمركز الحامل مدارٌ حول مركز المدير يسمى الفلكَ الحامل لمركز الفلك الحامل .

[١٠] والثالثة حركة الحامل وهي مثل ضعف حركة مركز الشمس إلى التوالي ـ لا حول مركزه ولا حول مركز العالم ولا حول مركز المدير بل حول نقطة سنذكرها . وتظهر في مركز التدوير .

[١١] ومركز التدوير يقارن موضع الشمس الوسطى دائماً . وإذا كان في أوج المدير كان في أوج الحامل أيضاً . ثم يفارقانه فيتحرك أوج الحامل إلى خلاف التوالي ويبعد عن أوج المدير بقدر حركة مركز الشمس ، ويتحرك مركز التدوير إلى التوالي ويبعد عن أوج المدير بقدر فضل حركته على حركة أوج الحامل وهو أيضاً مثل حركة مركز الشمس . فيكون أوج المدير دائماً في المنتصف بين أوج الحامل ومركز التدوير كما مرّ في القمر من توسّط مركز الشمس بين الأوج ومركز التدوير . وإذا قطع كل واحد منهما الربع انتهى المركز إلى حضيض الحامل وهما في تربيعي أوج المدير ، وبعد قطع ربع آخر يتلاقيان في مقابلة أوج المدير فيكون المركز في حضيض المدير وأوج الحامل ، ثم يتفارقان ويتقابلان في التربيعين ويعودان إلى الملاقاة عند أوج المدير . فالبعد الأبعد لمركز التدوير يكون عند كونه في أوجيه معاً . ولا يكون بعده الأقرب في مقابلة ذلك الموضع لكونه في أوج الحامل وحضيض المدير هناك ، ولا في التربيعين لأنّ البعدين المتقابلين اللذين في الأوج ومقابله ليسا / بمتساويين / ؛ بل يكون في موضعين بعدهما من أوج المدير أكثر من مقابلته وهما تثليثا الأوج بحسب ما يقتضيه تركّب الحضيضين .

[١٢] وتجتمع من هذه الحركة وحركة الأوج حركةُ وسط عطارد .

/21/ ...] M , β = متساويين] α .

[13] The fourth motion is that of the epicycle orb, which is 3°6′ per day. The planet moves with this [motion] in such a way that in the far segment [of the epicycle] it is sequential. The planet will undergo retrogradation on this epicycle in the near segment since the ratio of the two motions is such so as to bring about retrogradation. The planet will not move away from the sun—either ahead or behind—except by the amount brought about by its epicycle radius. At the apex and perigee [of the epicycle], it will be in conjunction with the [sun] since its [epicycle] center is permanently in conjunction with it. The radius of the epicycle is according to observation 22½ parts, the deferent radius being 60 parts.

[14] The dirigent center is 6 parts removed from the center of the World, this also according to the above units. The point about which / the motion of the epicycle center and the motion of the deferent / is always uniform is at the midpoint of this distance on the diameter that passes through these two [centers]; this point is called the **equant center**. About it one may imagine a circle, called the **equant orb**, whose size is that of the deferent equator and which is in the same plane. Thus the center of the epicycle will cut off from its circumference equal arcs in equal times. It is as if there were a line extending from the center of the equant to the center of the epicycle so as to turn it with uniform motion. The mean apex and perigee of the epicycle are also aligned with this point. The distance of the deferent center from the dirigent center is likewise equal to the distance of the equant center from the latter. It therefore follows that the deferent center will encounter the equant center once every revolution, and this will occur when the center of the epicycle is in opposition to the dirigent apogee. At that time the deferent equator will coincide with the equant orb; afterwards, they will become separate. When the epicycle center is at the two apogees, the four centers will be equally spaced on the diameter passing through the centers.

[15] Turning now to Mercury's anomalies that follow from its motions, the first is its anomaly resulting from the diameter of its epicyclic orb when it is at the mean distance. It is an angle

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/10–11/ the motion of the epicycle center and the motion of the deferent] β = the motion of the deferent] α, M.

[١٣] والحركة الرابعة حركة فلك التدوير ، كل يوم ثلاثة أجزاء وست دقائق . فيتحرك بها الكوكب على وجه يكون في القطعة البعيدة منه على التوالي . ويقع للكوكب في هذا التدوير رجوعٌ في القطعة القريبة لكون نسبة الحركتين على ما يقتضي الرجوع . ولا يبعد الكوكب من الشمس قدامها وخلفها إلا بقدر ما يقتضيه نصف قطر تدويره ، ويقارنها في الذروة والحضيض لكون / مركزه / مقارناً لها دائماً . ونصف قطر التدوير اثنان وعشرون جزءاً ونصف بالرصد على أنّ نصف قطر الحامل ستون جزءاً .

[١٤] ومقدار خروج مركز المدير عن مركز العالم ستة أجزاء بهذه الأجزاء أيضاً . وتكون النقطة التي تتشابه / حركة مركز التدوير وحركة الحامل / حولها أبداً عند منتصف هذا البعد على القطر المارّ بهما ، وتسمى مركز مُعدِّل المَسير . وتتوهم حولها دائرةٌ بقدر منطقة الحامل وفي سطحها تسمى فلك معدل المسير ؛ فإنّ مركز التدوير يقطع من محيطه في أزمنة متساوية قسياً متساوية كأنّ خطاً خرج من مركز معدل المسير إلى مركز التدوير ليُديره حركةً متشابهة . والذروة والحضيض الوسطيان من التدوير يحاذيان أيضاً هذه النقطة . ومقدار خروج مركز الحامل عن مركز المدير أيضاً بقدر بعد مركز معدل المسير عنه ؛ فلذلك يلزم أن يلاقي مركزُ الحامل في كل دورة مرةً مركزَ معدل المسير ، وذلك عند كون مركز التدوير في مقابلة أوج المدير ، وحينئذ تنطبق منطقة الحامل على فلك معدل المسير ثم يتفارقان . وعند كون مركز التدوير في الأوجين تكون المراكز الأربعة على القطر المار بالمراكز على أبعاد متساوية .

[١٥] وأما اختلافات عطارد اللازمة لحركاته فالأول اختلافه اللازم من جهة قطر فلك تدويره عند كونه في البعد الأوسط . وهو زاوية

/7/ ...] = β مركزها] α , M . /10–11/ ...] = β حركة الحامل] α , M .

that occurs at the center of the World from the production of two lines from it, one to the center of the epicycle and the other to the center of the planetary body. The maximum of this anomaly will be according to the size of the epicycle radius. It will be additive with respect to the position of the epicycle center in the descending half, subtractive in the ascending half. This anomaly is called the / **second / equation.**

[16] The second is the apparent increase [in size] of the epicycle radius when it is at a nearer distance over what is observed at the mean distance, and its decrease when it is at a farther distance. This anomaly will combine with the first anomaly according to the amount of the latter as determined by the radius, thus either decreasing or increasing the [first anomaly]. The resulting [quantity] will then follow [the above rule for the first anomaly] in being either additive or subtractive with respect to the center. This anomaly is called the **anomaly of the farthest and nearest distances**.

[17] The third is the anomaly that results from the motion of the epicycle center being uniform about a point other than the center of the World and from the difference between the apparent and the mean [epicyclic] apices. These two anomalies are one and the same since the diameter of the epicycle passing through the mean apex and perigee is aligned with this very same point. This [anomaly] is an angle occurring at the center of the epicycle from the production of two lines from it, one to the center of the World and the second to the equant center. The anomaly is subtractive from the center and additive to the proper motion as long as the epicycle center is descending on the dirigent; the opposite will hold when it is ascending. This anomaly is called the **equation of the center and of the proper** [motion].

[18] These then are [Mercury's] anomalies.

[19] The same difficulty that was mentioned in the chapter on the moon due to the uniformity of the motion of the epicycle center being about a point different from the center of its deferent is present here as well. However, the [difficulty] that was mentioned as arising on account of the anomaly in alignment is not present because the alignment is toward the point with respect to which the uniformity of motion occurs. There [also] results from the motions of the dirigent

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/5/ second] $\beta = -\alpha$, –M.

على مركز العالم تَحدث من خروج خطّين عنه : أحدهما إلى مركز التدوير ، والآخر إلى مركز جرم الكوكب . وغاية هذا الاختلاف بقدر نصف قطر التدوير . ويكون زائداً على موضع مركز التدوير في النصف الهابط ، ناقصاً في النصف الصاعد . ويسمى هذا الاختلاف بالتعديل / الثاني / .

[١٦] والثاني زيادة نصف قطر التدوير في الرؤية على ما يُرى في البعد الأوسط إذا صار في بعد أقرب منه ، ونقصانه من ذلك إذا صار في بعد أبعد / منه / . وهذا الاختلاف يلحق الاختلاف الأول بقدر ذلك الاختلاف من نصف القطر فينقص منه أو يزيد عليه ، ويكون بعد ذلك في الزيادة على المركز أو النقصان منه تابعاً له . ويسمى هذا الاختلاف اختلاف البعد الأبعد والأقرب .

[١٧] والثالث الاختلاف اللازم بحسب تشابه حركة مركز التدوير حول نقطة غير مركز العالم وبحسب اختلاف الذروتين المرئية والوسطى . وهذان الاختلافان شيء واحد لكون قطر التدوير المارّ بالذروة والحضيض الوسطيين محاذياً لتلك النقطة بعينها . وهو زاوية تَحدث على مركز التدوير من خطّين يخرجان منه : أحدهما إلى مركز العالم ، والثاني إلى مركز معدل المسير . ويكون الاختلاف ناقصاً من المركز ، زائداً على الخاصة ، ما دام مركز التدوير هابطاً في المدير ، وبالعكس ما دام صاعداً . ويسمى هذا الاختلاف تعديل المركز والخاصة .

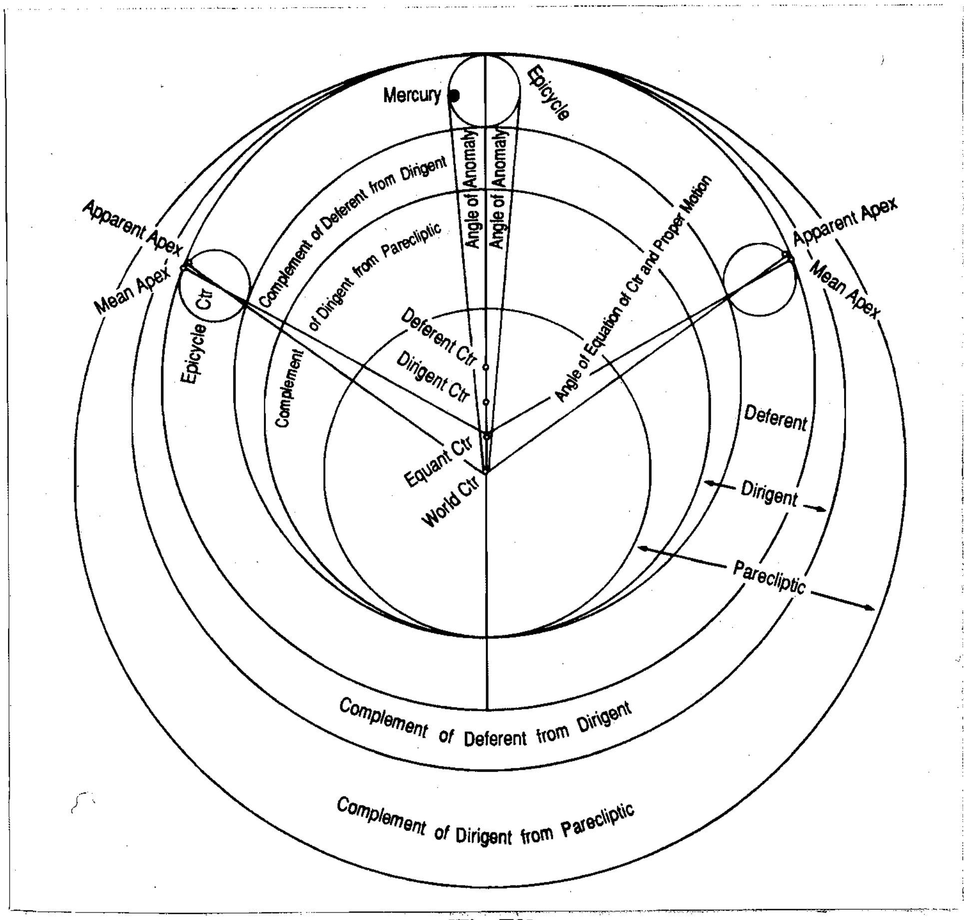
[١٨] فهذه اختلافاته .

[١٩] والإشكال المذكور في باب القمر بسبب تشابه حركة مركز التدوير حول نقطة خارجة عن مركز حامله واردٌ بعينه ههنا . وأما الذي ذُكر بحسب اختلاف المحاذاة فغير وارد لكون المحاذاة نحو النقطة التي بحسبها تتشابه الحركة . ويلزم من كون حركتي المدير

/5/ ...] β = M–, α– . /8/ ...] β = M–, α– .

and the deferent being about two different points an anomaly that was not mentioned with regard to the motion of the epicycle center, which is composed of these two.

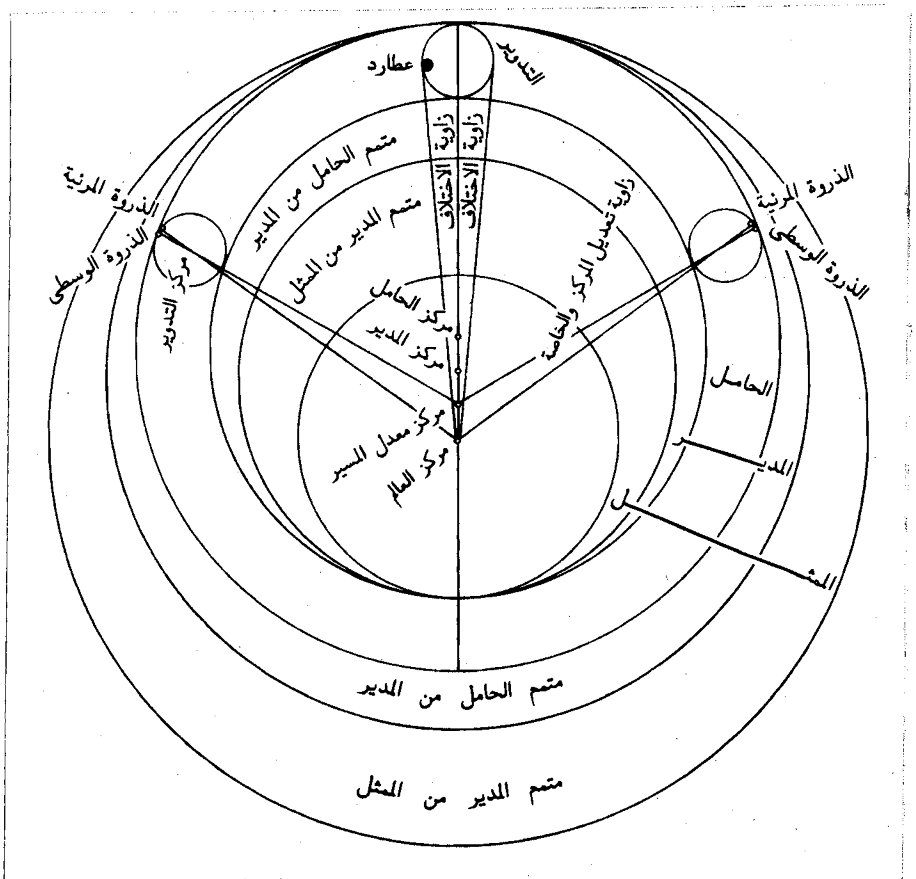
[20] This is the illustration of the orbs of Mercury:



[Fig. T8]

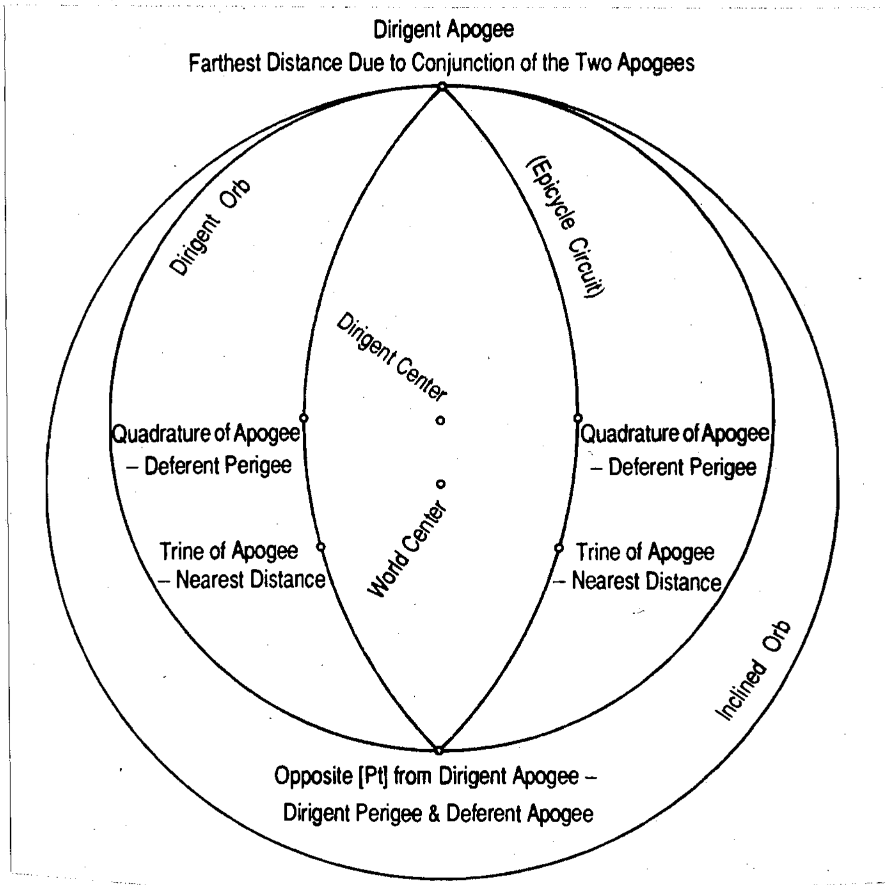
والحامل حول نقطتين مختلفتين اختلافٌ لم يُذكر في حركة مركز التدوير المركبة عنهما .

[٢٠] وهذه صورة أفلاك عطارد .



[شكل ٨]

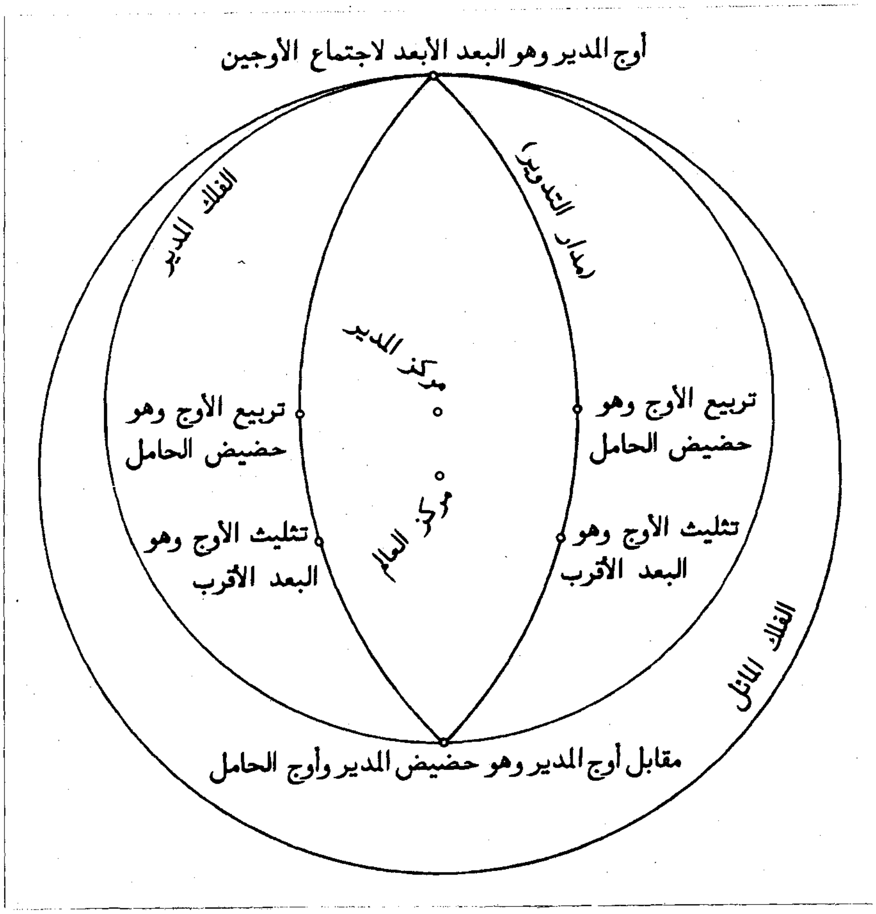
[21] Those who limit themselves to circles set forth six orbs: the parecliptic, the inclined, the deferent of the epicycle, the equant, the deferent of the deferent's center, and the epicycle. The shape of the circuit of the epicycle center in relation to the inclined orb and to the center of the World is thus:



[Fig. T9]

[22] The explication of terms is analogous to what was given for the moon. A discussion of the latitudes will be forthcoming in a separate chapter.

[٢١] والمقتصر على الدوائر يُورد ستة أفلاك : الممثل ، والمائل ، والحامل للتدوير ، ومعدل المسير ، وحامل مركز الحامل ، والتدوير . وشكل مدار مركز التدوير بالقياس إلى المائل وإلى مركز العالم يكون هكذا :



[شكل ٩]

[٢٢] وتفسير الألقاب يكون على قياس ما مرّ في القمر . والكلام في العروض يجيء في باب مفرد .

CHAPTER NINE

On the Orbs and Longitudinal Motions of the Remaining Planets

[1] They found that the three upper planets were slower in speed than the sun. So when the sun is in conjunction with one of them, it will move ahead of it; the [planet] will then appear in the east and be at its fastest speed. It then begins to slow until it stops when the sun is near its first trine or a little beyond. Thereupon the planet will retrograde and the sun will be in opposition to it at the middle part of its retrogradation. It will then stop a second time when the sun comes near the second trine or is a little / beyond it. / The [planet] will then proceed with direct motion and go from a slower to a faster speed until the sun approaches it; the planet will then disappear in the west, and the sun will be in conjunction with it at the middle part of its direct motion. If a certain circumstance of the [planet] is compared with another that corresponds to it, it is found to differ from it. Identical circumstances occurring in exactly the same parts of the ecliptic are found to shift with the movement of the fixed stars. The circumstances resulting from the nearest distance are found to occur in parts [of the ecliptic] directly facing those in which the opposite [set of circumstances] resulting from the farthest / distance / occurs. The [planet] does not travel on the same circuit as the sun but instead is north of it in one half of the ecliptic orb, sometimes approaching and sometimes receding from it, and south of it in the other half, similarly [approaching and receding]. The nodes shift with the movement of the fixed stars. They found Venus's situation in longitude and latitude to be similar to that of Mercury except that its nearest distance was directly facing its farthest distance, just as was the case for the upper planets. Its maximum elongation ahead of or behind the sun does not exceed 47°.

[2] They therefore established three orbs and three motions for each of the four.

/9/ beyond it] β, M = before it] α. /14/ distance] β, M = –α.

الفصل التاسع
في أفلاك الكواكب الباقية
وحركاتها الطولية

[١] وجدوا الكواكب الثلاثة العلوية أبطأ سيراً من الشمس . فإذا قارنتها الشمس سبقتها فظهرت مشرّقة وتكون في أسرع سيرها ، ثم تأخذ في البطء حتى إذا صارت الشمس إلى قريب من تثليثها الأول أو بَعده بقليل وقفت ، ثم رجعت وتقابلها الشمس في أواسط رجوعاتها ، ثم تقف ثانياً بقرب وصول الشمس إلى تثليثها الثاني أو / بعده / بقليل ، ثم تستقيم وتأخذ من البطء إلى السرعة إلى أن تقرب الشمس منها فتختفي مغرّبة وتقارنها الشمسُ في أواسط استقاماتها . وإذا قيست حال من أحوالها إلى نظير تلك الحال وُجدت مخالفة لها . والأحوال المتشابهة في أجزاء بأعيانها من فلك البروج تنتقل بانتقال الثوابت . ووُجدت الأحوال التي يقتضيها البعدُ الأقرب في أجزاء مقابلة للتي يقتضي فيها / البعدُ / الأبعد أضدادها . وهي لا تسير على مدار الشمس بعينه بل تكون شمالية عنه في نصف فلك البروج متقاربة إليه تارة ومتباعدة عنه أخرى ، وجنوبية عنه في النصف الآخر كذلك . والمَجازان ينتقلان انتقال الثوابت . ووجدوا الزُهَرَة شبيهة الأحوال بعطارد طولاً وعرضاً إلا أنّ أقرب أبعادها مقابل لأبعدها كما في العلوية . وغاية بعدها في الطول عن الشمس قداماً وخلفاً لا تتجاوز سبعاً وأربعين درجة .

[٢] فأثبتوا لكل من الأربعة ثلاثة أفلاك وثلاث حركات .

/9/ …] M , β = قبله] α . /14/ …] M , β = α− .

[3] The first orb is the parecliptic. For Saturn, its convex surface is contiguous with the concave surface of the eighth orb, and its concave surface is contiguous with the convex surface of Jupiter's parecliptic. The concave surface of Jupiter's parecliptic is contiguous with the convex surface of Mars's parecliptic. The concave surface of Mars's parecliptic is contiguous with the convex surface of the sun's parecliptic. The convex surface of Venus's parecliptic is contiguous with the concave surface of the sun's parecliptic, while its concave surface is contiguous with the convex surface of Mercury's parecliptic.

[4] The second is the eccentric deferent for the epicycle. It is located in the thickness of the parecliptic.

[5] The third is the epicycle, which is in the thickness of the deferent. The planet is embedded in the epicycle.

[6] The equator of the epicycle is not fixed in the plane of the deferent equator; rather, only its center is fixed therein. The deferent equator is inclined to the parecliptic equator, this inclination being fixed for the upper planets but not for Venus; its plane intersects the plane of the parecliptic equator, and there occurs on the parecliptic / a great circle whose center is the center of the World / called the **inclined orb** for the planet. It intersects the parecliptic equator in two places, which are the **head** and **tail** of that planet. The amounts of the inclinations are as we shall state in the chapter on latitudes.

[7] As for the motions, the first is the motion of the parecliptic [which moves] with the motion of the fixed stars. It manifests itself in the two [extreme] distances and / in the two nodes. /

[8] The second is the motion of the eccentric and is per day: 2 minutes for Saturn, 5 minutes for Jupiter, 31 minutes for Mars, and for Venus the same as the sun's mean motion of center. It manifests itself in the epicycle center and is therefore ascribed to it, thus being called the / epicycle's / **motion of center**.

[9] This motion is not uniform about the center of the World nor about the center of the eccentric but instead is uniform about a point removed from the eccentric center whose location is on the diameter passing through the two centers on the part that is toward the apogee and away from the eccentric center

/12/ a great circle whose center is the center of the World] β = a great circle] M = a great one] α. /17/ in the two nodes] β, M = the two nodes] α. /21/ epicycle's] β = planet's] α, M.

[٣] الفلك الأول الممثل : محدبه لزُحَل يماس مقعرَ الفلك الثامن ومقعره لمحدب ممثل المُشْتَري ، ومقعر ممثل المشتري لمحدب ممثل المِرّيخ ، ومقعر ممثل المريخ لمحدب ممثل الشمس ، ومحدب ممثل الزهرة لمقعر ممثل الشمس ومقعره لمحدب ممثل عطارد .

[٤] والثاني خارج المركز الحامل للتدوير وهو في ثخن الممثل .

[٥] والثالث التدوير وهو في ثخن الحامل . والكوكب مركوز في التدوير .

[٦] ومنطقة التدوير لا تثبت في سطح منطقة الحامل بل يثبت مركزُه فيه فقط . ومنطقة الحامل مائلة عن منطقة الممثل _ ثابتة الميل في العلوية ، غير ثابتة في الزهرة ؛ وسطحها يقاطع سطح منطقة الممثل وتَحدث في الممثل / دائرةٌ عظيمة مركزها مركز العالم / تسمى الفلك المائل لذلك الكوكب ، وتقاطع منطقةَ الممثل في موضعين هما الرأس والذنب لذلك الكوكب . ومقادير الميول على ما نوردها في باب العروض .

[٧] وأما الحركات فالأولى حركة الممثل بحركة الثوابت ، وتظهر في البعدين / وفي العقدتين / .

[٨] والثانية حركة الخارج المركز ، وهي كل يوم : لزحل دقيقتان ؛ وللمشتري خمس دقائق ؛ وللمرّيخ إحدى وثلاثون دقيقة ؛ وللزهرة مثل حركة مركز الشمس الوسطى . وهي تظهر في مركز التدوير ولذلك تُنسب إليه فتسمى حركةَ مركز / التدوير / .

[٩] وهذه الحركة لا تتشابه حول مركز العالم ولا حول مركز الخارج المركز ، بل تتشابه حول نقطة خارجة عن مركز الخارج المركز ، موضعها على القطر المارّ بالمركزين عمّا يلي الأوج من مركز

/12/ ... [β = دائرة عظيمة] M = عظيمة [α . /17/ ... [β , M = والعقدتين [α .
/21/ ... [β = الكوكب] α , M .

at a distance equal to the eccentricity. This [eccentricity] is: $(3 + \frac{1}{4} + \frac{1}{6})$ parts for Saturn, $2\frac{3}{4}$ parts for Jupiter, 6 parts for Mars, and for Venus approximately one-half the sun's eccentricity. All of these [values], which are based on the deferent radius for each planet being 60 parts, are known by observation. Twice this amount is the distance of that point from the center of the World. This point is called the **equant center**, and one may imagine a circle the size of the deferent equator with this point as center and it is called the **equant orb**.

[10] When the motion of the apogee is added to this [second] motion, there results the **mean motion** of the planet.

[11] The third motion is that of the epicycle orb. For the upper planets, it is equal to the excess of the mean motion of the sun over the mean motion of each one of the [planets]; for Venus it is 37 minutes per day. In the upper part of the epicycles, it is sequential. The starting point [for this motion] is the **mean apex**, which is aligned with the equant center as was the case for Mercury.

[12] Because the ratio of the two motions is a ratio that causes retrogradation in the epicycles, these planets will become retrograde in the segment nearest the Earth. The upper planets at the mean apices of their epicycles are always [aligned] with the mean sun. And on account of their motions on the epicycles being in the amount of the excess [of the motion] of the mean sun over their mean [motions], their distances along the epicycles from the apices are in the amount of the mean sun's distances along their orbs surrounding the Earth from the centers of their epicycles. Therefore the mean sun will be in opposition to the [planets] when they are at their mean [epicyclic] perigees during the middle of their periods of retrogradation; they will return to conjunction with [the mean sun] at the apices. As for Venus, the center of its epicycle is permanently in conjunction with the sun's center. For that reason, it will combust at the apex of its epicycle half-way through its period of direct motion and at its perigee half-way through its period of retrogradation. It will not move away from the [sun] beyond what is brought about by its epicycle radius.

الخارج على بعد مساوٍ لِما بين المركزين . وذلك لزحل ثلاثة أجزاء وربع وسدس جزء ؛ وللمشتري جزءان وثلاثة أرباع جزء ؛ وللمرّيخ ستة أجزاء ؛ وللزهرة قريب من نصف ما بين مركزي الشمس . جميع ذلك ، بحسب ما يكون نصف قطر حامل ذلك الكوكب ستين جزءاً ، عُرف بالرصد . وضعف هذا المقدار هو بعد تلك النقطة عن مركز العالم . وتسمى تلك النقطة مركز معدل المسير ، وتتوهم دائرةٌ بقدر منطقة الحامل مركزها هذه النقطة وتسمى فلك معدل المسير .

[١٠] وإذا أُضيفت حركةُ الأوجِ إلى هذه الحركة حصلت حركةُ وسط الكوكب .

[١١] والثالثة حركة فلك التدوير ، وهي للعلوية بقدر فضل حركة وسط الشمس على وسط كل واحد منها ، وللزهرة كل يوم سبع وثلاثون دقيقة . وهي تكون في أعالي التداوير إلى التوالي . ومبادئها الذروة الوسطى وهي محاذية لمركز معدل المسير كما في عطارد .

[١٢] ولكون نسبة الحركتين نسبة توجب الرجوع في التداوير تصير هذه الكواكب راجعة في القطعة القريبة من الأرض . والكواكب العلوية تكون في ذُرى تداويرها الوسطى مع وسط الشمس أبداً . ولكون حركاتها في التداوير بقدر فضل وسط الشمس على أوساطها تكون أبعادها في التداوير عن الذُرى بقدر أبعاد وسط الشمس عن مراكز تداويرها في أفلاكها المحيطة بالأرض . فإذن يقابلها وسطُ الشمس وهي في حضيضاتها الوسطى في أواسط أيام رجوعاتها ويعود إلى مقارنتها في الذرى . وأما الزهرة فمركز تدويرها مقارن لمركز الشمس أبداً ، ولذلك تحترق في ذروة تدويرها عند انتصاف مدة استقامتها وفي حضيضه عند انتصاف مدة رجوعها . ولا تبعد عنها فوق ما يقتضيه نصف قطر تدويرها .

[13] The size of the epicycle radius, by observation, is: 6½ parts for Saturn, 11½ parts for Jupiter, 39½ parts for Mars, and 43⅙ parts for Venus, which are based on the deferent radius being 60.

[14] It should be noted that the epicycles for Mars and Venus are very much larger than the remaining epicycles, and for this reason the difference between the sizes of their bodies at apex and perigee is greater than is the case for the remaining planets. It will be shown that the sphere of the epicycle of Mars is much larger than the pareclipic sphere of the sun including what is inside it. For this reason, they sometimes ask: how is it that Mars when it is at opposition to the sun and six zodiacal signs away from it is nearer to the [sun] than when it is at combust and in conjunction with it at the same minute [of arc]? This is so / because Mars / at combust is at the apex of its epicycle; thus the distance between them will be the diameter of its epicycle plus whatever occurs by way of the complementary bodies of their orbs. At opposition, Mars is at the perigee of its epicycle; thus the distance between them will be the diameter of the sun's pareclipic plus whatever occurs by way of the complementary bodies. This is another instance of something that is found to be strange in this science.

[15] As for the anomalies that result from these motions, they are three in number and are exactly the same as those for Mercury. The above-mentioned difficulty, which is due to the motion being uniform about a point other than the center of its equator but without there being the [difficulty] due to the alignment [of the epicycle diameter], occurs here just as it did for [Mercury].

/11/ because Mars] β = because it] α, M.

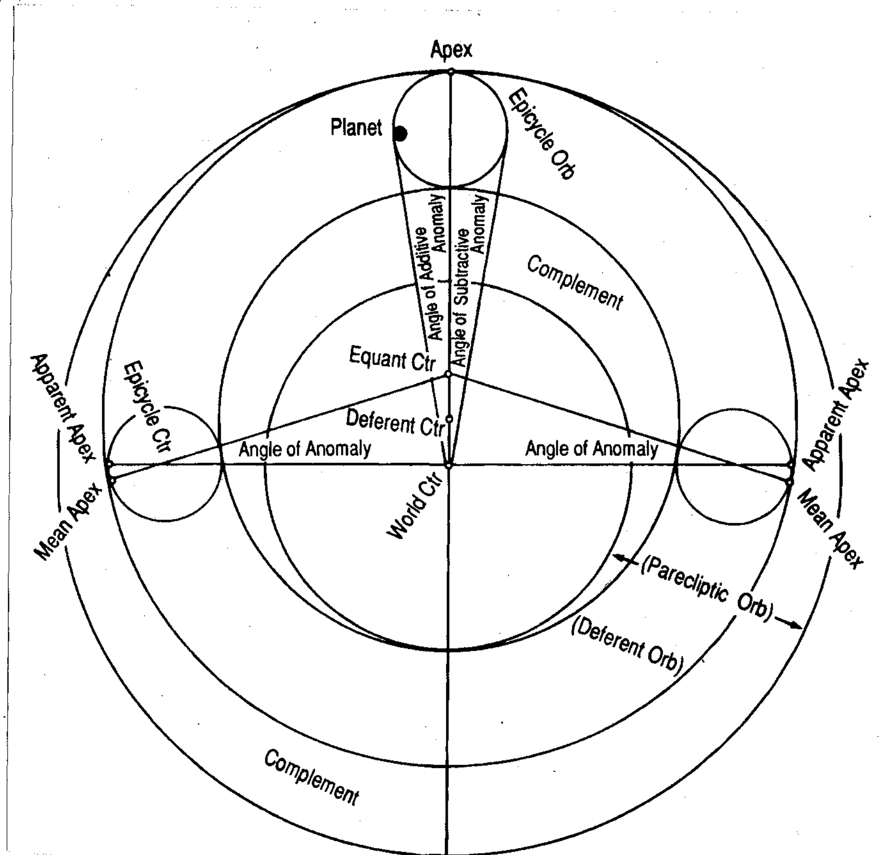
[١٣] ومقدار نصف قطر التدوير بالرصد : لزحل ستة أجزاء ونصف ؛ وللمشتري أحد عشر جزءاً ونصف ؛ وللمرّيخ تسعة وثلاثون جزءاً ونصف ؛ وللزهرة ثلاثة وأربعون جزءاً وسدس ، بحسب ما يكون نصف قطر الحامل ستين .

[١٤] وآعلم أنّ تدويري المرّيخ والزهرة أعظم جداً من سائر التدويرات ، ولذلك يكون الاختلاف بين جرميهما بالصغر والكبر في الذروة والحضيض أكثر مما يكون في سائر الكواكب . وسيتّضح أنّ كرة تدوير المرّيخ أعظم كثيراً من كرة ممثل الشمس بما فيها ، فلذلك ربما سألوا فقالوا : ما بال المرّيخ كان في مقابلة الشمس على بعد ستة بروج منها أقرب إليها منه في الاحتراق مجتمعاً معها في دقيقة واحدة ؟ وإنما يكون ذلك / لكون المريخ / في الاحتراق في ذروة تدويره فيكون البعد بينهما قطر تدويره مع ما يتّفق من متمّمات فلكيهما ، وفي المقابلة في حضيض تدويره فيكون البعد بينهما قطر ممثل الشمس مع ما يتّفق من المتمّمات . وهذا أيضاً مما يُستغرب في هذا العلم .

[١٥] وأما الاختلافات اللازمة لهذه الحركات فثلاثة وهي كما مرّ في عطارد بعينه . والإشكال المذكور بسبب كون الحركة متشابهة حول نقطة غير مركز منطقتها دون الذي بسبب المحاذاة واردٌ كما مرّ فيه .

/11/ ...] β = لكونه] α , M .

[16] This is the illustration of the orbs for each of the four planets:

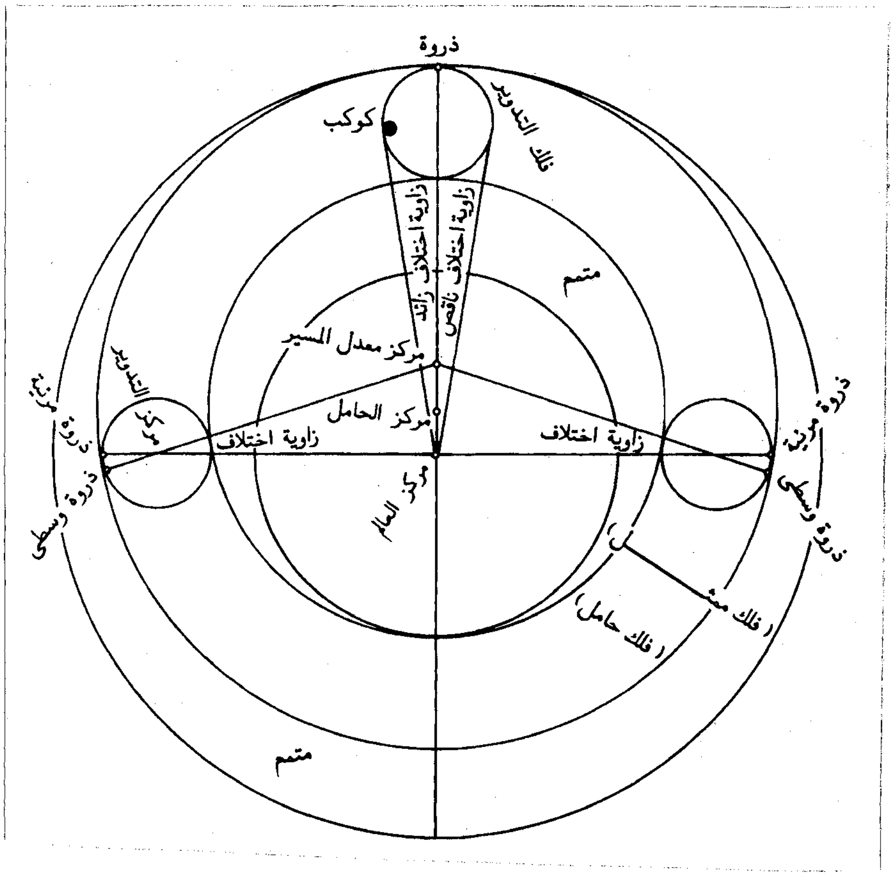


[Fig. T10]

[17] Those who limit themselves to circles set forth five orbs: the parecliptic, the inclined, the deferent, the equant, and the epicycle. The explication of terms is analogous to what was given previously.

[18] This then is what the practitioners of this science have said concerning the orbs of the planets. The total number of solid orbs that they have established for the seven planets is 22; for those who limit themselves to circles, it is 32.

[١٦] وهذه صورة أفلاك كل كوكب من الأربعة :



[شكل ١٠]

[١٧] والمقتصرون على الدوائر يوردون خمسة من الأفلاك : الممثل ، والمائل ، والحامل ، ومعدل المسير ، والتدوير . وتفسير الألقاب يكون على قياس ما مرّ .

[١٨] فهذا ما ذكر أهل هذا العلم في أفلاك الكواكب . وجميع الأفلاك المجسّمة التي أثبتوها للكواكب السبعة اثنان وعشرون ، وعند المقتصرين على الدوائر اثنان وثلاثون .

CHAPTER TEN

On the Latitudes of the Five Planets

[1] The maximum inclination of the inclined [orb] from the parecliptic is: 2½° for Saturn, 1½° for Jupiter, 1° for Mars, ⅙° for Venus, and (½ + ¼)° for Mercury. For the upper planets, this [inclination] is fixed in both directions, while in the case of the two lower planets it is not fixed. Rather, for Venus it is always northerly, whereas it is always southerly for Mercury. This is so because of the motion of the inclined equator toward the parecliptic equator; it will approach the [parecliptic] until it coincides with it whereupon it will separate from it in the other direction until it reaches its maximum distance from it. Then the [inclined equator] will again approach the [parecliptic] until it coincides with it once more. It will then separate from it until it reaches its maximum distance in the original direction. The two halves will exchange directions after each coinciding, the northern becoming southern and vice versa. This process is completed in each solar year.

[2] The epicycle centers of Venus and Mercury will always be at their heads / or tails / at the time of this coinciding. Thus if Venus's epicycle center is at its head and Mercury's epicycle center is at its tail and then they depart from them, the inclined [equator] will separate from the parecliptic / so that / Venus's center will come to be in the northern half and Mercury's center will be in the southern half. The inclination will increase little by little until the [centers] reach the midpoint between the two nodes whereupon the inclination attains its maximum. The two centers will then head toward the other node as the two inclinations begin to decrease until Venus's center reaches the tail and Mercury's center reaches the head. The inclined [equator] will thereupon coincide once again with the parecliptic. It will then separate from it after the [centers] depart from the nodes so that the half that had been northern becomes southern and vice versa; Venus comes to be in the half that had been southern but which, upon the arrival of its center there, is now northern,

/14–15/ or tails] β = and tails] α, M. /17/ so that] β = and] α, M.

الفصل العاشر
في عروض الكواكب الخمسة

[١] أما غاية ميل المائل عن المثل : فلزحل جزءان ونصف ، وللمشتري جزء ونصف ، وللمرّيخ جزء واحد ، وللزهرة سدس جزء ، ولعطارد نصف وربع جزء . وهي للعلوية ثابتة في الجهتين ، وللسفليين غير ثابتة بل إنما تكون للزهرة أبداً شمالية ولعطارد أبداً جنوبية . وذلك يكون كذلك بسبب حركة منطقة المائل نحو منطقة المثل ، فتقرب منها حتى تنتبق عليها ، ثم تفارقها في الجهة الأخرى إلى أن تبعد عنها غاية بعدها ، ثم ترجع متقاربة إليها إلى أن تنطبق عليها ثانياً ، ثم تفارقها إلى أن تبعد عنها غاية البعد في الجهة الأولى . ويتبادل النصفان في الجهتين بعد كل انطباق بأن يصير الشمالي جنوبياً وبالعكس ، وتتم الأحوال في كل سنة شمسية .

[٢] ومركزا تدويري الزهرة وعطارد يكونان مع رأسيهما / أو ذنبيهما / وقت الانطباق أبداً . فإذا كان مركز تدوير الزهرة مع رأسها ومركز تدوير عطارد مع ذنبه ثم فارقاهما ، فارق المائل المثل / فيصير / مركز الزهرة في النصف الشمالي ومركز عطارد في النصف الجنوبي . ويزداد الميل شيئاً بعد شيء إلى أن ينتهيا إلى منتصف ما بين العقدتين فيبلغ الميل غايته . ثم يتوجه المركزان نحو العقدة الأخرى ويأخذ الميلان في التناقص إلى أن ينتهي مركز الزهرة إلى الذنب ومركز عطارد إلى الرأس فينطبق المائل ثانياً على المثل . ثم يفارقه بعد مفارقتهما العقدة فيصير النصف الذي كان شمالياً جنوبياً وبالعكس ؛ والزهرة تصير إلى النصف الذي كان جنوبياً وصار عند وصول مركزها إليه شمالياً ،

/14–15/ …] β = وذنبيهما] α , M . /17/ …] β = ويصير] α , M .

and Mercury comes to be in the half that had been northern but which, upon the arrival of its center there, is now southern. They will then proceed in these halves as the inclination increases until they reach the midpoint between the nodes whereupon the inclination reaches its maximum. They will thereupon head toward the original node as the inclination begins to decrease until the [centers] reach the starting point from which they departed. The result of the above is that Venus's epicycle center will always be either to the north or else on the [parecliptic] equator at the node, and Mercury's epicycle center will always be either to the south or else on the [parecliptic] equator at the node. These two motions require two movers, which have not been referred to by [our] predecessors.

[3] Saturn's head is 140° before its apogee / while its tail is 40° past its apogee. / Jupiter's head is 70° before its apogee / while its tail is 110° past its apogee. / The heads of Mars and Venus are one-quarter revolution before their apogees. Mercury's head is one-quarter revolution past its apogee. In the case of the two lower planets, the head and the tail are indistinguishable except by assumption. Their divergent [values] for the positions of the apogees and nodes, with the stipulation of date, are given in the *zījes*.

[4] As for the epicyclic equators, their diameters that pass through the apices and [epicyclic] perigees are not fixed in the planes of their inclined orbs, and they will not be in them except when, for the upper planets, the epicycle centers are at the nodes and when, for the lower planets, they are at the two distances, *i.e.* the apogee and perigee. Afterwards the apices of the upper planets will always incline in the direction of the ecliptic equator, while their [epicyclic] perigees will incline in the opposite direction. They will reach their maximum [inclinations] at the midpoint between the nodes. At that point, the angle of intersection of the plane of the epicyclic equator and the plane of the inclined equator is: $4\frac{1}{2}°$ for Saturn, $2\frac{1}{2}°$ for Jupiter, and $2\frac{1}{4}°$ for Mars. Thus the inclination of Saturn when it is at its apex at the northern limit will be seen to be 26′

/10/ while its tail is 40° past its apogee] $\beta = -\alpha$, –M. /11–12/ while its tail is 110° past its apogee] $\beta = -\alpha$, –M.

وعطارد يصير إلى النصف الذي كان شمالياً وصار عند وصول مركزه إليه جنوبياً . فيسيران فيهما والميل متزايد إلى أن ينتهيا إلى منتصف ما بين العقدتين فيبلغ الميل غايته . ثم يتوجهان إلى العقدة الأولى ويأخذ الميل في التناقص إلى أن يبلغا المبدأ الذي فارقاه . ويحصل من ذلك كون مركز تدوير الزهرة دائماً إما في الشمال وإما على المنطقة مع العقدة ، وكون مركز تدوير عطارد دائماً إما في الجنوب وإما على المنطقة مع العقدة . وتحتاج هاتان الحركتان إلى مُحرِّكين لم يذكرهما المتقدمون .

[٣] ورأس زحل متقدم على أوجه بمائة وأربعين درجة ، / وذنب زحل متأخر عن أوجه بأربعين درجة / ؛ ورأس المشتري متقدم على أوجه بسبعين درجة ، / وذنب المشتري متأخر عن أوجه بمائة وعشر درجات / ؛ ورأسا المريخ والزهرة متقدمان على أوجيهما بربع دور ؛ ورأس عطارد متأخر عن أوجه بربع دور . والرأس والذنب في السفليين لا يتمايزان إلا بالفرض . ومواضع الأوجات والجوزهرات مذكورة في الزيجات مع قيد التواريخ على اختلافهم فيها .

[٤] وأما مناطق التداوير فأقطارها المارّة بالذُرى والحضيضات لا تثبت في سطوح أفلاكها المائلة ولا تكون فيها إلا عند كون مراكز التدويرات / للعلوية / في العقدتين وللسفليين في البعدين ، أعني الأوج والحضيض . وبَعد ذلك تميل ذُرى العلوية أبداً إلى جهة منطقة البروج وحضيضاتُها إلى خلاف تلك الجهة ، وتنتهي إلى غاياتها في منتصف ما بين العقدتين . وزاوية تقاطُع سطح منطقة التدوير وسطح منطقة المائل حينئذ تكون لزحلَ أربعة أجزاء ونصف ، وللمشتري جزءان ونصف ، وللمرّيخ جزءان وربع . ويُرى لذلك ميل زحل في ذروته في غاية البعد الشمالي ستاً وعشرين

/10/ ...] β = M– , α– . /11–12/ ...] β = M– , α– . /19/ ...] M , β = العلوية] α .

and at the southern 28′; when it is at its [epicyclic] perigee at the northern limit, the inclination will be 33′ and at the southern 35′. The inclination of Jupiter when it is at its apex at the northern limit will be 24′ and at the southern 25′; when it is at its [epicyclic] perigee at the northern limit, the inclination will be 35′ and at the southern 38′. The inclination of Mars when it is at its apex at the northern limit will be 22′ and at the southern 27′; when it is at its [epicyclic] perigee at the northern limit, the inclination will be 3°22′ and at the southern $6\frac{1}{10}$°. As for the two lower planets: as long as Venus's center is descending on the apogee orb, its apex will incline toward the north and its [epicyclic] perigee toward the south, and in the other half the opposite will hold. As for Mercury, as long as its center is descending its apex will incline toward the south and its perigee toward the north, and in the other half the opposite will hold. The angle of intersection of the two planes upon reaching the maximum is $2\frac{1}{2}$° for Venus and $6\frac{1}{4}$° for Mercury. Thus the inclination of Venus's apex at both maximum distances will be seen to be 1°2′ and the inclination of its [epicyclic] perigee will be 6°23′. The inclination of Mercury's apex at both maximum distances will be $1\frac{3}{4}$° and the inclination of its [epicyclic] perigee will be 4°4′. This latter latitude is known as the **deviation** [*mayl*]. The upper planets have only these two latitudes.

[5] As for the lower planets, the diameter that passes through the two mean distances, which intersects the first diameter at right angles, is not fixed in the planes of the inclined orbs nor is it in the parecliptic planes except when the centers of their epicycles are at one of the nodes. After the [centers] depart from the head, the posterior endpoint of that diameter, known as the **evening**, slants toward the north, while the anterior endpoint, known as the **morning**, slants toward the south until the two centers reach the midpoint between the head and the tail. At that point is the apogee for Venus and the point opposite [the apogee] for Mercury; the slants will thereupon reach the maximum. The centers will then cross the midpoint and the slants will decrease

دقيقة وفي الجنوبي ثمانياً وعشرين دقيقة ، وفي حضيضه في غاية البعد الشمالي ثلاثاً وثلاثين دقيقة وفي الجنوبي خمساً وثلاثين دقيقة ؛ وميل المشتري في ذروته في غاية البعد الشمالي أربعاً وعشرين دقيقة وفي الجنوبي خمساً وعشرين دقيقة ، وفي حضيضه في غاية البعد الشمالي خمساً وثلاثين دقيقة وفي الجنوبي ثمانياً وثلاثين دقيقة ؛ وميل المرّيخ في ذروته في غاية البعد الشمالي اثنتين وعشرين دقيقة وفي الجنوبي سبعاً وعشرين دقيقة ، وفي حضيضه في غاية البعد الشمالي ثلاثة أجزاء واثنتين وعشرين دقيقة وفي الجنوبي ستة أجزاء وعُشر جزء . وأما السفليان فالزهرة ما دام مركزها في فلك الأوج هابطاً مالت ذروتها إلى الشمال وحضيضها إلى الجنوب وفي النصف الآخر بالعكس ، وعطارد ما دام مركزه هابطاً مالت ذروته إلى الجنوب وحضيضه إلى الشمال وفي النصف الآخر بالعكس . وزاوية تقاطُع السطحين عند المنتهي إلى الغاية للزهرة جزءان ونصف ولعطارد ستة أجزاء وربع . ولذلك يُرى ميلُ ذروة الزهرة في غايتي البعدين جزءاً ودقيقتين وميل حضيضها ستة أجزاء وثلاثاً وعشرين دقيقة ، وميلُ ذروة عطارد في غايتي البعدين جزءاً وثلاثة أرباع وميل حضيضه أربعة أجزاء وأربع دقائق . وهذا العرض يُعرف بالميل . وليس للعلوية غير هذين العرضين .

[٥] وأما في السفليين فالقطر المارّ بالبعدين الأوسطين المقاطع للقطر الأول على قوائم لا يثبت في سطوح الأفلاك المائلة ولا يكون في سطوح المثلة إلا عند كون مركزي تدويريهما مع إحدى العقدتين . وبعد مفارقتهما الرأسَ فالطرف المتأخر من ذلك القطر – ويُعرف بالمسائي – ينحرف إلى الشمال والطرف المتقدم – ويعرف بالصباحي – إلى الجنوب إلى أن ينتهيا إلى منتصف ما بين الرأس والذنب ، وهناك يكون الأوج للزهرة ومقابلته لعطارد فينتهي الانحرافان إلى الغاية . ثم يجاوز المركزان المنتصف وينتقص

until they disappear when the [centers] arrive at the tail. After leaving the tail, the reverse will occur, namely the evening endpoint will slant toward the south, while the morning endpoint slants toward the north until the [centers] complete their revolutions. The size of the angle of intersection of the plane of the epicycle with a plane passing through its center and parallel to the ecliptic equator is, at the maximum slant, $3\frac{1}{2}°$ for Venus and 7° for Mercury. Therefore, the slant of Venus, due to this [angle], will appear at the apogee and perigee to be $2\frac{1}{2}°$ in either direction; the slant of Mercury at the apogee will be $2\frac{1}{4}°$ in either direction and at the perigee $2\frac{3}{4}°$. This latitude is known as the **slant** [*inḥirāf*], the **slope** [*wirāb*], the **twist** [*iltiwā'*], and the **winding** [*iltifāf*].

[6] Each of these motions requires the establishment of a mover, which was not referred to by the Ancients. We shall state [below] what has reached us concerning this matter from the teachings of recent [astronomers], if God / Most Exalted / so wills. The quantities given in this chapter are derived from observation and calculation based on what is stated in the *Almagest*.

CHAPTER ELEVEN

An Indication of the Solution—of That Which Is Amenable to Solution—of the Difficulties Referred to Previously That Arise from the Aforementioned Motions of the Planets

[1] As for the first difficulty, which was cited [in connection] with the configuration of the moon's orbs, no statement concerning it has reached me from my predecessors. In this matter, I myself have devised what I shall now present.

[2] Let us set forth for that [purpose] a lemma, which is as follows: if two coplanar circles, the diameter of one of which is equal to half the diameter of the other, are taken to be internally tangent at a point, and if a point is taken on the smaller circle—and let it be at the point of tangency—and if the two circles move with simple motions

/12/ Most Exalted] β, M = –α.

الانحرافان إلى أن ينعدما عند وصولهما إلى الذنب . وبعد مفارقتهما الذنب بالعكس من ذلك ، أعني ينحرف المسائي إلى الجنوب والصباحي إلى الشمال إلى أن تتم دورتهما . ومقدار الزاويةِ التي عليها يقاطع سطحُ التدوير سطحاً يمرّ بمركزه ويوازي منطقةَ البروجِ إذا كان الانحرافان في الغاية ثلاثةُ أجزاء ونصف للزهرة وسبعةُ أجزاء لعطارد . فيُرى بحسبها انحراف الزهرة في الجهتين عند الأوج والحضيض جزأين ونصفاً ، وانحراف عطارد في الجهتين عند الأوج جزأين وربعاً وعند الحضيض جزأين وثلاثة أرباع . وهذا العرض يعرف بالانحراف والوراب والالتواء والالتفاف .

[٦] وكل واحد من هذه الحركات محوج إلى إثبات مُحرك لها لم يذكره القدماء ، وسنذكر ما انتهى إلينا من أقوال المتأخرين فيها إن شاء الله / تعالى / . والمقادير المذكورة في هذا الفصل مستخرجة من الرصد والحساب على ما ذكر في المجسطي .

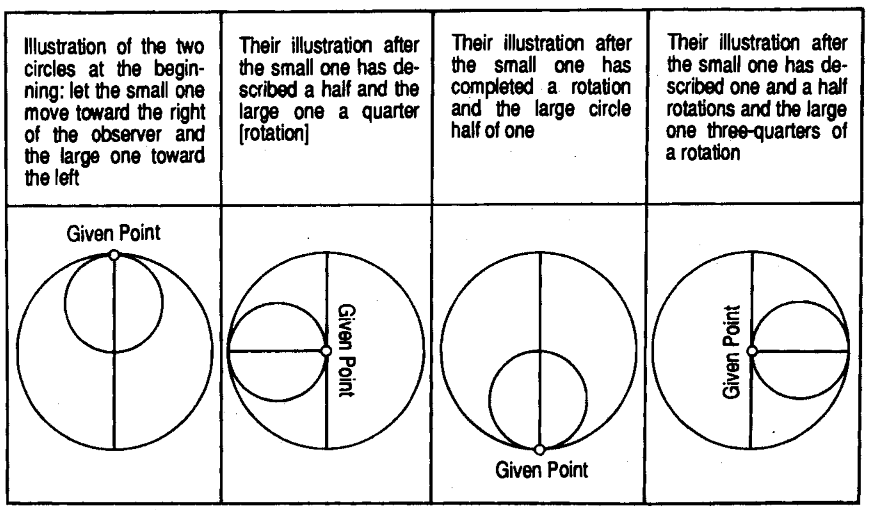
الفصل الحادي عشر في الإشارة إلى حلّ ما ينحلّ من الإشكالات الواردة على حركات الكواكب المذكورة التي سبقت الإشارة إليها

[١] أما الإشكال الأول المذكور في هيئة أفلاك القمر فلم يصل إليّ فيه ممن سبقني كلام ، وأنا استنبطتُ فيه ما أذكره ههنا .

[٢] ولنقدم لذلك مقدمة هي هذه : إذا كانت دائرتان في سطح واحد قطرُ إحداهما مساوٍ لنصف قطر /الأخرى/ ، وفُرضتا متماستين من داخل على نقطة ، وفُرضت نقطةٌ على الدائرة الصغيرة ولتكن عند نقطة التماسّ ، ثم تحركت الدائرتان حركتين

/12/ ...] M , β = α− . /21/ ...] M , β = الآخر] α .

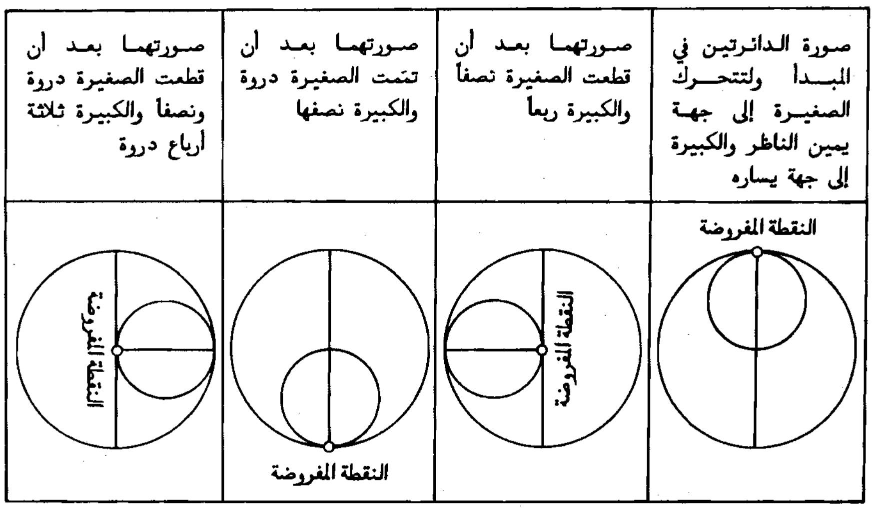
in opposite directions in such a way that the motion of the smaller [circle] is twice that of the larger so the smaller completes two rotations for each rotation of the larger, then that point will be seen to move on the diameter of the large circle that initially passes through the point of tangency, oscillating between its endpoints. Let us illustrate this with four drawings so that one may conceive from them how this [may occur], / and they are these: /



[Fig. T11]

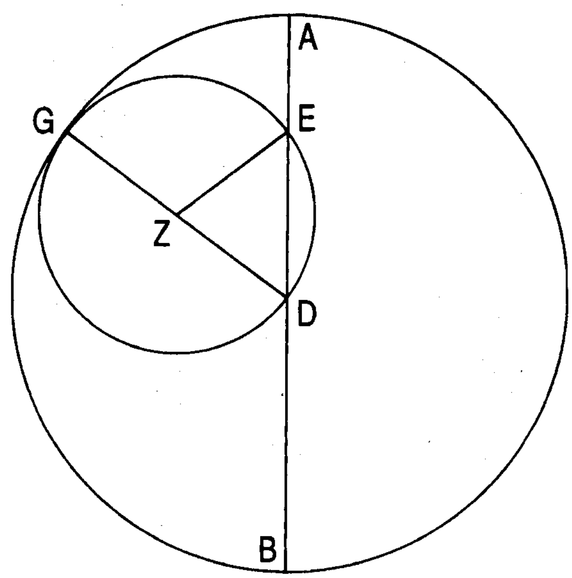
/5/ and they are these] $\beta = -\alpha$, –M.

بسيطتين متخالفتين في الجهة على أن تكون حركة الصغيرة ضعف حركة الكبيرة فتتم للصغيرة دورتان مع دورة واحدة للكبيرة ، رُئيت تلك النقطة متحركة على قطر الدائرة الكبيرة المارّ بنقطة التماسّ أولاً مترددة بين طرفيه . ولنُصوِّر لها صوراً أربعة يتوهم منها كيف ذلك ، / وهي هذه / :



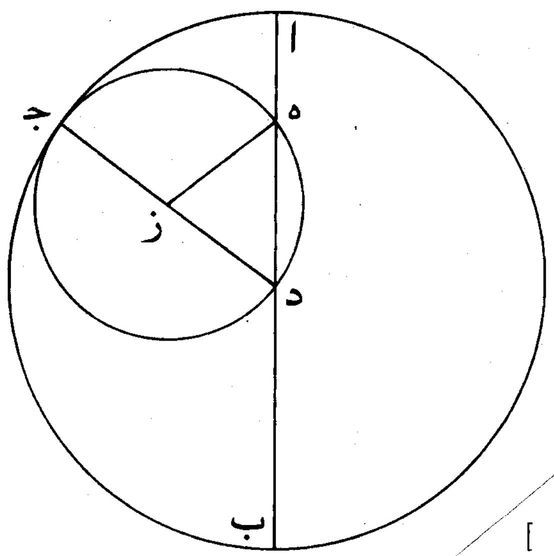
[شكل ١١]

. −α , −M = β [… /5/



[Fig. T12]

[3] In order to prove that the point does not deviate at all from the line (even though it was not our intention to provide geometrical proofs in this compendium), let the larger be circle ABG, its diameter AB, its center D, and the smaller be circle GED, its diameter GD, its center Z; the given point is E. Initially let diameter GD coincide with line AD and point G with point A, and let E be there with the two [points]. Then let circle GED move in the direction GE and let point E be moved along due to its motion until it has described, let us say, arc GE. And let circle AGB move [concurrently] in the direction AG with half the former's motion, and let the endpoint of diameter DG be moved along until it has described arc AG, which will then be equal [*lit.*, similar] to half of arc GE. We connect E to Z and E to D. Then angle GZE is twice angle GDA on account of the two motions, and it is also twice this [angle] because it is an exterior [angle] of triangle EZD and equal to the two interior [angles] ZED and ZDE, which are equal because the two legs ZE and ZD are equal. Thus the two angles GDE and GDA are equal, and line DE coincides with line DA. Therefore point E is on diameter BA, not deviating from it, and the same will be the case in all other positions. Hence point E will continually oscillate between the endpoints of line AB and will not deviate from it.



[شكل ١٢]

[٣] ولبيان أنّ النقطة لا تزول عن الخط أصلاً _ وإن لم نكن نقصد إيراد البراهين الهندسية في هذا المختصر _ فلتكن الكبيرةُ دائرةَ ا ب جـ وقطرها ا ب ومركزها د ، والصغيرة دائرة جـ ه د وقطرها جـ د ومركزها ز والنقطة المفروضة ه . ولينطبق أولاً قطر جـ د على خط ا د ونقطة جـ على نقطة ا ولتكن ه هناك معهما. ثم لتتحرك دائرة جـ ه د في جهة جـ ه ولتنتقل بحركتها نقطة ه إلى أن تقطع قوسَ جـ ه مثلاً . ولتتحرك معها دائرةُ ا جـ ب في جهة ا جـ نصف تلك الحركة ولينتقل طرف قطر د جـ إلى أن يقطع قوسَ ا جـ فهي شبيهة بنصف قوس جـ ه . ونصل ه ز ه د . فزاوية جـ ز ه ضعف زاوية جـ د ا لأجل الحركتين ، وهي أيضاً ضعفها لكونها خارجة من مثلث ه ز د ومساوية لداخلتي ز ه د ز د ه المتساويتين لتساوي ساقي ز ه ز د . فإذن زاويتا جـ د ه جـ د ا متساويتان وخط د ه منطبق على خط د ا ، فنقطة ه إذن على قطر ب ا غير زائلة عنه وكذلك في سائر الأوضاع . فإذن نقطة ه مترددة دائماً بين طرفي خط ا ب غير زائلة عنه .

[4] If we wish, we may make the two circles the [inner] equators of two solid orbs. / What must be intended by the small circle is the circuit of the epicycle center and by the large circle a circle whose radius measures the same as the diameter of the small circle. / Then if we replace the point with a given sphere and we wish the diameter of the given sphere to coincide always with the diameter of the large sphere and not deviate from its position, then we assume another sphere enclosing the given one and moving with exactly the same motion and direction as the large [sphere] so as to restore the diameter to its position in the amount by which the excess of the motion of the small [sphere] over the large [sphere] causes it to deviate. / It is [also] stipulated for it that the diameter of the small circle be half the diameter of the large circle and that it always pass through its center. / Thereupon the given sphere is found to move upon a straight line that coincides with its diameter, oscillating between its two endpoints and not deviating from that coincidence.

[5] When this lemma is established, let us then place the moon's epicycle in the position of the given sphere. / Its center is the point E and its circumference has the same dimension as the moon's epicycle. / Let us assume another sphere of any appropriate thickness that encloses it and maintains its position (it should not be large lest it occupy too much space), and two [other] spheres, one of which, the deferent for these two, takes the place of the small sphere and its diameter is the same size as the eccentricity, and the other takes the place of the large [sphere] and contains all the others / and its center is that of a circle that the epicycle center touches at its farthest and nearest distances. / Its diameter is / then / twice the eccentricity. Let us then assume that the large [sphere] is within the thickness of a concentric deferent that is enclosed by the inclined [orb] in such a way that the enclosing [orb] of the epicycle, which is inside it, is tangent to the convex surface of the deferent / at the [epicyclic] apex. / Let the diameter of the deferent that passes through the point of tangency be considered fixed.

/2–4/ What must be intended...small circle] β = What must be intended by the [inner] equator of the small one is the circuit of the epicycle center within it and by the equator of the large one a circle whose radius measures the same as the diameter of the equator of the small one] α, M. /9–10/ It is [also] stipulated...through its center] β = It is [also] stipulated for it that the diameter of the equator of the small one be half the diameter of the equator of the large one and that it always pass through its center] α, M. /14–15/ Its center is the point E...moon's epicycle] β = –α, –M. /19–20/ and its center...nearest distances] β = –α, –M. /20/ then] β = –α, –M. /22/ at the [epicyclic] apex] β = near the [epicyclic] apex] α, M.

[٤] وإن أردنا جعلنا الدائرتين منطقتي فلكين مجسّمين ، وينبغي أن يكون المرادُ من / الدائرة / الصغيرة مدارَ مركز / التدوير / ومن / الدائرة / الكبيرة دائرة نصف قطرها بقدر قطر / الدائرة / الصغيرة . ثم إن جعلنا بدل النقطة كرة مفروضة ، وأردنا أن يكون قطر الكرة المفروضة دائماً منطبقاً على قطر الكرة الكبيرة غير زائل عن وضعها ، فرضنا كرة أخرى محيطة بالمفروضة متحركة مثل حركة الكبيرة بعينها وفي جهتها لترد القطر إلى وضعه بقدر ما يزيله فضل حركة الصغيرة على الكبيرة . ويشترط فيها أن يكون قطر / الدائرة / الصغيرة نصف قطر / الدائرة / الكبيرة ماراً بمركزها أبداً . وحينئذ تُرى الكرة المفروضة متحركة على خط مستقيم منطبق على قطرها ، مترددة بين / طرفيه / غير زائلة عن ذلك الانطباق .

[٥] وإذا تقررت هذه المقدمة فلنُقِم تدوير القمر مكان الكرة المفروضة / مركزه نقطة ة ومحيطه بالبعد الذي يكون تدوير القمر / . ولنفرض كرة أخرى محيطة به حافظة لوضعه بأي قدر من الثخن يتّفق وينبغي أن لا تكون عظيمة لئلاّ تشغل مكاناً كثيراً ، وكرتين إحداهما حاملة لهما بدل الكرة الصغيرة قطرها بقدر ما بين المركزين والأخرى بدل الكبيرة متضمنة للجميع / مركزها مركز دائرة يماسّها مركز التدوير في بعديها الأبعد والأقرب فيكون / قطرها بقدر ضعف ما بين المركزين . ثم لنفرض الكبيرة في ثخن حامل موافق المركز يحيط به المائل بحيث يكون المحيط بالتدوير الذي فيه مماساً لمحدب الحامل / عند / الذروة . وليتوهم قطر الحامل ماراً بنقطة التماسّ ثابتاً .

/2/ ...] β = منطقة] M , α . /3/ التدوير] β = التدوير فيها] M , α . الدائرة] β = منطقة] M , α . /4/ ...] β = منطقة] M , α . /9/ ...] β = منطقة] M , α . /10/ ...] β = منطقة] M , α . /12/ ...] = طرفيها] M = طرفها] α . /14–15/ ...] β = –M , –α . /19–20/ ...] β = –M , –α . /22/ ...] β = بقرب من] M , α .

[6] We now assume their motion: the epicycle with its own proper motion, the enclosing and large [spheres] with motions [such that] their rotations are completed with / a rotation / of the deferent, and the small sphere with a motion [such that] its rotation is completed with a half rotation of the deferent. We assume the deferent to move with the motion of the moon's center sequentially and the inclined [orb] with the motion of the moon's apogee countersequentially as is the case with the parecliptic.

[7] If this be the case, the diameter of the epicycle will descend as it adheres to the diameter of the large sphere. The diameter of the large [sphere] departs from its coincidence with the diameter of the deferent that passes through the tangent point cited above; its endpoint, however, will always be tangent to the circumference of the deferent, and the apex of the epicycle will be adjacent to that endpoint. The deferent will, by its motion, turn all these spheres. The epicycle center will then undergo a motion on a circuit resembling the circumference of a circle. Thus if the deferent moves through half a rotation, the epicycle will reach the other endpoint of the large sphere's diameter, which will once more coincide with the diameter of the deferent passing through the tangent point, and the epicycle's enclosing [sphere] touches the concave [surface] of the deferent near the epicyclic perigee. The epicycle will then be at its nearest distance from the center of the World, and that diameter will pass through the farthest and nearest distances. As the orbs continue to move, the epicycle will begin to ascend on the above diameter and move away from the center of the World until it reaches the farthest distance, which is the starting point that it originally departed from. The epicycle [thus] completes its circuit, which takes the place of the eccentric insofar as the inclined [orb] touches one of its points, namely the farthest distance from the center of the World, and [another] point, namely the nearest distance from it, is directly opposite from it. The difference between the farthest and nearest distances is in the amount of twice the eccentricity. Despite this, the motion [of the epicycle center]

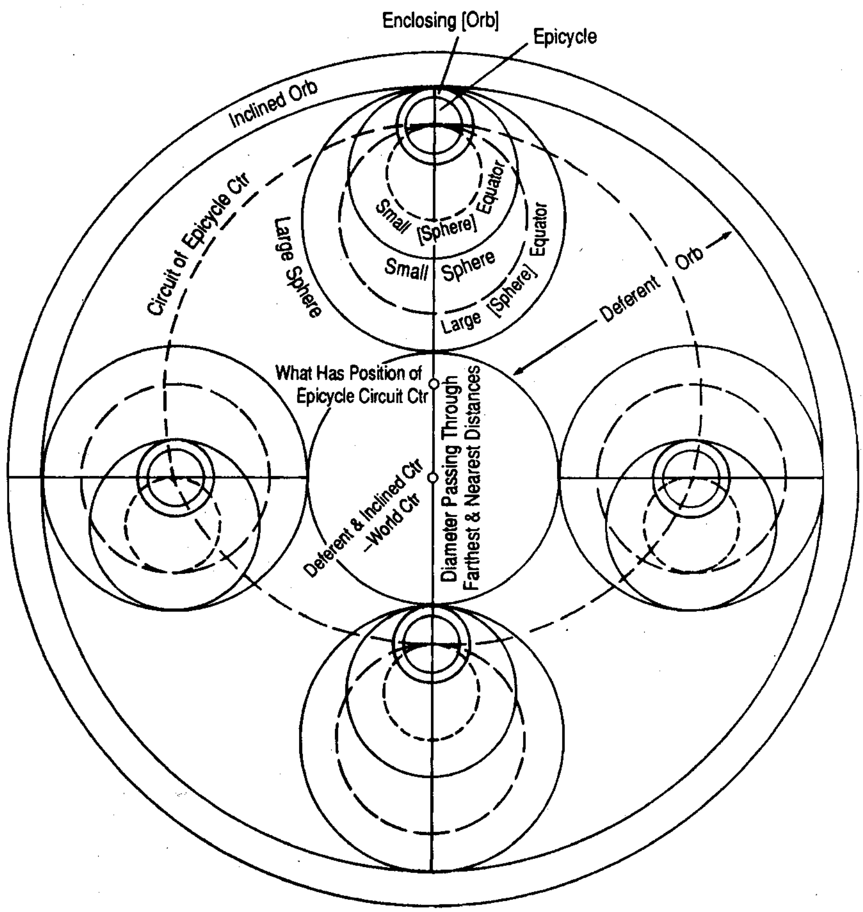
/2/ a rotation] β, M = a completion of a rotation] α.

[٦] ثم نفرضها متحركة : أما التدوير فبحركته الخاصة به ، والمحيطة والكبيرة بحركتين يتم دورهما مع / دورة / للحامل ، والصغيرة بحركة يتم دورها مع نصف دورة للحامل . ونفرض الحامل متحركاً بحركة مركز القمر إلى التوالي ، والمائل بحركة أوج القمر إلى خلافه كالممثل .

[٧] وإذا كان كذلك نزل قطر التدوير ملازماً لقطر الكرة الكبيرة ، وزال قطر الكبيرة عن انطباق قطر الحامل المارّ بنقطة التماسّ المذكورة لكن يكون طرفه مماساً لمحيط الحامل أبداً ، وتلي الذروةُ من التدوير ذلك الطرفَ . وأدار الحاملُ بحركته جميعَ تلك الكرات ، فحدث لمركز التدوير حركةٌ على مدار شبيه بمحيط دائرة حتى إذا تحرك الحامل نصف دورة وصل التدوير إلى الطرف الآخر من قطر الكرة الكبيرة ، وانطبق قطرها ثانياً على قطر الحامل المارّ بنقطة التماسّ ، وتماسّ المحيطةُ بالتدوير مقعرَ الحامل بقرب من حضيض التدوير ؛ فكان التدوير في البعد الأقرب من مركز العالم وكان ذلك القطر ماراً بالبعدين الأبعد والأقرب . ثم تتحرك الأفلاك ويأخذ التدوير في التصاعد على القطر المذكور والتباعد عن مركز العالم إلى أن ينتهي إلى البعد الأبعد وهو المبدأ الذي فارقه أولاً . ويتم للتدوير مداره وهو يقوم مقام الخارج المركز من حيث يماسّ المائل نقطة منه هي البعد الأبعد من مركز العالم وتقابلها نقطة هي البعد الأقرب منه ، ويكون الفضل بين البعد والقرب بقدر ضعف ما بين المركزين . وتكون مع ذلك حركتُه

/2/ دورة] M , β = تمام دورة] α .

about the center of the World is uniform. As was the case previously, the apogee will meet it due to the motion of the inclined [orb]. Its illustration is thus:

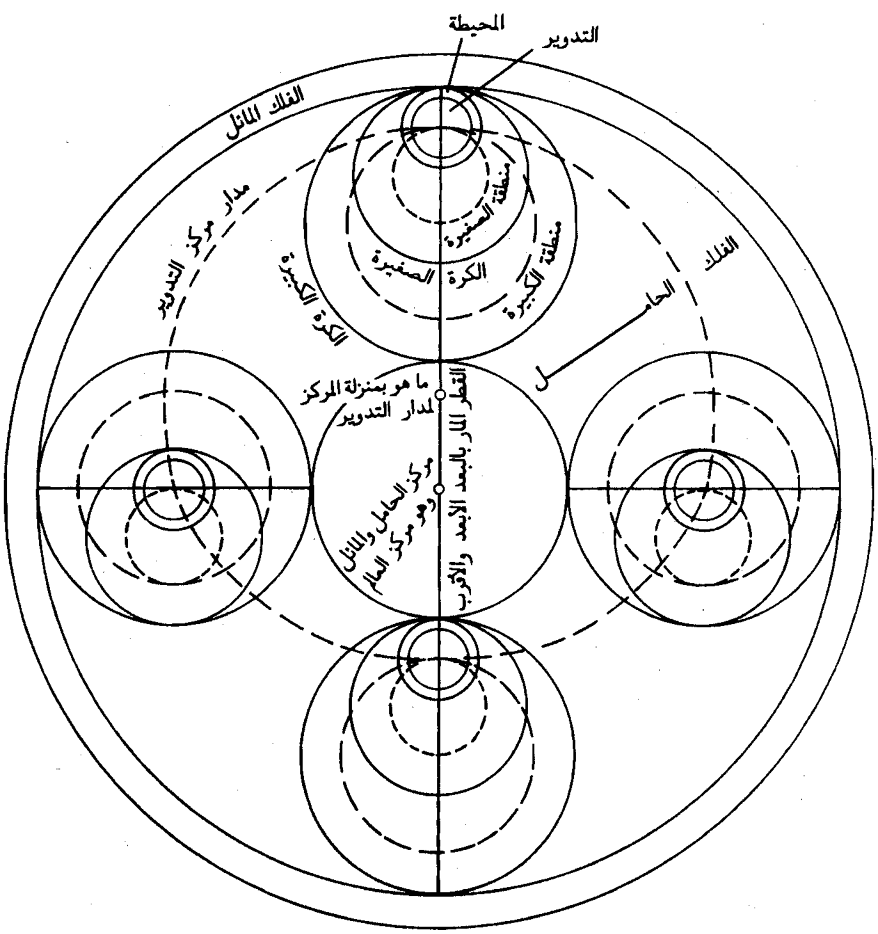


Solid spheres are in red; circles are in black.
[ed. note: solid lines = red, broken lines = black]

[Fig. T13]

[8] This then is my proposal concerning this matter. It is, however, brought about by three additional orbs over and above what has been stated before. The concentric deferent here takes the place of the eccentric orb that was used previously.

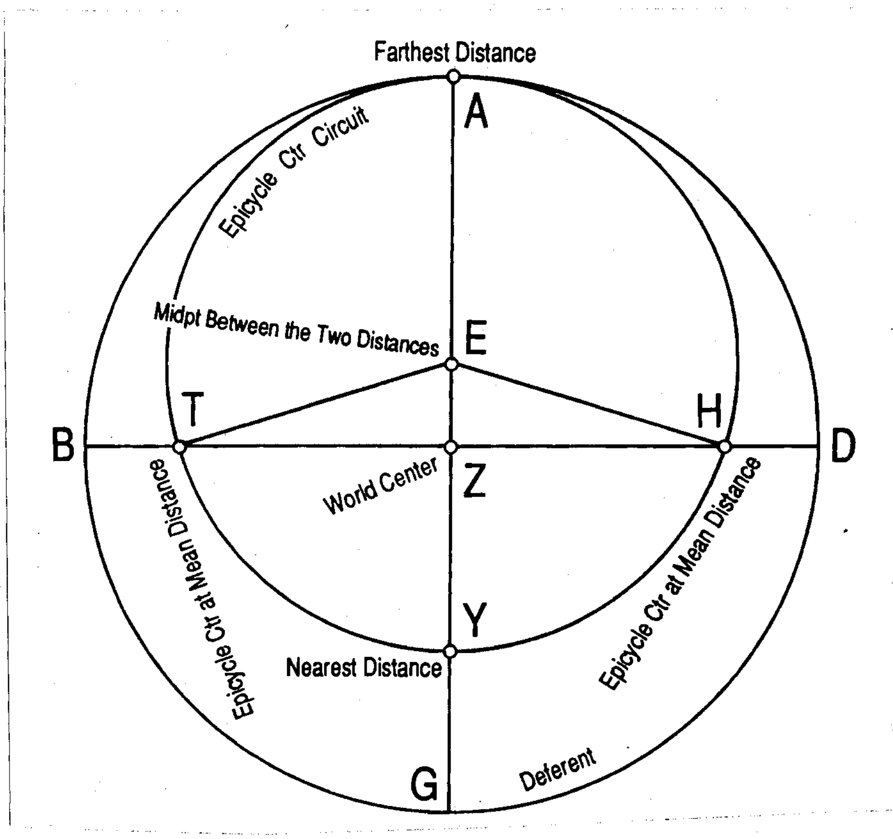
حول مركز العالم متشابهة . ويستقبله الأوجُ بحركة المائل كما كان أولاً . وصورته هكذا :



الكرات المجسمة ما هي بالحمرة والدوائر ما هي بالسواد

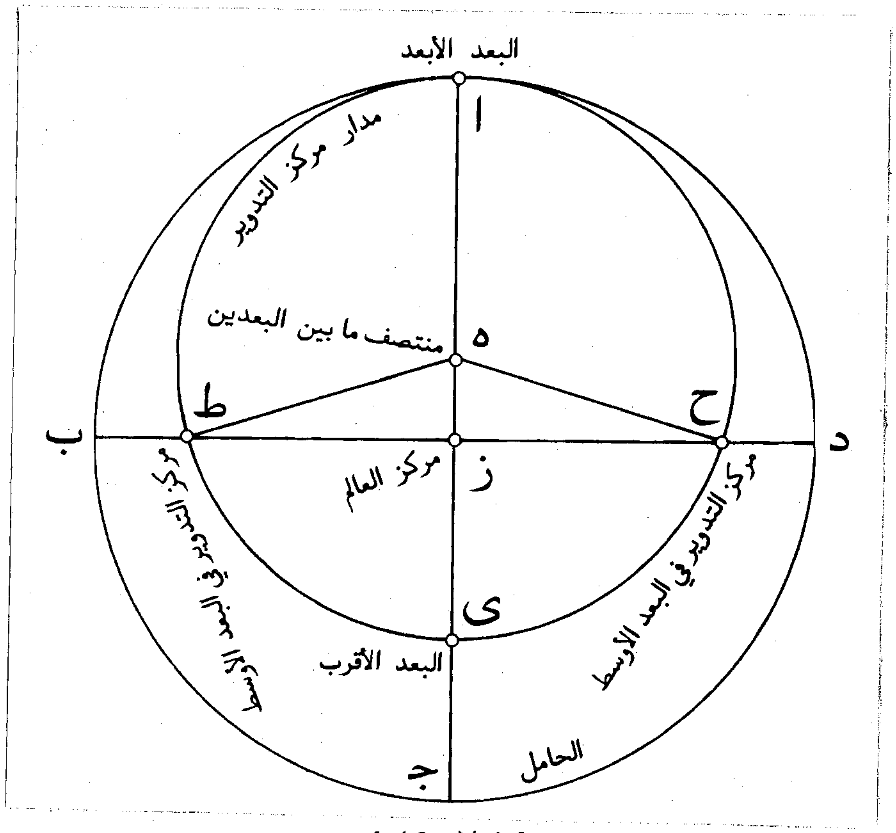
[شكل ١٢]

[٨] فهذا ما عندي فيه . وإنما يتم ذلك بثلاثة أفلاك زائدة على ما قيل ، ويكون الحامل الموافق المركز بدل الفلك الخارج المركز المذكور .



[Fig. T14]

[9] We have stated that the circuit of the epicycle center resembles a circle, but we did not say that it was a circle since it is not a true circle. The proof of this is [as follows]: at quadrature to the apogee, the epicycle will have descended along one-half of the line upon which it oscillates, which amount is equal to the eccentricity. At that point the distance between the center of the World and the center of the epicycle will come to half the distance between the farthest and nearest distances. But for the circuit to be a circle, it would be necessary for the [distance] from the midpoint between the farthest and nearest distances to the epicycle center to be that amount. Thus the aforementioned circuit



[شكل ١٤]

[٩] وإنما قلنا إنّ مدار مركز التدوير شبيه بدائرة ولم نقُل إنه دائرة لأنه لا يكون دائرة حقيقية . بيان ذلك أنّ التدوير ينزل عند تربيع الأوج نصف الخط الذي يتردد عليه ـ وهو بقدر ما بين المركزين ـ ويبقى البعد بين مركز العالم ومركز التدوير حينئذ بقدر نصف ما بين البعد الأبعد والأقرب ؛ وكان من الواجب أن يكون من منتصف ما بين البعد الأبعد والأقرب إلى مركز التدوير ذلك القدر حتى يكون المدار دائرة . فإذن المدار

is not a circle, and / the [distance] between the two mean distances on it is longer than the [distance] between the other two distances, / *i.e.* the farthest and the nearest. For this reason this method is not completely equivalent to the model they use; however, the difference between the calculations resulting from this method and the model they use does not come to $\frac{1}{6}$ of a degree. The maximum difference will occur at the midpoints between the quarter marks (*i.e.* conjunction, opposition, and quadratures). At those [points] this will be an imperceptible amount in the moon's true position.

[10] This same method may also be postulated for the upper planets and for Venus. We thus make the / diameter of the small circle / equal to the distance between the deferent and equant centers, and the / diameter of the large circle / twice that [amount]. We may then assume an eccentric orb in the thickness of the pareclíptic whose center is that of the equant. We also assume the large sphere, along with what is in it, to be in the thickness of this [latter] orb so that the motion about the equant center is uniform. The distances of the epicycle center from the center of the World are the same as resulted from the [Ptolemaic] deferent without there being a difference that might disturb any of the circumstances of these planets. The difficulty concerning them may then be resolved by the addition of three spheres for each one of them, with the solid equant orb replacing the eccentric deferent of the previous [model].

[11] As for Mercury, it has not yet been possible for me to conceive how it should be done. For it is difficult to conceive what could cause the motion to be uniform about a point whereby the moved object in its motion toward and away from it is composed of multiple motions. If God Most High should enable me to accomplish this, I will append it / to that place, / God willing.

[12] Concerning the moon's point of alignment, one of the practitioners of this science has said: Another orb for the moon must be established with this point as its center so that

/1–2/ the [distance]...the other two distances] α, β = the [distance] between the two mean distances on it and the midpoint of the other two distances is longer than half the [distance] between the other two distances] M. /9/ diameter of the small circle] β = diameter of the equator of the small sphere] α, M. /10/ diameter of the large circle] β = diameter of the equator of the large sphere] α, M. /21/ to that place] β = to this place] α, M.

المذكور ليس بدائرة ، / وما بين البعدين الأوسطين فيه أطول مما بين البعدين الآخرين / ، أعني الأبعد والأقرب . ولهذا السبب لا يكون هذا الوجه مطابقاً للأصل الذي يعملون عليه مطابقة تامة لكنّ التفاوت بين ما يُخرِجه الحساب في هذا الوجه وبين ما يخرجه الحساب على الأصل الذي يعملون عليه لا يبلغ سدس درجة ، وغايته تكون في منتصف الأرباع ، أعني الاجتماع والاستقبال والتربيعين ، وذلك غير محسوس في تقويم القمر هناك .

[١٠] وهذا الوجه بعينه يمكن أن يُفرض في الكواكب العلوية والزهرة . فنجعل قطر / الدائرة / الصغيرة بقدر ما بين مركزي الحامل ومعدل المسير ، وقطر / الدائرة / الكبيرة ضعف ذلك . ثم نفرض في ثخن الممثل فلكاً خارج المركز مركزها [!] مركز معدل المسير ، ونفرض الكرة الكبيرة بما فيه[!] في ثخن ذلك الفلك حتى تكون الحركة حول مركز معدل المسير متشابهة . وأبعاد مركز التدوير عن مركز العالم كما كان يقتضيه الحامل من غير تفاوت يختل به شيء من أحوال تلك الكواكب . فينحل الإشكال فيها بزيادة ثلاث أكر في كل واحد منها ، ويكون فلك معدل المسير المجسّم بدل الخارج المركز الحامل المذكور .

[١١] وأما في عطارد فلم يتيسر لي بعدُ توهُّمُ ذلك كما ينبغي ، فإنّ توهُّمَ السبب في تشابه الحركة حول نقطة تتركب حركة المتحرك في القرب إليها والبعد عنها تركباً كثيراً متعذّر . وإن يسّر الله تعالى ذلك ألحقتُه / بذلك / الموضع إن شاء الله تعالى .

[١٢] وأما في نقطة محاذاة القمر فقد قال بعض أهل هذا العلم : ينبغي أن يُثبَت فلكٌ آخر للقمر تكون النقطة مركزه ليحاذي

/1–2/ ...] α , β = وما بين البعدين الأوسطين فيه وبين منتصف البعدين الآخرين أطول من نصف ما بين البعدين الآخرين] M . /9/ ...] β = منطقة الكرة] α , M . /10/ ...] β = منطقة الكرة] α , M . /21/ ...] β = بهذا] α , M .

the diameter of the epicycle passing through the mean apex and perigee will, by means of that orb's motion, stay aligned with its center. He did not, however, show how this motion might occur so as not to disturb the existing motions of the moon.

[13] My own opinion is that just as one may imagine the diameters passing through the apices and the perigees of the epicycles of the five planets to have latitudinal inclinations by which the planes of the equators of their epicycles are displaced from the planes which they were in at the time they had no latitude, one may likewise conceive that diameter of the equator of the lunar epicycle to have a longitudinal inclination. The equator would not, however, become displaced by it from the plane that it is in but instead its parts would undergo a displacement as if they were twisting upon themselves. So as to make this completely clear, let us imagine a line passing through the point of alignment and perpendicular to the diameter passing through the moon's centers and the point of alignment. This [line] will thus divide the deferent into two segments, one of which, the larger, is bisected by the apogee, and the second, the smaller, is bisected by the perigee. When the aforementioned diameter of the epicycle has departed from the diameter passing through the centers, after having coincided with it on the side of the apogee, its apex endpoint will incline in the counter-sequence [of the signs], while the perigee endpoint will incline in the sequence [of the signs]. This inclination will continue to increase until this diameter coincides with the perpendicular passing through the point of alignment. At that point its inclination will be at its maximum. It will thereafter begin to decrease until it disappears when this [diameter of the epicycle] coincides with the diameter passing through the centers on the side of the perigee. When it departs from this [diameter], its apex endpoint will then incline in the sequence [of the signs], whereas its perigee endpoint will incline in the counter-sequence [of the signs] until the [diameter] once more coincides with the perpendicular passing through the point of alignment. At this point its inclination will [again] be at its maximum. It will then begin to decrease until it disappears when the [diameter] reaches the starting point from which it had departed originally, *viz.* when it coincides with the diameter passing through the centers on the side of the apogee. The apex endpoint of the [diameter] will thus move in the counter-sequence [of the signs] in the larger of the two previously mentioned segments, and its maximum speed will occur at the midpoint of the segment at the apogee. In the smaller segment, [its motion will be] in

قطرُ التدوير المارّ بالذروة والحضيض الأوسطين بحركة ذلك الفلك دائماً نحو مركزه . ولم يُبيّن كيفيةَ تلك الحركة على وجه لا يُخِل بالحركات الموجودة للقمر .

[١٢] وأنا أقول : كما توهّم لأقطار تداوير الكواكب الخمسة المارة بالذُرى والحضيضات ميولٌ عرضية تَخرج بها سطوحُ مناطق تداويرها عن السطوح التي كانت فيها وقت انعدام العرض ، فليتوهم لذلك القطر من منطقة تدوير القمر ميلٌ طولي لا تخرج المنطقة به عن / سطحها الذي هي / فيه لكن يحصل لأجزائها زوال عن مواضعها كأنها تلتوي على نفسها . وليتوهم لتمام تقرير ذلك خط يمرّ بنقطة المحاذاة ويكون عموداً على القطر المارّ بمراكز القمر وبنقطة المحاذاة ، فهو يفصل الحامل إلى قطعتين إحداهما أعظم وهي التي ينصّفها الأوجُ والثانية أصغر وهي التي ينصّفها الحضيض . فالقطر المذكور من التدوير إذا فارق القطرَ المارّ بالمراكز بعد انطباقه عليه في جانب الأوج مال طرف الذروة منه إلى خلاف التوالي وطرف الحضيض إلى التوالي . ولا يزال يزيد ذلك الميل إلى أن ينطبق القطر المذكور على العمود المارّ بنقطة المحاذاة فيكون ميله حينئذ في الغاية ، ثم يأخذ في التناقص إلى أن ينعدم عند انطباقه على القطر المارّ بالمراكز من جانب الحضيض ، ثم إذا فارقه مال طرف الذروة منه إلى التوالي وطرف الحضيض إلى خلاف التوالي إلى أن ينطبق على العمود المارّ بنقطة المحاذاة ثانياً ويصير حينئذ ميله في الغاية ، ثم يأخذ في التناقص إلى أن ينعدم عند انتهائه إلى المبدأ الذي فارقه أولاً وهو كونه منطبقاً على القطر المارّ بالمراكز من جانب الأوج . فكان طرف الذروة منه / متحركاً / إلى خلاف التوالي في القطعة العظمى من القطعتين المذكورتين وغاية سرعته في منتصف القطعة عند الأوج ، وفي القطعة الصغرى إلى

/8/ [...] β = سطحه الذي هو] α , M . /23/ [...] β = متحرك] α , M .

the sequence [of the signs], and its maximum speed will occur at the midpoint of this [segment] at the perigee. [The motion of] the perigee [endpoint] in the two [segments] will be the opposite of this.

[14] This diameter will therefore need a mover, and the discussion concerning it is the same as for the movers of the aforementioned diameters of the epicycles. Let us then set forth what has been said about the latter. Ptolemy states in the *Almagest* that the endpoints of the diameters passing through the apices and the perigees of the five [planetary] epicycles revolve along small circles whose planes are perpendicular to the planes of the equators of the epicycles. Their radii are equal to the maximum inclinations of these diameters and their motions are the same as the motions of the epicycle centers upon their deferents. And just as the motions of the epicycle centers are not uniform with respect to the centers of their deferents but rather with respect to other points, so also are these motions nonuniform with respect to the centers of the aforementioned small circles but instead are uniform about other points, the ratio of whose distances from the centers of the small circles to the radii of the small circles is the same as the ratio of the distances from the deferent centers of the points at which there occurs uniform motion of the epicycle centers to the radii of the deferents. This is in order that the arcs described by the endpoints of the diameters of the epicycles on the [small circles] be the same as those described by the centers of the epicycles on the orbs upon which they move. Thus there results a displacement of the endpoints of the epicycle diameters from the planes in which they have no inclination. This will be in each direction in the amount of the radii of the aforementioned small circles, which are equal to the maximum inclinations. He states that a similar situation must also be conceived for the endpoints of the epicycle diameters, known as the morning and evening endpoints, that pass through the mean distances in the case of the two lower planets.

[15] My own opinion is that this explanation is inadequate for our purposes on three counts: (1) it does not take into account the configuration [*hay'a*] of those bodies that are the principles for these motions; (2) it compounds the difficulty that we are expending all this effort to resolve [by making] the motion uniform about a point other than the center of its circuit; (3) just as the aforementioned small circles bring about latitudinal inclinations,

التوالي وغاية سرعته في منتصفها عند الحضيض . والحضيض فيهما بالضد منها .

[١٤] فإذن هذا القطر يحتاج إلى مُحرك . والقول فيه كالقول في المحرَكات التي تُحرك أقطارَ التداوير المذكورة ، فلنورد ما قيل في ذلك . أما بطلميوس فقد ذكر في المجسطي أنّ أطراف أقطار تداوير الخمسة المارة بالذُرى والحضيضات تدور على دوائر صغار سطوحها قائمة على سطوح مناطق التداوير ، وأنصاف أقطارها بقدر غايات ميول تلك الأقطار وحركاتها مساوية لحركات مراكز التداوير على حواملها . وكما أنّ حركات مراكز التداوير لا تتشابه عند مراكز حواملها وإنما تتشابه عند نقط غيرها ، كذلك تلك الحركات لا تتشابه عند مراكز الدوائر الصغار المذكورة وإنما تتشابه حول نقط غيرها ، نسبة أبعادها عن مراكز الدوائر الصغار إلى أنصاف أقطار الدوائر الصغار كنسبة أبعاد النقط التي تتشابه عندها حركات مراكز التداوير عن مراكز الحوامل إلى أنصاف أقطار الحوامل لتكون القسي التي تقطعها أطراف أقطار التداوير منها شبيهة بما تقطعها مراكز التداوير من الأفلاك التي تتحرك عليها . وحينئذ يلزم خروج أطراف أقطار التداوير عن السطوح التي تكون فيها عديمةَ الميول في الجهتين بقدر أنصاف أقطار الدوائر الصغار المذكورة المساوية لغايات الميول . قال : ومثل ذلك ينبغي أن يتوهم في أطراف أقطار التداوير المارّة بالأبعاد الوسطى المعروفة بالصباحية والمسائية للسفليين .

[١٥] أقول : وهذا البيان ليس بمفيد فيما نحن فيه من ثلاثة أوجه : الأول أنه ليس بمشتمل على هيئة الأجسام التي هي مبادئ تلك الحركات ؛ والثاني أنه يُضعّف الإشكال الذي نجهد جميع هذا الجهد في حلّه ، وهو تشابُه الحركة عند نقطة غير مركز مدارها ؛ والثالث أنّ الدوائر الصغار المذكورة كما تُحدِث الميولَ

they also cause inclinations to occur in longitude by which the positions of the apices and the perigees alter from what they should be with respect to the points with which they are aligned.

[16] Ibn al-Haytham has published a treatise in which he discusses the bodies that produce these motions. For each epicycle he adds two spheres to account for the deviation and, in the case of the two lower planets, two other spheres for the slant. He decided to assume a sphere that surrounds the epicycle having poles at a distance from the endpoints of the diameter passing through the apex and the perigee in the two alternate directions equal to the maximum deviation of this diameter for the planet from the plane in which it has no latitude. He [further] assumes this [sphere] to have a motion equal to that assumed for the previously discussed small circle of that planet so that the endpoints of the above diameter will, due to this motion, move upon a circuit identical to the small circle with a motion that is uniform with respect to a point other than its center as was the case for the small circle. However, as a result of this [sphere's] motion, all parts of the epicycle will also move. This includes the mean diameter, which will be displaced because of this motion; thus its morning endpoint will become the evening one and vice versa. The situation will be the same for the other parts of the epicycle. It is therefore necessary to assume another sphere between this sphere and the epicycle sphere whose poles are at the endpoints of the above diameter, *i.e.* the apex and the perigee. Its motion is assumed to be exactly the same as that given for the first sphere except that it is in the opposite direction so as to return to their proper positions all parts of the epicycle that would otherwise be displaced. There will then be no remaining effect of motion from the first sphere except what had resulted from the motion of the above diameter as well as those [parts] of the plane of the epicycle equator connected with it. One may assume in exactly the same way two other spheres for each of the lower planets to account for the slant so that one will slant the epicycle's mean diameter, while the other will preserve the position of the rest of the epicycle lest the apex become the perigee and the perigee the apex. The epicycle of each of the upper planets will then come to comprise

العرضية فهي تُحدِث ميولاً أيضاً في الطول تتغير بها أوضاعُ الذُرى والحضيضات عند النقط التي تحاذيها عما يجب .

[١٦] وقد أورد ابن الهيثم مقالة ذكر فيها الأجسام التي تحرك هذه الحركات ، فزاد في كل تدوير كرتين لأجل الميل وفي السفليين كرتين أخريين لأجل الانحراف . وتقريره أن يفرض كرة تحيط بالتدوير ويكون لها قطبان بعدهما عن طرفي القطر المارّ بالذروة والحضيض في جهتين متبادلتين بقدر غاية ميل ذلك القطر لذلك الكوكب عن السطح الذي هو فيه يكون عديم الميل . ويفرض لها حركة مثل / الذي / فرضت للدائرة الصغيرة المذكورة التي لذلك الكوكب ليتحرك بحركتها طرفا القطر المذكور على مدار مثل الدائرة الصغيرة بعينها حركة متشابهة عند نقطة غير مركزها كما فرضت للدائرة الصغيرة ؛ لكن تلزم من حركتها حركة جميع أجزاء التدوير حتى القطر الأوسط ، فإنه يزول بتلك الحركة عن وضعه فيصير طرفه الصباحي مسائياً وبالعكس ، وكذلك في سائر أجزاء التدوير . فيجب لذلك أن يفرض كرة أخرى بين هذه الكرة وبين كرة التدوير قطباها طرفا القطر المذكور ، أعني نقطتي الذروة والحضيض . ويفرض لها حركة مساوية للحركة المذكورة في الكرة الأولى بعينها لكنها إلى خلاف تلك الجهة لترد جميعَ أجزاء التدوير التي كادت أن تزول عن وضعها إلى وضعها الواجب ، ولا يبقى فيها من الكرة الأولى أثرُ حركة سوى ما كان يلزم بسبب حركة القطر المذكور وما يتّصل به من سطح منطقة التدوير . وتُفرض لكل واحد من السفليين كرتان أخريان لأجل الانحراف بهذه الصفة بعينها لتُحرّف إحداهما القطرَ الأوسط من التدوير وتحفظ الأخرى وضعَ باقي التدوير كي لا تصير الذروة حضيضاً والحضيض ذروة . فيصير تدوير كل واحد من العلوية مشتملاً على

/9/ ...] β = التي] M , α .

three spheres, while the epicycle of each of the lower planets will have five spheres. What Ptolemy has described will then be accomplished by establishing solid movers. Ibn al-Haytham states that one could reach the same result by assuming truncated orbs [*manāshīr*] instead of spheres, but to set forth something other than a sphere would not be appropriate for the models of this science.

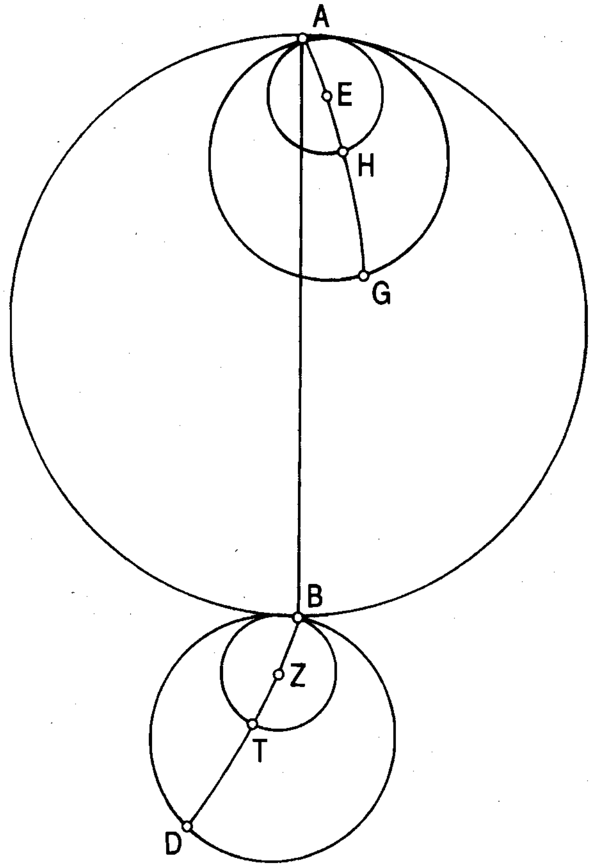
[17] One should note that his intended result could also have been accomplished had he placed the poles of the first assumed sphere at a distance from the poles of the epicycle equal to the distance that he assumed between them and the endpoints of the diameter of the epicycle.

[18] Furthermore, if one adds another sphere for each of the motions and if one conceives on the surface of the sphere a situation similar to what we have described above, namely the oscillation of a point between two endpoints of a straight line, the third of the three objections I have raised against what was stated by Ptolemy would be eliminated. This was the error in longitude resulting from the longitudinal inclination that follows from [his model].

ثلاث أكر ، وتدوير كل واحد من السفليين خمس أكر ؛ ويتم ما ذكره بطلميوس بحسب إثبات المحرّكات الجسمية . وذكر ابن الهيثم أنه لو فرض بدل الأكر مناشير لتم ذلك لكنّ إثبات غير الكرة لا يصح على أصول هذا العلم .

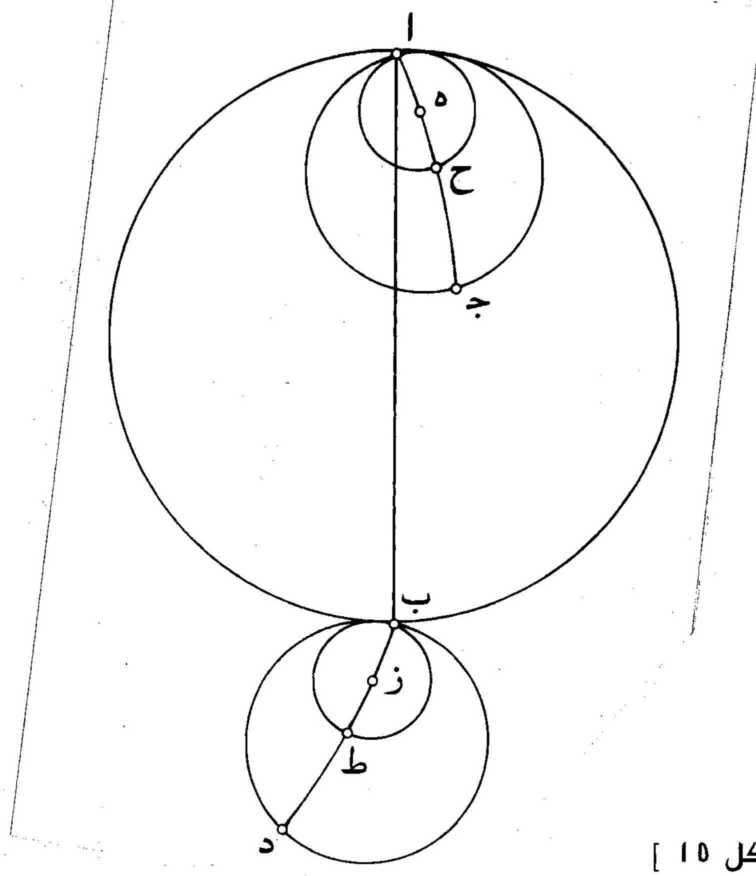
[١٧] واعلم أنه إن جعل قطبي الكرة التي فرضها أولاً على بعد من قطبي التدوير مساوٍ للبعد الذي فرضه بينهما وبين طرفي قطر التدوير لتم مقصوده بذلك أيضاً .

[١٨] وأيضاً إن زيد في كل حركة منها كرة أخرى وتوهّم على سطح الكرة مثل ما ذكرنا من قبل في تردد نقطة بين طرفي خط مستقيم ، زال ما ذكرتُ في الوجه الثالث من الوجوه الثلاثة التي أوردتُها على ما ذكر بطلميوس ، وهو الخلل الحادث في الطول بسبب الميل الطولي اللازم منه .



[Fig. T15]

[19] Let us present a lemma to explain this. Let the epicycle be a sphere with diameter AB. We take a great circle occurring on the epicycle that passes through the poles of the epicycle and through points A and B. Let two arcs AG and BD be on this [great circle]; we cut off AE and BZ from these [arcs] equal to half the maximum inclination in one direction in such a way that points E and Z are also endpoints of another diameter of the epicycle. We assume a sphere surrounding the epicycle, which we call the small [sphere], and we take it to move on two poles that are aligned with these two points. Points A and B will then move as a result of this motion; let their circuits [*lit.*, circuit] intersect arcs AG and BD at points H and T, which will also be at the endpoints of another diameter of the epicycle. We now take another sphere, which we shall call the large [sphere], that moves



[شكل ١٥]

[١٩] ولنورد لبيان ذلك مقدمة ، فليكن التدوير كرة قطرها اب . ونفرض دائرة من العظام التي تقع على التدوير تمرّ بقطبي التدوير وبنقطتي ا ب . ولتكن قوسا اج بد منها ، ونفصل اه بز منهما مساويتين لنصف غاية الميل في إحدى الجهتين على وجه تكون نقطتا ه ز أيضاً طرفي قطر آخر للتدوير . ونفرض كرة تحيط بالتدوير ونسميها الصغيرة ، ونفرضها متحركة على قطبين محاذيين لهاتين النقطتين . فتتحرك نقطتا ا ب بحركتها ، وليقطع مدارهما قوسي اج بد على نقطتي ح ط وهما أيضاً على طرفي قطر آخر للتدوير . ونفرض كرة أخرى نسميها الكبيرة تتحرك

on two poles aligned with these latter two points. The two circuits AH and BT will then move with its motion. Let the two circuits that are tangent to [AH and BT] be circuits AG and BD. We then assume the large sphere to move with a motion equal to that of the epicycle center on its orb surrounding the Earth on which it moves and the small sphere to move with a motion opposite the former in direction and equal to twice it in amount. The result of these two motions is that the endpoints of diameter AB will continually oscillate on arcs AG and BD between their two endpoints so that they will have no inclination in longitude whatsoever in either direction from these [arcs]. When the endpoint A reaches G, endpoint B will be at D. The inclinations of these [points] are in alternate directions. If one then adds to the two [spheres] the sphere enclosing the epicycle that preserves its position so that the morning endpoint of its diameter does not become the evening endpoint and vice versa, the aforementioned motion will be accomplished, and the above error in the third of the three cited objections will be eliminated. There remains only what was stated concerning the second objection, but I have been unable to conceive a manner in which this difficulty would be eliminated. By the above method, we add three spheres for each of the epicycles of the upper planets and six spheres for each of the epicycles of the lower planets.

[20] In exactly the same way, one may also conceive of how to move the equator of the inclined orb of the two lower planets in latitude until it coincides with the parecliptic equator and then inclines its maximum amount in the opposite direction. It will thereafter return and coincide once again [with the parecliptic] and then return to its original inclination. There will not occur any longitudinal inclination with it that would cause a change to result in what was assumed in the way of longitudinal motion. On this account we would add three spheres surrounding the Earth for each of the lower planets.

[21] Furthermore one may use this method to conceive of how to move the lunar epicycle so as to produce the longitudinal inclination by which its diameter passing through the mean apex and the perigee will remain permanently aligned with the point of alignment. This will occur without that diameter being displaced from the plane of the inclined orb. We also add here three other spheres that surround the epicycle over and above what has previously been presented. This method, however, requires that [the movement of] the inclination in the sequence and in the counter-sequence [of the signs] occur in equal times.

على قطبين محاذيين لهاتين النقطتين فيتحرك مدارا آح بط بحركتها . وليكن المداران اللذان يماسّانهما مداري آج بد . ثم لنفرض الكرة الكبيرة متحركة بحركة مساوية لحركة مركز التدوير على فلكه الذي يتحرك عليه محيطاً بالأرض ، والكرة الصغيرة متحركة بحركة مخالفة لها في الجهة ومساوية لضعفها في المقدار . ويلزم من الحركتين أن لا يزال طرفا قطر آب مترددين على قوسي آج بد بين طرفيهما بحيث لا يميلان في الطول عنهما إلى أحد الجانبين أصلاً ــ إذا انتهى طرف ا إلى ج انتهى طرف ب إلى د ، ويكونان بميلهما في الجهتين على التبادل . ثم إذا أضيف إليهما الكرة المحيطة بالتدوير الحافظة لوضعه حتى لا يصير طرف قطره الصباحي مسائياً ولا بالعكس تمت الحركة المذكورة وزال الخلل المذكور في الوجه الثالث من الوجوه الثلاثة المذكورة عنها . وبقي المذكور في الوجه الثاني وحده ، ولم يمكن لي توهُّم وجه يزول به ذلك الإشكال . وعلى هذا الوجه نزيد ثلاث أكر في كل واحد من تدويرات العلوية وست أكر في كل واحد من تدويري السفليين .

[٢٠] وبمثل هذا الوجه بعينه أيضاً يمكن توهُّم تحريك منطقة الفلك المائل للسفليين في العرض إلى أن تنطبق على منطقة الممثل وتميل إلى الجانب الآخر غاية ميلها ، ثم تعود فتنطبق ثانياً وترجع إلى ما كان عليه من الميل أولاً من غير أن يَحدث معه ميل طولي يُحدِث تغيّراً فيما فرض من الحركة الطولية . ونزيد بسببه ثلاث أكر محيطة بالأرض لكل واحد من السفليين .

[٢١] وأيضاً بمثل هذا الوجه يمكن توهُّم تحريك تدوير القمر على وجه يحدث الميل الطولي الذي به يصير قطره المارّ بالذروة والحضيض الوسطيين دائماً محاذياً لنقطة المحاذاة من غير أن يخرج ذلك القطر عن سطح الفلك المائل . ونزيد هناك أيضاً ثلاث أكر أخرى تحيط بالتدوير زائدة على ما مرّ ؛ إلا أنّ هذا الوجه يقتضي أن يكون الميل إلى التوالي وإلى خلافه في زمانين متساويين ،

But the existing state of affairs is different from this, for the inclination is counter-sequential as long as the epicycle center is in the larger of the two previously mentioned segments of the eccentric, while the inclination is sequential as long as it is in the smaller segment. The [epicycle center] will not traverse the two segments in equal times since its motion is uniform, while the two segments are of different sizes.

[22] The same method may also be used for trepidation and for the movement of the obliquity in latitude for the ecliptic orb if the fact of these two motions and their variability is ascertained.

[23] This then is my proposal concerning these difficulties. Perhaps God may grant a reader of this book success in devising a more complete means to solve them all or in eliminating the remaining shortcomings in what we have presented. For He is the One who inspires truth and the Guide to the straightest path.

CHAPTER TWELVE

On Parallax

[1] There may occur for the planets near the Earth, and especially for the moon, a difference between their true positions on the ecliptic orb and their apparent positions. This is because the Earth's radius has an appreciable size in relation to / the diameters of / their orbs. Thus the line extending from the center of the World to the center of the planet and from there to the ecliptic orb will terminate at its true position on it, whereas the line extending from the position of the observer to the center of the planet and from there to the ecliptic orb will terminate at its apparent position on it. The amount occurring between them is the **parallax** [*lit.*, divergence of sight] of the planet, and this is on the altitude circle since the planet's altitude circle passes through the endpoints of the two lines on the ecliptic orb. The apparent position will always be closer to the horizon. The angle occurring from the two lines at the center of the planet is called the **angle of divergence**.

/16/ the diameters of] $\beta = -\alpha$, –M.

والوجود بخلاف ذلك لأنّ الميل إلى خلاف التوالي يكون ما دام مركز التدوير في القطعة الكبرى من قطعتي الخارج المركز المذكورتين والميل إلى التوالي يكون ما دام في القطعة الصغرى ، وهو لا يقطع القطعتين في زمانين متساويين لتشابه حركته واختلافهما بالصغر والكبر .

[٢٢] وبمثل هذا الوجه أيضاً يتم كل واحدة من حركة الإقبال والإدبار وحركة الميل في جهة العرض لفلك البروج إن تحقق وجودهما واختلافهما .

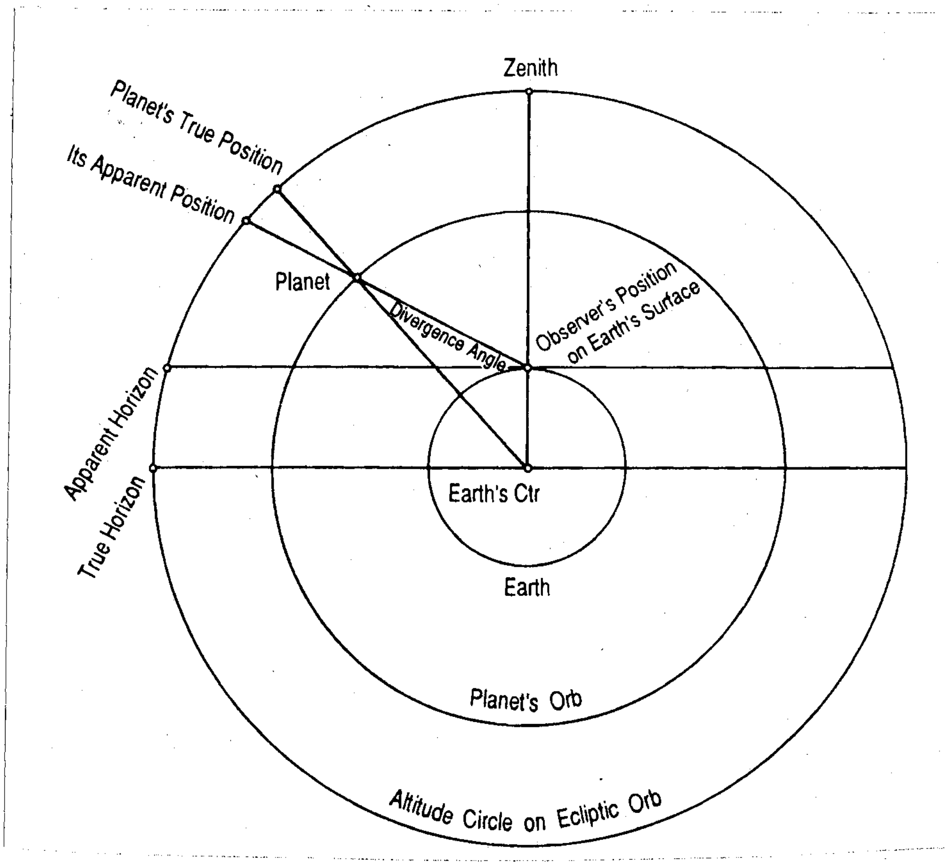
[٢٣] فهذا ما عندي في هذه الإشكالات . ولعل الله يوفّق الناظر في هذا الكتاب أن يستنبط وجهاً تاماً لحلّ جميعها أو يُزيل الخلل الباقي فيما ذكرناه ، إنه ملهم الصواب والهادي إلى سواء الصراط .

الفصل الثاني عشر
في اختلاف المَناظِر

[١] قد يعرض للكواكب القريبة من الأرض وخصوصاً للقمر أن تُخالِف مواضعُها الحقيقية من فلك البروج مواضعَها المرئية ، وذلك لكون نصف قطر الأرض ذا قدر محسوس عند / أقطار / أفلاكها . فإنّ الخط الخارج من مركز العالم إلى مركز الكوكب ومنه إلى فلك البروج ينتهي إلى موضعه الحقيقي منه ، والخط الخارج من موضع الناظر إلى مركز الكوكب ومنه إلى فلك البروج ينتهي إلى موضعه المرئي منه . والقدر الواقع بينهما هو اختلاف مَنْظَر الكوكب في دائرة الارتفاع لأنّ دائرة ارتفاع الكوكب تمرّ بطرفي الخطّين في فلك البروج . ويكون الموضع المرئي إلى الأفق أقرب دائماً . وتسمّى الزاوية الحادثة على مركز الكوكب من الخطّين زاوية الاختلاف .

/16/ ...] β = M– ,α– .

And this is its illustration:



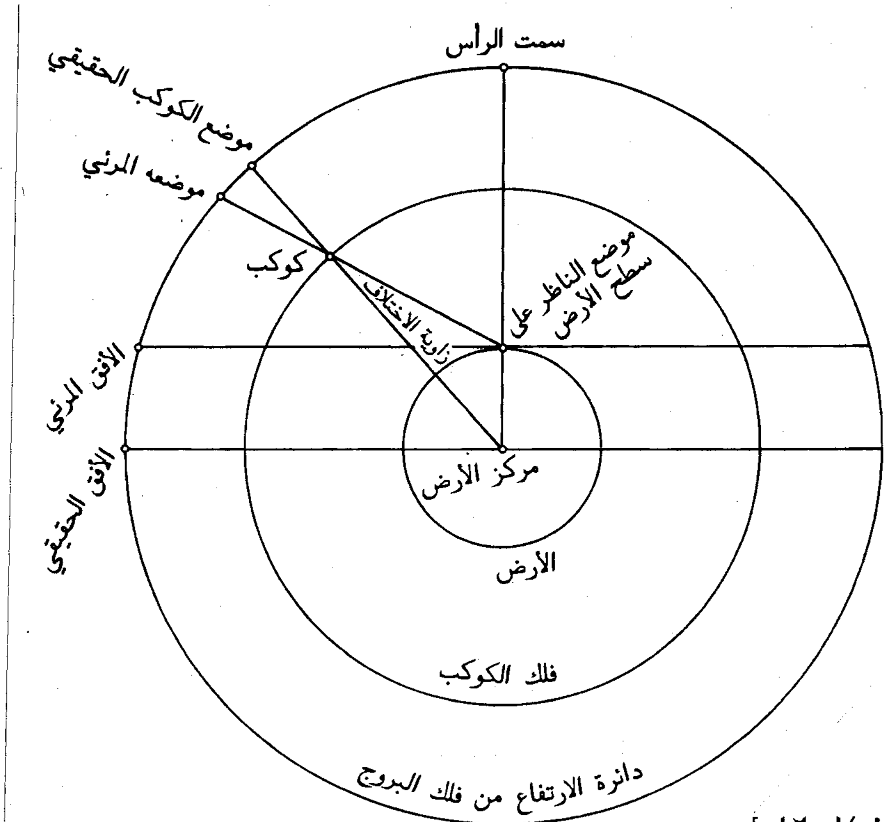
[Fig. T16]

[2] The planet does not have a parallax if it is at the zenith since the two lines are one and the same. Its divergence increases the closer it is to the horizon with the maximum occurring at its rising or setting. The visible part of the planet's orb will be less than half [of the entire orb] by the amount of the difference between the apparent and the true horizon.

[3] As for the planets that are far from the Earth, one does not perceive these divergences. The lines extending from the position of the observer and from the center of the World will be [virtually] the same since the difference with respect to the / diameters of the / orbs of these planets is so small.

/8/ diameters of the] β = [margin of M] = $-\alpha$.

وهذه صورته :



[شكل ١٦]

[٢] ولا يكون للكوكب اختلاف منظر إذا كان على سمت الرأس لاتّحاد الخطّين . ويزيد اختلافه كلما صار إلى الأفق أقرب ، وأكثره عند طلوعه أو غروبه . ويكون الظاهر من فلك الكوكب أقل من نصفه بقدر التفاوت بين الأفق المرئي والأفق الحقيقي .

[٣] وأما في الكواكب البعيدة من الأرض فلا يُحس بهذه الاختلافات . وتكون الخطوط الخارجة من موضع الناظر ومن مركز الأرض متّحدة لقلّة التفاوت بالقياس إلى / أقطار / أفلاك تلك الكواكب .

. –α = Mها = β [… /8/

[4] The above divergence requires that the true positions of the planet in longitude and latitude differ from their apparent positions. This is so since when we conceive two latitude circles passing through the endpoints of the two lines, then what is between the two points at which the latitude circles fall on the ecliptic orb will be the **difference in longitude**. And if the two arcs occurring on the circles between the endpoints of the lines and the ecliptic orb are not the same, the difference [in length] will be the **difference in latitude**. This is because the two points are the true and apparent positions of the planet, and the two arcs are its true and apparent latitudes.

[5] If the planet is on the ecliptic meridian circle, it will not have a difference in longitude since the two points on the ecliptic orb are one and the same, and its divergence on the altitude circle will be precisely the difference in latitude. / In other positions, / the [planet's] difference in longitude will be additive to the true position in the visible eastern quarter of the ecliptic orb and subtractive from it in the visible western quarter. This is because the apparent position is always nearer the horizon and the sequence of the signs is from west to east.

[6] Furthermore if the ecliptic equator passes through the zenith, then a planet that has no latitude will have no difference in latitude and its divergence on the altitude circle will be precisely the difference in longitude. / In other positions, / the [planet's] difference in latitude will be additive to the true latitude occurring in the direction of the invisible ecliptic pole and subtractive from the true latitude occurring in the opposite direction unless the planet and the ecliptic orb are in opposite directions from the zenith—thereupon the difference in latitude will also be additive to the true latitude. Now if the planet has no [true] latitude or if its true latitude is less than its difference [in latitude], then the direction

/12/ In other positions] β = Under other circumstances] α, M. /18/ In other positions] β = Under other circumstances] α, M.

[٤] والاختلاف المذكور يقتضي أن يكون موضعا الكوكب في الطول والعرض في الحقيقة مخالفين لموضعيهما المرئيين . وذلك لأنّا إذا توهّمنا دائرتي عرض تمرّان بطرفي الخطّين فهما إن وقعتا على نقطتين من فلك البروج كان ما بينهما اختلاف الطول . وإن اختلف[!] القوسان الواقعتان من الدائرتين بين طرفي الخطّين وبين فلك البروج كان التفاضل اختلاف العرض ، وذلك لأنّ النقطتين هما موضعا الكوكب الحقيقي والمرئي والقوسين عرضاه الحقيقي والمرئي .

[٥] وإذا كان الكوكب على دائرة وسط سماء الرؤية فلا يكون له اختلاف الطول لأنّ نقطتيه تتّحدان على فلك البروج ، ويكون اختلافه في دائرة الارتفاع اختلاف العرض بعينه . وفي غير ذلك / الموضع / يكون له اختلاف في الطول زائد على الموضع الحقيقي في الربع الشرقي الظاهر من فلك البروج ، وناقص عنه في الربع الغربي الظاهر منه . وذلك لكون الموضع المرئي إلى الأفق أقرب دائماً وكون توالي البروج من المغرب إلى المشرق .

[٦] وأيضاً إذا كانت منطقة البروج مارة بسمت الرأس فلا يكون للكوكب الذي لا عرض له اختلاف العرض ، ويكون اختلافه في دائرة الارتفاع اختلاف الطول بعينه . وفي غير ذلك / الموضع / يكون له اختلاف في العرض زائد على العرض الحقيقي الكائن في جهة القطب الخفي من قطبي فلك البروج ، ناقص من العرض الحقيقي الكائن في خلاف تلك الجهة ؛ اللّهم إلاّ أن يكون الكوكب وفلك البروج في جهتين متخالفتين عن سمت الرأس فإنّ اختلاف العرض هناك يكون أيضاً زائداً على العرض الحقيقي . فإن كان الكوكب عديم العرض أو كان عرضه الحقيقي أقل من اختلافه فجهة

/12/ ...] الوضع = β [α , M . /18/ ...] الوضع = β [α , M .

of the difference [in the first case] or the direction of the excess of the difference over the true latitude [in the second case] will be toward the invisible pole, likewise for exactly the same reason as before.

[7] By observation of the moon's parallax, one may find its distances from the Earth, as will later be explained.

[8] As far as the solar parallax is concerned, it is imperceptible even though calculation / yields / a small divergence not exceeding 3 minutes. The divergence for the two lower planets has not been found because of the difficulty in obtaining their true positions in longitude and latitude.

CHAPTER THIRTEEN

On the Variation in the Moon's Illumination and on Lunar and Solar Eclipses

[1] The variation in shape of the moon in accordance with its changing position vis-à-vis the sun indicates that its body is dark, thick and smooth; due to its thickness, it receives light from the sun, which is reflected from it due to its smoothness. Thus approximately half of its spherical body will always be illuminated. A great circle (or one nearly so) on its body separates the illuminated and the dark, and another great circle (or one nearly so) separates that which is apparent to the observer from that which the light of the eye does not reach. The two circles coincide at conjunction when the dark half is discerned, this state being new moon, and at opposition when the illuminated half is seen, which is full moon. At other positions [the two circles] will intersect: at the quadratures at right angles, whereupon that quarter facing the sun of the half that is toward us is illuminated; otherwise at acute and obtuse angles, whereupon in the first and last quarters what is toward the sun will be the part on the side of the acute angle and thus is crescent-shaped, while in the other two quarters it will be the part that is on the side of the obtuse angle and thus is oval-shaped.

/6/ yields] β = yields for it] α, M.

الاختلاف أو جهة فضل الاختلاف على العرض الحقيقي هي جهة القطب الخفي للعلّة المذكورة أيضاً بعينها .

[٧] وبرَصَد اختلاف منظر القمر يُتوصّل إلى معرفة أبعاده من الأرض كما يجيء بيانه .

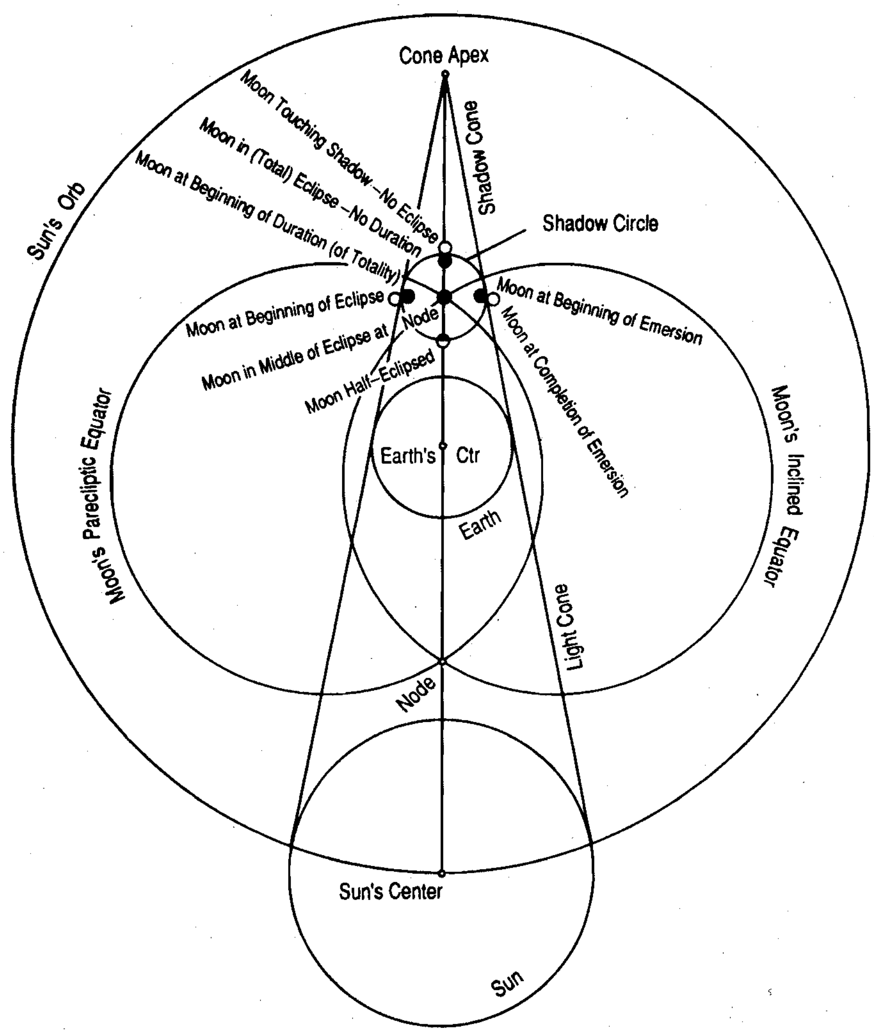
[٨] وأمّا اختلاف منظر الشمس فغير محسوس لكنّ الحساب / يُخرِج / اختلافاً قليلاً لا يزيد على ثلاث دقائق . والسفليان لا يوقف على اختلافهما لتعذّر الوقوف على مواضعهما الحقيقية في الطول والعرض .

الفصل الثالث عشر
في اختلاف نور القمر وفي الخسوف والكسوف

[١] اختلاف تشكّلات القمر بحسب اختلاف وضعه من الشمس يدلّ على أنّ جرمه مُظلِم كثيف صقيل يقبل من الشمس الضوء لكثافته وينعكس عنه لصقالته . فيكون أبداً المُضيء من جرمه الكري قريباً من نصفه . وتفصل بين المضيء والمظلم دائرة عظيمة أو قريبة من العظيمة على جرمه ، وتفصل بين المرئي منه عند الناظر وبين ما لا يصل إليه نور البصر أيضاً عظيمة أو قريبة منها . والدائرتان تتطابقان في الاجتماع ، ويكون المُبصَر منه النصف المظلم وتلك الحالة هي المُحاق ، وفي الاستقبال ، ويكون المبصر منه النصف المضيء وهو البَدْر . وتتقاطعان في سائر الأوضاع : أما في التربيعين فعلى زوايا قائمة ، ويكون الربع الذي يلي الشمسَ من النصف الذي يلينا مضيئاً ؛ وفي غيرهما على زوايا حادّة ومنفرجة ، والذي يلي الشمسَ في الربعين الأوّل والأخير هو القسم الذي يلي الزاوية الحادّة فيكون هِلالي الشكل وفي الربعين الآخرين هو القسم الذي يلي الزاوية المنفرجة فيكون إهْلِيلَجِي الشكل .

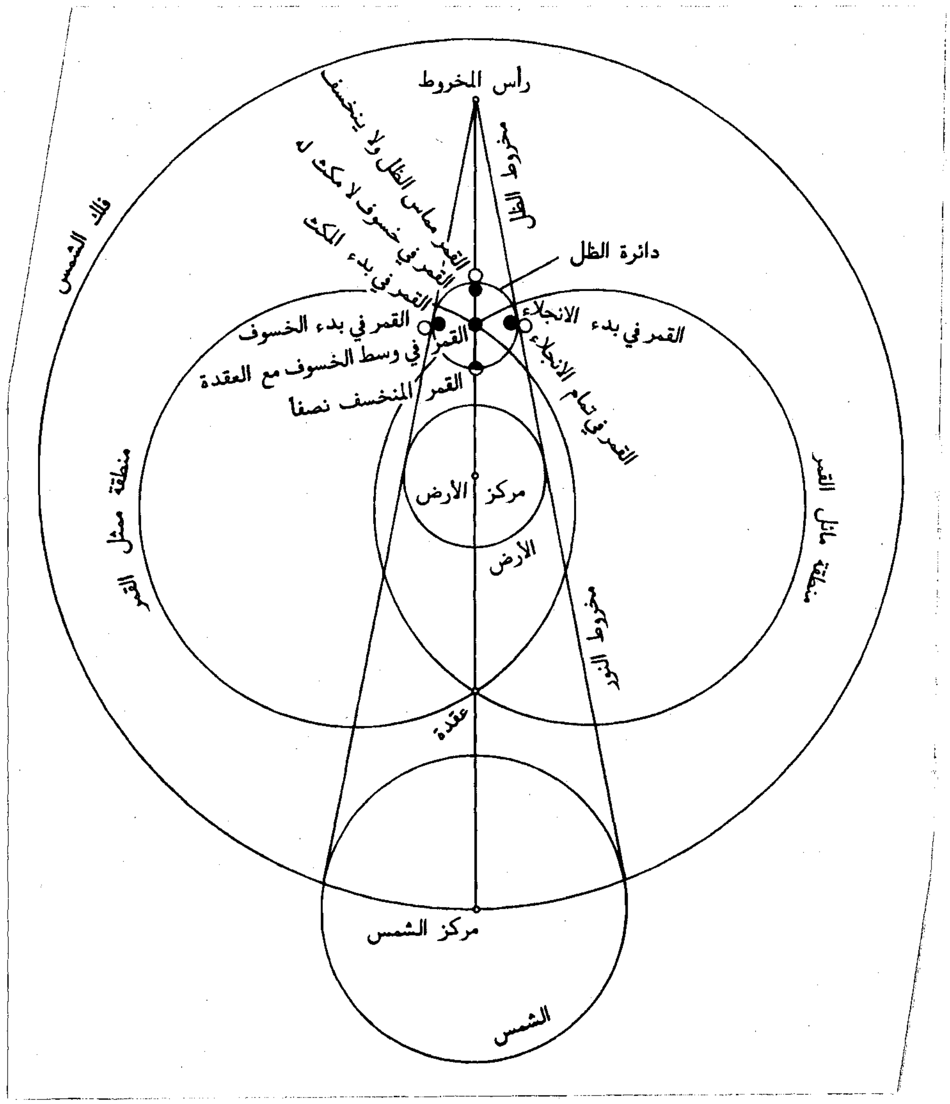
/6/ ...] β = يخرج له] M , α .

[2] The Earth is also a thick, dark, spherical body that blocks the light of the sun and so it has a shadow. When it is aligned with the two luminaries at the time of opposition, it blocks the light of the sun from the moon and the moon will fall in its shadow. The moon will thereupon be eclipsed, which can be observed if it is night. The following is an illustration of the **lunar eclipse**:



[Fig. T17]

[٢] والأرض أيضاً جسم كثيف مظلم كري يحجب نور الشمس فيقع له ظلّ . وإذا صارت مقاطرة للنيّرين وقت الاستقبال حجبت نور الشمس عن القمر ووقع القمر في ظلّها ، فانخسف القمر ورُئيَ إن كان ذلك ليلاً . وهذه صورة الخُسوف :



[**شكل ١٧**]

[3] The farther away the moon is from the Earth, the shorter in duration is its eclipse. It was concluded from this that the shadow narrows with increasing distance from the Earth, which [in turn] indicates that the sun is larger than the Earth. For if the sun were smaller than the Earth, then the shadow would widen with increasing distance from the Earth and thus the moon's duration during a [lunar] eclipse would increase as its distance from the Earth increased, which is contrary to what is found [to be the case]. If the [sun] were the same [size] as the Earth, the shadow would be cylindrical and the duration at all distances would be the same which is also not the case. It is thus clear that the sun is larger than the Earth, that the Earth's shadow is in the shape of a circular cone that vanishes to a point, and that the moon is smaller than the Earth since the latter's shadow, which becomes much smaller than the [Earth] upon [reaching] the moon, conceals it.

[4] The center of the **shadow cone** is always on the [plane of the] ecliptic equator since the sun is always on it and since the center of the Earth is the center [of this equator]. If one imagines the plane of the apparent lunar body as a circle extending until it intersects the shadow cone, it will produce a circle parallel to its base called the **shadow circle** whose center is on [*i.e.* in the plane of] the equator. Now if the latitude of the moon at the time of opposition is greater than the radius of its disk plus the radius of the shadow circle, then the moon will not undergo an eclipse; if its latitude is equal to them, the moon will touch the shadow but will not undergo an eclipse; if it is less than them and equal to the radius of the shadow, the shadow circle will pass through the center of the lunar disk and half its diameter will be eclipsed; if it is equal to the excess of the shadow radius over the radius of the moon, the entire moon will be eclipsed and its surface will touch the shadow circle but it will have no duration in the eclipse; and if it is less than [this excess], it will be eclipsed with duration depending on the extent to which it is in the shadow.

[5] The eclipse limits have been measured as 12° in terms of the distance of the moon from one of the two nodes; for when the moon crosses this limit, its latitude will exceed the two radii. And just as the shadow circle varies according to distance, the circle of the lunar disk will likewise vary according to distance. The two have been compared and the shadow circle's diameter was found to be two and three-fifths times the diameter of the lunar disk

[٣] وكلما كان القمر أكثر بعداً من الأرض كان خسوفه أقلّ مَكْثاً ، فاستُدلّ بذلك على أنّ الظل يستدق بازدياد بعده من الأرض . ويدلّ ذلك على كون الشمس أكبر من الأرض . وذلك لأنّ الشمس لو كانت أصغر من الأرض لكان الظلّ يستغلظ بازدياد بعده من الأرض ، فكان كلّما زاد بعد القمر من الأرض زاد مكثه في الخسوف على ضدّ ما يوجد ؛ ولو كانت مساوية للأرض لكان الظلّ أُسطوانياً والمكث في جميع الأبعاد متساوياً وليس أيضاً كذلك . فإذن ظهر أنّ الشمس أكبر من الأرض ، وأنّ ظلّ الأرض على هيئة مخروط مستدير ينعدم على نقطة ، وأنّ القمر أصغر من الأرض لستر ظلّها الذي صار أصغر منها كثيراً عند القمر إيّاه .

[٤] ومركز مخروط الظلّ يكون دائماً على منطقة البروج لكون الشمس دائماً عليها وكون مركز الأرض مركزاً لها . وإذا توهّم سطح جرم القمر المرئي كدائرة خارجاً إلى أن يقطع مخروط الظلّ أحدث دائرةً موازية لقاعدته تُسمى دائرة الظلّ ويكون مركزها على المنطقة . فإن كان عرض القمر وقت الاستقبال أكثر من نصفي قطر صفحته وقطر دائرة الظلّ لم يقع للقمر خسوف ؛ وإن كان عرضه مساوياً لهما ماسّ القمرُ الظلّ ولم يقع له خسوف ؛ وإن كان أقلّ منهما وكان مساوياً لنصف قطر الظلّ مرّت دائرة الظلّ بمركز صفحة القمر وانخسف نصف قطره ؛ وإن كان مساوياً لفضل نصف قطر الظلّ على نصف قطر القمر انخسف القمر كلّه وماسّ سطحه دائرةَ الظلّ فلم يكن له مكث في الخسوف ؛ وإن كان أقلّ من ذلك انخسف ومَكَثَ بحسب ما يقع في الظلّ .

[٥] وإنّما قُدّرَ حدود الخسوف باثني عشر جزءاً من بعد القمر عن إحدى العقدتين لأنّ عرضه إذا جاوز هذا الحدّ زاد على نصفي القطرين . وكما أنّ دائره الظلّ تختلف بحسب الأبعاد فدائرة صفحة القمر أيضاً تختلف بحسب الأبعاد . وقد قيس بينهما فوجد قطر دائرة الظلّ مثلي قطر صفحة القمر وثلاثة أخماسه

at all distances. Each of the diameters of the two luminaries and their bodies is divided into 12 equal parts called **digits**, those of the diameters being designated as absolute, while those of the bodies as adjusted.

[6] Since a lunar eclipse is possible at a distance of less than 12° from one of the nodes, then if opposition occurs after the crossing of the node with an eclipse taking place at the edge of the [eclipse] limit, and then opposition occurs five months later at the edge of the eclipse limit before the other node is reached, it will be possible for the moon to be eclipsed once again. This is due to the counter-sequential motion of the node and to its meeting up with the position of the eclipse. If opposition for an eclipse occurs before reaching the first node at the edge of the limit and the other opposition after crossing the second node seven months later, it is not possible [for the moon] to be in the eclipse limit on account of the node, by its counter-sequential motion, having crossed beyond the range necessary for an eclipse. Thus two lunar eclipses cannot be seven months apart. It is most frequently the case that they are six months apart.

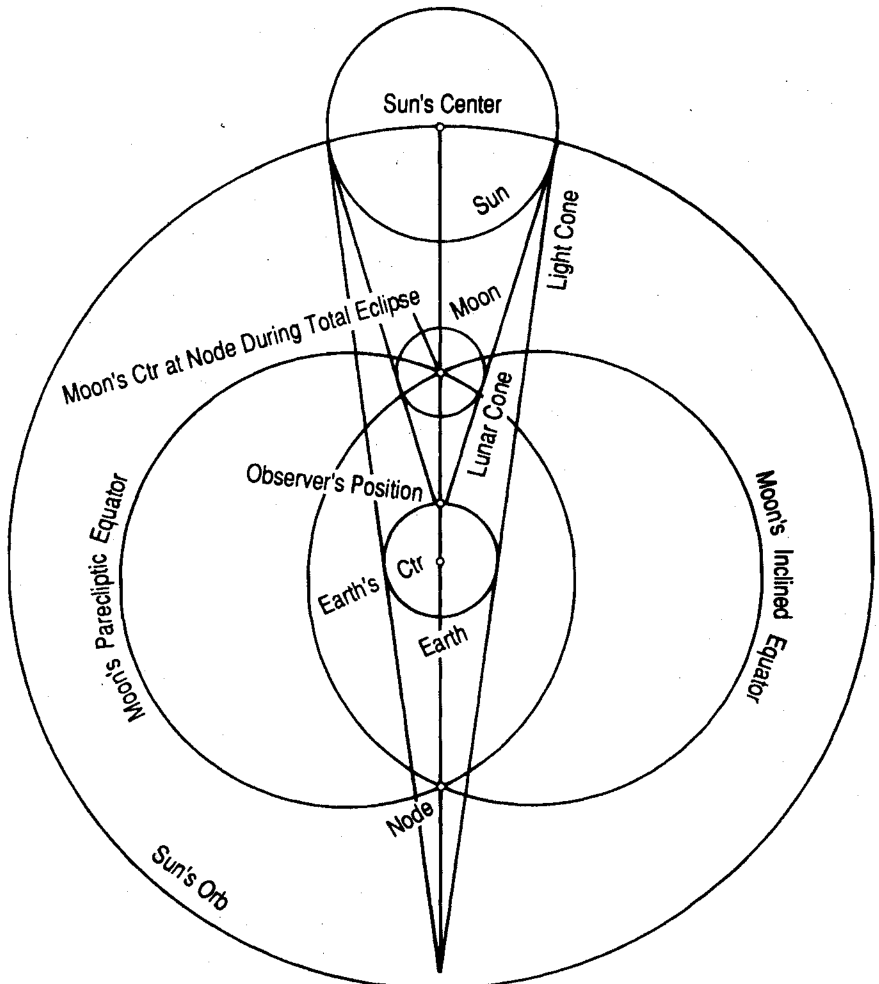
[7] Furthermore, if the moon blocks the light of the sun from the observer due to its falling on the line extending from the eye to the sun, the sun will be seen eclipsed, without light; for the obstruction is dark and the [part] of it facing us at that time is not illuminated. This will be during an apparent rather than a true conjunction occurring in the daytime. For this reason parallax is taken into account for solar but not for lunar eclipses, and it is possible for a solar eclipse to occur with respect to one group of people but not another. In order for a solar eclipse to take place, it is necessary that the apparent lunar latitude (that is, adjusted in latitude for parallax) at the time of the apparent conjunction (that is, adjusted in longitude for parallax) be less than the radii of the disks of the two luminaries. For if it is equal to them, the [disks] will touch but the sun will not be eclipsed, and if it is greater than them, then all the more reason [for it not to be eclipsed]; if it is less than them, the eclipse will occur commensurate with that amount.

في كلّ بعد . ويُجزّأ كلّ واحد من قطري النيّرين وجرميهما إلى اثني عشر جزءاً متساوية تسمى الأصابع ، وتقيّد القطرية بالمطلقة والجرمية بالمعدّلة .

[٦] ولمّا كان الخسوف على بعد أقلّ من اثنتي عشرة درجة من إحدى العقدتين ممكناً ، فإن كان الاستقبال بعد التجاوز عن العقدة ووقع خسوف على طرف الحدّ ، ثم وقع استقبال بعد خمسة أشهر قبل الانتهاء إلى العقدة الأخرى على طرف حدّ الخسوف ، أمكن أن ينخسف القمرُ مرة ثانية . وذلك لحركة العقدة إلى خلاف التوالي واستقبالها لموضع الخسوف . وإن كان الاستقبال الخسوفي قبل الوصول إلى العقدة الأولى على طرف الحدّ ، والاستقبال الآخر بعد التجاوز عن العقدة الثانية بعد سبعة أشهر ، لم يمكن أن يقع في حدّ الخسوف لمجاوزة العقدة بحركته[!] إلى خلاف التوالي عن المقدار المقتضي للخسوف ؛ فلا يكون خسوفان بينهما سبعة أشهر . وأمّا بعد ستة أشهر فأكثريّ الوقوع .

[٧] وأيضاً إذا حجب القمرُ نور الشمس عن الناظرين لوقوعه على الخط الخارج من الأبصار إلى الشمس رُئيت الشمس منكسفة عديمة النور ؛ فإنّ الحاجب مظلم والذي يلينا منه غير مضيء في ذلك الوقت . وذلك يكون في الاجتماع الواقع نهاراً المرئي لا الحقيقي ؛ ولذلك يعتبر اختلاف المناظر في الكسوفات دون الخسوفات ، ويمكن أن يقع كسوف بالقياس إلى قوم دون قوم . وينبغي أن يكون العرض المرئي للقمر ، أعني المعدّل باختلاف المنظر في العرض ، وقت الاجتماع المرئي ، أعني المعدّل باختلاف المنظر في الطول ، أقلّ من نصفي قطري صفحتي النيّرين حتى يقع كسوف . فإنّه إن ساواهما تماسّا ولم تنكسف الشمس ، وإن كان أكثر منهما فبالأولى ، وإن كان أقل منهما يقع الكسوف بقدر ذلك .

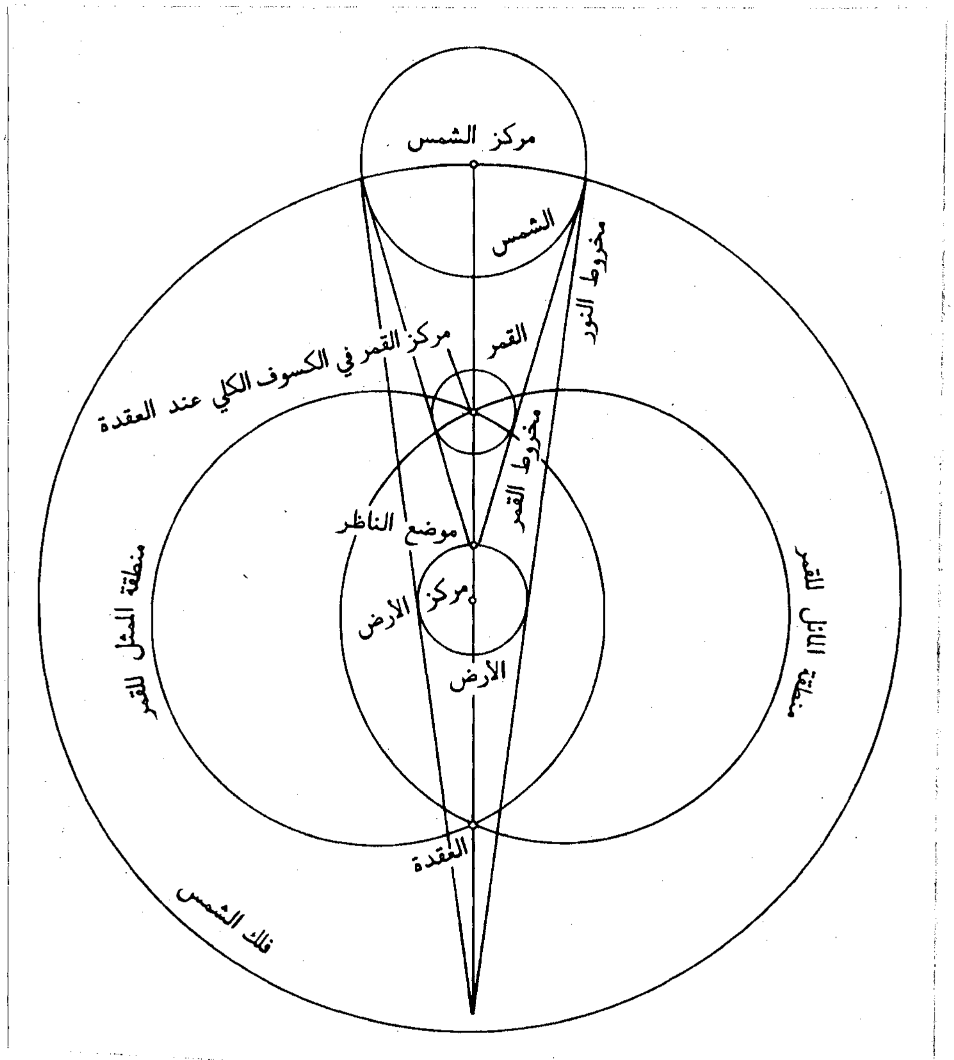
And this is the illustration of a **solar eclipse**:



[Fig. T18]

[8] The solar diameter between its two [extremal] distances was found [to vary] from 31 to 34 minutes, while the lunar diameter was found [to vary] from 29 to 36 minutes. Thus if the two centers fall on the line

وهذه صورة الكسوف :



[شكل ١٨]

[٨] وقطر الشمس فيما بين بعديه وُجد من إحدى وثلاثين دقيقة إلى أربعة[!] وثلاثين . وأمّا قطر القمر فقد وُجد من تسع وعشرين دقيقة إلى ست وثلاثين . فإن وقع المركزان على الخط

extending from the eye to the sun, and if the two diameters are equal, the sun will be completely eclipsed and there will be no duration. If the sun's diameter is greater, there remains a luminous ring called the **ring of light**. If it is smaller the eclipse will have a slight duration commensurate with the difference between the two diameters. This is so inasmuch as the moon also has a shadow cone whose apex will be at the eye at a distance that results in the equality of the two diameters and higher than the eye at a distance that results in the ring of light; the eye will fall within a shadow circle that intersects the cone when the distance results in a duration [of totality].

[9] As regards the solar eclipse limits: since with respect to the true latitude the divergence in latitude will sometimes be added to and sometimes subtracted from it in order to arrive at the apparent [latitude], it follows that the limits on the two sides of the nodes will vary according to change in local position. In the fourth clime, / a solar eclipse is / possible at a maximum distance of 18° after the node of the head or before the node of the tail, or at a maximum distance of 7° before the node of the head or after the node of the tail. It is therefore possible for two solar eclipses to occur at the endpoints of a five-month [interval], one of which is after the head and the other before the tail, or at / the endpoints of / a seven-month [interval], one of which is before the tail and the other after the head. As for the endpoints of six months, this possibility is beyond doubt nor is [there any doubt concerning] the occurrence of a lunar and a solar eclipse during a consecutive opposition and conjunction. It is not possible for two lunar eclipses to be one month apart nor two solar eclipses unless [the latter] occur in two locations with different directions in latitude. And because the moon is / that which [can] become eclipsed as well as be the occulting [body], / what disappears first in a lunar eclipse will always be its eastern part while that which is occulted [first] in a solar eclipse is the western part of the sun. The same [will hold true] for the first parts to reappear.

/11/ a solar eclipse is] β = the possibility of a solar eclipse is] α, M. /15/ the endpoints of] β = –α, –M. /19–20/ that which [can] become eclipsed...occulting [body]] β = the occulting [body] as well as that which [can] become eclipsed] α, M.

الخارج من البصر إلى الشمس وكان القطران متساويين انكسفت الشمس كلّها ولم يكن هناك مكث ؛ وإن كان قطر الشمس أكبر بقي حلقة نورانية وتسمى حلقة النور ؛ وإن كان أصغر كان للكسوف مكث قليل بقدر الفضل بين القطرين . وذلك أنّ للقمر أيضاً مخروط ظلّ يكون رأسه عند الأبصار في بعد يقتضي تساوي القطرين ، وأعلى من الأبصار في بعد يقتضي حلقة النور ؛ وتقع الأبصار في دائرة من الظلّ قاطعة للمخروط في بعد يقتضي المكث .

[٩] ولاعتبار حدود الكسوفات إذا اعتُبر العرض الحقيقي وكان اختلاف العرض تارة يزاد عليه وتارة ينقص منه ليصير مرئياً لزم أن تكون الحدود عن جانبي العقدتين مختلفة بحسب اختلاف البقاع . ففي الإقليم الرابع / يكون / الكسوف على بعد غايته بَعد عقدة الرأس أو قبل عقدة الذنب إلى ثماني عشرة درجة أو على بعد غايته قبل عقدة الرأس أو بَعد عقدة الذنب إلى سبع درجات ممكناً . ولذلك يمكن كسوفان على طرفي خمسة أشهر أحدهما بعد الرأس والآخر قبل الذنب أو على / طرفي / سبعة أشهر أحدهما قبل الذنب والآخر بعد الرأس . وأمّا على طرفي ستة أشهر فلا اشتباه في إمكانه ولا في وقوع خسوف وكسوف في استقبال واجتماع متواليين . ولا يمكن خسوفان بينهما شهر ولا كسوفان إلاّ في بقعتين مختلفتي جهة العرض . ولكون القمر هو / الداخل في الخسوف والكاسف / يكون المنخسف أولاً أبداً شرقيَهُ والمنكسف غربي الشمس ، وكذلك المنجلي أولاً .

/11/ يكون] β = يكون إمكان α , M [. /15/ ...] β = −α , −M . /19–20/ ...] β = الكاسف والداخل في الخسوف α , M [.

CHAPTER FOURTEEN

/ On Sectors and Conjunctions and the Situation of Visibility and Invisibility /

[1] The first and third initial points of the [planetary] **sectors** are the apogee and apex and the two perigees. These are the farthest and nearest distances from the center of the World, and the positions at which the fastest and slowest motions occur. The initial points of the remaining two [sectors] on the two sides are either according to distance, so that for the apogee orb it is where the two lines produced to it from the center of the World and the [center] of that orb are equal, while for the epicyclic orb [the initial points] are where the circumferences of the epicycle and the deferent intersect; or else, they are according to speed, so that for the apogee orb it is where a line passing through the center of the World perpendicular to the diameter passing through the centers reaches the [orb], while for the epicyclic orb it is where a line produced from the center of the World to it is tangent to its circumference. Something moving in the two orbs is ascending in the third and fourth [sectors], descending in the first and second; elevated in the fourth and first [sectors], depressed in the others. [Sector] measurements are provided in the practical handbooks.

[2] With respect to visibility and invisibility, for the stars these vary first, according to their differing sizes; second, according to differences in [both] their measure and direction in latitude; and third, according to different horizons. Thus some stars do not become invisible at all, whereas some of them become invisible for a long period. Venus does not become invisible in the fourth clime when it is in Pisces; during its day of combust, it may be seen in the evening and following morning while retrograding. But it will be invisible for a considerable period when it combusts in Virgo / while retrograding. / Mercury is not visible in the evenings round about the autumnal point in the vicinity of its apogee; nor [is it visible] in the mornings round about the vernal point in the vicinity of [the point] opposite its apogee. When the upper planets

/2–3/ On Sectors...Invisibility] β = On Sectors and Conjunctions and the Situation of Visibility and Invisibility and Conjunctions] M = On Sectors, the Situation of Visibility and Invisibility, and Conjunctions] α. /21/ while retrograding] β = while undergoing direct motion] α, M.

الفصل الرابع عشر
في النطاقات / والاقترانات
وأحوال الظهور والاختفاء /

[١] مبادئ الأول والثالث من النطاقات هي الأوج والذروة والحضيضان ، وهي الأبعاد البعيدة والقريبة من مركز العالم والمواضع التي يكون هناك أسرع الحركات وأبطؤها . ومبادئ الباقيين في الجانبين إمّا بحسب البعد ، ففي فلك الأوج حيث يتساوى الخطّان الخارجان من مركزي العالم وذلك الفلك إليه ، وفي فلك التدوير حيث يتقاطع محيطا التدوير والحامل ؛ وإمّا بحسب السير ، ففي فلك الأوج حيث ينتهي إليه العمود المارّ بمركز العالم القائم على القطر المارّ بالمراكز ، وفي فلك التدوير حيث يماسَ محيطه الخطّ الخارج إليه من مركز العالم . والسائر في الفلكين صاعد في الثالث والرابع ، هابط في الأول والثاني ؛ مستعلٍ في الرابع والأول ، منخفض في الباقيين . ومقاديرها تُورَد في كتب العمل .

[٢] وأما الظهور والاختفاء فيختلف في الكواكب أولاً بحسب كبرها وصغرها ، وثانياً بحسب اختلاف مقادير عروضها وجهاتها ، وثالثاً بحسب اختلاف الآفاق . ولذلك لا يختفي بعض الكواكب أصلاً ويختفي بعضها مدّة طويلة . والزهرة لا تختفي في الإقليم الرابع في الحوت ، تُرى يوم احتراقها راجعة بُكرة وعشياً ؛ وتختفي إذا احترقت في السُنبلة / راجعة / مدّة كثيرة . وعطارد لا يظهر بالعشيات حوالى النقطة الخريفية وحدود أوجه ، ولا بالغدوات حوالى النقطة الربيعية وحدود مقابلة أوجه . والكواكب

/2–3/ ...] β = والاقترانات وأحوال الظهور والاختفاء والقرانات] M = وأحوال الظهور والاختفاء والاقترانات] α . /21/ ...] β = مستقيمة] α , M .

become visible after the sun departs from them, they may be seen rising in the mornings in the east until the sun crosses its quadratures with them. They will then be seen rising in the evenings until the sun is in opposition to them, after which they may be seen setting in the mornings until the second quadratures. They are then seen setting in the evenings in the west, and thereafter they become invisible. When the lower planets go ahead of the sun, they will be visible in the evenings in the west, thus setting in the evenings, until they retrograde and become invisible in the evenings. They then become visible, rising in the mornings in the east, until they become invisible in the mornings. As for the moon, one may add to the previously cited reasons for variability [the moon's] parallax and its variable distance from the sun that leads to increases and decreases in the light[ed part] of its body. The minimum that it is invisible is two nights and the maximum is three nights. [The matter] has been subjected to testing and the limits of visibility and invisibility of the six wandering [planets] have been found whereby the altitude at the [time of the] rising or setting of the sun is: 11° for Saturn; 10° for Jupiter; 11½° for Mars; 5° for Venus; 10° for Mercury; and 8° for the moon, which, in its case only, is apparent.

[3] As for the conjunction of two planets, [it occurs when] they fall on the same latitude circle in the same direction from one of the two poles. A **true latitudinal conjunction** is when a single line produced from the center of the World passes through them. An **apparent latitudinal conjunction** is when a single line produced to the two [planets] from the position of the / observer / passes through them.

/18/ observer] β, M = observers] α.

العلوية إذا فارقتها الشمسُ وظهرت فهي تُرى تطلع بالغدوات مُشرِّقة إلى أن تجاوز الشمس تربيعاتها ؛ ثم تُرى تطلع بالعشيّات إلى أن تقابلها الشمسُ ؛ وبعد ذلك تُرى تغرب بالغدوات إلى التربيعات الثانية ؛ ثم تُرى تغرب بالعشيات مُغرّبة ؛ ثم تختفي . والسفليان إذا سبقا الشمسَ ظهرا بالعشيات مُغرّبين فيغربان بالعشيات إلى أن يرجعا ويختفيا بالعشيّات ؛ ثم يظهران ويطلعان بالغدوات مشرّقين إلى أن يختفيا بالغدوات . وأمّا القمر فينضاف فيه إلى أسباب الاختلافات المذكورة اختلاف مَنظره واختلاف بعده من الشمس المقتضي لزيادة نور جرمه ونقصانه . وأقلّ ما يختفي ليلتان وأكثره ثلاث ليالٍ . وقد امتُحِنَ فوُجد حدود ظهور السيّارات الستّة وخفائها حيث يكون الارتفاع عند طلوع الشمس أو غروبها : لزحل أحد عشر جزءاً ، وللمشتري عشرة أجزاء ، وللمريّخ أحد عشر جزءاً ونصفاً ، وللزهرة خمسة أجزاء ، ولعطارد عشرة أجزاء ، وللقمر ثمانية أجزاء مرئية له فقط .

[٣] وأمّا اقتران الكوكبين فهو وقوعهما على دائرة عرض واحدة في جهة واحدة من أحد القطبين . والاقتران العرضي الحقيقي هو أن يمرّ بهما خط واحد خارج من مركز العالم ، والاقتران العرضي المرئي أن يمرّ بهما خط واحد خارج من موضع / الناظر / إليهما .

/18/ ...] M , β = الناظرين] α .

BOOK III

On the Configuration of the Earth and the [Consequences] Accruing to It Due to the Changing Positions of the Celestial Bodies

Twelve Chapters

CHAPTER ONE

A General Summary of the Configuration and Circumstances of the Earth

[1] It was shown in the first part of the book that the Earth is as a whole spherical and that in all directions the head of someone standing upon it will be toward the circumference, which is up, while his feet will be toward the center, which is down, and that the surface of the Earth, which is its convexity, is parallel to the concavity of the orb surrounding [the surface]. The direction of the head of someone traveling upon the Earth is necessarily at each instant [toward] another part of the [celestial] orb. If it were possible to travel around the entire Earth and then three individuals were assumed to become separated at some location, one of them traveling toward the west, the second traveling toward the east, and the third staying in place until the two travelers had circled the Earth—the traveler who went west returning to him from the east and the traveler who went east returning to him from the west—then the first [traveler] will have one fewer than the total [number] of days that have been generally counted because he has lengthened [the period for each of] the revolutions of the orb due to his travel so that he distributes a revolution among their total [number]. The second will have one more because he has shortened [the period for each of] the revolutions due to his motion so that a revolution accumulates for him from the decreases. This is also something that is asked about and found to be strange.

[2] The great circle / that is / on the surface of the Earth occurring in the plane of the equinoctial is called the **equator**. If another great circle is imagined

/19/ that is] β, M = –α.

الباب الثالث
في هيئة الأرض وما يلزمها بحسب اختلاف أوضاع العلويات اثنا عشر فصلاً

الفصل الأول
في جمل من هيئة الأرض وأحوالها

[١] قد تبيّن في أول الكتاب أنّ الأرض بجملتها مستديرة ، وأنّ الواقف عليها من جميع الجوانب رأسه إلى ما يلي المحيط وهو الفوق ورجله إلى ما يلي المركز وهو التحت ، وأنّ سطح الأرض وهو محدّبه موازٍ لمقعر الفلك المحيط به . والسائر على الأرض يجب أن يصير سمت رأسه في كل وقت جزءاً آخر من الفلك . ولو كان السير على جميع الأرض ممكناً ، ثم فُرض تفرّق ثلاثة أشخاص عن موضع ، فسار أحدهم نحو المغرب والثاني نحو المشرق وأقام الثالث حتى دار السائران دوراً من الأرض ورجع السائر في المغرب إليه من الشرق والسائر في المشرق من الغرب ، نقص من الأيّام التي عدّوها جميعاً للأول واحد لأنّه زاد بسيره في أدوار الفلك فوزّع دوراً على جملتها ، وزاد للثاني واحد لأنّه نقص بسيره عن الأدوار فاجتمع له من النقصانات دور . وهذا أيضاً مما يُسأل عنه ويُستغرب .

[٢] والدائرة العظيمة / التي / على سطح الأرض الكائنة في سطح معدّل النهار تسمى خط الاستواء . وإذا توهّمت عظيمة

/19/ …] M , β = α− .

that passes through its two poles, the Earth will be divided by them into fourths. One of the two northern [fourths] is the **populated quarter**, and the others are either submerged in the seas and not populated or else their circumstances are unknown. One should then conceive the division of the Earth's surface to be in longitude according to the division of the equinoctial and in latitude to the poles according to the divisions of the circles of declination. One should also conceive on the [surface] circles that are exactly aligned with the day-circles in order to allow one location to be differentiated from another. Distances and quantities are measured just as they are on the orb.

[3] The **inhabited world** has been determined to be a quarter because observations of celestial phenomena, such as lunar eclipses, have not been found [to occur] for those living in the farthest eastern [regions] earlier than 12 hours ahead [of their occurrence] for those in the farthest western regions. From this they discerned that the length of the populated area does not exceed one-half revolution of the orb. It was [further] determined that [this] quarter is in the north because the shadows at noon of the equinoxes are not southerly in any part of [the inhabited world] except that it has been reported that they are southerly in a few populated regions at the edge of Zanzibar [*al-Zanj*], Abyssinia [*al-Ḥabasha*], and some other [areas]; in any case their latitudes do not exceed a few degrees. In the northern region as well, it is not possible to live beyond the latitude that is the complement of the obliquity due to the intensity of the cold.

[4] The sea surrounds most sides of the aforementioned area of the Earth. This is well-established for the western side, the north, and most of the south, especially the eastern part of it. As for the southwest, it has been stated that persons traveling in the direction of the sources of Egypt's Nile have reached locations whose southern latitude is not more than 10° to 20°. They saw mountains at a distance to their south white with snow, which were named for the moon, from which [arise] the headwaters of the Nile. They did not reach a body of water. Furthermore we do not have definitive knowledge about the sea in the northeast.

[5] In the area that has been uncovered to allow for habitation, there are also numerous bodies of water some of which are connected to the encompassing [ocean] such as: that between the Maghrib and Andalusia and that between Andalusia and Syria; the southern sea, which is connected to the eastern region, from which extends four gulfs into the middle of the inhabited world: the Gulf of Barbary [Gulf of Aden], which is the nearest of them to

أخرى تمرّ بقطبيها انقسمت الأرض بهما أرباعاً : أحد الشماليين هو الربع المسكون ، والباقية إمّا غامرة في البحار غير مسكونة وإما غير معلومة الأحوال . فينبغي أن يتوهّم تجزئة سطح الأرض طولاً بحسب تجزئة معدّل النهار وعرضاً إلى القطبين بحسب تجزئة دوائر الميول ، وتتوهّم عليه مدارات محاذية للمدارات اليومية بعينها لتمكّن امتياز بعض المواضع عن بعض . وتُقدّر المسافات والمقادير كما على الفلك .

[٣] وإنّما حُكم بأنّ المعمور ربع لأنّه لم يوجد في أرصاد الحوادث الفلكية كالخسوفات تقدّم ساعات الواغلين في المشرق لها على ساعات الواغلين في المغرب زائداً على اثنتي عشرة ساعة ؛ فعلموا من ذلك أنّ طول المسكونة لا يزيد على نصف دور الفلك . وإنّما حُكم بأنّ الربع شمالي لأنّه لم توجد أظلال أنصاف نهار الاعتداليين في شيء منها جنوبية إلاّ قليل من مساكن على أطراف الزنج والحبشة وغيرها حُكي أنّها جنوبية لكنّ لا تزيد عروضها على نيف درجات . وفي جانب الشمال أيضاً لا يمكن أن يُسكن فيما جاوز عرضه تمام الميل الكلي لشدّة البرد .

[٤] والبحر محيط بأكثر جوانب القدر المذكور من الأرض . أمّا من جانب المغرب والشمال وأكثر الجنوب لا سيّما الشرقي منه فمعلوم . وأمّا جنوب المغرب فقد ذُكر أنّ السائرين على سمت منابع نيل مصر انتهوا إلى مواضع زاد عرضها الجنوبي على بضع عشرة درجة ، وشاهدوا الجبال البيض من الثلج المنسوبة إلى القمر التي منها منابع النيل في جنوبهم من بعيد ؛ ولم يصلوا إلى بحر . وأيضاً ليس لنا على البحر الذي في شمال المشرق وقوف يقيني .

[٥] وفي القدر المكشوف للعمارة أيضاً بحار كثيرة بعضها متّصل بالمحيط كالذي بين المغرب وأندلس والذي بين أندلس والشام ؛ والبحر الجنوبي المتّصل بالجانب الشرقي الذي خرج منه أربع خليجات إلى وسط العمارة : الخليج البربري وهو أقربها إلى

the Maghrib, the Red Gulf [Red Sea], the Persian Gulf, and the Green Gulf, for each of which the longitude and latitude are well-established; and such as the Sea of *Warank* [Baltic Sea] in the northern region. Some [of the bodies of water] are unconnected [to the encompassing ocean] such as the Sea of Ṭabaristān [Caspian Sea], Lake Khwārazm [Aral Sea], and other channels and basins. In addition to the seas, there are numerous other obstacles to habitation such as deserts, mountains, hills, sands, jungles, and so on, which the learned have imparted in the geographical placebooks [*al-masālik*] and which travelers and others [have also made known].

[6] Some practitioners of this science have stated that the reason for the lack of habitation in the southern region is that on account of its proximity to the day-circle of the sun's perigee it is warmer since the sun will there be larger in size and more intense in its rays and its effects because it is nearer. This is unconvincing because the difference between the smallest size of the sun from its being at the apogee and its largest size from being at the perigee is imperceptible to the senses. It is thus far-fetched that its effect would reach an extent whereby one of two similarly positioned locations would be populated, while the other would not be populated. Furthermore, if this were the reason, then what is beyond [this day-circle] in the south, namely the [potential] populated regions whose latitude is greater than the maximum declination [*i.e.* obliquity], would be inhabited. In addition some have stated that the southern region is generally warmer than the northern region during the period that the perigee is in the southern signs; the heat draws the moisture and so the seas are drawn to the southern hemisphere and the uncovered part of the Earth is in the northern hemisphere. The inhabited world would then move as the apogee moved. This is likewise unconvincing because the existence of seas north of the inhabited world contradicts this judgment. Also some have said that the region that is beneath those southern day-circles occurring between the "falls" of the two luminaries is not populated; it is called the combust way whence they named what on the orb is between the two "falls" by this name as well. This is one of the fairy tales of the astrologers. In general there is no known cause for the above-mentioned amount of uncovering of the Earth except divine providence; otherwise, why would one of the northern quarters be characterized by [habitation] and not the other despite their situation with respect to the heavens being the same?

المغرب ، والخليج الأحمر ، وخليج فارس ، والخليج الأخضر ، ولكلّ واحد منها طول وعرض صالحان ؛ وكبحر ورنك من جانب الشمال . وبعضها غير متّصل كبحر طبرستان ، وبحيرة خوارزم ، وغيرهما من البطائح والمغائض . وغير البحار من موانع العمارة كالبراري والجبال والتلال والرمال والآجام وغيرها أيضاً كثير ، يعرّفها أهل العلم بالمسالك والسيّاح وغيرهم .

[٦] وقد قال بعض أهل هذا العلم في علّة عدم العمارة في الناحية الجنوبية أنّها لقربها من مدار حضيض الشمس تكون أحرّ إذ الشمس توجد هناك لقربها أعظم جرماً وأشدّ شعاعاً وأثراً . وهذا ليس بيقيني لأنّ التفاوت بين صغر الشمس من جهة كونها في الأوج وكبرها من جهة كونها في الحضيض ليس ببيّن عند الحسّ ، فمن البعيد أن يبلغ تأثيرها إلى حدّ يصير أحد موضعين متساويين في الوضع مسكوناً والآخر غير مسكون . وأيضاً لو كان السبب ذلك لكان ما جاوزه في الجنوب من المساكن التي يزيد عرضها على غاية الميل معموراً . وذكر أيضاً بعضهم أنّ ناحية الجنوب بالجملة أحرّ من ناحية الشمال مدّة كون الحضيض في البروج الجنوبية . والحرارة تجذب الرطوبة فلذلك انجذبت البحار إلى النصف الجنوبي ، وصار المنكشف من الأرض في النصف الشمالي . وتنتقل العمارة بانتقال الأوج . وهذا أيضاً ليس بيقيني لأنّ وجود البحار في شمال العمارة ينافي ذلك الحكم . وقال بعضهم أيضاً أنّ المواضع التي تكون تحت المدارات الجنوبية التي تقع بين هبوطي النيّرين غير مسكونة ، وتُسمّى بالطريقة المحترقة ولذلك سمّوا ما بين الهبوطين من الفلك بهذا الاسم أيضاً . وهذا من خرافات الأحكاميين . وبالجملة ليس لانكشاف القدر المذكور من الأرض سبب معلوم غير العناية الإلهية ؛ وإلاّ لِما اختُصّ أحد الربعين الشماليين بها دون الآخر مع تساوي أوضاعهما بالقِياس إلى السماويّات ؟

[7] Most of the inhabited world on the northern side falls between the [area] beyond 10 degrees in latitude up to 50 [degrees]. The practitioners of the science have divided it into **seven climes** [stretching] lengthwise so that each clime is beneath a day-circle, the conditions of the places in it then being similar. Thus each clime extends from east to west in longitude, while its [extent in] latitude is a small amount, which results in a difference of half an hour in lengths of longest daylight. The majority have made the initial point for the longitudes on the western side so that the longitude increases in magnitude in the direction of the sequence of the signs; the starting point for the latitudes is the equator since it, rather than something else, is determined by nature. They have stated that the beginning of the inhabited world in the west is in the islands named the Eternals (which at the present time are uninhabited) so some have made them the initial point for longitude. Another group made the coast of the western ocean the initial point. Between them is 10 degrees of revolution of the equinoctial. Among their learned the end of the inhabited world on the eastern side is Kangdezh, which is the initial point for those who make it be from the eastern side. What is on the equator between the two ends they named the **cupola** of the Earth, and it is at a distance of one-quarter revolution from the initial western point. There will thus be a divergence attendant upon [its location] because of the difference concerning [the initial point].

[8] Turning to the initial and midpoints of the climes according to latitudes and the longest periods of daylight, they are as follows: the initial point of the first is where the longest day is ($12 + \frac{1}{2} + \frac{1}{4}$) hours and its latitude is $12\frac{2}{3}$ degrees. Its midpoint is where the day is 13 hours and its latitude is ($16 + \frac{1}{2} + \frac{1}{8}$) degrees. The initial point of the second is where the day is $13\frac{1}{4}$ and the latitude is ($20 + \frac{1}{4} + \frac{1}{5}$); its midpoint is where the day is $13\frac{1}{2}$ and the latitude is ($24 + (\frac{1}{2}$ of $\frac{1}{6}$)) [*sic*]. The initial point of the third is where the day is ($13 + \frac{1}{2} + \frac{1}{4}$) and the latitude is $27\frac{1}{2}$; its midpoint is where the day is 14 and the latitude is $30\frac{2}{3}$. The initial point of the fourth is where the day is $14\frac{1}{4}$ and the latitude is ($33 + \frac{1}{2} + \frac{1}{8}$); its midpoint is

[٧] ومعظم العمارة في طرف الشمال يقع بين ما يجاوز عشر درجات في العرض إلى حدود الخمسين . فقسمها أهل الصناعة بالأقاليم السبعة طولاً ليكون كل إقليم تحت مدار ، فتتشابه أحوال البقاع التي فيه . فإذن كل إقليم يمتدّ ما بين الخافقين طولاً ، ويكون عرضه قدراً قليلاً وهو ما يوجب تفاضل نصف ساعة في مقادير النهار الأطول . والجمهور جعلوا مبدأ الأطوال من جانب المغرب ليكون ازدياد عدد الطول في جهة توالي البروج ، ومبدأ العروض خط الاستواء لأنّه بالطبع متعيّن دون ما عداه . وقد ذكروا أنّ بداية العمارة في المغرب كانت في جزائر منسوبة إلى الخالدات ـ وهي الآن غير معمورة ـ فجعلها بعضهم مبدأ الطول . وقوم آخر جعلوا ساحل البحر الغربي مبدأه ، وبينهما عشر درجات من دور معدّل النهار . ونهاية العمارة من الجانب الشرقي عند علمائهم كَنْكْدِز ، وهي المبدأ عند من يجعله من جانب المشرق . وسمّوا ما بين النهايتين على خط الاستواء قبّة الأرض ، وهي على بعد ربع الدور من المبدأ الغربي ، فيلزمها الاختلاف بسبب الاختلاف فيه .

[٨] وأمّا مبادئ الأقاليم وأوساطها بحسب العروض وساعات النهار الأطول فهي هذه : أمّا الأول فمبدؤه حيث النهار الأطول اثنتا عشرة ساعة ونصف وربع ، وعرضه اثنتا عشرة درجة وثُلثا درجة ؛ ووسطه حيث النهار ثلاث عشرة ساعة ، وعرضه ستّ عشرة درجة ونصف وثُمن . وأمّا الثاني فمبدؤه حيث النهار ثلاث عشرة وربع ، والعرض عشرون ورُبع وخُمس ؛ ووسطه حيث النهار ثلاث عشرة ونصف ، والعرض أربع وعشرون ونصف سُدس . وأمّا الثالث فمبدؤه حيث النهار ثلاث عشرة ونصف وربُع ، والعرض سبع وعشرون ونصف ؛ ووسطه حيث النهار أربع عشرة ، والعرض ثلاثون وثُلثان . وأمّا الرابع فمبدؤه حيث النهار أربع عشرة وربع ، والعرض ثلاث وثلاثون ونصف وثُمن ؛ ووسطه

where the day is $14\frac{1}{2}$ and the latitude is $(36 + \frac{1}{5} + \frac{1}{6})$. The initial point of the fifth is where the day is $(14 + \frac{1}{2} + \frac{1}{4})$ and the latitude is 39 less $\frac{1}{10}$; its midpoint is where the day is 15 and the latitude is $41\frac{1}{4}$. The initial point of the sixth is where the day is $15\frac{1}{4}$ and the latitude is $(43 + \frac{1}{4} + \frac{1}{8})$; the midpoint is where the day is $15\frac{1}{2}$ and the latitude is $(45 + \frac{1}{4} + \frac{1}{10})$. The initial point of the seventh is where the day is $(15 + \frac{1}{2} + \frac{1}{4})$ and the latitude is $47\frac{1}{5}$; its midpoint is where the day is 16 and the latitude is $(48 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8})$; its endpoint is where the day is $16\frac{1}{4}$ and the latitude is $50\frac{1}{3}$. The end of each clime other than the [last] is the beginning of the following one. One group has made the initial point of the first clime the equator. The end of the seventh is the termination of the inhabited world.

[9] The longest [period] of daylight reaches 17 hours where the latitude is 54 degrees plus a fraction; it reaches 18 where the latitude is 58; it reaches 19 where the latitude is 61; it reaches 20 where the latitude is 63; it reaches 21 where the latitude is $64\frac{1}{2}$; it reaches 22 where the latitude is 65 plus a fraction; it reaches 23 where the latitude is 66; and it reaches 24 where the latitude is equal to the colatitude of the obliquity. It reaches 1 month where the latitude is $67\frac{1}{4}$, 2 months where the latitude is 70 less $\frac{1}{4}$, 3 months where the latitude is $73\frac{1}{2}$, 4 months where the latitude is $78\frac{1}{2}$, 5 months where the latitude is 84, and half a year where the latitude is a quarter revolution.

[10] Let us now go into the characteristics of the day-circles.

حيث النهار أربع عشرة ونصف ، والعرض ستّ وثلاثون وخُمس وسُدس . وأمّا الخامس فمبدؤه حيث النهار أربع عشرة ونصف وربُع ، والعرض تسع وثلاثون إلاّ عُشر ؛ ووسطه حيث النهار خمس عشرة ، والعرض إحدى وأربعون وربع . وأمّا السادس فمبدؤه حيث النهار خمس عشرة وربع ، والعرض ثلاث وأربعون وربع وثُمن ؛ ووسطه حيث النهار خمس عشرة ونصف ، والعرض خمس وأربعون وربع وعُشر . وأمّا السابع فمبدؤه حيث النهار خمس عشرة ونصف وربع ، والعرض سبع وأربعون وخُمس ؛ ووسطه حيث النهار ستّ عشرة ، والعرض ثمان وأربعون ونصف وربع وثُمن ؛ وآخره حيث النهار ستّ عشرة وربع ، والعرض خمسون وثُلث . وآخر كل إقليم ما عداه أوّل الذي يليه . وقوم جعلوا مبدأ الإقليم الأول خط الاستواء . وآخر السابع منتهى العمارة .

[٩] والنهار الأطول يبلغ سبع عشرة ساعة حيث العرض أربع وخمسون درجة وكسر ؛ ويبلغ ثماني عشرة حيث العرض ثمان وخمسون ؛ ويبلغ تسع عشرة حيث العرض إحدى وستّون ؛ ويبلغ عشرين حيث العرض ثلاث وستّون ؛ ويبلغ إحدى وعشرين حيث العرض أربع وستّون ونصف ؛ ويبلغ اثنتين وعشرين حيث العرض خمس وستّون وكسر ؛ ويبلغ ثلاثاً وعشرين حيث العرض ستّ وستّون ؛ ويبلغ أربعاً وعشرين حيث العرض مثل تمام الميل كلّه . ويبلغ شهراً حيث العرض سبع وستّون وربع ، وشهرين حيث العرض سبعون إلاّ ربع ، وثلاثة أشهر حيث العرض ثلاث وسبعون ونصف ، وأربعة أشهر حيث العرض ثمان وسبعون ونصف ، وخمسة أشهر حيث العرض أربع وثمانون ، ونصف السنة حيث العرض ربع الدور .

[١٠] ولنشرع الآن في خواصّ المدارات .

CHAPTER TWO

On the Characteristics of the Equator

[1] Horizon circles of localities that are on the equator bisect all the day-circles since they pass through the poles of the equinoctial. Therefore / night and day / during the entire year are equal. Also the period of visibility of each point on the orb is equal to the period of its invisibility. If there is a difference, it is because of variability in speed in the two halves due to the second[ary] motion; this will not be perceptible. The sun will in a year pass twice over their zenith, this being when it is at the / point / of the two equinoxes. It moves away from their zeniths only to the extent of the ecliptic orb's maximum declination from the equinoctial; thus [the sun's] maximum altitude is never less than the complement of the obliquity. The sun is in each direction for half the year, the noon shadow being in the opposite direction. The poles of the ecliptic are at the horizon when one of the two equinox points are at the zenith; thereupon the intersection of the ecliptic equator and the horizon is at right angles. During the passage of the northern half of the [ecliptic] equator across the meridian, the southern of the two ecliptic poles is visible; during the passage of the southern half, the northern is visible. The altitude [of each pole] does not exceed the magnitude of the obliquity. Since [by definition] the beginning of summer is the time in which the sun is nearest the zenith and the beginning of winter is the time in which it is farthest from it, the time at which it is at the two points / of the two equinoxes / is the beginning of their summer and the time at which it is at the solstitial points is the beginning of their winter. The beginnings of the other two seasons are [at] the midpoints of the quarter [divisions of the ecliptic]. It follows from this that they have eight seasons in a year. The turning of the orb there is wheel-like because the planes

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/5/ night and day] β = day and night] α, M. /9/ point] β = two points] α, M. /20/ of the two equinoxes] β = of the equinox] α, M.

الفصل الثاني
في خواصّ خط الاستواء

[١] دوائر آفاق البقاع التي تكون على خط الاستواء تنصّف جميع المدارات اليومية لكونها مارّة بقطبي معدّل النهار ؛ فلذلك يكون / الليل والنهار / في جميع السنة متساويين . وأيضاً يكون زمان ظهور كل نقطة على الفلك مساوياً لزمان خفائه ؛ فإن كان تفاوت كان بسبب اختلاف السير بالحركة الثانية في النصفين ، وذلك لا يكون محسوساً . وتمرّ الشمس في السنة مرّتين بسمت رؤوسهم ، وذلك عند كونها في / نقطة / الاعتدالين . ولا تبعد عن سمت رؤوسهم إلاّ بقدر غاية ميل فلك البروج عن معدّل النهار ، فلا تنقص غاية ارتفاعها عن تمام الميل كلّه . وتكون الشمس نصف السنة في كل جهة وظل نصف النهار إلى خلاف تلك الجهة . وقطبا البروج يكونان على الأفق عند كون إحدى نقطتي الاعتدالين على سمت الرأس ، وهناك يكون قطع منطقة البروج للأفق على قوائم . وفي مدّة مرور النصف الشمالي من المنطقة على نصف النهار يكون الظاهر من قطبي البروج جنوبيّهما ، وفي مدّة مرور النصف الجنوبي يكون الظاهر شماليّهما ؛ ولا يزيد ارتفاعهما على قدر الميل الكلي . ولكون مبدأ الصيف الوقت الذي تكون الشمس فيه إلى سمت الرأس أقرب ومبدأ الشتاء الوقت الذي تكون فيه منه أبعد ، يكون وقت كونها في نقطتي / الاعتدالين / مبدأ صيفهم ووقت كونها في نقطتي الانقلاب مبدأ شتائهم . وتكون مبادئ الفصلين الآخرين أوساط الأرباع . فيلزم على ذلك أن يكون لهم في سنة ثمانية فصول . ويكون دور الفلك هناك دولابياً لأنّ سطوح

/5/ ...] β = النهار والليل] M , α . /9/ ...] β = نقطتي] M , α . /20/ ...] β = الاعتدال] M , α .

of all the day-circles intersect the horizon at right angles. Their horizons are therefore called the **horizons of the right orb**. And since the horizon circle [at the equator] is one of the circles of declination, the **ortive amplitude** of each point, which is the arc along the horizon between its rising place and the rising place of the equinoctial, is in the amount of its declination; the same [holds] for the **occasive** [setting] **amplitude**.

[2] The Grand Master Abū ᶜAlī ibn Sīnā judged the [equator] to be the most temperate locality. He stated: Because the sun does not linger there long at the zenith, but rather it passes by it at the times of its crossing from one of the directions to the other and its motion in declination will there be at its fastest, then the heat of their summer will therefore not be intense. This is because even though being directly overhead leads to heating, nevertheless duration of this [state] is more effective for that than the [state] itself. It is because of this that summer is warmer than spring and the afternoon is warmer than before [noon] despite the equality of alignments in each case. Furthermore since the periods of their day and their night are equal, the severity of each of the weather conditions arising from them will be quickly broken by the other; thus [each] period will be temperate. He also argued that the hottest localities in summer are those whose latitudes are equal to the obliquity. For the sun will be directly overhead and will linger near this alignment for nearly two months. Their days at that time will be long and their nights short.

[3] The eminent Imām Fakhr al-Dīn al-Rāzī rejected the first argument, saying: Even though the sun lingers on the equator only briefly, it nonetheless is never too far from being directly overhead; it is thus virtually overhead for the length of the year. We see localities whose maximum solar altitudes do not greatly exceed the minimum [solar] altitude at the equator, and the heat of their summers is extremely intense. / One learns / from this that the heat of the winter at the equator is many times the heat of the summer of those localities. He judged the most temperate clime to be the fourth.

/23/ One learns] β = Let one learn] α, M.

جميع المدارات تقطع سطح الأفق على قوائم ، وتسمى لذلك آفاقها بآفاق الفلك المستقيم . ولكون دائرة الأفق إحدى دوائر الميول تكون سعة مشرق كل نقطة ، وهي القوس التي تكون من الأفق بين مطلعها ومطلع معدّل النهار ، بقدر ميلها ؛ وكذلك سعة المغرب .

[٢] والشيخ الرئيس أبو علي بن سينا حكم بأنّها أعدل البقاع . قال : لأنّ الشمس لا تلبث على سمت الرؤوس هناك كثيراً بل إنّما تمرّ به وقتي اجتيازها عن إحدى الجهتين إلى الأخرى وتكون هناك حركتها في الميل أسرع ما تكون ، فلا تكون لذلك حرارة صيفهم شديدة . وذلك لأنّ المسامتة ــ وإن كانت مقتضية للتسخين ــ لكنّ المكث عليها أبلغ في ذلك من نفسها ؛ ولذلك يكون الصيف أحرّ من الربيع وبعد الزوال أحرّ من قبله مع تساوي المسامتة فيهما . وأيضاً لتساوي زماني نهارهم وليلهم تنكسر سَورتا كل واحدة من الكيفيتين الحادثتين منهما بالآخر[!] سريعاً ، فيعتدل الزمان . وحكم أيضاً بأنّ أحرّ البقاع صيفاً هي التي تكون عروضها مساوية للميل الكلي : فإنّ الشمس تسامتها وتلبث في قرب مسامتتها قريباً من شهرين ، ونهارها حينئذ يطول وليلها يقصر .

[٣] وردّ الإمام الفاضل فخر الدين الرازي عليه الحكم الأول بأن قال : لبثُ الشمس في خط الاستواء ــ وإن كان قليلاً ــ لكنّها لا تبعد كثيراً عن المسامتة ، فهي طول السنة في حكم المسامتة . ونحن نرى بقاعاً أكثر ارتفاعات الشمس / بها / لا يزيد كثيراً على أقلّ ارتفاعاتها بخط الاستواء وحرارة صيفها في غاية الشدّة . / فيُعلم / من ذلك أنّ حرارة شتاء خط الاستواء تكون أضعاف حرارة صيف تلك البقاع . وحكم بأنّ أعدل البقاع الإقليم الرابع .

/21/ بها] β = بها ما] M , α . /23/ ...] β = فليعلم] M , α .

[4] The truth of the matter is that if one means by temperate a uniformity in the conditions, then there is no doubt that it is most so at the equator as stated by the Master. But if one means by it a balancing of the two [extreme] weather conditions, then there is no doubt that the equator is not this way; this is indicated by the extreme blackness in color of its inhabitants among the peoples of Zanzibar and Abyssinia, the extreme frizziness of their hair, and other things that are brought about by the heat of the air. The opposite of this among the people of the fourth clime indicates that the state of its air is more temperate; indeed, the general reason for the profusion of habitations and the magnitude of propagation and reproduction in the seven climes, but not in the rest of the uncovered parts of the Earth, indicates that they are more temperate than other [places]. And what is nearer to the middle [of the climes] is most certainly nearer [to being temperate] than what is at their fringes; for becoming seared and remaining immature, which result from the two [extreme] weather conditions, are clearly evident at the two fringes.

CHAPTER THREE

On the Characteristics of Locations Having Latitude Which Are Called the Oblique Horizons

[1] For every location that is beneath one of the day-circles between the equator and one of the poles of the / first / motion, the turning of the orb there is slanted. The altitude of the pole that is in the direction to which the location is inclined is in the amount of the local latitude. The distance from the equinoctial of the day-circles that are permanently visible or permanently invisible is greater than the local colatitude; the distance of the largest of them, which touches the horizon, is equal to it. The remaining day-circles are divided by the horizon into two unequal parts—the largest visible ones being closest to the visible pole, the [largest] invisible ones being farthest away. The two parts are conversely equal for any two day-circles equidistant from and on opposite sides of the equinoctial. There is an increase in daylight up to the apex of the solstice that is adjacent to the visible pole, and a decrease in it until

/15/ first] $\beta = -\alpha$, –M.

[٤] والحق في ذلك أنّه إن عُني بالاعتدال تشابه الأحوال فلا شكّ أنّه في خط الاستواء أبلغ كما ذكره الشيخ . وإن عُني به تكافؤ الكيفيتين فلا شكّ أنّ خط الاستواء ليس كذلك ـ يدلّ عليه شدّة سواد لون سكّانه من أهل الزنج والحبشة وشدّة جُعودة شعورهم وغير ذلك مما تقتضيه حرارة الهواء . وأضداد ذلك في أهل الإقليم الرابع تدل على كون هوائه أعدل ؛ بل السبب الكلّي في توفّر العمارات وكثرة التوالُد والتناسُل في الأقاليم السبعة دون سائر المواضع المنكشفة من الأرض يدلّ على كونها أعدل من غيرها . وما يقرب من وسطها يكون لا محالة أقرب مما يكون على أطرافها ، فإنّ الاحتراق والفَجاجة اللازمين من الكيفيتين ظاهران في الطرفين .

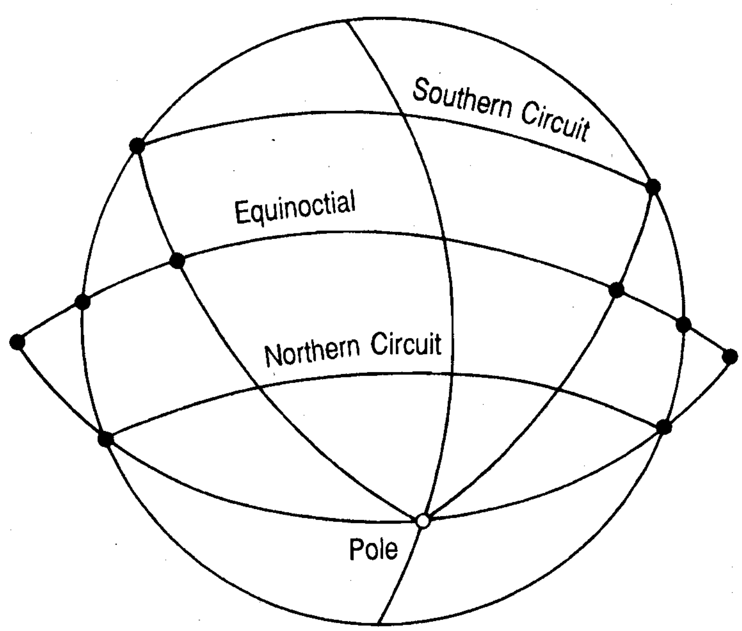
الفصل الثالث
في خواصّ المواضع التي يكون لها عرض وتُسمّى بالآفاق المائلة

[١] كل موضع يكون تحت أحد المدارات اليومية بين خط الاستواء وأحد قطبي الحركة / الأولى / يكون دور الفلك هناك حمائلياً . ويكون ارتفاع القطب الذي يكون في الجهة التي مال الموضع إليها بقدر عرض البلد . وكان بُعد المدارات الأبدية الظهور والأبدية الخفاء عن معدّل النهار أكثر من تمام عرض البلد ؛ وبُعد أعظمها الذي يماسّ الأفق مساوياً له . وسائر المدارات ينقسم بالأفق إلى مختلفين : أعظمهما الظاهر فيما هو إلى القطب الظاهر أقرب ، والخفي فيما هو أبعد . ويتساوى القسمان على التبادل في كل مدارين متساويي البعد عن معدّل النهار على جنبتيه . وتزايد النهار يكون إلى رأس المنقلب الذي يلي القطب الظاهر وتناقصه إلى

/15/ ...] β = M−, α− .

the apex of the other solstice. Daylight will only be equal to night when the sun is at the equinox points.

[2] If two circles of declination are assumed to pass through the two points at which the day-circle of the sun or of some star and the horizon intersect, there will occur two triangles between / those two circles, / the horizon, and the equinoctial; one is easterly and the other westerly. One of the sides of each [triangle] is the declination of the sun or the distance of the star from the equinoctial, which is along the declination circle; the second of them is the ortive amplitude of the sun or of the star, which is along the horizon circle; and the third [of the sides] is the **equation of daylight** of the sun or of the star, which is along the equinoctial, and it is half the difference between the daylight of the sun or of the star and the daylight of the equator. [When] this triangle is in the direction of the visible pole, it is below the Earth [*i.e.* horizon], while in the direction of the invisible pole it is above it.

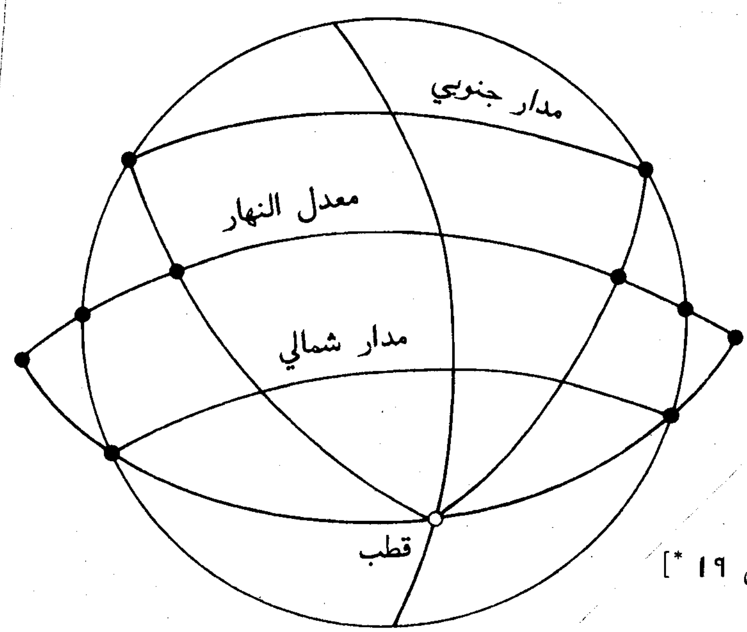


[Fig. T19*]

/5/ those two circles] β = that circle] α, M. [* Figure T19 occurs only in MS L.]

رأس المنقلب الآخر . ولا يكون النهار مساوياً للّيل إلاّ عند كون الشمس في نقطتي الاعتدالين .

[٢] وإذا فُرضت دائرتا ميل تمرّان بالنقطتين اللتين عليهما يتقاطع مدار الشمس أو كوكب من الكواكب والأفق حدث مثلّثان بين / تينك الدائرتين / والأفق ومعدّل النهار ، أحدهما شرقي والآخر غربي : أحد أضلاع كل واحد منهما ميل الشمس أو بُعد الكوكب عن معدّل النهار وهو الذي يكون من دائرة الميل ، وثانيها سعة مشرق الشمس أو الكوكب وهو الذي يكون من دائرة الأفق ، وثالثها تعديل نهار الشمس أو الكوكب وهو الذي يكون من معدّل النهار وهو نصف الفضل بين نهار الشمس أو الكوكب وبين نهار خط الاستواء . ويكون ذلك المثلّث في جانب القطب الظاهر تحت الأرض وفي جانب القطب الخفي فوقها .



[شكل ١٩ *]

/5/ ...] β = تلك الدائرة] α , M . [*لا يوجد الشكل إلاّ في مخطوط ل.]

[3] For every day-circle / whose elongation / from the equinoctial is on the side of the invisible pole, what revolves on [the part of the circle] above the Earth will not reach the prime vertical circle. Every day-circle whose elongation from the equinoctial in the direction of the visible pole is equal to the local latitude will pass through the zenith and be tangent to the prime vertical circle above the Earth; every [day-circle] whose elongation is greater than that will pass beyond the zenith in the direction of the visible pole, and it will not meet the prime vertical circle; and every one whose elongation is less than that will intersect the prime vertical at two points, one of which is east and the other west. The star, as long as it is between the two points, is beyond the prime vertical circle in the direction of the invisible pole.

CHAPTER FOUR

On the Characteristics of Locations Whose Latitude Does Not Exceed the Complement of the Obliquity

[1] They are divided into four divisions:

[2] The first is that whose latitude is less than the obliquity. In those locations the sun will pass over the zenith at two points whose declination is equal to the local latitude in the direction of the visible pole. At that time the ecliptic equator will be perpendicular to the horizon, its poles will be on the horizon, and objects will not have shadows at noon. As long as the [sun] is on the arc between the two points in the direction of the visible [equinoctial] pole, the [noon] shadow will fall toward the invisible pole; [in this case] the visible one of the ecliptic orb's poles will be that which is adjacent to the invisible pole of the equinoctial, and the invisible will be that which is adjacent to the visible pole. As long as the sun is on the other arc, *i.e.* the one between the two points in

/1/ whose elongation] $\beta = -\alpha$, –M.

[٣] وكل مدار يكون / بُعده / من معدّل النهار في جانب القطب الخفي فلا يصل ما يدور عليه فوق الأرض إلى دائرة أوّل السموت . وكل مدار يكون بعده عن معدّل النهار في جهة القطب الظاهر مثل عرض البلد فهو يمرّ بسمت الرأس ويماسّ دائرة أول السموت فوق الأرض ؛ وكل ما يكون بُعده أكثر من ذلك فهو يمرّ عن سمت الرأس في جهة القطب الظاهر ولا يلاقي دائرة أول السموت ؛ وكل ما يكون بُعده أقلّ من ذلك فهو يقطع أول السموت على نقطتين ، إحداهما شرقية والأخرى غربية ، ويكون الكوكب ما دام بين النقطتين عن دائرة أول السموت في جهة القطب الخفي .

الفصل الرابع
في خواصّ المواضع التي عرضها لا يجاوز تمام الميل الكلّي

[١] وهي تنقسم أربعة أقسام :

[٢] الأول ما يكون عرضها أقلّ من الميل الكلّي . وفي تلك المواضع تمرّ الشمس بسمت الرأس في نقطتين ميلهما يساوي عرض البلد في جهة القطب الظاهر ؛ وحينئذ تقوم منطقة البروج على الأفق على قوائم ، ويكون قطباها على الأفق ، ولا يكون للأشخاص في انتصاف النهار ظل . وما دامت في القوس التي بين النقطتين في جهة القطب الظاهر يقع الظل إلى جهة القطب الخفي ؛ ويكون القطب الظاهر من قطبي فلك البروج هو الذي يلي القطب الخفي من معدل النهار ، والخفي هو الذي يلي القطب الظاهر . وما دامت الشمس في القوس الأخرى ، أعني التي تكون بين النقطتين في

/1/ ...] β = M– , α– .

the direction of the invisible [equinoctial] pole, the [noon] shadow will fall toward the visible pole; [in this case] the visible one of the ecliptic orb's poles will be that which is adjacent to the visible pole of the equinoctial, and the invisible will be that which is adjacent to the invisible. The seasons of the year in those regions [*lit.*, horizons] are not equal; even if [their number] were increased beyond the four, they would still not be uniform.

[3] The second division is that whose latitude is equal to the obliquity. In those locations the sun will pass once a year over the zenith. One of the ecliptic orb's poles will be permanently visible, while the second will be permanently invisible; in their revolution, they will not touch the horizon except once, this being when the solstice point that is in the direction of the visible [equinoctial] pole reaches the zenith. At that time only, the ecliptic equator will be perpendicular to the horizon. Throughout the year, shadows will be toward the visible pole. The altitude of the sun increases from one of the solstices to the other; it then returns, [the altitude] decreasing until [the sun] comes back to [the original solstice]. The seasons of the year will be four—no more, no less.

[4] The third division is that whose latitude is greater than the obliquity and less than its complement. There the sun will not reach the zenith; it will have a highest altitude, which is equal to the sum of the obliquity and the local colatitude, and a lowest, which is equal to the excess of the local colatitude over the obliquity. The remaining conditions are as we have [already] explained. Where the local latitude does not exceed the obliquity by [more than] the amount of the latitudes of the other wandering [planets], those whose latitude is greater than the excess of the local latitude over the obliquity will pass over the zenith twice, while those whose latitude is equal to the excess [will pass over] once. In these latitudes, the equation of daylight and the ortive and occasive amplitudes increase with increasing latitude.

جهة القطب الخفي ، يقع الظل إلى جهة القطب الظاهر ؛ ويكون القطب الظاهر من قطبي فلك البروج هو الذي يلي القطب الظاهر من معدل النهار ، والخفي هو الذي يلي الخفي . ولا تكون فصول السنة في تلك الآفاق متساوية ؛ وإن زادت على الأربعة لم تكن متشابهة .

[٣] القسم الثاني ما يكون عرضها مساوياً للميل الكلي . وفي تلك المواضع تمرّ الشمس في السنة مرة واحدة بسمت الرأس . ويصير أحد قطبي فلك البروج أبدي الظهور والثاني أبدي الخفاء ، لا يماسّان الأفق في دورتهما إلاّ مرة واحدة وذلك عند انتهاء نقطة المنقلب الذي يكون في جهة القطب الظاهر إلى سمت الرأس ؛ وحينئذ تقطع منطقة البروج الأفق على قوائم فقط . وتصير الأظلال في جميع السنة إلى جهة القطب الظاهر . وارتفاعات الشمس تتزايد من أحد الانقلابين إلى الآخر ، ثم ترجع وتتناقص إلى أن تعود إليه . وتصير فصول السنة أربعة لا غير .

[٤] القسم الثالث ما يكون عرضها زائداً على الميل الكلي وناقصاً من تمامه . وهناك لا تنتهي الشمس إلى سمت الرأس ، ويكون / لها / ارتفاعان : أعلى وهو يكون بقدر مجموع الميل الكلي وتمام عرض البلد ، وأسفل وهو يكون بقدر فضل تمام عرض البلد على الميل الكلي . ويكون سائر الأحوال كما بيّنّا . فإن كان عرض البلد لا يزيد على الميل الكلي بقدر عروض سائر السيّارة مرّ منها بسمت الرأس مرّتين ما زاد عرضه على فضل عرض البلد على الميل الكلي ، ومرة ما ساوى عرضه الفضل . وفي هذه العروض يزداد تعديل النهار وسعة المشرق والمغرب بازدياد العرض .

. α , M [له = β [... /17/

[5] The fourth division is that whose latitude is equal to the complement of the obliquity. The day-circle of the solstice that is in the direction of the visible pole becomes permanently visible there, while the day-circle of the other solstice is permanently invisible. The day-circle of the visible pole of the ecliptic orb passes over the zenith, and the day-circle of the other pole over the nadir [*lit.*, that opposite it]. Then when the visible solstice comes to touch the horizon, it will do so at the point of the pole of the prime vertical that is in the direction of the visible pole, while the invisible solstice will touch it at the other pole; the two poles [of the ecliptic] will thereupon be at the zenith and the nadir [*lit.*, that opposite it], and the ecliptic equator will coincide with the horizon. Then when the pole departs from the zenith, and the visible solstice rises from [the horizon], the eastern half of the [ecliptic] equator rises in one stroke from the horizon. The point subsequent to the invisible solstice will then be upon the pole of the prime vertical, being about to set, and the point subsequent to the visible solstice will be at its other pole, being about to rise. The visible half is what is between them, *i.e.* the half that the vernal equinox is in the middle of if the visible pole is northerly, or the autumnal if it is southerly; the invisible half is the other half. Then the invisible half will rise point by point in all parts of the eastern half of the horizon, and the visible half will similarly set point by point, during the period of a nychthemeron [*lit.*, the day with its night] until the position of the orb returns to its original condition. [The maximum for] each of the ortive amplitude and the equation of daylight will there be a quarter revolution. Daylight will increase until the measure of the nychthemeron becomes entirely daylight; night will thereafter occur, increasing until the measure of the nychthemeron becomes entirely night. The altitude of the sun increases until it reaches twice the obliquity; it will then begin to decrease, decreasing until it becomes zero [whereupon] the sun will touch the horizon. The rising of a half revolution of the ecliptic equator occurs with a revolution of the equinoctial; the rising of the other half of the ecliptic equator does not [require] time.

[٥] القسم الرابع ما يكون / عرضها / مساوياً لتمام الميل الكلي . وهناك يصير مدار المنقلب الذي يكون في جهة القطب الظاهر أبدي الظهور ومدار المنقلب الآخر أبدي الخفاء . ويمرّ مدار قطب فلك البروج الظاهر بسمت الرأس ومدار القطب الآخر بمقابله . فإذا وافى المنقلبُ الظاهرُ مماسّةَ الأفق ، ماسّةً على نقطة قطب أول السموت التي في جهة القطب الظاهر ، وماسّه المنقلب الخفي على القطب الآخر ، وصار القطبان على سمت الرأس ومقابله ، وانطبقت منطقة البروج على الأفق . ثم إذا زال القطب عن سمت الرأس وارتفع المنقلب الظاهر عنه ، ارتفع النصف الشرقي من المنطقة دفعة عن الأفق ؛ فيكون الجزء التالي للمنقلب الخفي على قطب أول السموت يريد الغروب ، والجزء التالي للمنقلب الظاهر على قطبه الآخر يريد الطلوع . ويكون النصف الظاهر ما بينهما ، أعني النصف الذي يتوسّطه الاعتدال الربيعي إن كان القطب الظاهر شمالياً أو الخريفي إن كان جنوبياً ؛ والنصف الخفي هو النصف الآخر . ثم يطلع النصف الخفي جزءاً بعد جزء في جميع أجزاء نصف الأفق الشرقي ، ويغيب النصف الظاهر جزءاً بعد جزء كذلك ، في مدة اليوم بليلته إلى أن يعود وضع الفلك إلى حاله الأولى . ويكون هناك كل واحد من سعة المشرق وتعديل النهار ربعاً من الدور . وزيادة النهار إلى أن يصير مقدار يوم بليلته نهاراً كله ؛ ثم يحدث ليل ويزيد إلى أن يصير مقدار يوم بليلته ليلة كله . ويزيد ارتفاع الشمس إلى أن يبلغ ضعف الميل الكلي ؛ ثم يأخذ في التناقص ويتناقص إلى أن يفنى ، وتماسّ الشمس الأفق . ويكون طلوع نصف دور من منطقة البروج مع دور من معدل النهار ؛ وطلوع النصف الآخر من منطقة البروج لا في زمان .

/1/ ...] β = عرضه] M , α .

CHAPTER FIVE

On the Characteristics of Locations Whose Latitude Exceeds the Complement of the Obliquity But Does Not Reach One-Quarter Revolution

[1] In these locations, the largest permanently visible day-circle will intersect the ecliptic equator at two points whose declinations in the direction of the visible pole are equal. The largest permanently invisible day-circle will intersect it at two points opposite these two in the direction of the invisible pole. The ecliptic equator is divided into four arcs: one of them is permanently visible at the middle of which is the solstice that is in the direction of the visible pole; the second is permanently invisible at the middle of which is the other solstice. The endpoints of the first arc touch the horizon but do not disappear, and the endpoints of the second arc touch it but do not rise. As for the two remaining arcs, the one at the middle of which is the first of Aries rises in reverse order and sets in regular order if the visible pole is northerly, and it rises in regular order and sets in reverse order if the visible pole is southerly; and the one at the middle of which is the first of Libra is the opposite of this. The visible solstice has a highest altitude, which is equal to the sum of the obliquity and the local colatitude along the meridian circle in the direction of the invisible pole, and a lowest [altitude], which is equal to the excess of the local latitude over the complement of the obliquity along the meridian circle in the direction of the visible pole. The visible pole of the ecliptic orb also has a highest altitude, which is equal to the sum of the local colatitude and the complement of the obliquity, and a lowest, which is equal to the excess of the local latitude over the obliquity; the pole will be simultaneously with the solstice on the meridian, but [they are] in opposite directions from the zenith and their altitudes are at opposite [extremes]. One may draw analogous conclusions from this for the situation of the invisible solstice and the invisible pole.

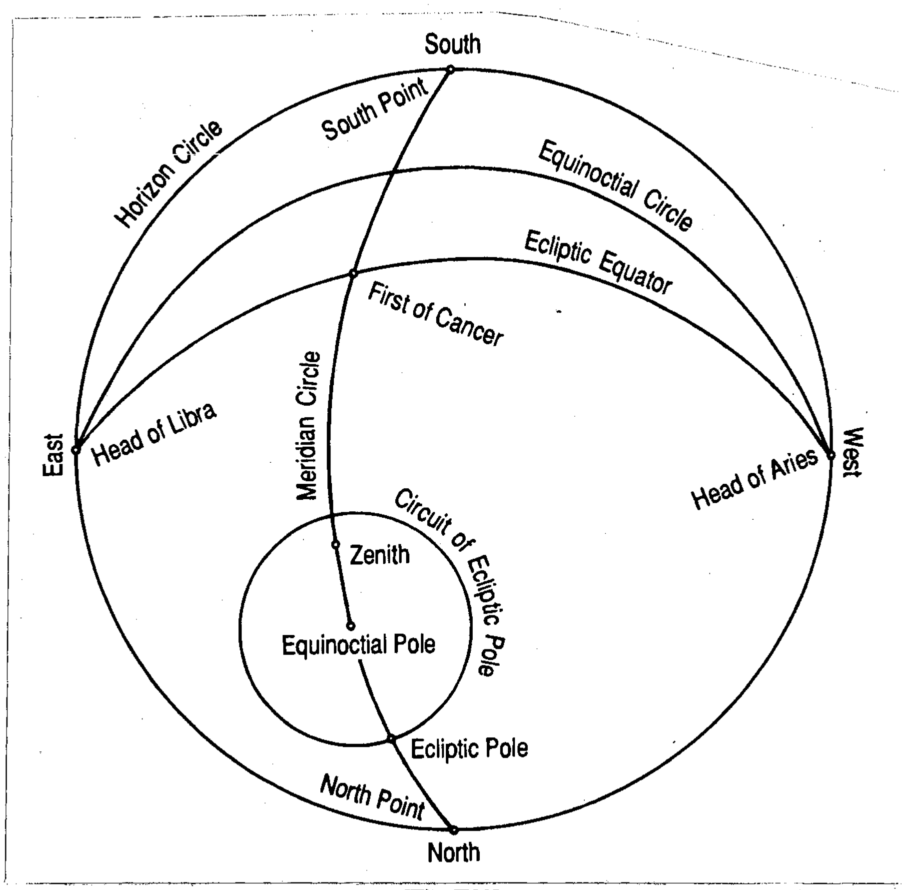
[2] In order to conceive positions in these latitudes, we shall take an example. Let the latitude in the north be 70: the arc that is permanently visible will be Gemini

الفصل الخامس
في خواصّ المواضع التي يجاوز عرضها تمام الميل الكلي ولا يبلغ ربع الدور

[١] في هذه المواضع يكون أعظم المدارات الأبدية الظهور قاطعاً لمنطقة البروج على نقطتين يتساوى ميلهما في جهة القطب الظاهر ؛ وأعظم المدارات الأبدية الخفاء قاطعاً لها على نقطتين متقابلتين لهما في جهة القطب الخفي . وتنقسم منطقة البروج إلى أربع قسي : إحداها أبدية الظهور ، وهي التي يتوسطها المنقلب الذي يكون في جهة القطب الظاهر ؛ والثانية أبدية الخفاء ، وهي التي يتوسطها المنقلب الآخر . وطرفا القوس الأولى يماسّان الأفق ولا يغيبان ، وطرفا القوس الثانية يماسّانه ولا يطلعان . وأما القوسان الباقيتان فالتي يتوسطها أول الحمل تطلع معكوسة وتغرب مستوية إن كان القطب الظاهر شمالياً ، وتطلع مستوية وتغرب معكوسة إن كان القطب الظاهر جنوبياً ؛ والتي يتوسطها أول الميزان تكون بالضد من ذلك . ويكون للمنقلب الظاهر ارتفاعان : أعلى ، وهو يكون بقدر مجموع الميل الكلي وتمام عرض البلد على دائرة نصف النهار في جهة القطب الخفي ؛ وأسفل ، وهو يكون بقدر فضل عرض البلد على تمام الميل الكلي على دائرة نصف النهار في جهة القطب الظاهر . ويكون لقطب فلك البروج الظاهر أيضاً ارتفاعان : أعلى ، وهو يكون بقدر مجموع تمام عرض البلد وتمام الميل الكلي ؛ وأسفل ، وهو يكون بقدر فضل عرض البلد على الميل الكلي . ويكون القطب مع المنقلب على نصف النهار معاً ولكن في الجهتين المتقابلتين عن سمت الرأس والارتفاعين المتبادلين . وقِس عليه حال المنقلب الخفي والقطب الخفي .

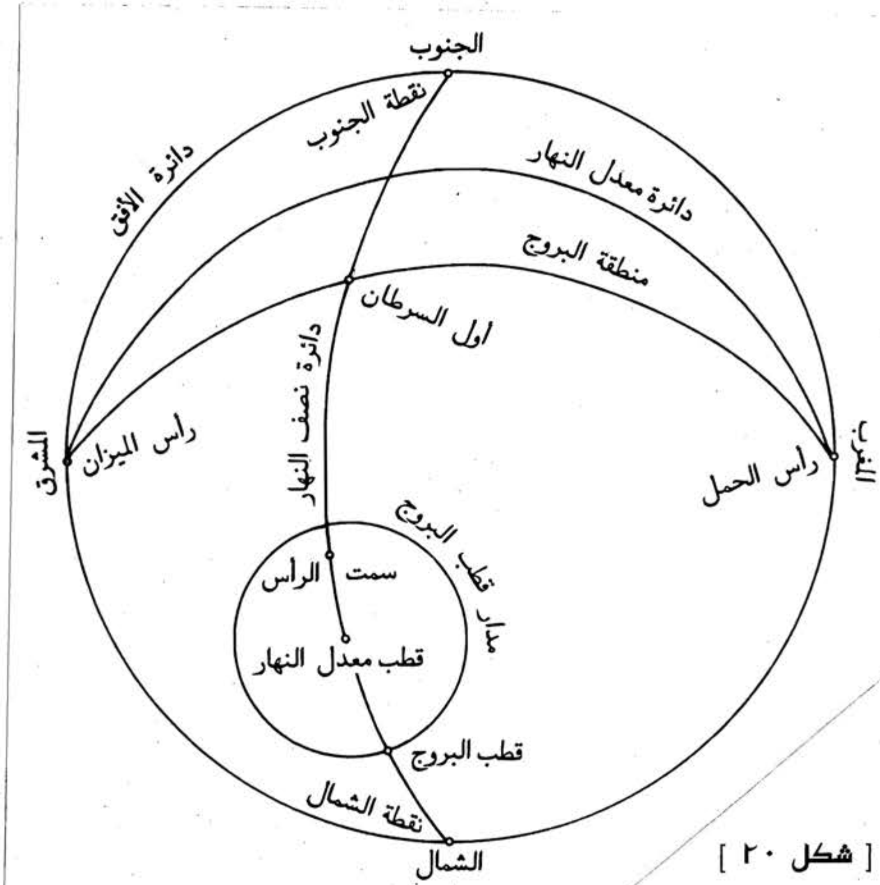
[٢] ولكي تتصور الأوضاع في هذه العروض نمثّل له مثالاً . وليكن العرض في الشمال سبعين : والقوس الأبدية الظهور الجوزاء

and Cancer, and the arc that is permanently invisible will be Sagittarius and Capricornus; the arc that rises in reverse order and sets in regular order is from the first of Aquarius to the last of Taurus, and that which rises in regular order and sets in reverse order is from the first of Leo to the last of Scorpius. Then when the first of Cancer is at the meridian on the southern side, its altitude being at its maximum [value], namely $(43 + \frac{1}{3} + \frac{1}{4})°$, the visible pole of the ecliptic orb is on the northern side on the meridian as well and its altitude is at its minimum [value], which is $(46 + \frac{1}{4} + \frac{1}{6})°$. At the rising place of the equinox is the first of Libra, which is on the verge of rising, and at its setting place is the first of Aries, which is on the verge of setting. The visible half of the ecliptic orb [extends] from west to east in the south as in this illustration:



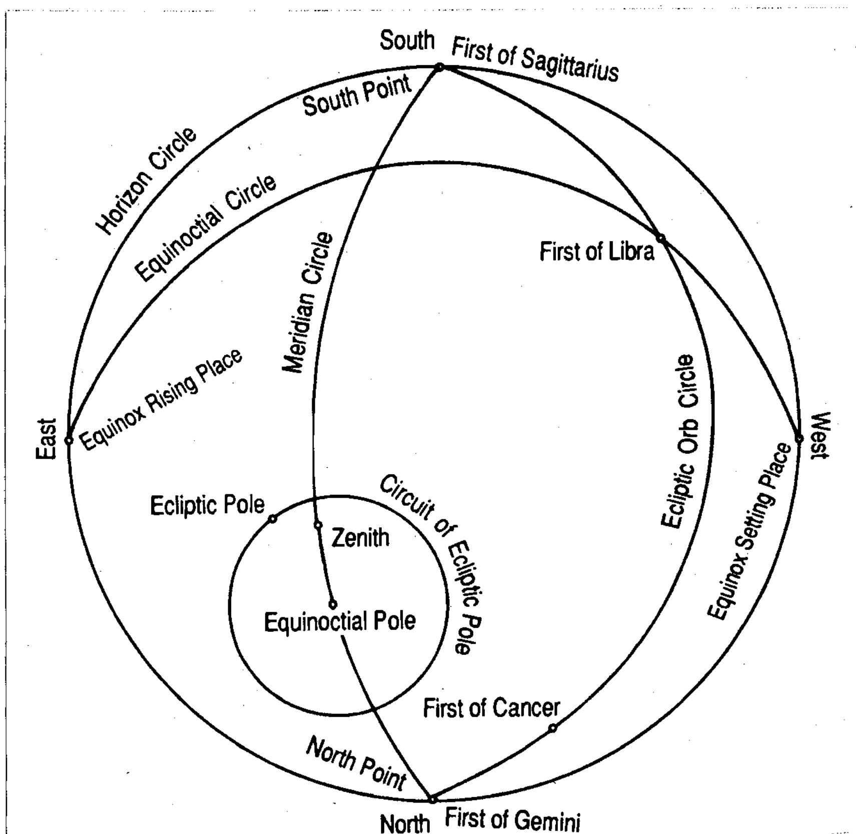
[Fig. T20]

والسرطان ، والقوس الأبدية الخفاء القوس والجدي ؛ والقوس التي تطلع معكوسة وتغرب مستوية من أول الدلو إلى آخر الثور ، والتي تطلع مستوية وتغرب معكوسة من أول الأسد إلى آخر العقرب . فإذا كان أول السرطان على نصف النهار من جانب الجنوب وارتفاعه في غاية الزيادة وهو ثلاث وأربعون درجة وثُلث وربع ، كان قطب فلك البروج الظاهر من جانب الشمال أيضاً على نصف النهار وارتفاعه في غاية النقصان وهو ست وأربعون درجة وربع وسدس . ويكون على مطلع الاعتدال أول الميزان يريد الطلوع ، وعلى مغيبه أول الحمل يريد الغروب . ونصف فلك البروج الظاهر من المغرب إلى المشرق في الجنوب على هذه الصورة :



[شكل ٢٠]

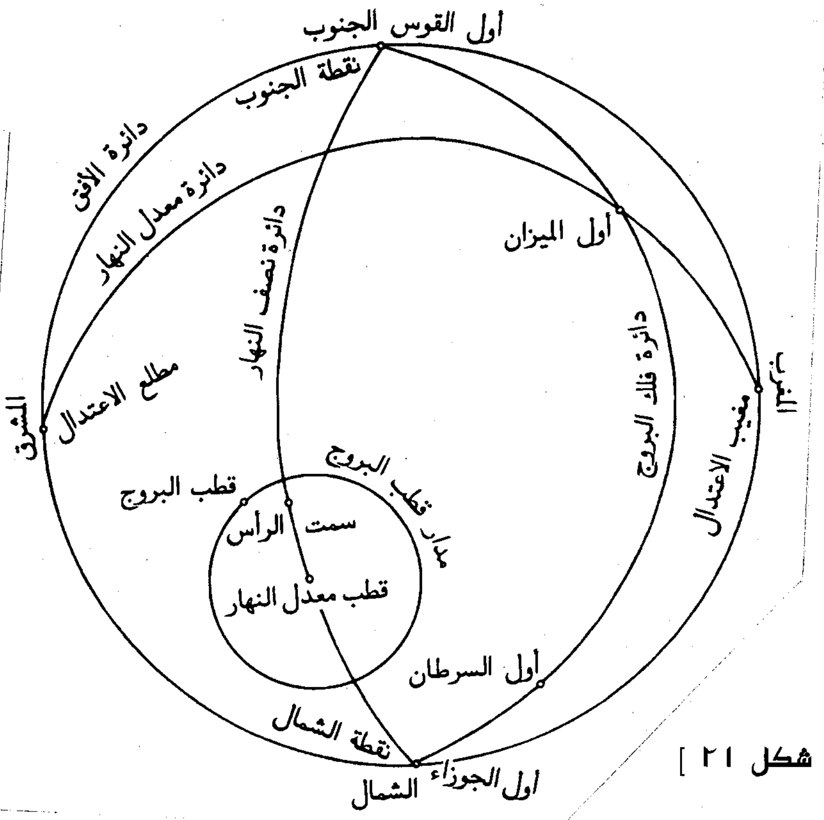
[3] Then let the orb move with the first motion so that Libra and Scorpius will rise in regular order, and their ortive amplitude will span the southeastern quarter [of the horizon]. Aries and Taurus will set likewise in regular order, and their occasive amplitude will span the northwestern quarter [of the horizon]. The first of Cancer will begin its descent toward the west and the pole of the ecliptic orb its ascent toward the east until the first of Sagittarius comes to touch the horizon at the south point and the first of Gemini [comes] to touch the horizon at the north point. The visible half of the ecliptic equator comes to be on the western side [extending] from south to north as in this illustration:



[Fig. T21]

[4] Then let the orb move so that the first of Gemini begins its rise toward the east, and the last of Taurus, which is contiguous with it, rises little by little until Taurus has risen.

[٣] ثم ليتحرك الفلك بالحركة الأولى : فيطلع الميزان والعقرب مستويين وليستغرق الربع الشرقي الجنوبي سعة مشرقهما ؛ ويغرب الحمل والثور أيضاً مستويين وليستغرق الربع الغربي الشمالي سعة مغربهما . وليأخذ أول السرطان في / الانحطاط / نحو المغرب وقطب فلك البروج في / الارتفاع / نحو المشرق إلى أن ينتهي أول القوس إلى مماسّة الأفق على نقطة الجنوب وأول الجوزاء إلى مماسّة الأفق على نقطة الشمال . ويصير النصف الظاهر من منطقة البروج في الجانب الغربي من الجنوب إلى الشمال على هذه الصورة :

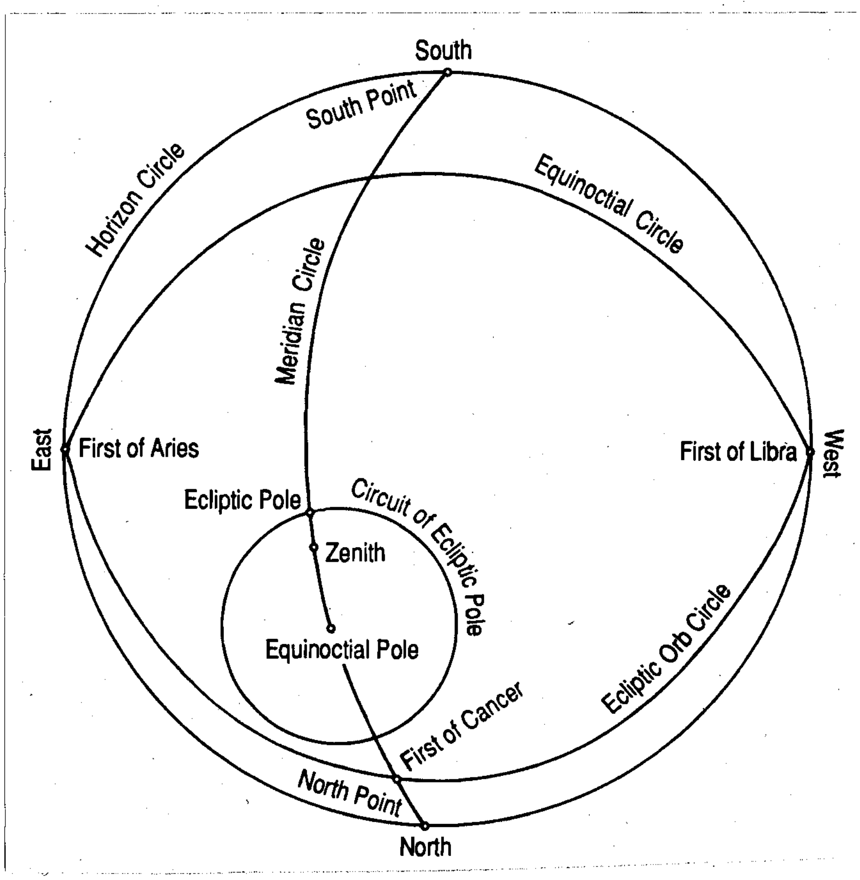


[شكل ٢١]

[٤] ثم ليتحرك الفلك : فيأخذ أول الجوزاء في الارتفاع نحو المشرق ويطلع آخر الثور المتّصل به شيئاً بعد شيء إلى أن يطلع

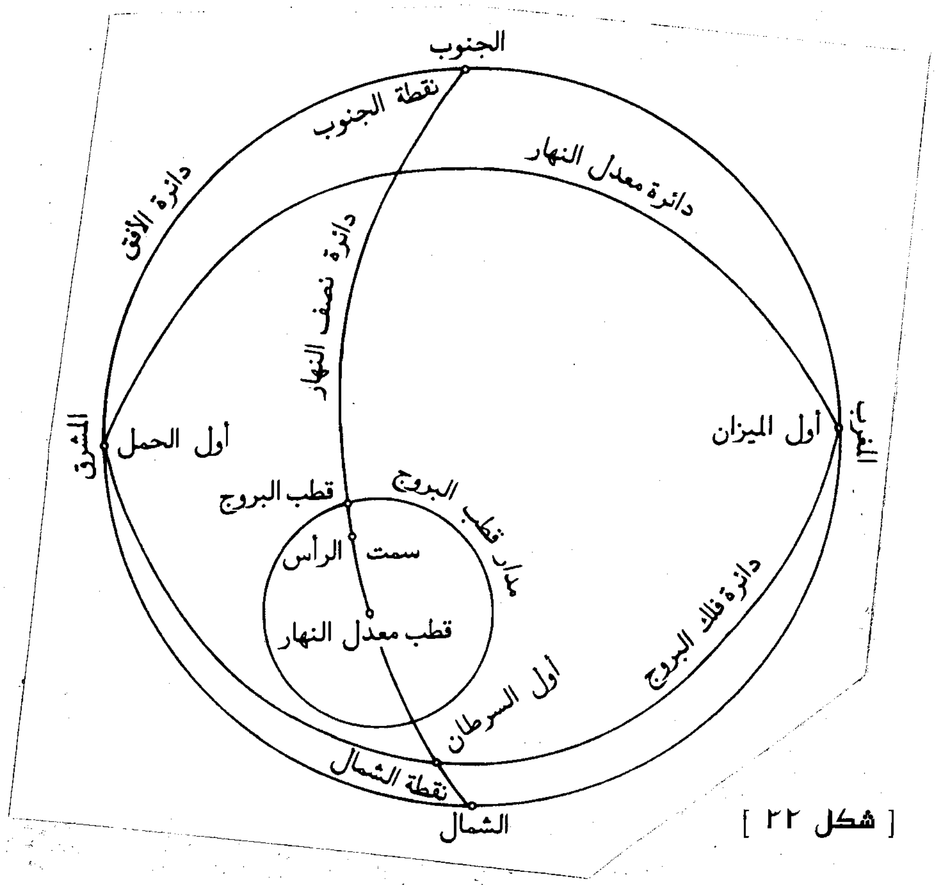
/4/ ...] β = انحطاط] M , α . /5/ ...] β = ارتفاع] M , α .

Then Aries rises from its last to its first. The ortive amplitude of these two signs spans the northeastern quarter [of the horizon], and the first of Aries reaches its rising place. Directly opposite this, the first of Sagittarius begins its descent below the horizon, and the last of Scorpius, which is contiguous with it, sets little by little until Scorpius has disappeared; then Libra sets from its last to its first. Their occasive amplitude spans the southwestern quarter [of the horizon], and the first of Libra reaches its setting place. The first of Cancer reaches the meridian circle on the northern side; it is at its lowest altitude, which is $(3 + \frac{1}{3} + \frac{1}{4})°$. The pole of the ecliptic orb [reaches] its highest altitude [which is] on the southern side, and this is $(86 + \frac{1}{4} + \frac{1}{6})°$. The visible half of the ecliptic orb is on the northern side between the rising and setting places of the equinox, the directional sequence [of the signs here being] opposite the conventional one, as in this illustration:



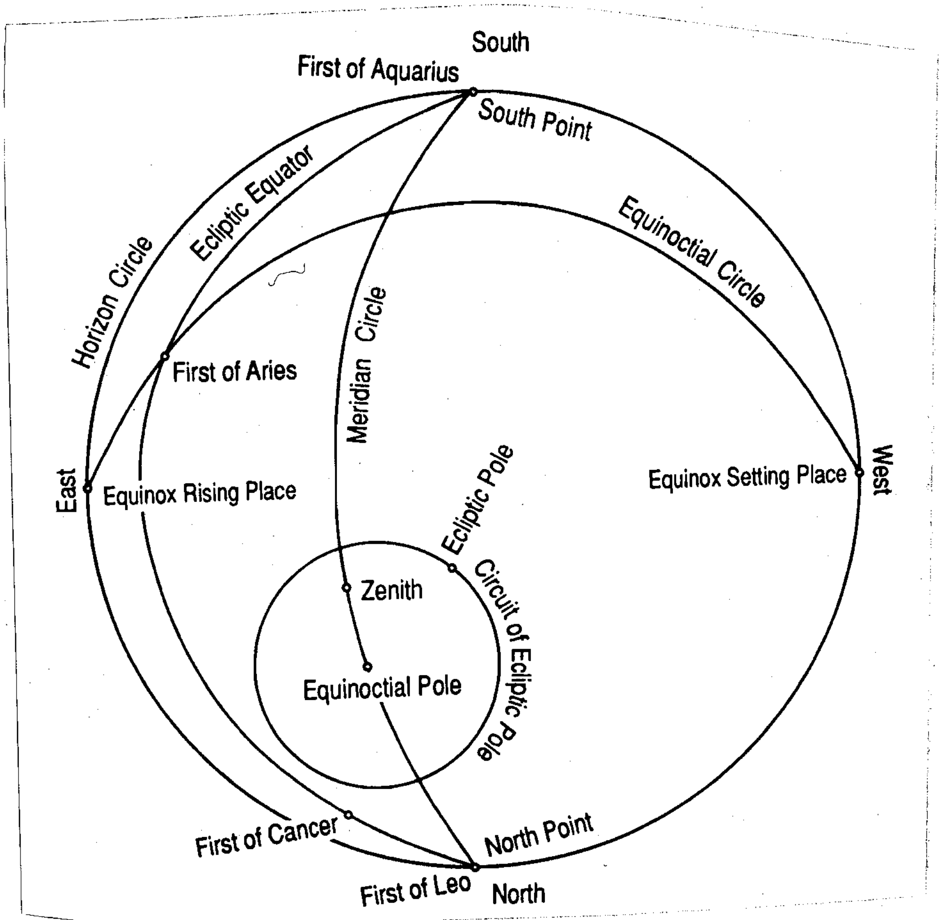
[Fig. T22]

الثور ، ثم يطلع آخر الحمل إلى أوله ؛ ويستغرق الربع الشمالي الشرقي سعة مشرق هذين البرجين ، وينتهي أول الحمل إلى مطلعه . ويأخذ بإزاء ذلك أول القوس في الانحطاط تحت الأفق ويغرب آخر العقرب المتّصل به شيئاً بعد شيء إلى أن يغيب العقرب ، ثم يغرب آخر الميزان إلى أوله ؛ ويستغرق الربع الجنوبي الغربي سعة مغربهما ، وينتهي أول الميزان إلى مغيبه . وينتهي أول السرطان إلى دائرة نصف النهار في جانب الشمال ويكون في ارتفاعه الأسفل وهو ثلاث درجات وثُلث وربع ، وقطب فلك البروج إلى ارتفاعه الأعلى في جانب الجنوب وهو ست وثمانون درجة وربع وسدس . ويكون النصف من فلك البروج الظاهر في جانب الشمال بين مطلع الاعتدال ومغيبه على توالي مخالف للمعهود على هذه الصورة :



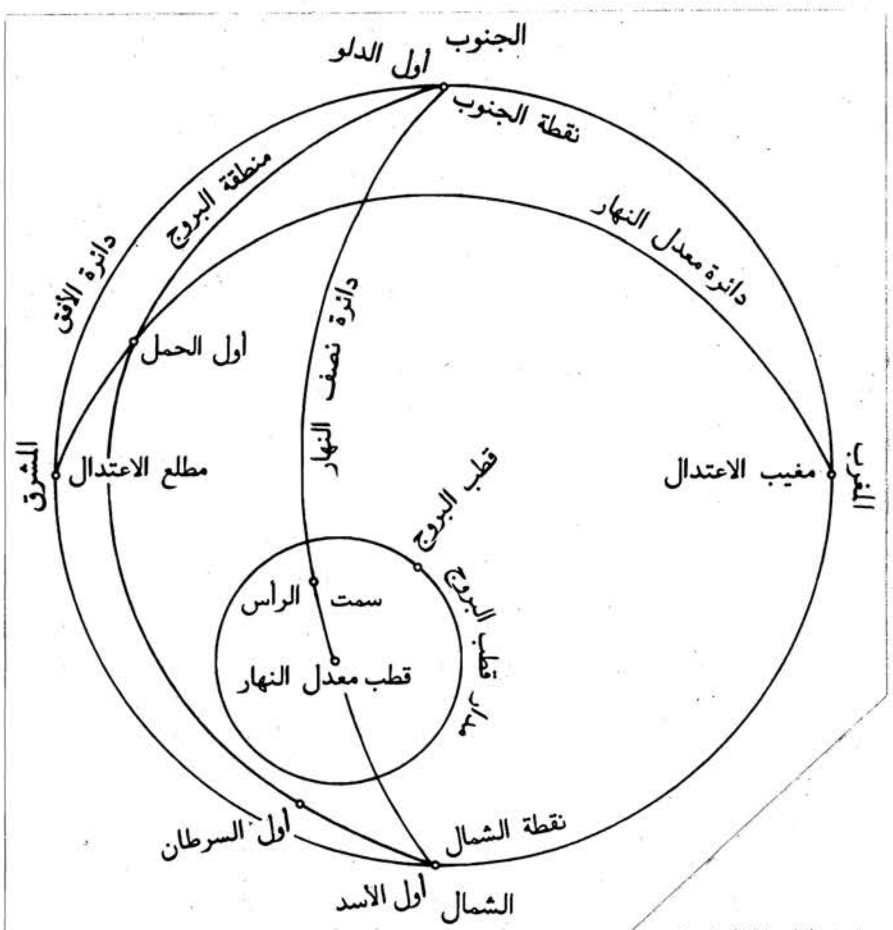
[شكل ٢٢]

[5] Then let the orb move so that Pisces rises from its last to its first, then Aquarius from its last to its first. Their ortive amplitude spans the southeastern quarter [of the horizon]. Directly opposite them, Virgo disappears from its last to its first, then Leo from its last to its first. Their occasive amplitude spans the northwestern quarter [of the horizon]. The first of Aquarius comes to touch the horizon at the south point, and the first of Leo comes to touch the horizon at the north point; the visible half of the ecliptic circle is in between them in the eastern direction. The first of Cancer has risen higher on the eastern side, and the pole has begun its descent on the western side as in this illustration:



[Fig. T23]

[٥] ثم ليتحرك الفلك : فيطلع آخر الحوت إلى أوله ، ثم آخر الدلو إلى أوله ؛ ويستغرق الربع الشرقي الجنوبي سعة مشرقهما . ويغيب بإزائهما آخر السنبلة إلى أولها ، ثم آخر الأسد إلى أوله ؛ ويستغرق الربع الغربي الشمالي سعة مغربهما . ويصير أول الدلو على نقطة الجنوب مماساً للأفق ، وأول الأسد على نقطة الشمال مماساً للأفق ، ونصف دائرة البروج الظاهر فيما بينهما من جهة المشرق . وأول السرطان قد ارتفع في جانب المشرق ، والقطب قد أخذ في الانحطاط في جانب المغرب على هذه الصورة :



[شكل ٢٢]

[6] Then let the orb move so that the first of Leo rises from the horizon moving toward the eastern half; the points of Leo will then rise sequentially until its last, then the points of Virgo. Their ortive amplitude spans the northeastern quarter [of the horizon]. Directly opposite this, the first of Aquarius will drop below the horizon to beneath the Earth. Thus Aquarius will set followed by Pisces sequentially. Their occasive amplitude spans the southwestern quarter [of the horizon]. The [action of] rising will then have reached the first of Libra and that of setting the first of Aries. At that time the first of Cancer will have come to be at the meridian circle at its highest altitude, and the visible pole of the ecliptic orb will be at its lowest altitude on the meridian. The visible half of the ecliptic orb comes to be on the southern side, and the situation returns to what we assumed originally. So the revolution is completed and what we have [earlier] described should become clear. We have been excessively verbose in this chapter only because of the difficulty of imagining these positions.

[7] In these regions [*lit.*, horizons] when the local latitude approaches the extreme and the altitude of the equinoctial from the horizon is small, then a planet may move, due to the extreme proximity of its day-circle from the horizon, to another day-circle by [means of] its secondary motion; thus it might disappear while in the eastern half after having been visible, or it might become visible while in the western half after having been invisible. So it would then have set in the east or risen in the west. And this too is one of the things asked about that is found to be strange.

CHAPTER SIX

On the Characteristics of Locations Whose Latitude Is Exactly One-Quarter Revolution

[1] This does not occur on the Earth except at two points at which one of the poles of the equinoctial is there at the zenith. The equinoctial circle becomes

[٦] ثم ليتحرك الفلك : وارتفع أول الأسد عن الأفق آخذاً نحو النصف الشرقي ، فتطلع أجزاء الأسد على التوالي إلى آخره ، ثم أجزاء السنبلة ؛ ويستغرق الربع الشمالي الشرقي سعة مشرقهما . وبإزاء ذلك ينخفض أول الدلو عن الأفق إلى تحت الأرض ، فيغرب الدلو ثم الحوت على التوالي ؛ ويستغرق الربع الجنوبي الغربي سعة مغربهما . ثم ينتهي الطلوع إلى أول الميزان والغروب إلى أول الحمل . ويصير حينئذ أول السرطان إلى دائرة نصف النهار وارتفاعه الأعلى ، والقطب الظاهر من فلك البروج إلى ارتفاعه الأسفل من نصف النهار . ويصير النصف الظاهر من فلك البروج في جانب الجنوب ، ويعود الوضع إلى ما فرضناه مبدأً ؛ فيتم الدور ويتّضح ما وصفناه . وإنما أطنبنا القول في هذا الفصل لعسر تصور هذه الأوضاع .

[٧] وفي هذه الآفاق إذا قرب عرض البلد من النهاية وصار ارتفاع معدّل النهار من الأفق قليلاً ، فربما ينتقل كوكب / لقرب / مداره من الأفق جداً إلى مدار آخر بحركته الثانية ؛ فيغيب بعد ما كان ظاهراً وهو في النصف الشرقي أو يظهر بعد ما كان خفياً وهو في النصف الغربي . فيكون قد غرب في المشرق أو طلع من المغرب . وهذا أيضاً من الأسئلة المستغربة .

الفصل السادس
في خواصّ المواضع التي يكون عرضها ربعاً من الدور سواءً

[١] وذلك لا يكون على الأرض إلا عند نقطتين يكون أحد قطبي معدل النهار على سمت الرأس هناك . وتصير دائرة معدل

/14/ ...] M , β = بقرب [α .

coincident with the horizon, and the orb turns with the first motion in a spinning manner; there is no longer on the horizon an east or a west. Thus the half of the orb that is in the direction of the visible pole from the equinoctial is permanently visible, and the other half is permanently invisible.

[2] As long as the sun is in the visible half of the ecliptic orb, it will be daytime; and as long as it is in the invisible half of it, it will be night. Thus its [the sun's?] entire year will be a nychthemeron [*lit.*, a day with its night], with one [*i.e.* daytime or night] exceeding the other due to the variability in speed [*lit.*, the slowness and fastness] of the [sun's] motion. Beneath the northern pole at the present time, their daytime is greater than their night by seven of our nychthemerons. This is because the apogee of the sun is at the end part of Gemini and its perigee is at the end part of Sagittarius. The period for the setting of dusk or the rising of dawn occurs for them in 50 of our days as we shall explain when describing these two [terms] later on. The maximum altitude of the sun and its maximum depression is in the amount of the maximum declination [*i.e.* the obliquity]. Neither the rising nor the setting of the sun and planets that is due to the secondary motion will occur at the same place on the horizon.

[3] Stars whose latitude is less than the obliquity will have a rising and a setting; the periods of visibility and invisibility will vary depending on how far or how near their day-circle is from the ecliptic orb [*sic*]. Stars whose latitude is equal to the obliquity will touch the horizon once during one revolution of the second motion; neither they nor those [stars] whose latitude exceeds the obliquity will have a rising or a setting, but instead they will be either permanently visible or invisible. Let it be recalled what we said regarding the positions of the orb due to the first two motions, and let [the situation] be determined here on that basis.

[4] This is the last of the descriptions of the localities that are beneath the day-circles and what is comparable to them.

النهار منطبقة على الأفق ، ويدور الفلك بالحركة الأولى رَحَوية ولا يبقى في الأفق مشرق ولا مغرب . فيكون النصف من الفلك الذي يكون من معدل النهار في جهة القطب الظاهر أبدي الظهور والنصف الآخر أبدي الخفاء .

[٢] والشمس ما دامت في النصف الظاهر من فلك البروج يكون نهاراً ، وما دامت في النصف الخفي منه يكون ليلاً ؛ فتكون سنتها كلها يوماً بليلته ويفضل أحدهما على الآخر من جهة بطء حركتها وسرعتها . فيكون تحت القطب الشمالي في هذا التأريخ نهارهم أكبر من ليلهم بسبعة أيام بلياليها من أيامنا ، وذلك لكون أوج الشمس في أواخر الجوزاء وحضيضها في أواخر القوس . وتكون مدة غروب الشفق أو طلوع الصبح لهم في خمسين يوماً من أيامنا على ما تتبين عند وصفهما فيما بعد . وتكون غاية ارتفاع الشمس وغاية انحطاطها بقدر غاية الميل . ويكون طلوع الشمس والكواكب بالحركة الثانية وغروبها لا في موضع بعينه من الأفق .

[٣] ويكون للكواكب التي عرضها ينقص من الميل كله طلوع وغروب ؛ وتختلف مدّتا الظهور والخفاء بحسب بُعد مدارها عن فلك البروج وقربها إليه . والكواكب التي عرضها مساوٍ للميل كله تماس الأفق في دور واحد من الحركة الثانية مرة واحدة ؛ ولا يكون لها ولا للتي يزيد عرضها على الميل الكلي طلوع ولا غروب بل تكون إما ظاهرة وإما خفية أبداً . وليتذكر ما قُلنا في أوضاع الفلك بسبب الحركتين الأوليين ، وليحكم هاهنا بحسب ذلك .

[٤] وهذا آخر أوصاف البقاع التي تحت المدارات اليومية وما يجري مجراها .

CHAPTER SEVEN

On the Co-ascensions of the Ecliptic

[1] The arc on the equinoctial that rises with a given arc on the ecliptic orb [*sic*] is called the **co-ascension** of that arc; the arc on the ecliptic orb is referred to as the **equal degrees**.

[2] The co-ascension varies according to the changing horizons. At the [Earth's] equator, each quarter that is bounded by two of the four [cardinal] points will rise with a quarter because when the equinox point, which is simultaneously one of the two boundaries for two quarters for [each of] the two equators, reaches the zenith, the solstitial colure will coincide with the horizon; thus the solstice point will be on the horizon and the two other boundaries [*i.e.* other than the equinox] of the two quarters will both be on the horizon. One should draw analogous conclusions from this for the remaining quarters. But 30 time units, *i.e.* (½ of ⅙) of the equinoctial, will not rise with a zodiacal sign, for example one that adjoins one of the four [cardinal] points, which is / (⅙ of ½) / of the ecliptic equator. This is so because if the zodiacal sign is one that adjoins the equinox point, then one of the boundaries, namely this point, is common to both [the sign and its co-ascension]. When the zodiacal sign's other boundary reaches the horizon, there occurs a triangle [formed] from the sign, the arc that rises with it along the equinoctial, *i.e.* the co-ascension, and what is between them along the horizon; its angle that is bounded by the equinoctial and the horizon is right, and the remaining two are acute. Since the sign is the subtense of a right [angle], and its co-ascension is the subtense of an acute angle, the sign is larger than its co-ascension; the same will hold for two signs that adjoin the equinox point and their co-ascension. Now if the zodiacal sign is one that adjoins the solstitial point, its co-ascension will be larger than it. This is because what remains [after taking] the co-ascension of the [above] two signs, which is less than ⅙ of a revolution, from a full quarter will be greater than (½ of ⅙) of a revolution, and [this remainder] will rise with the remaining sign.

/12/ ⅙ of ½] β, M = ½ of ⅙] α.

الفصل السابع
في مَطالِع البروج

[١] القوس من معدل النهار التي تطلع مع قوس مفروضة من فلك البروج يقال لها مَطالِع تلك القوس ، ويقال للقوس من فلك البروج الدرَج السَوَاء .

[٢] والمطالع تختلف بحسب اختلاف الآفاق . أما في خط الاستواء فكل ربع يتحدّد بنقطتين من النقط الأربع يطلع مع ربع لأنّ نقطة الاعتدال التي هي أحد حدّي الربعين من المنطقتين معاً إذا انتهت إلى سمت الرأس انطبقت الدائرة المارة بالأقطاب الأربعة على الأفق ؛ فتكون على الأفق نقطة الانقلاب ويكون الحدان الآخران للربعين معاً على الأفق . وقِس عليه سائر الأرباع . ولا يطلع مع برج مثلاً يلي إحدى نقط الأرباع ، وهو / سدس نصف / منطقة البروج ، ثلاثون زماناً أعني نصف سُدس معدل النهار ؛ وذلك لأنّ البرج إن كان مما يلي نقطة الاعتدال كان أحد حدّيهما مشتركاً وهو تلك النقطة . وإذا انتهى الحدّ الآخر للبرج إلى الأفق حدث من البرج والقوس الطالعة معه من معدل النهار – أعني مطالعه – ومما يقع بينهما من الأفق مثلث زاويته التي يحيط بها معدل النهار والأفق قائمة والباقيتان حادّتان . فلكون البرج وترَ قائمةٍ ومطالعه وترَ حادةٍ ، يكون البرج أعظم من مطالعه ؛ وكذلك القول في برجين يليان نقطة الاعتدال ومطالعهما . أما إن كان البرج مما يلي نقطة الانقلاب فيكون مطالعه أعظم منه ، وذلك لأنّ الباقية من مطالع البرجين التي هي أصغر من سدس الدور إلى تمام الربع تكون أعظم من نصف سدس الدور ، وهي تطلع مع البرج الباقي .

/12/ ...] M , β = نصف سدس] α .

It is apparent from the above that for any two equal arcs that are equidistant from one of the four [cardinal] points, *i.e.* the two equinoxes and the two solstices, their co-ascensions at the equator will be equal. The ecliptic equator may be divided into four segments whose initial [points] are at the midparts of the quarters [of the ecliptic]. Each segment at the midpart of which falls one of the equinoxes will be greater than its co-ascension; each segment at the midpart of which falls one of the solstices is smaller than its co-ascension.

[3] The passage of the equinoctial and the ecliptic equator across the meridian circles in all localities is as their rising for the equator since each one of [these meridian circles] is one of the horizons for the equator; the same conclusion holds for all the declination circles. The co-descensions are as the co-ascensions for these horizons [*i.e.* those of the equator].

[4] As for the oblique horizons, a quarter will not rise with a quarter since the plane of the equinoctial is not perpendicular to the plane of the horizon. A half will rise with a half when they are bounded by the two equinox points. When an arc adjoining the equinox point that is away from the equinoctial in the direction of the visible pole rises, it will be larger than its co-ascension because in the above-mentioned triangle it will be the subtense of an obtuse [angle] and its co-ascension will be the subtense of an acute [angle]. And if it is away from the equinoctial in the direction of the invisible pole, then its co-ascension is greater than [the arc] because the disposition becomes the opposite of what it was [before]. It is apparent from the above that for equal arcs that are equidistant from one of the equinox points their co-ascensions are equal. The orb [*i.e.* the ecliptic equator] may be divided into two segments: one of them is bisected by that equinox which is such that a star, when crossing it, comes to be in the direction of the visible pole; the other is that bisected by the other equinox. The first is larger than its co-ascension, while the other is smaller. The co-ascensions of northern arcs for northern horizons are like [*i.e.* equal to] the co-ascensions of the corresponding [equivalent arcs] opposite to them in the south for the [equivalent] southern horizons; the same holds for the southern [arcs]. The co-descension of any arc for any horizon is like [*i.e.* equal to] the co-ascension of the arc directly opposite.

[5] As for those regions [*lit.*, horizons] in which the day-circles of the two solstice points are the largest of the permanently visible and invisible day-circles, we have shown that a half of the ecliptic orb

وقد ظهر من ذلك أنّ كل قوسين متساويتين متساويتي البعد عن إحدى النقط الأربع ـ أعني الاعتدالين والانقلابين ـ فمطالعهما في خط الاستواء متساوية . ومنطقة البروج تنفصل إلى أربع قطع تكون مبادئها أواسط الأرباع . ويكون كل قطعة يقع في وسطها أحد الاعتدالين أعظم من مطالعها ، وكل قطعة يقع في وسطها أحد الانقلابين أصغر من مطالعها .

[٣] ومرور معدل النهار ومنطقة البروج على دوائر أنصاف النهار في جميع البقاع يكون كطلوعها في خط الاستواء لأنّ كل واحدة منها أفق من آفاق خط الاستواء ؛ وكذلك الحكم في جميع دوائر الميول . والمغارب كالمطالع في تلك الآفاق .

[٤] وأما في الآفاق المائلة فلا يطلع ربع مع ربع لكون سطح معدل النهار غير قائم على سطح الأفق ؛ ويطلع نصف مع نصف إذا كانا متحدَدين بنقطتي الاعتدالين . وإذا طلعت قوس تلي نقطة الاعتدال وكانت من معدل النهار في جهة القطب الظاهر ، فهي أعظم من مطالعها لأنها في المثلث المذكور تكون وترَ منفرجةٍ ومطالعها وترَ حادةٍ . وإن كانت من معدل النهار في جهة القطب الخفي فمطالعها أعظم منها لأنّ الحكم يصير بضدّ ما كان . ويظهر من ذلك أنّ القسي المتساوية التي تتساوى أبعادها عن إحدى نقطتي الاعتدال تكون مطالعها متساوية . والفلك ينقسم إلى قطعتين : إحداهما التي يتوسطها الاعتدال الذي إذا جاوزه الكوكب صار في جهة القطب الظاهر ، والأخرى التي يتوسطها الاعتدال الآخر . والأولى تكون أعظم من مطالعها والأخرى تكون أصغر . ومطالع القسي الشمالية في الآفاق الشمالية كمطالع نظائرها من الجنوبية في الآفاق الجنوبية ، وكذلك في الجنوبية . ومغارب كل قوس في كل أفق تكون كمطالع نظير تلك القوس .

[٥] وأما في الآفاق التي يكون فيها مدارا نقطتي الانقلابين أعظم المدارات الأبدية الظهور والخفاء ، فقد بيّنَا أنّ نصفاً من فلك

rises with the entire equinoctial and the other half rises without [requiring] time; in setting, the two halves exchange [properties].

[6] As for the regions [*lit.*, horizons] in which there are arcs of the ecliptic orb that are permanently visible and invisible: let the example be the one in the northern region we have [previously] used, namely the region whose latitude is 70 where Gemini and Cancer are permanently visible and Sagittarius and Capricornus are permanently invisible. When the vernal equinox point rises, Pisces will rise after it in reverse order from last to first, then Aquarius in reverse order from last to first; then will begin the rising of Leo from its first in regular order, then Virgo, then Libra, then Scorpius, likewise [in regular order]. Now when the first of Sagittarius is reached, the last of Taurus will begin to rise reversed, and Taurus and Aries will rise in reverse order. Thereupon the vernal equinox point will return to the horizon. One should draw analogous conclusions from this for the remaining regions [*lit.*, horizons], and from rising for setting.

CHAPTER EIGHT

On the Lengths of the Nychthemerons

[1] The **nychthemeron** [*lit.*, a day with its night] is the time that falls from either the sun's occurrence on the horizon—whether rising or setting—or else [from its occurrence] on the meridian until its return there after one complete revolution by the first motion. Its length is one revolution of the equinoctial increased by the amount of it that rises with the arc traversed by the sun during that nychthemeron.

[2] Since what is traversed by the sun is variable, the lengths of the nychthemerons will [also] be variable; for [the sun] will traverse in the far half from the Earth smaller arcs, while in the near half larger arcs, and furthermore [those arcs] that rise along the equinoctial with the arcs from the ecliptic orb are variable, [the former] being sometimes smaller, sometimes larger than [the latter]. However, the variation [in the nychthemerons] is imperceptible in one or two days due to the smallness of the difference; one does perceive it over many days.

البروج يطلع مع جميع معدل النهار والنصف الآخر يطلع لا في زمان ؛ وفي الغروب يتبادل النصفان .

[٦] وأما في الآفاق التي تكون فيها قسي من فلك البروج أبدية الظهور والخفاء _ وليكن الأفق ما تمثّلنا به من الآفاق الشمالية ، وهو أفق عرضه سبعون ، والجوزاء والسرطان فيه أبديا الظهور ، والقوس والجدي أبديا الخفاء _ فإذا طلعت نقطة الاعتدال الربيعي طلع بَعدها الحوت معكوساً من الآخر إلى الأول ، ثم الدلو معكوساً من الآخر إلى الأول ؛ ثم يبتدئ طلوع الأسد من أوله مستوياً ، ثم السنبلة ، ثم الميزان ، ثم العقرب كذلك . فإذا انتهى إلى أول القوس ابتدأ آخر الثور بالطلوع المعكوس ويطلع الثور والحمل معكوسين ، فتعود نقطة الاعتدال الربيعي إلى الأفق . وقِس عليه في سائر الآفاق ، والغروب على الطلوع .

الفصل الثامن
في مقادير الأيّام بلياليها

[١] اليوم بليلته هو الزمان الذي يقع بين كون الشمس إمّا على الأفق طالعة أو غاربة وإمّا على نصف النهار وبين عودها إلى هناك بعد دورة واحدة تامة بالحركة الأولى . ومقداره دورة من أدوار معدل النهار مع زيادة تطلع منه مع القوس التي تقطعها الشمس في ذلك اليوم بليلته .

[٢] ولكون ما تقطعها الشمس مختلفاً _ فإنّها تقطع في النصف البعيد من الأرض قسياً أصغر وفي النصف القريب قسياً أكبر ، وأيضاً ما يطلع من معدل النهار مع القسي من فلك البروج مختلف فإنّه تارة يكون أصغر منها وتارة يكون أكبر _ تكون مقادير الأيّام بلياليها مختلفة ؛ لكنّ اختلافها غير محسوس في يوم أو يومين لصغر التفاوت ، ويحسّ به في أيّام كثيرة . وأهل

The calculators, being obliged to use nychthemerons of equal size in order to find the mean and other motions, have taken the above increase to be in the amount of the mean motion of the sun during a nychthemeron. They named those days that were taken to be equal the **mean days**, each day of which is the measure of a revolution of the equinoctial plus the amount traveled by the mean sun in a day. As for the true state of affairs, one needs to find out all about each of the two differences.

[3] As for the difference that is due to the sun's variable speed: in the period that the sun moves from the apogee to the mean distance that follows it, it will be the increase of the sun's mean over its true position, [which is] in the amount of the maximum anomaly; in the period that it moves from the other mean distance to the apogee, it will be the same as this. Thus the increase of the mean over the true position in the far segment from the Earth of the sun's orb is in the amount of twice the anomaly; in the near segment, the increase of the true position over the mean is the same as this as well. The difference between the two segments is four times the anomaly.

[4] As for the difference due to the co-ascension: if the beginning of the day is made to be when the sun reaches the horizon, this difference will vary according to the changing horizons, and it will not be the exact same thing for all localities. This will be the case whether the beginning is [upon] its reaching the eastern horizon, at that place being according to the difference between the equal degrees and their co-ascension, or whether [the beginning] is upon its reaching the western horizon, at that place being according to the difference between the equal degrees and the co-ascension of the [arc] directly opposite them. But if the beginning of the day is made to be upon its reaching the meridian, the difference agrees for all horizons, and that is according to the co-ascension for the equator. They thus chose this rather than the first alternative.

[5] It has been mentioned above that the ecliptic orb may be divided into four segments. The two that are bisected by the equinoxes are greater than their co-ascensions, and they are from the middle part of Aquarius to the middle part of Taurus, and from the middle part of Leo to the middle part of Scorpius. The amount of excess of each one of them over its co-ascension is five degrees for the [Earth's] equator.

الحساب ، لمّا اضطرّوا إلى استعمال أيّام بلياليها متساوية الأقدار لمعرفة حركات الأوساط وغيرها ، أخذوا تلك الزيادة مقدار حركة الشمس الوسطى في يوم بليلته . وسمّوا تلك الأيّام المأخوذة بالتساوي الأيّام الوسطى ، كل يوم منها يكون مقدار دور من معدّل النهار مع سير وسط الشمس ليوم . وأمّا التحقيق فمحوج إلى معرفة جملة كل واحد من التفاوتين .

[٣] أمّا التفاوت الذي يكون بسبب اختلاف سير الشمس فيكون في المدّة التي تسير الشمس من الأوج إلى البعد الأوسط الذي يليه زيادةَ وسط الشمس على تقويمها بقدر غاية الاختلاف ، وفي المدة التي تسير من البعد الأوسط الآخر إلى الأوج مثل تلك . فتكون زيادة الوسط على التقويم في القطعة البعيدة من الأرض من فلك الشمس بقدر ضعف الاختلاف ؛ وتكون في القطعة القريبة زيادة التقويم على الوسط أيضاً بمثل ذلك . ويكون الفضل بين القطعتين بأربعة أمثال الاختلاف .

[٤] وأمّا التفاوت الذي يكون بسبب المطالع : فإن جعل مبادئ الأيّام انتهاءَ الشمس إلى الأفق اختلف ذلك التفاوت بحسب اختلاف الآفاق ، ولم يكن في جميع البقاع شيئاً واحداً بعينه ؛ ويكون ذلك إن كان المبدأ انتهاءها إلى الأفق الشرقي بحسب التفاوت بين درج السواء ومطالعها في ذلك الموضع ، وإن كان انتهاءها إلى الأفق الغربي بحسب التفاوت بين درج السواء ومطالع نظيرها في ذلك الموضع . وإن جعل مبادئ الأيّام انتهاءها إلى نصف النهار اتّفق التفاوت في جميع الآفاق ، ويكون ذلك بحسب مطالع خطّ الاستواء ؛ فاختاروا ذلك دون الوجه الأول .

[٥] وقد مرّ أنّ فلك البروج ينقسم إلى أربع قطع : اثنتان منها اللتان يتوسّطهما الاعتدالان تزيدان على مطالعيهما ، وهما من أواسط الدلو إلى أواسط الثور ومن أواسط الأسد إلى أواسط العقرب ؛ ومقدار زيادة كل واحدة منهما على مطالعها بخط

The other two segments, which are bisected by the solstices, are less than their co-ascensions, and they are from the middle part of Taurus to the middle part of Leo and from the middle part of Scorpius to the middle part of Aquarius. The amount of deficiency of each one of them from its co-ascension is likewise five degrees for the [Earth's] equator.

[6] When one additively combines the two differences when they are both additive or both subtractive, or takes the difference when they differ, there results the total amount of difference between the mean days and the true days during the year.

[7] One day must necessarily be taken as the initial one from which the other days are measured. Noon of that day is then the initial [time] for both mean and true days. For any day of the year that is taken to be the initial one, the difference between elapsed mean and elapsed true days [measured] from that day is sometimes additive and sometimes subtractive, except for [initial days at] the end part of Aquarius and the beginning part of Scorpius; for if the initial [point] is made the end part of Aquarius, the true days will always be shorter than the mean ones, and if it is made the beginning part of Scorpius, the true days will always be longer than the mean ones. / The practitioners / of the profession have agreed on making [the initial point] the end part of Aquarius.

/16/ The practitioners] β = Therefore the practitioners] α, M.

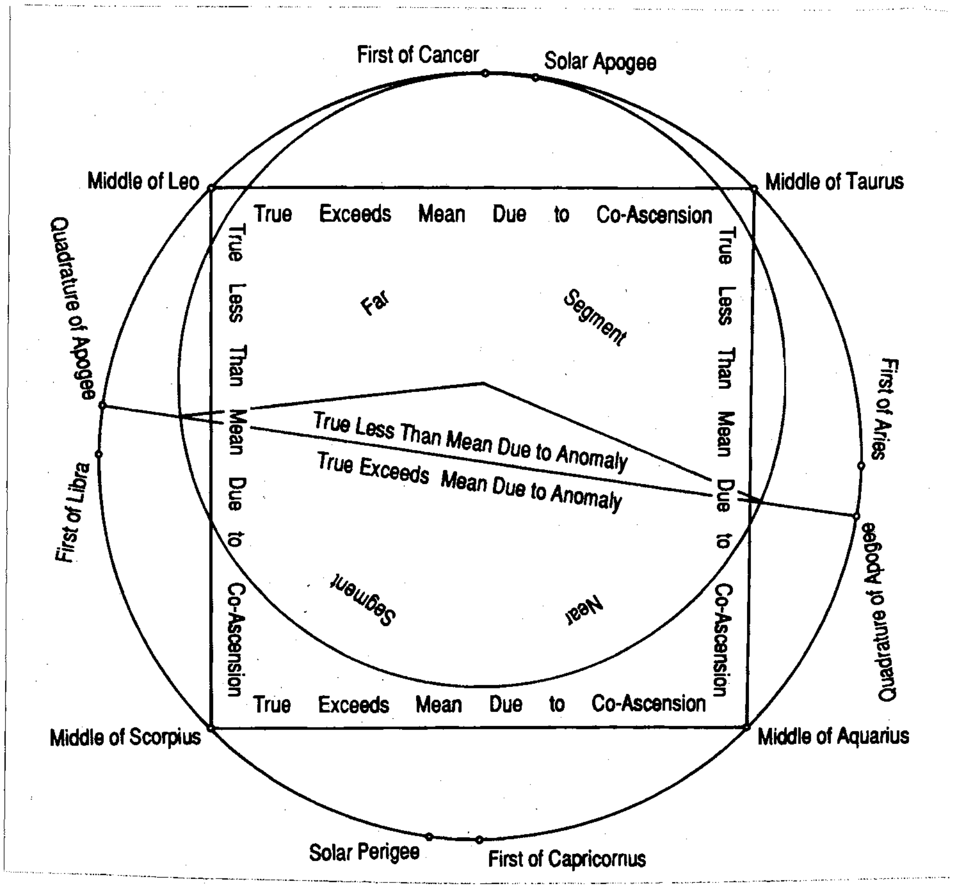
الاستواء خمس درجات . والقطعتان الأخريان وهما اللتان يتوسّطهما الانقلابان تنقصان عن مطالعيهما ، وهما من أواسط الثور إلى أواسط الأسد ومن أواسط العقرب إلى أواسط الدلو ؛ ومقدار نقصان كل واحدة منهما من مطالعها بخط الاستواء أيضاً خمس درجات .

[٦] وإذا تركّب التفاوتان بالجمع إذا كانا زائدين معاً أو ناقصين معاً أو بالتفريق إذا اختلفا ، حصل مقدار التفاوت بين الأيّام الوسطى والأيّام الحقيقية جملة في السنة .

[٧] ولا بدّ من يوم يُفرض مبدأً ويقاس سائر الأيّام إليه ؛ فيكون نصف نهار ذلك اليوم مبدأً للأيّام الوسطى والحقيقية جميعاً . وكل يوم من السنة يفرض مبدأً يكون التفاوت بين الأيّام الماضية الوسطى والحقيقية الماضية من ذلك اليوم تارة زائداً وتارة ناقصاً إلاّ أواخر الدلو وأوائل العقرب ؛ فإنّ المبدأ إذا جعل أواخر الدلو كانت الأيّام الحقيقية دائماً ناقصة من الوسطى ، وإذا جعل أوائل العقرب كانت الأيّام الحقيقية دائماً زائدة على الوسطى . / واتفق / أهل الصناعة على جعلة أواخر الدلو .

/16/ ... [β = فاتفق] M , α .

This is an illustration of the segments, the apogee being / in the end part / of Gemini.



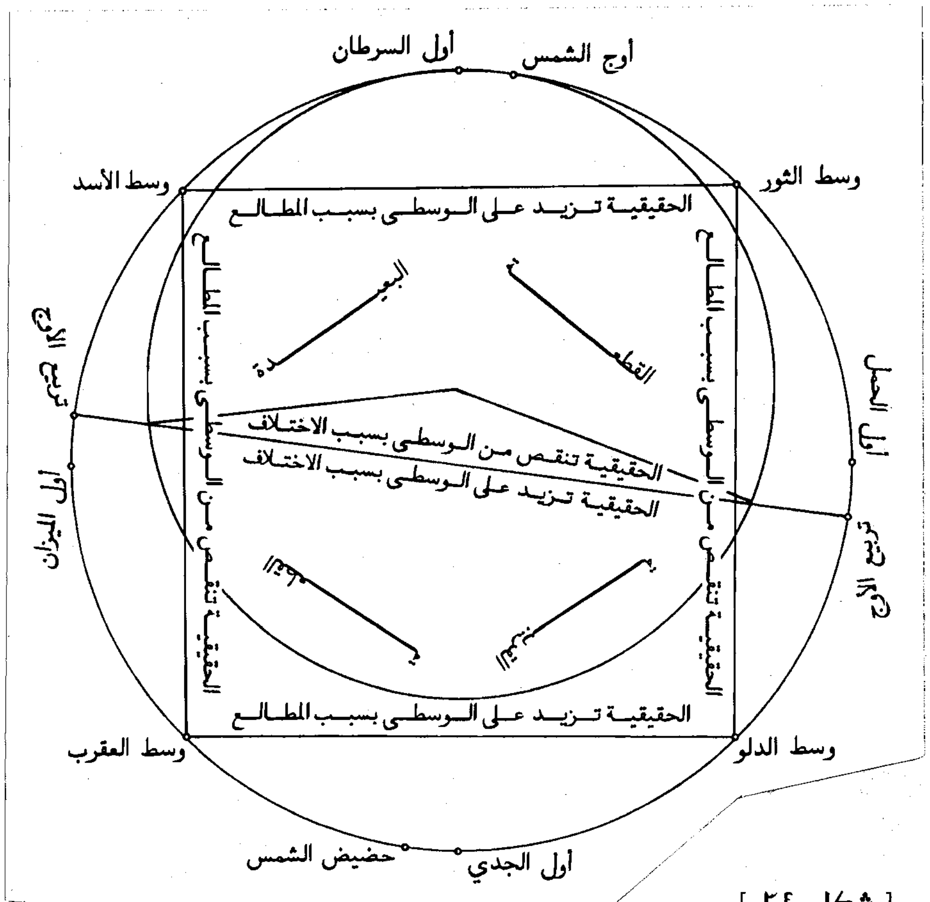
[Fig. T24]

The difference of the anomaly changes due to the motion of the apogee, but over a long period.

[8] This then is the explanation of the difference in the lengths of the days. Finding the [actual] amounts for all times pertains to the practical handbooks. This difference is called the **equation of the nychthemeron**. When there is a complete revolution [of the sun through the ecliptic], the true and mean days become equal, and this ceases to be a consideration.

/1/ in the end part] β = at the end] α, M.

وهذه صورة القطع على أنّ الأوج في / أواخر / الجوزاء :



[شكل ٢٤]

ويتغيّر تفاوت الاختلاف بسبب حركة الأوج ولكنّ في مدة طويلة .

[٨] فهذا بيان التفاوت في مقادير الأيّام . ووجود المقادير في كل وقت يتعلّق بكتب العمل ، ويُسمّى هذا التفاوت تعديل الأيّام بلياليها . وإذا تمّ الدور تساوت الأيّام الحقيقية والوسطى وسقط هذا الاعتبار .

/1/ ...] آخر = β] M , α .

CHAPTER NINE

On Dawn and Dusk

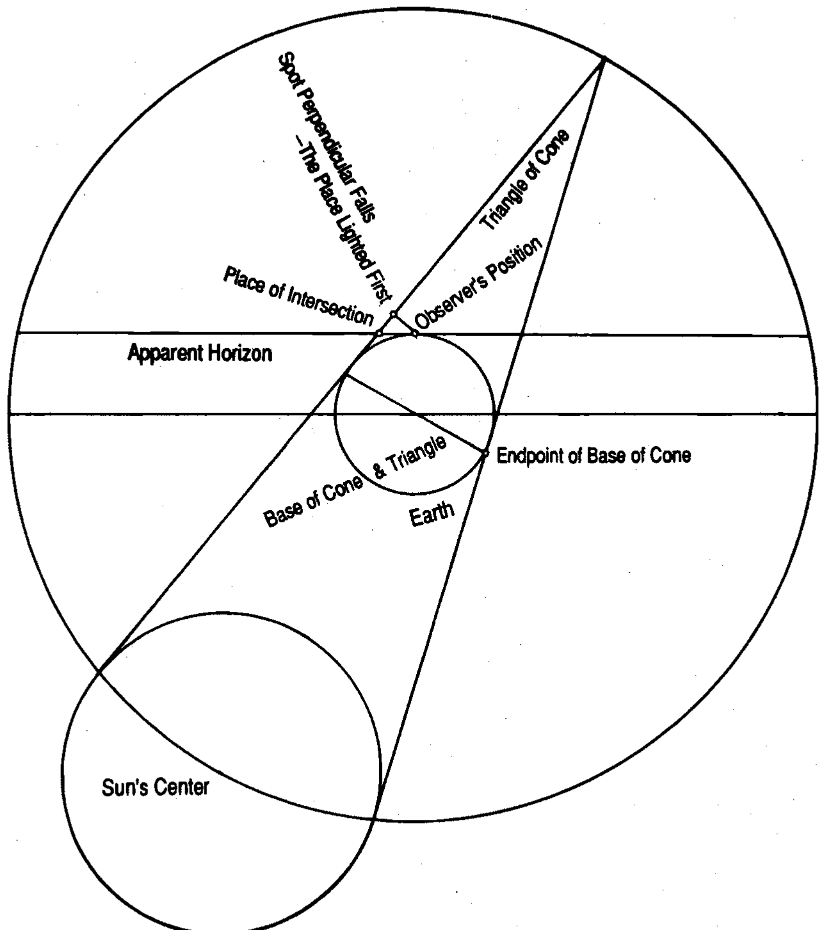
[1] When the sun approaches the eastern horizon, the Earth's shadow cone inclines toward the west. Then of the rays surrounding it, what is visible first is that which is nearest the eye, and the nearest of the sides of the cone to the eye is the side that is toward the sun. Let a plane pass through the centers of the sun and Earth and through the axis of the cone, and let there occur in it a triangle whose angles are acute, whose base is on the horizon, and whose two sides are on the surface of the cone. There is no doubt that the [point] that is nearest the observer on the side [of the triangle] toward the sun is the spot on which the perpendicular extending from the eye falls on that side, not the place of intersection of the side with the horizon. Therefore the first observed light of the sun is seen above the / Earth / as a straight line that falls on the above side, but what is near the horizon is still dark. For this reason that light is called **first dawn** and **false dawn**. Its designation as first is obvious. As for the designation false, this is due to the horizon being dark; in other words, for it to be believed that it is truly the light of the sun, then [one would expect that] what is illuminated should be toward the sun rather than farther away from it. This is an illustration of the horizon, the triangle, the perpendicular, the sun, and the Earth:

/11/ Earth] β, M = horizon] α.

الفصل التاسع
في الصبح والشفق

[١] إذا قربت الشمس من الأفق الشرقي مال مخروط ظلّ الأرض نحو المغرب ؛ فيكون المرئي من الشعاع المحيط به أوّلاً ما هو أقرب إلى البصر ، والأقرب من جوانب المخروط إلى البصر هو الجانب الذي يلي الشمس . وليمرّ سطح بمركزي الشمس والأرض وبسهم المخروط ، وليحدث منه مثلّث حادّ الزوايا قاعدته على الأفق وضلعاه على سطح المخروط . ولا شكّ أنّ الأقرب من الضلع الذي يلي الشمس إلى الناظر يكون موقع العمود الخارج من النظر الواقع على ذلك الضلع ، لا موضع اتّصال الضلع بالأفق . فإذن أوّل ما يُرى نور الشمس يُرى فوق / الأرض / كخط مستقيم منطبق على الضلع المذكور ، ويكون ما يقرب من الأفق بعدُ مظلماً . فلذلك يسمّى ذلك النور بالصبح الأول والصبح الكاذب . أمّا تسميته بالأول فظاهر ؛ وأمّا تسميته بالكاذب فلكون الأفق مظلماً ، أي لو كان يصدُق أنّه نور الشمس لكان المنير ما يلي الشمس دون ما يبعد منه . وهذه صورة الأفق والمثلّث والعمود والشمس والأرض :

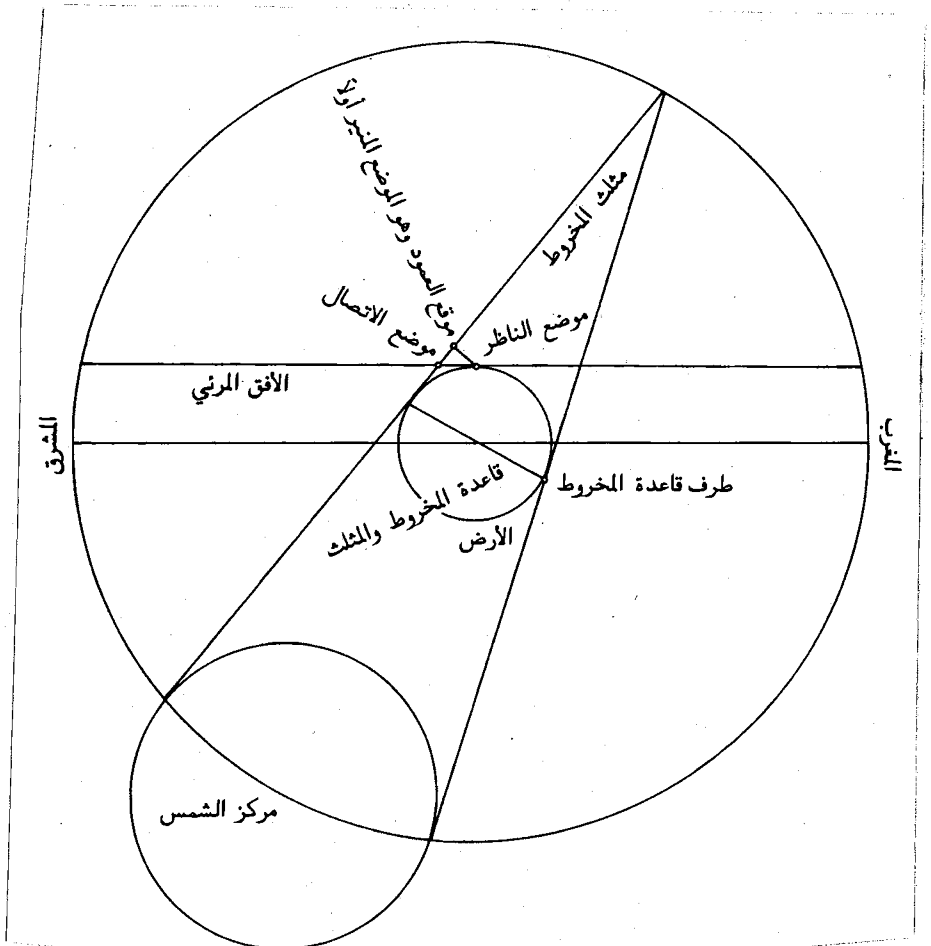
/11/ ...] M , β = الأفق] α .



[Fig. T25]

[2] Then when the sun comes quite near, the light will spread; the horizon will then become lighted and the **dawn** will be true. **Dusk** will be the reverse of dawn. It has become known by trial and error that the depression of the sun below the horizon at the first rising of dawn and the final setting of dusk is 18°. Thus in the lands whose latitude is 48½ / degrees, / dusk will be continuous

/5/ degrees] $\beta = -\alpha, -M$.



[شكل ٢٥]

[٢] ثم إذا قربت الشمس جداً انبسط النور ، فصار الأفق منيراً ويصير الصبح صادقاً . والشفق يكون بعكس الصبح . وقد عُرف بالتجربة أنّ انحطاط الشمس من الأفق عند أوّل طلوع الصبح وآخر غروب الشفق يكون ثمانية عشر جزءاً . ففي البلاد التي تكون عروضها ثمانية وأربعين / درجة / ونصفاً يتّصل الشفق

. −α , −M = β [… /5/

with dawn when the sun is at the summer solstice. In the [regions] whose latitudes exceed that amount, this [continuity of dawn and dusk] will occur for a longer period in accordance with the decrease in the above value for the sun's depression below the horizon. From what we have described, the reason for the stated delimitation [of the extent] of dawn and dusk in the earlier discussion of the spinning horizon should be clear.

CHAPTER TEN

On Understanding the Units of the Day, Namely Hours, and What Is Composed of Days, Namely Months and Years

[1] It is commonly held that the **arc of daylight** is the sum of half a revolution plus twice the equation of daylight or the excess of half a revolution over twice the equation of daylight (when there is an equation of daylight). The true state of affairs requires that the arc of daylight be that which turns along the equinoctial from the time half the solar body rises on the horizon until half of it sets on the horizon. It will be greater / or less / than the first by the amount of the co-ascension of [the arc] upon which the sun has moved during that day for a [given] locality. The **arc of night** is in accordance with the above. Now if each of the two arcs is divided by 15, one obtains the [number of] **equal hours** for the day and for the night. If each is divided by 12, one obtains the [number of] degrees in the **seasonal** [*lit.*, temporal] or **unequal** [*lit.*, distorted] **hours** [for the day and for the night]. The difference between them is that the length [*lit.*, length and shortness] of the days and of the nights is [measured] by the number of equal hours [or else] by the degrees in the unequal [hours] since it is the degrees in an equal [hour] and the number of unequal [hours] that are invariable.

[2] The **month** is derived from the illuminated shapes of the moon, which have been shown really to be according to its position from the sun. Its cycle is completed when

/13–14/ or less] $\beta = -\alpha$, –M.

بالصبح إذا كانت الشمس في المنقلب الصيفي . وفيما جاوزت عروضها ذلك المقدار يكون ذلك في زمان أكثر بحسب تناقص انحطاط الشمس عن الأفق القدر المذكور . ويتبيّن مما وصفنا السبب في تحديد الصبح والشفق المذكور فيما مرّ للأفق الرحوي .

الفصل العاشر
في معرفة أجزاء الأيام
وهي الساعات وما يتركب من الأيام
وهي الشهور والسنون

[١] المشهور أنّ قوس النهار هي مجموع نصف الدور وضعف تعديل النهار أو فضل نصف الدور على ضعف تعديل النهار إن كان تعديل نهار . والحقيقة تقتضي أن تكون قوس النهار هو[!] ما يدور من معدل النهار من وقت طلوع نصف جرم الشمس من الأفق إلى وقت غروب نصفه في الأفق ؛ وهو أزيد من الأول / أو أنقص / بقدر مطالع ما تسيره الشمس في ذلك اليوم لتلك البقعه . وقوس الليل بحسب ذلك . فإذا قُسم كل واحد من القوسين على خمسة عشر حصلت ساعات النهار والليل المستوية ؛ وإذا قُسم على اثني عشر حصلت أجزاء ساعاتهما الزمانية والمعوّجة . والفرق بينهما أنّ طول الأيام والليالي وقصرهما يكونان بعدد الساعات المستوية وأجزاء المعوّجة لأنّ أجزاء المستوية وعدد المعوّجة لا يختلفان .

[٢] وأما الشهر فمأخوذ من تشكّلات القمر النورية ، وقد تبيّن أنها إنما تكون بحسب أوضاعه من الشمس . ويتم دورُه إذا

/13–14/ أو أنقص] $\beta = -M$, $-\alpha$.

the excess of the moon's true motion over the sun's true motion becomes one revolution. Finding this is difficult. Compounding the difficulty is that it varies due to the irregularity of their motions. Those employing the [month] who [rely] on appearances take it to be from one day of conjunction to another, or from one night of [first] visibility of the crescent to another, or from some other shape to its like, depending on what they have conventionally adopted. Those employing the [month] who [rely] on calculation derive a cycle from the difference between the two mean motions; they find it to be in 29½ days plus a fraction. They then take one month to be 30, one month to be 29. / They add the accumulated fractions that exceed a half day [which is] 11 days in every 30 years; / thus in a period of 30 years, 11 months that should be 29 become 30 each. These are called **intercalary days**. Or else they add the intercalary days to the months in some other way. These months are **lunar**, of which there are true and of which there are mean.

[3] As for the **year**, it is derived from the return of the sun to its location on the ecliptic orb that results in a return of the [same] yearly condition determined by the seasons. This occurs in 365¼ days minus a fraction. During [a year], there are 12 complete mean lunar months plus an excess of 11 days minus a fractional part. Of those employing [the year] who do not take into account the lunar months, some take it to be from a day on which the sun reaches a certain point, such as the vernal equinox, to the same [point]. They take its months to be from the days on which [the sun] reaches points along the ecliptic analogous to the [initial point]. Or they reckon the months to be 30 each and add at the end [of the year] 5 or 6 [days]. The 5 are called the stolen or / the supplementary / [*i.e.* the epagomenal days] and the 6th intercalary. For these [people], their years are **true solar**, while their months are either true solar or conventional.

/8/ They add…in every 30 years] β = For the accumulated fractions that exceed a half day, they add 11 days in every 30 years] α, M. /22/ the supplementary] β = supplementary] α, M.

صار فضل حركة القمر على حركة الشمس الحقيقيتين دوراً ؛ ووجوده متعذر ومع تعذُره مختلف لاختلاف حركتيهما . فمستعملوه من أهل الظاهر يأخذونه من يوم الاجتماع إلى يومه أو من ليلة رؤية الهلال إلى ليلتها أو من تشكّل آخر إلى مثله بحسب ما يصطلحون عليه . ومستعملوه من أهل الحساب يأخذون الدور من الفضل بين الحركتين الوسطيين ، فيجدونه في تسعة وعشرين يوماً ونصف وكسر ؛ فيأخذون لشهر ثلاثين ولشهر تسعة وعشرين . ويزيدون / الكسور / المجتمعة التي تزيد على نصف يوم في كل ثلاثين سنة أحد عشر يوماً فيصير أحد عشر شهراً مما يجب أن يكون تسعة وعشرين في مدة ثلاثين سنة ثلاثين ثلاثين ، وتسمى تلك الأيام كبائس ؛ أو يزيدون الكبائس في الشهور على وجه آخر . وهذه الشهور قمرية ، فمنها حقيقية ومنها وسطية .

[٣] وأما السنة فمأخوذة من عود الشمس إلى موضعها في فلك البروج المقتضي لعود حال السنة بحسب الفصول ؛ ويحصل ذلك في ثلاثمائة وخمسة وستين يوماً وربع يوم إلا كسراً . ويتم فيها من الشهور القمرية الوسطى اثنا عشر ، ويزيد عليها أحد عشر يوماً غير شيء من الكسور . ومستعملوها ، إن لم يعتبروا الشهور القمرية ، فربما يأخذونها من يوم تحلّ الشمس فيه نقطة بعينها ــ كالاعتدال الربيعي ــ إلى مثله . ويأخذون شهورها من الأيام التي تحلّ فيها أمثال تلك النقطة من البروج ؛ أو يعدّون الشهور ثلاثين ثلاثين ويزيدون في آخرها خمسة أو ستة ، وتسمى الخمسة المسترَقة / واللواحق / والسادس كبيسة . وهؤلاء سنوهم شمسية حقيقية وشهورهم إما شمسية حقيقية وإما اصطلاحية . وربما

/8/ ...] β = للكسور] M , α . /22/ ...] β = ولواحق] M , α .

Some others take [the year to begin] from a day agreed upon without regard to the position of the sun, and they have conventionally adopted months that are around 30 since the lunar months are approximately that. The fraction in excess of 365 is taken by some to be a quarter exactly, and they intercalate 1 day in every 4 years, while others drop it completely. These years are **conventional solar**. For those who do wish to take into account lunar months, they make the year solar and the months lunar. In every 3 years or in every 2 years, they add a month to the year to reconcile the previously mentioned 11 days minus a fraction in accordance with what they have conventionally adopted. One people has made every 12 lunar months a year, which they call a **lunar year**. Every people has an epoch to which they refer the years of their history; understanding the details of that does not pertain to this science.

CHAPTER ELEVEN

On the Degrees of Transit of the Stars on the Meridian and on Their [Degrees of] Rising and Setting

[1] When the poles of the ecliptic orb are on the meridian circle, this being when the two solstice points are also on it and the two equinox points are on the horizon, then the transit of the stars at that time is at their degrees in longitude since the meridian circle is their latitude circle. When the visible pole of the ecliptic orb is to the east of the meridian (this occurring, if the visible pole is northerly, during the transiting of the half of the ecliptic orb bisected by the autumnal equinox and the rising of the southern half [of the ecliptic], or, if it is southerly, [during] the transiting of the [corresponding] other half and the rising of the [corresponding] other half), then the star whose latitude is in the direction of the visible pole will transit the meridian circle after its degree [in longitude has done so] since its latitude circle extending from the pole meets the star before its [longitudinal] degree; thus when its [longitudinal] degree reaches the meridian, the star will be

يأخذونها من يوم يتّفق من غير ملاحظة موضع الشمس ، ويصطلحون على شهور تدور حول الثلاثين لكون الشهور القمرية قريبة منه . والكسر الزائد على ثلاثمائة وخمسة وستين ربما يأخذونه ربعاً تاماً ويكبسون في كل أربع سنين بيوم ، وربما يحذفونه مطلقاً ؛ وهذه السنون شمسية اصطلاحية . وإن أرادوا اعتبار الشهور القمرية جعلوا السنة شمسية والشهور قمرية ، وزادوا في كل ثلاث سنين أو في كل سنتين شهراً في السنة لاجتماع الأحد عشر يوماً غير كسر المذكور على حسب ما يصطلحون عليه . وقوم يجعلون كل اثني عشر من الشهور القمرية سنة ويسمونها سنين قمرية . ولكل قوم مبدأ ينسبون سني تأريخهم إليه ، ومعرفة تفاصيل ذلك غير متعلقة بهذا العلم .

الفصل الحادي عشر
في درجات ممرّ الكواكب بنصف النهار وطلوعها وغروبها

[١] إذا كان قطبا فلك البروج على دائرة نصف النهار _ وذلك يكون عند كون نقطتي الانقلابين أيضاً عليها ونقطتي الاعتدالين على الأفق _ فمرور الكواكب حينئذ يكون مع درجاتها الطولية لأنّ دائرة نصف النهار تكون دائرة عرضها . وإذا كان القطب الظاهر من فلك البروج شرقياً عن نصف النهار _ وذلك يكون عند مرور النصف من فلك البروج الذي يتوسطه الاعتدال الخريفي وطلوع النصف الجنوبي منه إن كان القطب الظاهر شمالياً ، أو مرور النصف الآخر وطلوع النصف الآخر إن كان جنوبياً _ فالكوكب الذي يكون عرضه في جهة القطب الظاهر يمرّ على دائرة نصف النهار بعد درجته لأنّ دائرة عرضه الخارج[!] من القطب تلاقي الكوكب قبل درجته ؛ فإذا وافى درجته نصف النهار كان الكوكب

away from [its degree] in the direction of the pole, *i.e.* it will still be to the east. The star whose latitude is in the direction opposite the visible pole transits [the meridian circle] before its [longitudinal] degree since the latitude circle mentioned above first meets the star's [longitudinal] degree that is on the meridian; it then meets the star, which has already transited and become westerly. When the visible pole is to the west (this occurring, if the pole is northerly, during the transiting of the half of the ecliptic orb bisected by the vernal equinox and the rising of the northern half [of the ecliptic], or, if it is southerly, [during] the transiting of the [corresponding] other half and the rising of the [corresponding] other half), then the star whose latitude is in the direction of the visible pole will transit before its degree [in longitude has done so] and that whose latitude is in the opposite direction will transit after it for exactly [the reasons] we have stated.

[2] The rising and setting of the stars on the horizons of the [Earth's] equator are similar to their transit across the meridian for the rest of the horizons. Thus a star that reaches the horizon along with the pole and the solstice rises or sets with its [longitudinal] degree; one that is in the direction of the visible pole rises in advance of its [longitudinal] degree and sets after it; and one that is in the direction of the invisible pole rises after its [longitudinal] degree and sets in advance of it. The northern pole will be visible there during the period of the rising of the half bisected by the vernal equinox and the transit of the southern half across the meridian from above [the horizon]; the southern pole will be visible during the period of the rising of the [corresponding] other half and the transit of the [corresponding] other half.

[3] As for the rising and setting of the stars in the remaining horizons, it is as we have described for the equator except for the transit and the rising of the halves of the ecliptic orb. For that will vary: it may be that one of the poles is visible and what transits or rises will be an arc that is smaller or larger than a half. In those horizons whose latitude exceeds the obliquity, one of the ecliptic poles will be permanently visible and the [above] rule concerning the stars may be uniformly applied without any variation [due to which ecliptic pole is visible].

منها في جهة القطب ، أعني يكون شرقياً بعدُ . والكوكب الذي يكون عرضه في خلاف جهة القطب الظاهر يمرّ عليها قبل درجته لأنّ دائرة العرض المذكورة تلاقي درجة الكوكب الكائنة على نصف النهار أولاً ، ثم تلاقي الكوكب وقد مرّ وصار غربياً قبل ذلك . وإذا كان القطب الظاهر غربياً ـ وذلك يكون عند مرور النصف من فلك البروج الذي يتوسطه الاعتدال الربيعي وطلوع النصف الشمالي منه إن كان القطب شمالياً ، أو مرور النصف الآخر وطلوع النصف الآخر إن كان جنوبياً ـ فالكوكب الذي يكون عرضه في جهة القطب الظاهر يمرّ قبل درجته والذي يكون عرضه في خلاف تلك الجهة يمرّ بعدها لِما ذكرنا بعينه .

[٢] وطلوع الكواكب وغروبها في آفاق خط الاستواء يكون كمرورها على نصف النهار في سائر الآفاق . فالكوكب الذي يوافي الأفق مع القطب والانقلاب يطلع أو يغرب مع درجته ، والذي يكون في جهة القطب الظاهر يطلع قبل درجته ويغيب بعدها ، والذي يكون في جهة القطب الخفي يطلع بعد درجته ويغيب قبلها . ويكون هناك القطب الشمالي ظاهراً مدة طلوع النصف الذي يتوسطه الاعتدال الربيعي ومرور النصف الجنوبي على نصف النهار من فوق ، والقطب الجنوبي ظاهراً مدة طلوع النصف الآخر ومرور النصف الآخر .

[٣] وأما طلوع الكواكب وغروبها في سائر الآفاق فكما وصفناه في خط الاستواء إلا في مرور الأنصاف وطلوع الأنصاف من فلك البروج ؛ فإنّ ذلك يختلف ، وربما يكون أحد القطبين ظاهراً والمارّة أو الطالعة قوس أصغر من النصف أو أكبر . وفي الآفاق التي يزيد عرضها على الميل الكلي يكون أحد قطبي البروج أبدي الظهور ، ويطّرد الحكم في الكواكب من غير اختلاف .

CHAPTER TWELVE

On Finding the Meridian Line and the *qibla* Bearing

[1] Two equal altitudes of the sun are observed on the same day from the two sides of its maximum altitude, and the directions of their shadows from the same gnomon are marked on level land. Then the angle occurring between them is bisected with a line. This line will then be in the plane of the meridian circle and is called the **meridian line**. The perpendicular to it is in line with the prime vertical circle.

[2] By another method, a gnomon is erected vertically to a plane of level land. A circle whose radius is equal to twice the gnomon is drawn, and the entry of the shadow into the circle and its emergence from it is observed before and after noon. The two places are marked and the arc that falls between them is bisected. The midpoint and the center are joined with a straight line; this then is the meridian line. The perpendicular to it that passes through the center of the circle is the east-west line. The two [lines] divide the circle into fourths. Each fourth is then divided into 90 equal parts in order to find the measures of the azimuths from the shadow lines falling on the circumference since [the number of] these parts between the east and west points and the shadow line is an azimuth. This circle is known as the **Indian.**

[3] As for the ***qibla* bearing**, let it be noted that the longitude of Mecca—may God Most High protect it—is $77\frac{1}{6}°$ from the Eternal Islands and $67\frac{1}{6}°$ from the coast of the western sea. Its latitude is $21\frac{2}{3}°$. / For / every locality

/22/ For] β = Thus for] α, M.

الفصل الثاني عشر
في معرفة خط نصف النهار
وسمت القبلة

[١] يُرصد ارتفاعان متساويان للشمس في يوم واحد عن جنبتي غاية ارتفاعها ؛ ويُخط على أرض مستوية سمتا ظليهما عن مقياس واحد ؛ ثم تُنصّف الزاويةُ الحادثة بينهما بخط . فيكون ذلك الخط في سطح دائرة نصف النهار ويسمى خط نصف النهار. والقائم عليه عموداً يكون في سمت دائرة أول السموت .

[٢] وبوجه آخر يُقام مقياس قائم على سطح أرض مستوية ؛ وتُرسم دائرة نصف قطرها بقدر ضعف المقياس ؛ ويُرصد دخول الظل / في / الدائرة وخروجها عنها قبل نصف النهار وبعده ؛ ويُعلَّم على الموضعين وتُنصّف القوسُ التي تقع بينهما ؛ ويوصَل بين المنتصف وبين المركز بخط مستقيم ، فهو خط نصف النهار . والقائم عليه عموداً المارّ بمركز الدائرة خط المشرق والمغرب . ويُربّعان الدائرةَ ، ثم يُقسم كلُ ربع / بتسعين / قسماً متساوية ليعرف مقادير السموت من خطوط الظل الواقعة على المحيط لأنّ ما بين نقطتي المشرق والمغرب وخط الظل من تلك الأقسام سمت . وهذه الدائرة يُعرف بالهندية .

[٣] وأما سمت القبلة فليعلم أنّ طول مكّة _ حماها الله تعالى _ عن جزائر الخالدات سبع وسبعون جزءاً وسدس جزء ، وعن ساحل البحر الغربي سبع وستون جزءاً وسدس جزء . وعرضها أحد وعشرون جزءاً وثُلثا جزء . / وكل / بلدة يكون

/11/ … [β = –M , –α . /15/ … [β = تسعين [M , α . /22/ … [β = فكل [M , α .

whose longitude is less than the longitude of Mecca, Mecca is to the east of it; for every locality whose longitude is greater than the longitude of Mecca, Mecca is to the west of it. If their longitudes are equal, then Mecca is on its meridian line—to the south if the latitude of Mecca is less than its latitude, to the north if it is greater. Every locality / whose latitude equals that of Mecca / is beneath the same day-circle as Mecca; then if its longitude is less than the longitude of Mecca, Mecca is to the left of the rising place of the equinox for that locality, and if its longitude is greater, Mecca is to the right of the setting place of the equinox.

[4] There are many ways to determine the *qibla* bearing, but it would not be appropriate to present them here. Let us instead limit ourselves to one simple method, which is [as follows]. The sun transits the zenith of Mecca when it is in degree 8 of Gemini and in [degree] 23 of Cancer at noontime there. The difference between its noon and the noon of other localities is measured by the difference between the two longitudes. Let this [latter] difference be taken and let an hour be assumed for each 15 degrees and 4 minutes for each degree. The resulting total is the interval in hours from noon [for that locality]. Let an observation be made on that day at that time—before noon if Mecca is to the east or after if it is to the west; the direction of the shadow at that time is the *qibla* bearing.

/4–5/ whose latitude equals that of Mecca] β = whose latitude and that of Mecca are equal] α, M.

طولها أقل من طول مكّة فمكّة شرقية عنها ؛ وكل بلدة يكون طولها أكثر من طول مكّة فمكّة غربية عنها . وإن تساوى طولاهما فمكّة على خط نصف نهارها – جنوبية إن كان عرض مكّة أقل من عرضها ، وشمالية إن كان أكثر . وكل بلدة / يساوي عرضها عرض مكّة / كانت مع مكّة تحت مدار واحد يومي ؛ فإن كان طولها أقل من طول مكّة فمكّة عن يسار مشرق الاعتدال لتلك البلدة ، وإن كان طولها أكثر فمكّة عن يمين مغرب الاعتدال .

[٤] ولمعرفة سمت القبلة طرق كثيرة لا يليق إيرادها هاهنا . فلنقتصر على وجه سهل وهو أنّ الشمس تكون مارّة بسمت مكّة عند كونها في الدرجة الثامنة من الجوزاء والثالثة والعشرين من السرطان وقت انتصاف النهار هناك . والفضل بين نصف نهارها وبين نصف نهار سائر البُلدان يكون بقدر التفاوت بين الطولين . فليؤخذ التفاوت وتؤخذ لكل خمسة عشر جزءاً ساعة ولكل جزء أربع دقائق ؛ فيكون ما اجتمع ساعات البعد عن نصف النهار . وليُرصد في ذلك اليوم ذلك الوقت قبل نصف النهار إن كانت مكّة شرقية أو بَعده إن كانت غربية . فسمت الظل ساعتئذ يكون سمت القبلة .

/4–5/ ... [= β تساوى عرضها وعرض مكة] α , M .

BOOK IV

On Finding the Measurements of the Distances and the Bodies

Seven Chapters

CHAPTER ONE

On the Measure [*misāḥa*] of the Earth

[1] In this book one has need of preliminary propositions other than those that have been stated. Among these is what was proven by Archimedes regarding the measure of a circle and a sphere, namely: that the circumference of every circle is approximately equal to $3\frac{1}{7}$ times its diameter; that the surface enclosed by the radius times half the circumference is equal to the area [*taksīr*] of the circle; that the surface enclosed by the diameter of the sphere times the circumference of the largest circle occurring in it is equal to the surface enclosing the sphere; and that each portion on the surface of the sphere bounded by two great circles [*i.e.* a lune] is equal to the surface enclosed by the diameter times the maximum inclination between them.

[2] Having presented these lemmas, we say: if a person travels along the meridian line on level land in an amount whereby the local latitude increases, or decreases, by one degree, the amount that he has traveled is a one-degree unit of a great circle that occurs upon the Earth. The great circle is 360 times that amount; the diameter of the Earth is 1 part in $3\frac{1}{7}$ parts, this being the total circumference of that great circle. Many people have undertaken to determine this among whom is a group of scientists during the reign of al-Ma'mūn, may God be pleased with him. By his decree, they came to the Plain of Sinjār and obtained the measure of 1° of the 360° along the meridian line; they found it to be $22\frac{2}{9}$ parasangs, each

الباب الرابع
في معرفة مقادير الأبعاد والأجرام
سبعة فصول

الفصل الأول
في مساحة الأرض

[١] يُحتاج في هذا الباب إلى مصادرات غير ما ذُكر . من ذلك ما بيّنه أرشميدس في مساحة الدوائر والأكر وهو : أنّ محيط كل دائرة مثل ثلاثة أمثال قطرها ومثل سُبع قطرها بالتقريب ؛ وأنّ السطح الذي يحيط به نصف القطر في نصف المحيط مساوٍ لتكسير الدائرة ؛ وأنّ السطح الذي يحيط به قطر الكرة في محيط أعظم دائرة تقع فيها مساوٍ للسطح المحيط بالكرة ؛ وأنّ كل قطعة من سطح الكرة تحيط بها دائرتان عظيمتان فهي مساوية لسطح يحيط به القطر في غاية الميل بينهما .

[٢] وبعد تقديم هذه المقدمات نقول : إذا سار سائر على خط نصف النهار في أرض مستوية بقدر ما يزيد جزء واحد في عرض البلد أو ينقص فالقدر الذي ساره يكون حصّة درجة واحدة من الدائرة العظيمة التي تقع على الأرض . والدائرة العظيمة تكون ثلاثمائة وستين مرة مثل ذلك القدر ، وقطر الأرض يكون جزءاً من ثلاثة أجزاء وسُبع جزء وهي مجموع محيط تلك العظيمة . وقد قام بتحقيق ذلك قوم كثير منهم طائفة من الحكماء في عهد المأمون – رضي الله عنه – حضروا بأمره بريّة سِنجار وحصلوا مقدار الجزء الواحد من ثلاثمائة وستين جزءاً من خط نصف النهار ؛ فوجدوه اثنين وعشرين فرسخاً وتُسعي فرسخ على أنّ كل

parasang being 3 miles, each mile being 4000 cubits, each cubit being 24 digits, and each digit being the measure of six average barleycorns laid side by side. When the parasangs plus the fractional [part] are multiplied by 360, one obtains the size of the circumference of the great circle on the Earth, which is 8000 parasangs. If this amount is divided by $3\frac{1}{7}$, one obtains for the size of the diameter 2545 / and a half / parasangs, approximately. Its radius is then 1273 parasangs, approximately. This is the standard with which the distances are measured; similarly the sphere of the Earth is the body with which the [other] bodies are measured.

[3] If the diameter is multiplied by the circumference of the great circle, one obtains the surface area of the Earth, which is twenty thousand thousands three hundred sixty thousand [20,360,000] parasangs. A fourth of this is the area of the populated quarter; the length of the / populated / quarter is half the circumference, while its width is a fourth of it. As for the amount that is [actually] inhabited, which is between the equator and the location whose latitude is equal to the complement of the obliquity, its length is likewise 4000 parasangs and its width, obtained by multiplying the parasangs in 1° by $(66 + \frac{1}{4} + \frac{1}{6})°$, is 1476 parasangs, / approximately. / Its area, obtained by multiplying this by the parasangs in the diameter, is / three thousand thousands seven hundred fifty-six thousand four hundred twenty [3,756,420] parasangs, / which is nearly $(\frac{1}{6} + (\frac{1}{6} \text{ of } \frac{1}{10}))$ of the entire surface of the Earth. If one wishes to know this in miles, he should multiply the linear parasangs by 3 and the square ones by 9. Likewise if he wishes the amount in cubits, digits or barleycorns, he should multiply [the parasangs] by the number of them in a linear or a square parasang. This then is how to find the measure of the Earth.

/7/ and a half] $\beta = -\alpha$, –M. /12/ populated] $\beta = -\alpha$, –M. /17/ approximately] $\beta = -\alpha$, –M. /18–19/ three thousand thousands seven hundred fifty-six thousand four hundred twenty [3,756,420] parasangs] β = three thousand thousands seven hundred sixty-five thousand four hundred twenty [3,765,420] parasangs] α, M.

فرسخ ثلاثة أميال ، وكل ميل أربعة آلاف ذراع ، وكل ذراع أربعة وعشرون أصبعاً ، وكل أصبع مقدار ست شعيرات مضمومة بُطون بعضها إلى بعض من الشعيرات المعتدلة . فإذا ضُرب الفراسخ مع الكسر في ثلاثمائة وستين حصل مقدار محيط الدائرة العظمى من الأرض وهو ثمانية آلاف فرسخ . وإذا قُسِمَ هذا المبلغ على ثلاثة وسُبع حصل مقدار قطرها ألفين وخمسمائة وخمسة وأربعين فرسخاً / ونصف فرسخ / بالتقريب . فيكون نصف قطرها ألفاً ومائتين وثلاثة وسبعين فرسخاً تقريباً ، وهو المقدار الذي تقدّر به الأبعادُ كما أنّ كرة الأرض هي الجرم الذي تقدّر به الأجرام .

[٣] وإذا ضرب القطر في محيط الدائرة العظمى حصل تكسير سطح الأرض وهو عشرون ألف ألف وثلاثمائة وستون ألف فرسخ . وربع ذلك تكسير الربع المسكون ، ويكون طول الربع / المسكون / نصف المحيط وعرضه ربعه . وأما القدر المعمور ، وهو ما بين خط الاستواء والموضع الذي عرضه بقدر تمام الميل ، فيكون طوله أيضاً أربعة آلاف فرسخ وعرضه الحاصل من ضرب فراسخ الجزء الواحد في ستة وستين جزءاً وربع وسدس جزء ألف وأربعمائة وستة وسبعون فرسخاً / تقريباً / . وتكسيره الحاصل من ضرب ذلك في فراسخ القطر / ثلاثة آلاف ألف وسبعمائة وستة وخمسون ألفاً وأربعمائة وعشرون فرسخاً / ، وهو قريب من سدس جميع سطح الأرض وسدس عُشره . وإن أراد مريد أن يعرف ذلك بالأميال ضرب الفراسخ الطولية في ثلاثة والتكسيرية في تسعة ؛ وكذلك إن أراد مقاديرها بالذُرعان والأصابع والشعيرات ضربها في أعدادها لفرسخ طولي أو تكسيري . فهذه معرفة مساحة الأرض .

/7/ ...] β = M– , α– . /12/ ...] β = M– , α– . /17/ ...] β = M– , α– .
/18–19/ ...] β = ثلاثة آلاف ألف وسبعمائة وخمسة وستين ألفاً وأربعمائة وعشرين فرسخاً] M , α .

[4] Abū al-Rayḥān has another method for determining the measure of the Earth that is found by observing the depression of the horizon from the peak of a high mountain whose height is possible to ascertain. We shall not present it here, however, inasmuch as it contains geometrical proofs.

[5] As for what we promised to show in the first part of this book, namely how to find the ratio of a mountain whose height is half a parasang to the diameter of the Earth, the way we do this is to double the parasangs in the diameter, which comes to 5090 parasangs. The ratio of half a parasang to the diameter is the same as the ratio of one to this amount. Then we take the barleycorns in a cubit, which is 144, and we divide the above amount by it; the result is then 35. The ratio of one part of this, which is ⅕ of ⅐ of the width of a barleycorn, to a cubit is the same as the ratio of half a parasang to the diameter.

CHAPTER TWO

On Finding the Distances of the Moon from the Center of the World

[1] The distances of the moon and the other wandering stars from the center of the World are known for any time based upon the radii of their orbs being 60 parts as is stated in calculating their true positions by the method of geometry. The ratio of one to the other, [however], is not known; thus finding that is required. One needs to assume a standard with which all of them may be measured, and so the radius of the Earth has been made that [standard].

[2] In order to find the lunar distances with this standard, Ptolemy observed the moon at the time of its minimum / altitude / on the meridian circle, and he found its apparent altitude to be exactly (39 + (½ of ⅙))°. Its true altitude by calculation for that time and place was 40⅕°. Thus he found the difference between them to be 1 degree and 7 minutes, which is the lunar parallax.

/20/ altitude] β = altitudes] α, M.

[٤] ولأبي الريحان طريق آخر في معرفة مساحة الأرض تعرف برصد انحطاط الأفق عن رأس جبل مرتفع يمكن الوقوف على ارتفاعه ؛ وإنما لم نورده هاهنا لاشتماله على براهين هندسية .

[٥] وأما ما وعدنا بيانه في صدر هذا الكتاب ، وهو معرفة نسبة جبل يكون ارتفاعه نصف فرسخ إلى قطر الأرض ، فالوجه فيه أن نضعّف فراسخ القطر فيصير خمسة آلاف وتسعين فرسخاً ؛ وتكون نسبة نصف فرسخ إلى القطر كنسبة الواحد إلى هذا القدر . ثم نأخذ شعيرات الذراع وهي مائة وأربع وأربعون ونقسم ذلك المبلغ عليها ، فتخرج خمسة وثلاثون . وتكون نسبة جزء منها وهو خُمس سُبع عرض شعيرة إلى ذراع كنسبة نصف فرسخ إلى القطر .

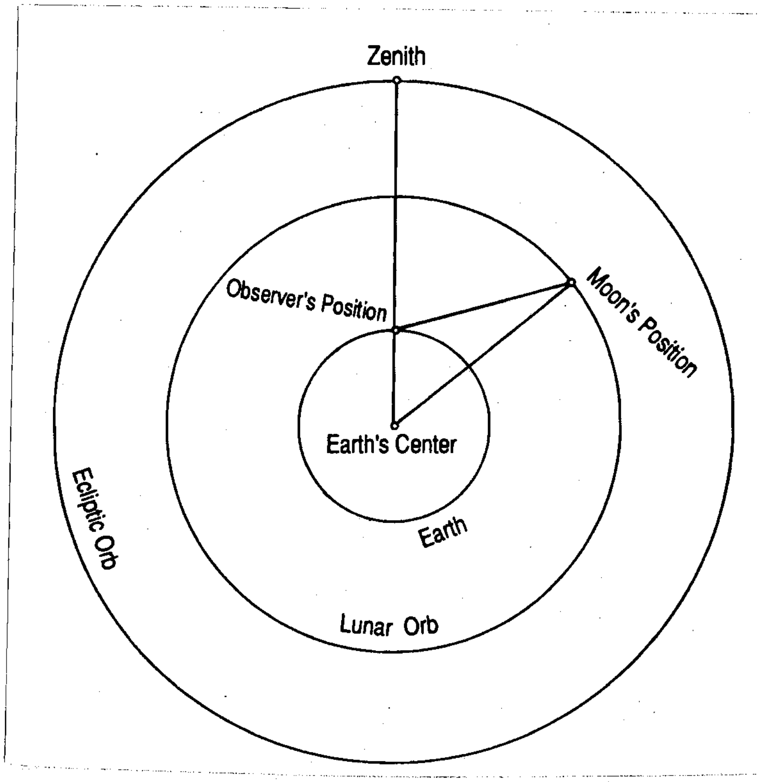
الفصل الثاني
في معرفة أبعاد القمر
من مركز العالم

[١] كان أبعاد القمر وغيره من الكواكب السيّارة من مركز العالم معلومة في كل وقت بحسب كون أنصاف أقطار أفلاكها ستين جزءاً على ما يُذكر في حساب تقويماتها بطريق الهندسة ؛ ولم تكن نسبة البعض إلى البعض معلومة ، فطلب معرفة ذلك . واحتيج إلى فرض مقدار يقدّر به الجميع ، فجُعل ذلك نصف قطر الأرض .

[٢] ولمعرفة أبعاد القمر بذلك المقدار رصد بطلميوس القمر في وقت كان في أقل / ارتفاعه / على دائرة نصف النهار ؛ فوجد ارتفاعه المرئي بالتدقيق تسعة وثلاثين جزءاً ونصف سدس جزء . وكان ارتفاعه الحقيقي بالحساب لذلك الوقت في تلك البقعة أربعين جزءاً وخُمس جزء ؛ فوجد التفاوت بينهما جزءاً وسَبع دقائق ، وهو اختلاف منظر القمر .

/20/ ... [ارتفاعه] = β ارتفاعاته [M , α .

[3] It may be shown in the science of geometry that if the sizes of two angles and a side of a triangle with straight sides are known, the sizes of its remaining sides and angles are known. If one pictures the figure for parallax, which is below, then in the triangle in which one of the angles is the parallax

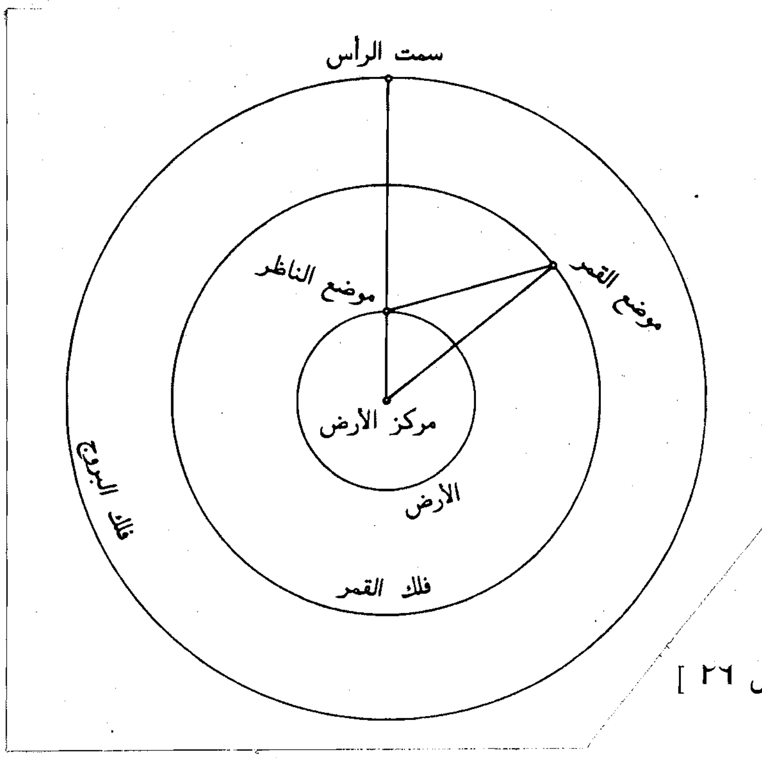


[Fig. T26]

at which is the position of the moon, the second the complement of the true altitude at which is the center of the Earth, and the third that at which is the position of the observer, there are two known angles, *i.e.* the parallax and the complement of the altitude. If the side that is the radius of the Earth is assumed to be 1, two angles and a side become known, and it is possible to find the remaining angle and the two remaining sides of the [triangle]. There results / by calculation / for the size of the side that is the distance of the moon from the center of the Earth (39 + ½ + ¼) parts, the radius of the Earth assumed to be 1 part.

/9/ by calculation] β = from calculation] α, M.

[٢] وقد تبيّن في علم الهندسة أنه إذا كانت مقادير زاويتين وضلع من مثلث مستقيم الأضلاع معلومة كانت مقادير الباقية من أضلاعه وزواياه معلومة . وإذا صُوِّر شكل اختلاف المنظر ـ وهو هذا ـ كان ، في المثلث الذي إحدى زواياه اختلاف المنظر وهي



[شكل ٢٦]

التي عندها موضع القمر والثانية تمام الارتفاع الحقيقي وهي التي عندها مركز الأرض والثالثة التي عندها موضع الناظر ، زاويتان معلومتان أعني اختلاف المنظر وتمام الارتفاع . وإذا فرض الضلع الذي هو نصف قطر الأرض واحداً صارت زاويتان وضلع معلومة وأمكن معرفة الزاوية الباقية والضلعين الباقيين منه . وقد خرج / بالحساب / مقدار الضلع الذي هو بعد القمر عن مركز الأرض تسعة وثلاثين جزءاً ونصف وربع جزء على أنّ نصف قطر الأرض جزء واحد .

/9/ ...] β = من الحساب] M , α .

[4] By calculating the true positions with the scale whereby the inclined radius is 60, the epicyclic radius is 5¼, and the eccentricity is 10 parts and 19 minutes, the distance of the moon from the center of the World for the above time is (40 + ¼ + ⅙) parts. When the same quantity is known in two units of measure, it is possible to convert anything that is measured in one of these two units to the other unit since they will all be according to the ratio [of the units]. Thus Ptolemy converted the above quantities into a unit of measure by which the radius of the Earth is 1: for the inclined radius the result is then 59, for the epicyclic radius 5⅙ parts, and for the eccentricity 10 parts and 9 minutes. The farthest distance of the moon, this being when it is at the apex and the epicycle is at the apogee, is 64⅙ parts; its nearest distance, this being when it is at the epicyclic perigee and the epicycle is at the perigee, is 33 parts and 33 minutes.

CHAPTER THREE

On the Sizes of the Diameters of the Moon, the Sun and the Shadow, and the Distances of the Sun and the Shadow from the Earth

[1] Ptolemy observed two lunar eclipses during which the moon was at the epicyclic apex. During one of them, one-quarter of its diameter was eclipsed and, during the other, half of it. By calculation, its latitude during the first eclipse was 48½ minutes and, during the second, 40⅔ minutes. / He took / the difference between them, namely (7 + ½ + ⅓) minutes, which is obviously a quarter of the diameter. He thus knew that the diameter of the moon at its farthest distance was four times this [amount], namely 31⅓ minutes, and that the latitude for the second eclipse was in the amount of the radius of the shadow since the shadow circle was passing through the center of the lunar disk. This [radius] was approximately 2 times the radius of the moon plus / ⅗ times its radius. / During numerous lunar eclipses at various distances, he found this same ratio between them.

/19/ He took] β = He thus took] α, M. /24/ ⅗ times its radius] β = ⅗ times its diameter] α, M.

[٤] وكان بحساب التقاويم بالقدر الذي يكون نصف قطر المائل ستين ونصف قطر التدوير خمسة وربعاً وما بين المركزين عشرة أجزاء وتسع عشرة دقيقة بُعدُ القمر عن مركز العالم في ذلك الوقت أربعين جزءاً وربع وسدس جزء . وإذا عُرف مقدار واحد بتقديرين أمكن أن يحوّل كل ما يقدّر بواحد من ذينك التقديرين إلى التقدير الآخر لكون الجميع على نسبتهما . فحوّل بطلميوس المقادير المذكورة إلى التقدير الذي به نصف قطر الأرض واحد ؛ فخرج نصف قطر المائل تسعة وخمسين ، ونصف قطر التدوير خمسة أجزاء وسدس ، وما بين المركزين عشرة أجزاء وتسع دقائق . ويكون أبعد بعد القمر ، وذلك عند كونه في الذروة والتدوير في الأوج ، أربعة وستين جزءاً وسدس جزء ؛ وأقرب بعده ، وذلك عند كونه في حضيض التدوير والتدوير في الحضيض ، ثلاثة وثلاثين جزءاً وثلاثاً وثلاثين دقيقة .

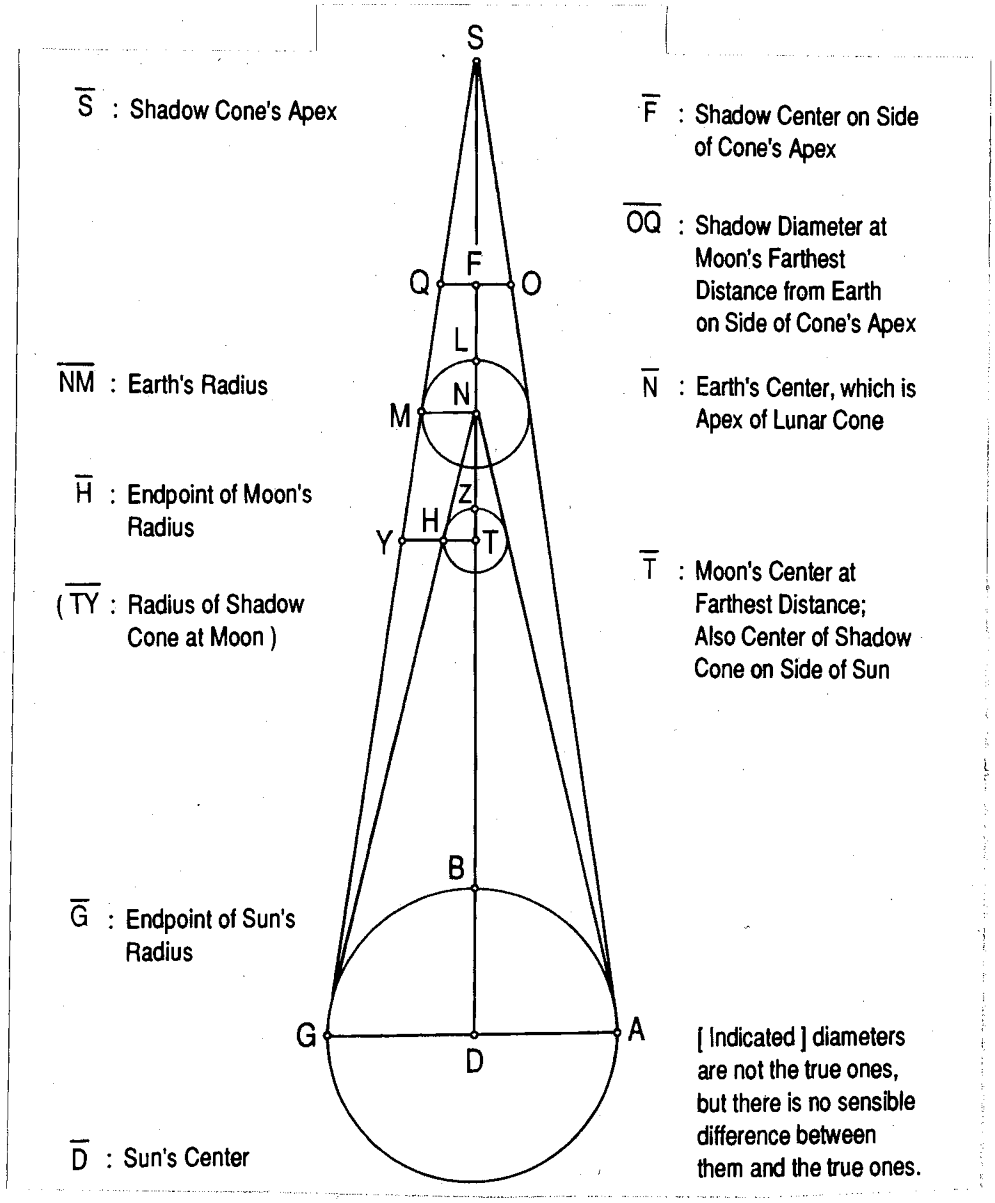
الفصل الثالث
في مقادير أقطار القمر والشمس والظل وأبعاد الشمس والظل عن الأرض

[١] رصد بطلميوس خسوفين للقمر كان القمر فيهما في ذروة التدوير ؛ وقد انخسف من قطره في أحدهما ربعه وفي الآخر نصفه . وكان بالحساب عرضه في الخسوف الأول ثمانية[!] وأربعين دقيقة ونصف وفي الثاني أربعين دقيقة وثُلثي دقيقة . / وأخذ / الفضل بينهما ، وهو سبع دقائق ونصف وثُلث دقيقة ، ويكون لا محالة ربع القطر ؛ فعرف أنّ قطر القمر في أبعد بعده أربعة أمثال ذلك ، وهو أحد[!] وثلاثون دقيقة وثُلث ، وأنّ العرض في الخسوف الثاني هو مقدار نصف قطر الظل لكون دائرة الظل مارّة بمركز صفحة القمر ، وهو بالتقريب مِثْلاَ نصف قطر القمر ومثل ثلاثة أخماس / نصف / قطره . وقد وجد في خسوفات كثيرة في أبعاد مختلفة النسبة بينهما هذه النسبة .

/19/ ... [فأخذ = β [M , α . /24/ ... [β = M– , α– .

[2] Furthermore, he found the diameter of the sun under most circumstances to be equal in appearance to the diameter of the moon at its farthest distance. He thus determined that the diameter of the sun at its mean distance is equal to the diameter of the moon at its farthest distance.

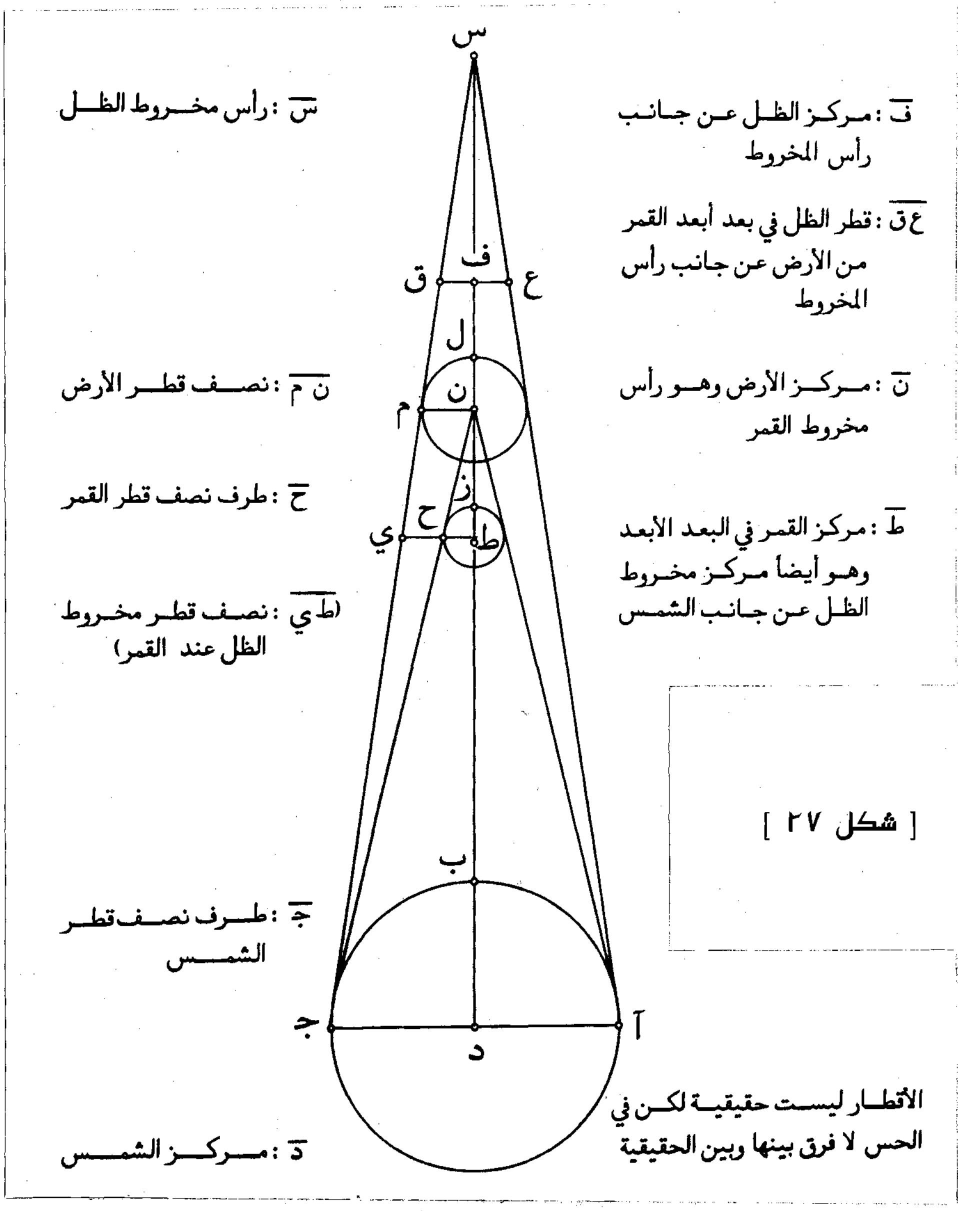
[3] He then set forth in the plane passing through them the figures of the two luminaries, the Earth, the two shadow cones, and the moon according to this illustration:



[Fig. T27]

[٢] وأيضاً وَجَد قطر الشمس في أكثر الأحوال مساوياً في النظر لقطر القمر في بعده الأبعد ؛ فحكم بأنّ قطر الشمس في بعدها الأوسط مساوٍ لقطر القمر في بعده الأبعد .

[٣] ثم وضع في السطح المارّ بالنيّرين والأرض ومخروطي الظل والقمر أشكالها على هذه الصورة :



[شكل ٢٧]

He assumed the moon to be at its farthest distance and a diameter of the shadow on the other side to be at the moon's [other] farthest distance; the distance between the centers of the shadow and the Earth and [the distance] between the centers of the moon and the Earth would then be equal, each one of them being $64\frac{1}{6}$. In the triangle that occurs in the moon's cone between the centers of the moon and the Earth and the endpoint of the moon's radius, the angle that is at the center of the Earth, which is in the amount of the moon's radius, and the angle that is at the center of the moon, which is right, are known. And since the angles of every triangle are equal to two right [angles], the third angle, which is at the endpoint of the moon's diameter, is known. And because the ratio of any side to another is as the ratio of the sine of the angle subtended by the first side to the sine of the angle subtended by the other side, as is shown in geometry, the ratio of the moon's radius to the distance of its center from the center of the Earth is as the ratio of $16\frac{2}{5}$ minutes to 60 parts less a small, imperceptible amount. Now the distance of the center of the moon from the center of the Earth, with the Earth's radius being 1, is $64\frac{1}{6}$ parts. The moon's radius using this measure is then known, namely 17 minutes and 33 seconds. The radius of the shadow using this measure is 45 minutes and 38 seconds.

[4] Because the distance between the centers of the moon and the shadow is twice the distance between the centers of the Earth and the shadow, the excess of the shadow cone's radius at the moon over the radius of the shadow is twice the excess of the Earth's radius over the radius of the shadow. For that reason the sum of the radius of the shadow and the radius of the shadow cone at the moon is equal to twice the radius of the Earth, *i.e.* to the diameter of the Earth. When the radius of the shadow and the radius of the moon are added together, which is 1 part, 3 minutes and 11 seconds, and this sum is subtracted from the diameter of the Earth, there remain 56 minutes and 49 seconds, which is the amount of the excess of the radius of the cone at the moon

وفَرَض القمر في بعده الأبعد وقطرَ الظل عن الجانب الآخر في بعد القمر الأبعد ؛ فيكون البعد بين مركزي الظل والأرض وبين مركزي القمر والأرض متساويين ، كل واحد منهما أربعة وستون وسدس . وتكون في المثلث الذي حدث في مخروط القمر بين مركزي القمر والأرض وطرف نصف قطر القمر الزاويةُ التي على مركز الأرض ، وهي بقدر نصف قطر القمر ، والزاويةُ التي على مركز القمر ، وهي قائمة ، معلومتين . ولكون زوايا كل مثلث مساوية لقائمتين تصير الزاوية الثالثة ، وهي التي على طرف قطر القمر ، معلومة . ولأنّ نسبة كل ضلع إلى آخر تكون كنسبة جيب الزاوية التي يوترها الضلع الأول إلى جيب الزاوية التي يوترها الضلع الآخر ــ على ما بُيّن في الهندسة ــ تكون نسبة نصف قطر القمر إلى بعد مركزه عن مركز الأرض كنسبة ست عشرة دقيقة وخُمسي دقيقة إلى ستين جزءاً إلاّ شيء قليل غير محسوس . وكان بعد مركز القمر عن مركز الأرض ــ على أنّ نصف قطر الأرض واحد ــ أربعة وستين جزءاً وسدس جزء ؛ فنصف قطر القمر بذلك المقدار يكون معلوماً ، وهو سبع عشرة دقيقة وثلاث وثلاثون ثانية ، ويكون نصف قطر الظل بذلك المقدار خمساً وأربعين دقيقة وثمانياً وثلاثين ثانية .

[٤] ولأنّ البعد بين مركزي القمر والظل ضعف البعد بين مركزي الأرض والظل تكون زيادة نصف قطر مخروط الظل الذي عند القمر على نصف قطر الظل ضعف زيادة نصف قطر الأرض على نصف قطر الظل ؛ ويكون لذلك مجموع نصفي قطر الظل وقطر مخروط الظل عند القمر مساوياً لضعف نصف قطر الأرض ، أعني لقطر الأرض . وإذا جمع نصف قطر الظل ونصف قطر القمر ، وهما جزء وثلاث دقائق وإحدى عشرة ثانية ، ونقص المجموع من قطر الأرض ، بقيت ست وخمسون دقيقة وتسع وأربعون ثانية ، وهي مقدار فضل نصف قطر المخروط عند القمر

over the radius of the moon. The ratio of the Earth's radius to this is as the ratio of the distance between the centers of the Earth and the sun to the distance between the centers of the two luminaries, which is as the ratio of 1 to 56 minutes and 49 seconds. Thus when the distance of the sun from the center of the Earth is 1, the distance between the two luminaries is 56 minutes and 49 seconds, and there / remain / 3 minutes and 11 seconds for the distance of the moon from the Earth; with the radius of the Earth being 1, this distance is $64\frac{1}{6}$ parts. Then by this [measure], the distance of the sun from the center of the Earth at its mean distance is 1210 times the Earth's radius.

[5] Furthermore the ratio of the Earth's radius to the radius of the shadow, which is 45 minutes and 38 seconds, is as the ratio of the distance of the cone's apex from the center of the Earth to its distance from the center of the shadow. Therefore when the distance of the cone's apex from the center of the Earth is 1, its distance from the center of the shadow is 45 minutes and 38 seconds, and there remain ($14 + \frac{1}{5} + \frac{1}{6}$) minutes for the distance of the shadow's center from the center of the Earth, which is, with the radius of the Earth being 1, $64\frac{1}{6}$. Then by this [measure], the distance of the cone's apex from the center of the / Earth / is ($203 + \frac{1}{2} + \frac{1}{3}$) times the Earth's radius.

/6/ remain] β = are] α, M. /19/ Earth] α, β = shadow] M (see commentary).

على نصف قطر القمر . وتكون نسبة نصف قطر الأرض إليه كنسبة البعد بين مركزي الأرض والشمس إلى البعد بين مركزي النيّرين ، وهي كنسبة الواحد إلى ست وخمسين دقيقة وتسع وأربعين ثانية . فإذن إذا كان بعد الشمس عن مركز الأرض واحداً كان البعد بين النيّرين ستاً وخمسين دقيقة وتسعاً وأربعين ثانية ؛ / ويبقى / بعد القمر عن الأرض ثلاث دقائق وإحدى عشرة ثانية ، وكان هذا البعد – على أنّ نصف قطر الأرض واحد – أربعة وستين جزءاً وسدس جزء . فبحسب ذلك يكون بعد الشمس عن مركز الأرض في / بعده / الأوسط ألفاً ومائتين وعشرة أمثال نصف قطر الأرض .

[٥] وأيضاً نسبة نصف قطر الأرض إلى نصف قطر الظل ، وهو خمس وأربعون دقيقة وثمان وثلاثون ثانية ، كنسبة بعد رأس المخروط عن مركز الأرض إلى بعده عن مركز الظل . فلذلك إذا كان بعد رأس المخروط عن مركز الأرض واحداً كان بعده عن مركز الظل خمساً وأربعين دقيقة وثمانياً وثلاثين ثانية ؛ ويبقى بُعد مركز الظل عن مركز الأرض أربع عشرة دقيقة وخُمس وسدس دقيقة ، وكان – على أنّ نصف قطر الأرض واحد – أربعة وستين وسدساً . فبحسب ذلك يكون بعد رأس المخروط عن مركز / الأرض / مائتين وثلاثة أمثال ونصف وثُلث مثل لنصف قطر الأرض .

/6/ ...] β = وكان] M , α . /9/ بعده] β , α = بعدها] M . /19/ الأرض] β , α = الظل] M (انظر شرحنا) .

CHAPTER FOUR

On the Volume of the Two Luminaries

[1] It has been established in the science of optics that for any two bodies that are equal in appearance but are at different distances, the ratio of the nearer of the two to the one farther away with respect to the size of the diameter of the body is as the ratio of the distance of the nearer to the distance of the farther. For that reason, the ratio of the moon's radius, which is 17 minutes and 33 seconds, to the sun's radius is as the ratio of the distance of the moon from the Earth, which is $64\frac{1}{6}$, to the distance of the sun from the Earth, which is 1210. Thus the radius of the sun is also known, namely $5\frac{1}{2}$, with the Earth's radius being 1. If the diameter of the moon is assumed to be 1, the diameter of the Earth becomes $3\frac{2}{5}$ and the diameter of the sun $18\frac{4}{5}$.

[2] Euclid has proven that the ratio of one sphere to [another] sphere is as the ratio of the cube of the [first] diameter to the cube of the [other] diameter. Then when the above quantities are multiplied by themselves twice so they become cubed, one finds that the sun is ($166 + \frac{1}{4} + \frac{1}{8}$) times the Earth and 6644 times / and $\frac{2}{3}$ times / the moon, and that the Earth is $39\frac{1}{4}$ / and ($\frac{1}{2}$ of $\frac{1}{10}$) / times the moon.

/17/ and $\frac{2}{3}$ times] $\beta = -\alpha$, –M. /18/ and $\frac{1}{2}$ of $\frac{1}{10}$] $\beta = -\alpha$, –M.

الفصل الرابع
في مقدار جرم النيّرين

[١] ثبت في علم المناظر أنّ كل جرمين متساويين في الرؤية ومختلفين في البعد تكون نسبة أقربهما إلى أبعدهما في مقدار قطر الجرم كنسبة بعد الأقرب إلى بعد الأبعد . ولذلك تكون نسبة نصف قطر القمر ، الذي هو سبع عشرة دقيقة وثلاث وثلاثون ثانية ، إلى نصف قطر الشمس كنسبة بعد القمر عن الأرض ، الذي هو أربعة وستون وسدس ، إلى بعد الشمس عن الأرض ، الذي هو ألف ومائتان وعشرة . فيكون نصف قطر الشمس أيضاً معلوماً ، وهو خمسة ونصف على أنّ نصف قطر الأرض واحد . وإن فرض قطر القمر واحداً صار قطر الأرض ثلاثة وخُمسَين وقطر الشمس ثمانية عشر وأربعة أخماس .

[٢] وقد بيّن أقليدس أنّ نسبة الكرة إلى الكرة تكون كنسبة مكعّب القطر إلى مكعّب القطر . فإذا ضربت هذه المقادير في أنفسها مرتين لتصير مكعّبة علم أنّ الشمس مائة وستة وستون مثلاً وربع وثمن مثل / الأرض / وستة آلاف وستمائة وأربعة وأربعون مثلاً / وثلثي مثل / القمر ، وأنّ الأرض تسعة وثلاثون مثلاً وربع / ونصف عشر / مثل / القمر / .

/16/ [...] = β] للأرض M , α . /17/ وثلثي مثل] β = M– , α– .. /18/ ونصف عشر] β = M– , α– . /18/ القمر] M , β = للقمر] α .

CHAPTER FIVE

On the Rest of the Distances of the Sun and the Distances and Body [Sizes] of the Two Lower Planets

[1] The known distance for the sun that was given above was only made on the assumption that it was at its mean distance. Its elongation from there at the two other distances is according to the amount between the centers of the two [orbs of the sun], which is, based on Ptolemy's observations, 2½ of those parts by which the radius of its eccentric orb is 60; therefore, it is 1 part in 24 of its mean distance. When we divide the known distance of the sun, which is 1210, by 24, the result is 50 and a fraction, which is the size of the eccentricity. The farthest distance of the sun is then approximately 1260 times the radius of the Earth, and its nearest distance is 1160 times it.

[2] Since there is no void between the orbs of the planets, and there is no known body other than their orbs, the farthest distance of each planet has been made the nearest distance of the planet above it so that the adopted distances are the minimum possible. Thus the nearest distance of the sun is the farthest distance of Venus.

[3] As for Venus, one finds in calculating true positions that its eccentricity is 1¼ parts, and the radius of its epicycle is 43⅙ of those parts by which the radius of / its deferent / is 60. Thus its farthest distance is (104 + ¼ + ⅙) parts, and its nearest distance is (15 + ⅓ + ¼) of those parts, which is approximately (1⁄10 + (½ of 1⁄10)) of the farthest distance.

/19/ its deferent] β, M = the deferent] α.

الفصل الخامس في سائر أبعاد الشمس وأبعاد السفليين وجرميهما

[١] البعد المعلوم للشمس المذكور إنما فرض عند كونها في بعدها الأوسط . ويكون تباعدها عنه في البعدين الآخرين بقدر ما بين مركزيهما ؛ وكان ذلك ، بحسب أرصاد بطلميوس ، جزأين / ونصفاً / من الأجزاء التي بها نصف قطر فلكها الخارج المركز ستون . فإذن هو جزء من أربعة وعشرين من بعدها الأوسط . وإذا قسمنا بُعد الشمس المعلوم ، وهو ألف ومائتان وعشرة ، على أربعة وعشرين خرج خمسون وكسر ، وهو مقدار خروج المركز . فيكون بعد الشمس الأبعد ألفاً ومائتين وستين مثلاً لنصف قطر الأرض بالتقريب ، وبُعدها الأقرب ألفاً ومائة وستين مثلاً له .

[٢] ولمّا لم يكن بين أفلاك الكواكب خلاء ولا جرم معلوم غير أفلاكها ، جُعل البعد الأبعد لكل كوكب البعد الأقرب للكوكب الذي فوقه لتكون الأبعاد المأخوذة هي التي لا يمكن أن تكون أقل منها ؛ فيكون البعد الأقرب للشمس البعد الأبعد للزهرة .

[٣] أما الزهرة فقد عُلم في حساب التقاويم أنّ ما بين مركزيها جزء وربع ، ونصف قطر تدويرها ثلاثة وأربعون وسدس من الأجزاء التي بها نصف قطر / حاملها / ستون . فيكون بعدها الأبعد مائة وأربعة أجزءا وربع وسدس ، وبعدها الأقرب خمسة عشر جزءاً وثُلث وربع بتلك الأجزاء وهو عُشر البعد الأبعد ونصف عُشره بالتقريب .

/7/ ...] ونصف = β] M , α . /19/ ...] الحامل = β , M] α .

[4] Moreover Mercury's eccentricity is 3 parts, which is equal to the distance between each of the centers of its orbs and the one adjacent to it. The radius of its epicycle is 22½ of those parts by which the radius of its deferent is 60. Its farthest distance is 91½ parts, and its nearest distance is 33 parts 4 minutes. This is known only by successive approximation [*istiqrā'*] because its nearest distance is not directly opposite its farthest distance. Its nearest distance is therefore (⅕ + ⅙) of its farthest distance or, [with respect] to the farthest distance of Venus, 11 : 200 parts, which is nearly 1 : 18 parts of it.

[5] The farthest distance of the moon to the nearest distance of the sun was found above also to be nearly 1 : 18 parts. So it seemed likely to them that their two orbs were between the orbs of the two luminaries since there was no way to leave this distance between the orbs unoccupied. This is the basis for our statement earlier [in the book] that the distance of the sun from the Earth is consistent with Venus and Mercury being below [the sun].

[6] To return to what we were doing: when we take (⅒ + (½ of ⅒)) of Venus's farthest distance, one obtains 174 times the radius of the Earth, which is then Venus's nearest distance and Mercury's farthest distance. It was previously stated that the height of the shadow cone is 203 plus a fraction times the radius of the Earth. It is then known that the Earth's shadow will disappear in the orb of Venus between its near and mean distances. Furthermore it is clear from the [above] that the thickness of Venus's orb is 1000 less 14 times the Earth's radius, and that the thickness of Mercury's orb plus what is contained inside it is 348 times, which is approximately ⅓ of [Venus's orb]. We then take (⅕ + ⅙) of Mercury's farthest distance with the result being 64 times the Earth's radius, which is the nearest of Mercury's distances and the farthest of the moon's distances, in accord with the result of the earlier calculation.

[٤] وأيضاً ما بين مركزي عطارد ثلاثة أجزاء ويساويه البعد بين كل مركز من مراكز أفلاكه وبين الذي يليه ، ونصف قطر تدويره اثنان وعشرون جزءاً ونصف بالأجزاء التي بها نصف قطر حامله ستون . وبعده الأبعد أحد وتسعون جزءاً ونصف ، وبُعد أقربه ثلاث[!] وثلاثون جزءاً وأربع دقائق . وإنما عُرف ذلك بالاستقراء لأنّ بعده الأقرب لا يقابل بعده الأبعد . فيكون بعده الأقرب خُمساً وسدساً من بعده الأبعد أو أحد عشر جزءاً من مائتي جزء هي أجزاء بعد الزهرة الأبعد ، وهي قريبة من جزء من ثمانية عشر منه .

[٥] ووُجد بعد القمر الأبعد من بعد / الشمس الأقرب / أيضاً قريباً من جزء من ثمانية عشر كما مرّ . فغلب على ظنونهم كون فلكيهما بين فلكي النيّرين إذ لا وجه لتعطيل هذا البعد بين الأفلاك . وهذا هو الوجه لقولنا فيما مرّ أنّ بعد الشمس من الأرض يناسب كون الزهرة وعطارد تحتها .

[٦] ونعود إلى ما كُنّا فيه . فإذا أخذنا العُشر ونصف العُشر من بعد الزهرة الأبعد حصل مائة وأربعة وسبعون مثلاً لنصف قطر الأرض ، فهو البعد الأقرب للزهرة والبعد الأبعد لعطارد . وقد مرّ أنّ ارتفاع مخروط الظلّ مائتان وثلاثة أمثال نصف قطر الأرض وكسر ، فعُلم أنّ ظلّ الأرض ينعدم في فلك الزهرة بين بعديه الأقرب والأوسط . وأيضاً يتبيّن منه أنّ ثخن فلك الزهرة ألف مثلاً لنصف قطر الأرض غير أربعة عشر مثلاً ، وأنّ ثخن فلك عطارد بما في ضمنه ثلاثمائة وثمانية وأربعون مثلاً وهو قريب من ثُلثه . ثم أخذنا الخُمس والسدس من بعد عطارد الأبعد فحصل أربعة وستون مثلاً لنصف قطر الأرض ، وهو أقرب أبعاد عطارد وأبعد أبعاد القمر موافقاً لما خرج من الحساب الأول .

/10/ ...] M , β = أقرب الشمس] α .

[7] Turning now to the body [size] of Venus and of Mercury: they have stated that the diameter of Venus at its mean distance is approximately equal to $\frac{1}{10}$ the diameter of the sun, and that the diameter of Mercury to the diameter of the sun is as 1 : 15. Then the [mid-distance] between Venus's two [extreme] distances is taken, and one obtains 667, which is its mean distance. Its ratio to the sun's mean distance is as the ratio of Venus's diameter to $\frac{1}{10}$ the sun's diameter. Venus's mean distance to the sun's mean distance is as 1 : 1;49 [*lit.*, one to one and forty-nine minutes], which is the size of Venus's diameter to $\frac{1}{10}$ the sun's diameter. When one multiplies 1;49 by 10, the result is $18\frac{1}{6}$. Thus the diameter of Venus to the diameter of the sun is as 1 : $18\frac{1}{6}$ parts. When 2 parts in 11 is taken of this, one obtains $3\frac{3}{10}$ parts; thus the diameter of Venus to the diameter of the Earth is as 1 : $3\frac{3}{10}$ parts. / If / the two quantities are cubed, it becomes approximately 1 : 35;56 [*lit.*, one to thirty-five and fifty-six minutes]; therefore, the body of the Earth is approximately 36 times the body of Venus.

[8] Furthermore Mercury's mean distance occurring between its two [extreme] distances is 119 times the radius of the Earth. To the sun's mean distance this is approximately as 1 : $10\frac{1}{6}$ parts, which is the size of Mercury's diameter to ($\frac{1}{3}$ of $\frac{1}{5}$) the diameter of the sun. Multiplied by 15, the result is 153. Thus the size of Mercury's diameter to the sun's diameter is 1 : 153. When 2 parts in 11 is taken of this, [the result] is approximately 28; thus the size of Mercury's diameter to the Earth's diameter is as 1 : 28 parts. The cube of 28 is 21,952. Thus the Earth's body is approximately equal to 22,000 times Mercury's body.

/13/ If] β = When] α, M.

[٧] وأما جرم الزهرة وعطارد فذكروا أنّ قطر الزهرة في بعدها الأوسط يكون مثل عُشر قطر الشمس تقريباً وأنّ قطر عطارد من قطر الشمس يكون كواحد من خمسة عشر . فأخذ ما بين بعدي الزهرة فحصل ستمائة وسبعة وستون ، وهو بعدها الأوسط . وتكون نسبتها إلى بعد الشمس الأوسط كنسبة قطر الزهرة إلى عُشر قطر الشمس . وبُعد الزهرة الأوسط من بعد الشمس الأوسط كواحد من واحد وتسع وأربعين دقيقة ، وهي قدر قطر الزهرة من عُشر قطر الشمس . وإذا ضرب واحد وتسع وأربعون دقيقة في عشرة بلغ ثمانية عَشر وسدساً ، فيكون قطر الزهرة من قطر الشمس كواحد من ثمانية عشر جزءاً وسدس جزء . وإذا أخذ منها جزءان من أحد عشر حصل ثلاثة أجزاء وثلاثة أعشار جزء ؛ فقطر الزهرة من قطر الأرض كواحد من ثلاثة أجزاء وثلاثة أعشار . / وإن / كُعّب المقداران يصير واحداً من خمسة وثلاثين وست وخمسين دقيقة بالتقريب ؛ فإذن جرم الأرض ستة وثلاثون مثلاً / لجرم الزهرة بالتقريب / .

[٨] وأيضاً بُعد عطارد الأوسط الكائن بين بعديه مائة وتسعة عشر مثلاً لنصف قطر الأرض . وهو من بعد الشمس الأوسط كواحد من عشرة أجزاء وسدس بالتقريب ، وهو قدر قطر عطارد من ثُلث خُمس قطر الشمس . ضرب في خمسة عشر بلغ مائة وثلاثة وخمسين . فقدر قطر عطارد من قطر الشمس واحد من مائة وثلاثة وخمسين . وإذا أخذ منه جزءان من أحد عشر كان ثمانية وعشرين بالتقريب ؛ فقدر قطر عطارد من قطر الأرض كجزء من ثمانية وعشرين . ومكعّب ثمانية وعشرين أحد وعشرون ألفاً وتسعمائة واثنان وخمسون ؛ فجرم الأرض مثل جرم عطارد اثنان وعشرون ألف مرة بالتقريب .

/13/ ...] β = وإذا] M , α . /15/ ...] β , M = بالتقريب لجرم الزهرة] α .

CHAPTER SIX

On the Distances of the Upper Planets and Their Body [Sizes]

[1] Ptolemy found the eccentricity of Mars to be 6 parts and the radius of its epicycle 39½ parts, based on the deferent radius being 60. Its farthest distance is then 105½ parts, and its nearest distance is 14½ parts, which is to the farthest distance as 1 : 7, approximately. Then multiplying the farthest distance of the sun, which is 1260, by 7 results in 8820 times the radius of the Earth, which is then the farthest distance of Mars.

[2] They have stated that the diameter of Mars at its mean distance is [with respect] to the diameter of the sun as 1 : 20 parts. They then took its mean distance, *i.e.* the mid-distance between its two [extreme] distances; this is equal to 5040 times the radius of the Earth, which is 4⅙ times the sun's mean distance. When one takes (½ of ⅒) the diameter of the sun, one obtains 16½ minutes; multiplying by 4⅙ results in 1;9 [*lit.*, one and nine minutes], which is the diameter of Mars when the diameter of the Earth is 1. By taking its cube, which is then 1;31 [*lit.*, one and thirty-one minutes], one finds that the volume of Mars is approximately equal to 1½ times the volume of the Earth.

[3] It is evident that the thickness of Mars's orb is 7560 times the Earth's radius and that the diameter of the sun's sphere is 2520 times it. Therefore the thickness of Mars's orb is 3 times the breadth of [the combination of] the sun's orb and the orbs and the elements [contained] inside it. This is an elucidation of what we stated in the chapter on the configurations [*hay'āt*] of the orbs of the upper planets.

الفصل السادس
في أبعاد الكواكب العلوية وأجرامها

[١] وجد بطلميوس ما بين مركزي المرّيخ ستة أجزاء ونصف قطر تدويره تسعة وثلاثين جزءاً / ونصفاً / على أنّ نصف قطر الحامل ستون . فيكون بعده الأبعد مائة وخمسة أجزاء ونصف وبعده الأقرب أربعة عشر جزءاً ونصف ، وهو من بعده الأبعد كواحد من سبعة تقريباً . فضَرْب أبعد بعد الشمس ، وهو ألف ومائتان وستون ، في سبعة بلغ ثمانية آلاف وثمانمائة وعشرين مثلاً لنصف قطر الأرض ، فهو بعد المرّيخ الأبعد .

[٢] وذكروا أنّ قطر المرّيخ في بعد أوسطه يكون من قطر الشمس كجزء من عشرين . فأخذوا بعده الأوسط ـ أعني منتصف ما بين بعديه ـ فكان خمسة آلاف وأربعين مثلاً لنصف قطر الأرض ، وهو أربع مرات وسدس مرة مثل بعد الشمس الأوسط . وإذا أخذ نصف عُشر قطر الشمس خرج ست عشرة دقيقة ونصف ؛ ضرب في أربعة وسدس بلغ واحداً وتسع دقائق ، وهو قطر المرّيخ إذا كان قطر الأرض واحداً . أخذ مكعّبه فكان واحداً وأحداً[!] وثلاثين دقيقة ، فعلم أنّ جرم المرّيخ مثل جرم الأرض مرة ونصف بالتقريب .

[٣] وقد ظهر أنّ ثخن فلك المرّيخ سبعة آلاف وخمسمائة وستون مثلاً لنصف قطر الأرض ، وقطر كرة الشمس يكون ألفين وخمسمائة وعشرين مثلاً له . فثخن فلك المرّيخ ثلاثة أمثال غلظ فلك الشمس مع ما فيه من الأفلاك والعناصر . وهذا بيان ما ذكرناه في باب هيئات أفلاك الكواكب العلوية .

/4/ ...] β = ونصف] α , M .

[4] Turning to Jupiter: by calculation Ptolemy found its eccentricity to be $(2 + \frac{1}{2} + \frac{1}{4})$ parts and the radius of its epicycle $11\frac{1}{2}$ parts, based on the radius of its deferent being 60. Thus its farthest distance is $74\frac{1}{4}$ parts, and its nearest distance is $(45 + \frac{1}{2} + \frac{1}{4})$ parts. So the first is $(1 + \frac{1}{4} + \frac{1}{5} + \frac{1}{6})$ times the second. When one takes the farthest distance of Mars times $(1 + \frac{1}{4} + \frac{1}{5} + \frac{1}{6})$, the result is 14,259 times the Earth's radius, which is then Jupiter's farthest distance.

[5] They have stated that [Jupiter's] diameter is equal to ($\frac{1}{2}$ of $\frac{1}{6}$) the diameter of the sun when both are at their mean distances. When one then takes the mid-distance of its two [extreme] distances, it is 11,540 times / the Earth's radius, / which is equal to $(9 + \frac{1}{3} + \frac{1}{5})$ times the sun's mean distance. When one takes ($\frac{1}{2}$ of $\frac{1}{6}$) the diameter of the sun, it is $27\frac{1}{2}$ minutes. If [this] is then multiplied by $(9 + \frac{1}{3} + \frac{1}{5})$, the result is $(4 + \frac{1}{5} + \frac{1}{6})$ units. The diameter of the Earth to the diameter of Jupiter is therefore as $1 : (4 + \frac{1}{5} + \frac{1}{6})$ units. When these two [quantities] are cubed, [one finds] the volume of Jupiter to be equal to / $83\frac{1}{4}$ / times the volume of the Earth.

[6] As for Saturn: by calculation Ptolemy found its eccentricity to be $(3 + \frac{1}{4} + \frac{1}{6})$ parts and the radius of its epicycle $6\frac{1}{2}$ parts, with the parts such that the radius of its deferent is 60 parts. Thus its farthest distance is $(69 + \frac{2}{3} + \frac{1}{4})$ parts, and its nearest distance is $(50 + (\frac{1}{2}$ of $\frac{1}{6}))$ parts; the farthest [distance] is equal to $1\frac{2}{5}$ the nearest. Then multiplying Jupiter's farthest distance by $1\frac{2}{5}$ results in 19,963 times the Earth's radius, which is Saturn's farthest distance.

/11/ the Earth's radius] β, M(?) = the Earth] α. /17/ $83\frac{1}{4}$] β = $82\frac{1}{4}$] α, M.

[٤] وأما المشتري فقد وجد بطلميوس بالحساب ما بين مركزيه جزأين ونصف وربع جزء ونصف قطر تدويره أحد عشر جزءاً ونصف على أنّ نصف قطر حامله ستون . فيكون بعده الأبعد أربعة وسبعين جزءاً وربع جزء وبعده الأقرب خمسة وأربعين جزءاً ونصف وربع جزء ؛ ويكون الأول من الثاني مثله ومثل ربعه وخُمسه وسدسه . وإذا أخذ مثل بعد المرّيخ الأبعد ومثل ربعه وخُمسه وسدسه بلغ أربعة عشر ألفاً ومائتين وتسعة وخمسين مثلاً لنصف قطر الأرض ، فهو البعد الأبعد للمشتري .

[٥] وذكروا أنّ قطره مثل نصف سدس قطر الشمس إذا كانا في بعديهما الأوسطين . فإذا أخذ منتصف بعديه كان أحد عشر ألفاً وخمسمائة وأربعين مثلاً / لنصف قطر الأرض / ، وهو تسع مرات مثل بعد الشمس الأوسط وثُلث وخُمس مرة . وإذا أخذ نصف سدس قطر الشمس كان سبعاً وعشرين دقيقة / ونصفاً / . فإذا ضرب في تسعة وثُلث وخُمس بلغ أربعة وخُمس وسدس واحدٍ ، فقطر الأرض من قطر المشتري كواحد من أربعة وخُمس وسدس واحدٍ . وإذا كُعّبا كان جرم المشتري مثل جرم الأرض / ثلاثة[!] / وثمانين مرة وربع مرة .

[٦] وأما زحل فقد وجد بطلميوس بالحساب ما بين مركزيه ثلاثة أجزاء وربع وسدس جزء ونصف قطر تدويره ستة أجزاء ونصف بالأجزاء التي بها نصف قطر حامله ستون جزءاً . فيكون بعده الأبعد تسعة وستين جزءاً وثُلثي جزء وربعه وبعده الأقرب خَمسين جزءاً ونصف سدس جزء ؛ والأبعد مثل الأقرب ومثل خُمسيه . فضرب بعد المشتري الأبعد في واحد وخُمسَين بلغ تسعة عشر ألفاً وتسعمائة وثلاثة وستين مثلاً لنصف قطر الأرض ، وهو البعد الأبعد لزحل .

/11/ ...] β, M(?) = للأرض [α. /13/ ...] β = ونصف [α, M. /17/ ...] β = اثنين [α, M.

[7] They have stated that [Saturn's] diameter is to the sun's diameter as 1 : 18 when both are at their mean distances. When one takes the mid-distance of its two [extreme] distances, it is 17,111 times the Earth's radius, this then being Saturn's mean distance, and it is approximately equal to 14 times the sun's mean distance. When one takes 1 part in 18 of the sun's diameter, it is $18\frac{1}{3}$ minutes. When multiplied by 14, the result is approximately $4\frac{1}{4}$ parts. The Earth's diameter is then to Saturn's diameter as 1 : $4\frac{1}{4}$ parts, approximately. When these two [quantities] are cubed, [one finds] the volume of Saturn to be approximately equal to 77 times the volume of the Earth.

CHAPTER SEVEN

On the Distance of the Fixed Stars and Their Body [Sizes] and a Concluding Discussion Regarding This Section

[1] The farthest distance of Saturn was made the distance of the fixed stars from the Earth—inasmuch as [the amount] by which it exceeds it is not known—in order that the [assigned] boundary not be beyond what exists.

[2] They have stated that the diameter of average-size stars of first magnitude in relation to the diameter of the sun is about ($\frac{1}{2}$ of $\frac{1}{10}$) of it. Their distance is approximately $16\frac{1}{2}$ times the mean distance of the sun. 1 : 20 parts of the sun's diameter is $16\frac{1}{2}$ minutes; when this is multiplied by $16\frac{1}{2}$, the result is ($4 + \frac{1}{3} + \frac{1}{5}$) units. Thus the diameter of average stars of first magnitude is equal to ($4 + \frac{1}{3} + \frac{1}{5}$) times the diameter of the Earth. When the two [quantities] are cubed, [one finds] their volume to be approximately equal to 93 times the Earth's volume.

[٧] وذكروا أنّ قطره من قطر الشمس كواحد من ثمانية عشر عند كونهما في بعديهما الأوسطين . وإذا أخذ منتصف بعديه كان سبعة عشر الفاً ومائة وأحد عشر مثلاً لنصف قطر الأرض ، فهو بعد زحل الأوسط ، وهو أربع عشرة مرة مثل بعد الشمس الأوسط تقريباً . وإذا أخذ جزء من ثمانية عشر من قطر الشمس كان ثماني عشرة دقيقة / وثُلثاً / . فإذا ضرب في أربعة عشر بلغ أربعة أجزاء وربع جزء بالتقريب ، فقطر الأرض من قطر زحل كجزء واحد من أربعة أجزاء وربع / تقريباً / . وإذا كُعّبا كان جرم زحل مثل جرم الأرض سبعاً وسبعين مرة بالتقريب .

الفصل السابع
في بعد الثوابت وأجرامها
وتمام القول في هذا الباب

[١] جُعل أبعد بعد زحل بعد الثوابت من الأرض إذ لم تكن الزيادة عليه معلومة لئلاّ يكون المحدود أكثر من الموجود .

[٢] وذكروا أنّ قطر أوسط كواكب القدر الأول جرماً يكون من قطر الشمس بالقياس قريباً من نصف عُشره . وكان بعدها ستة عشر مثلاً ونصفاً لبعد الشمس الأوسط بالتقريب . والجزء من عشرين من قطر الشمس ست عشرة دقيقة ونصف . وإذا ضرب في ستة عشر ونصف بلغ أربعة وثُلث وخُمس واحدٍ . فقطر أوسط كواكب القدر الأول أربع مرات مثل قطر الأرض ومثل ثُلثه وخُمسه . وإذا كُعّبا كان جرمه ثلاثاً وتسعين مرة بالتقريب مثل جرم الأرض .

/6/ ...] وثُلث = β , M] α . /8/ ...] بالتقريب = β , M] α .

[3] One should divide this amount by 6, and this sixth is made the difference between the mean of any magnitude and the mean of the magnitude that follows it. The sixth is divided by 3, and this third of the sixth is made the difference between the largest and the mean [values] for any magnitude or between the mean and the smallest [values]. Thus the largest of the fixed stars is $98\frac{1}{6}$ times the Earth, and the smallest is $10\frac{1}{3}$ times it.

[4] It has become clear from this study that the largest of these bodies is the sun; then there are the fixed stars of first magnitude, then Jupiter, then Saturn, then the remaining fixed stars, then Mars, then the Earth, then Venus, then the moon, and then Mercury, which is the smallest star.

[5] Whoever wishes to convert the distances to parasangs, miles, and other [units] may do so. We have converted two of these distances to parasangs. The first is the nearest, which is the nearest distance of the moon from the center of the Earth, *i.e.* the radius of the world of generation and corruption; it is 42,709 parasangs. As for [the distance] from the surface of the Earth to the part of the moon's orb closest to us, it is 41,436 parasangs. The second is the most distant, which is the distance of the fixed stars from the center of the Earth; it is twenty-five thousand thousands four hundred twelve thousand eight hundred ninety-nine [25,412,899] parasangs.

[6] Let us end the book here, praising God Most High and praying for His prophet, the chosen one. God is for us sufficient, how wonderful is the Entrusted One!

[٣] وينبغي أن يقسم هذا القدر على ستة ويجعل السدس التفاضل بين أوسط كل قدر وأوسط القدر الذي يليه . ويقسم السدس على ثلاثة ويجعل ثُلث السدس التفاضل بين أكبر كل قدر وبين أوسطه أو بين أوسطه وأصغره . فيكون أكبر الثوابت ثمانية وتسعين مثلاً وسدس مثل للأرض ، وأصغرها عشرة أمثالها وثُلث مثلها .

[٤] وقد بان من هذا البحث أنّ أعظم هذه الأجرام الشمس ؛ ثم كواكب القدر الأول من الثوابت ، ثم المشتري ، ثم زحل ، ثم باقي الكواكب الثابتة ، ثم المرّيخ ، ثم الأرض ، ثم الزهرة ، ثم القمر ، ثم عطارد ، وهو أصغر الكواكب .

[٥] ومَن أراد أن يحوِّل الأبعاد إلى الفراسخ والأميال وغيرهما فله ذلك . ونحن حوّلنا بُعدين منها إلى الفراسخ . الأول أقربها ، وهو بعد القمر الأقرب من مركز الأرض ، أعني نصف قطر عالم الكون والفساد ؛ فكان اثنين وأربعين ألفاً وسبعمائة وتسع[!] فراسخ . وأما من سطح الأرض إلى ما هو أقرب إلينا من فلك القمر فأحد وأربعون ألفاً وأربعمائة وستة وثلاثون فرسخاً . والثاني أبعدها ، وهو بعد الثوابت عن مركز الأرض ؛ فكان خمسة وعشرين ألف ألف وأربعمائة واثني عشر ألفاً وثمانمائة وتسعة وتسعين فرسخاً .

[٦] ولنختم الكتاب هاهنا حامدين لله تعالى ومصلّين على نبيّه المصطفى . والله حسبنا ونعم الوكيل .

Part III

Commentary Figures

343 e

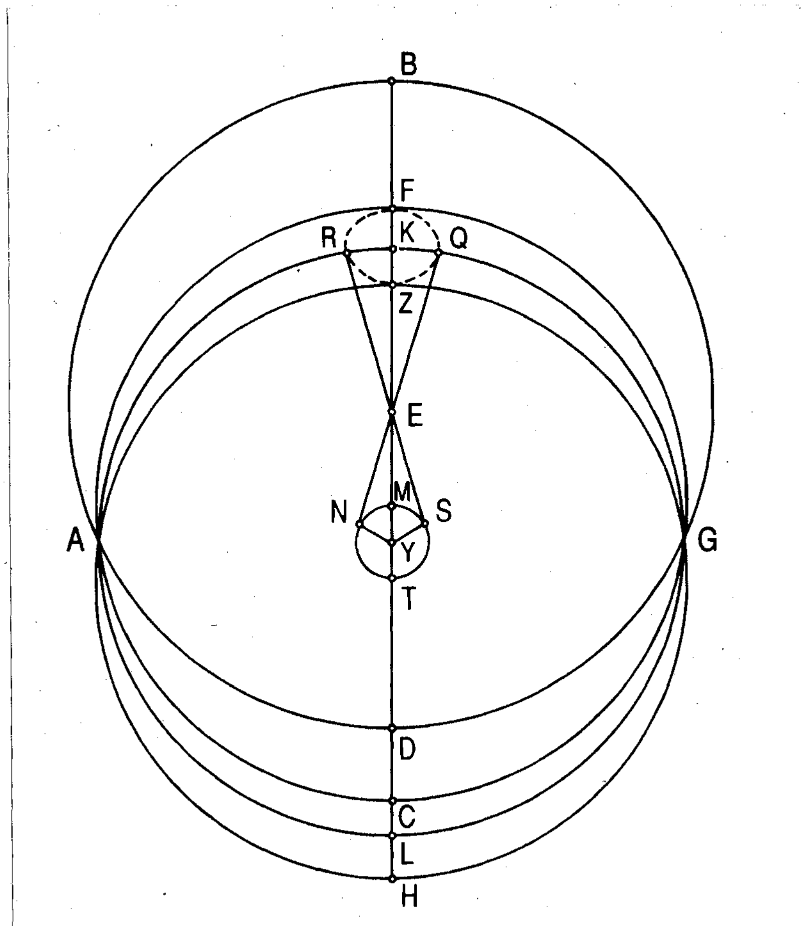


Fig. C1

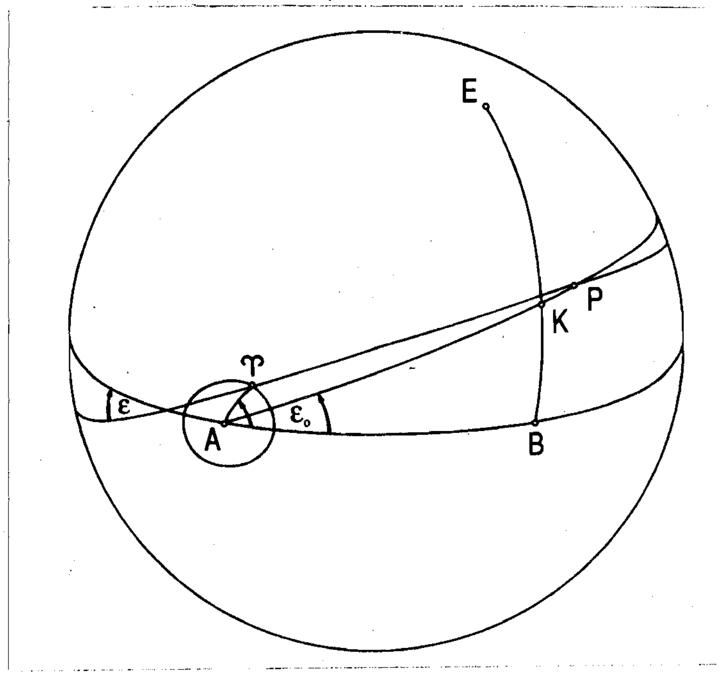


Fig. C2

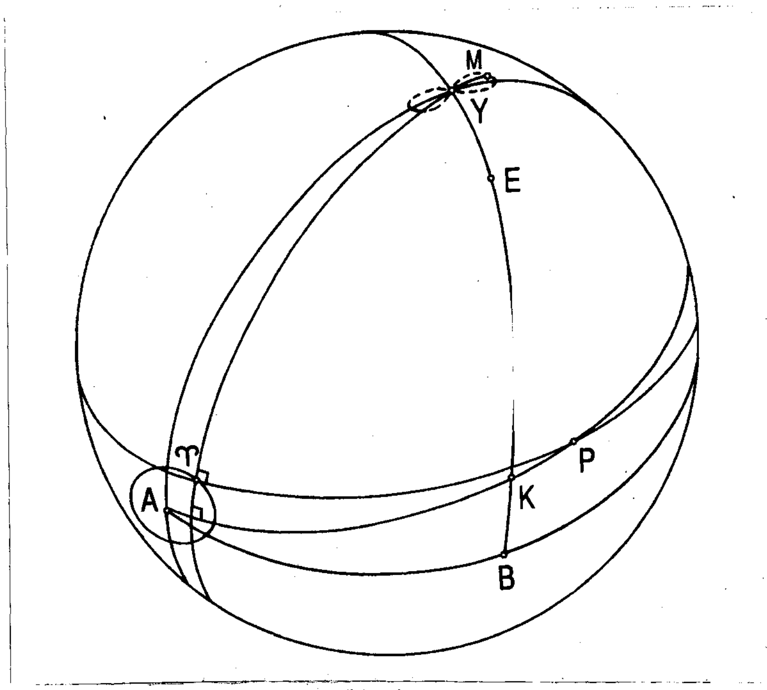
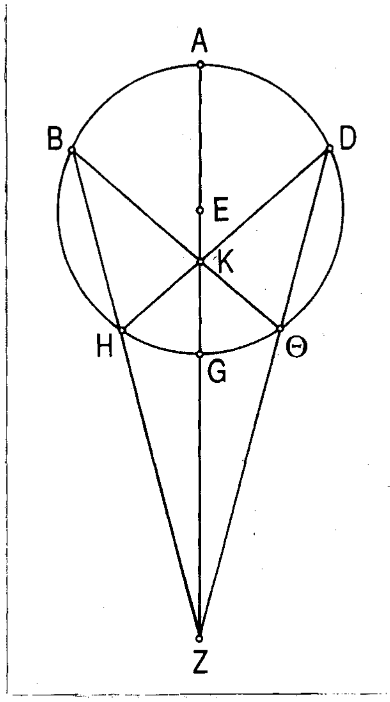
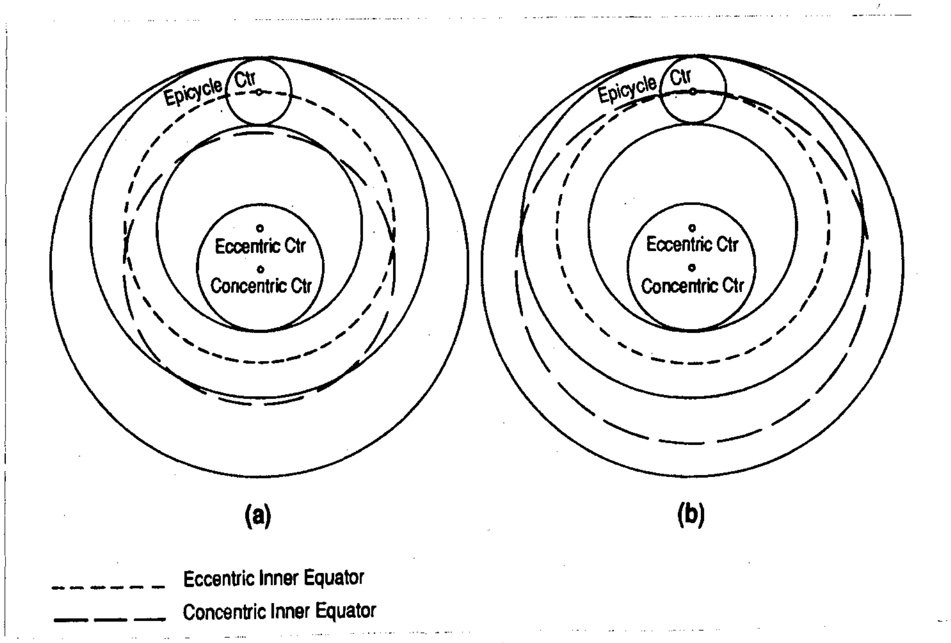


Fig. C3



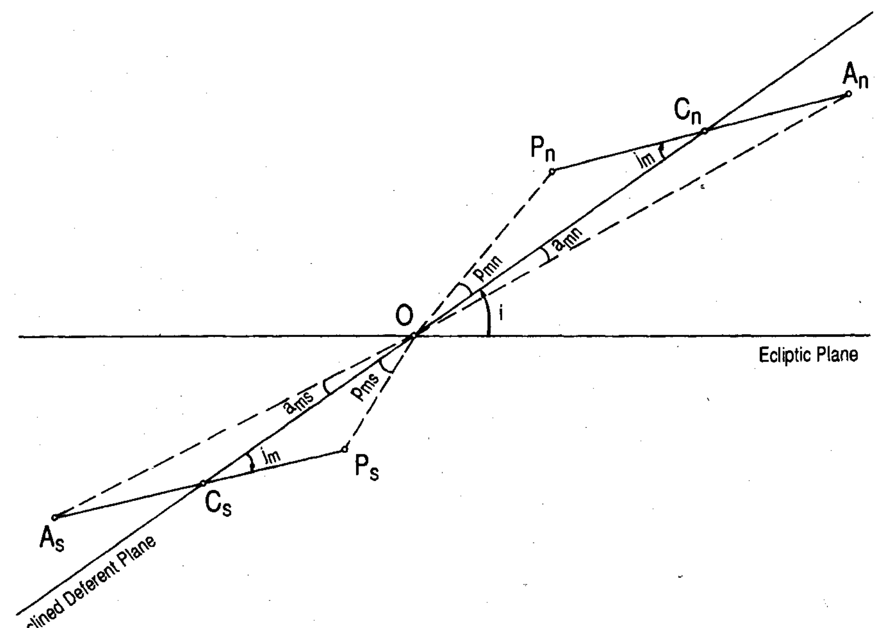
A
B
D
E
K
H
Θ
G
Z

Fig. C4

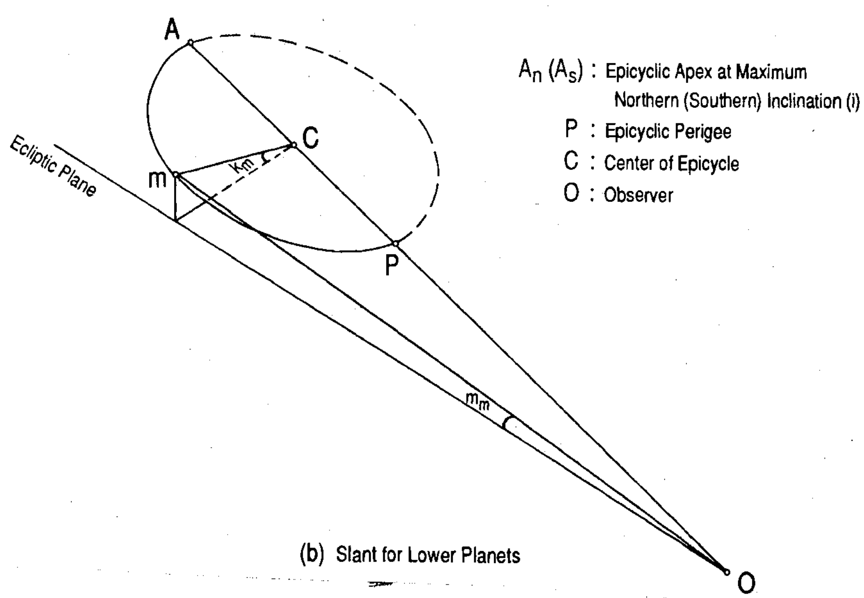


Epicycle
Ctr
Eccentric Ctr
Concentric Ctr
(a)
(b)
Eccentric Inner Equator
Concentric Inner Equator

Fig. C5



(a) Deviation for Upper Planets



(b) Slant for Lower Planets

Fig. C6

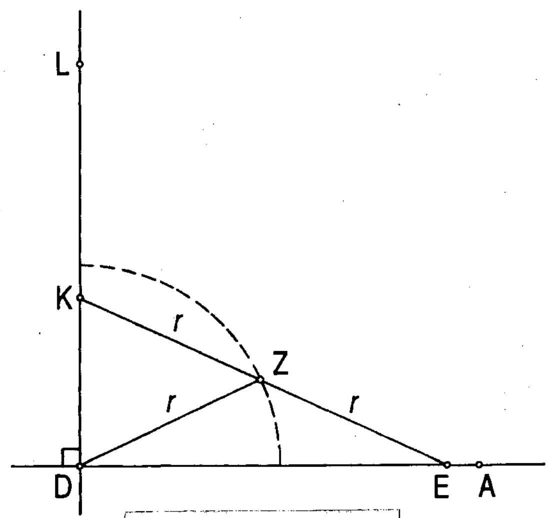


Fig. C7

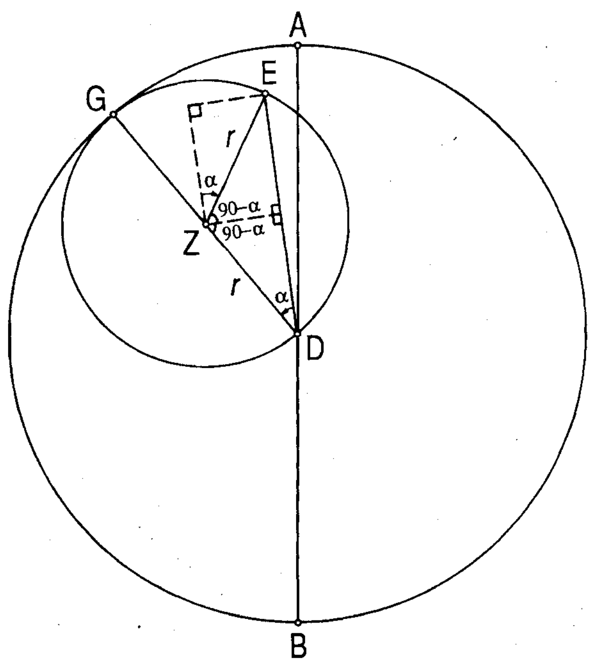


Fig. C8

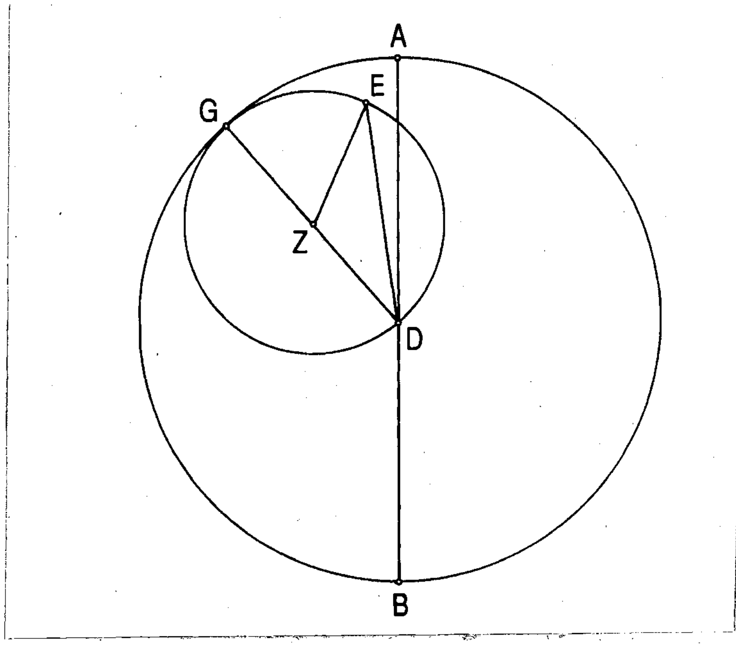


Fig. C9

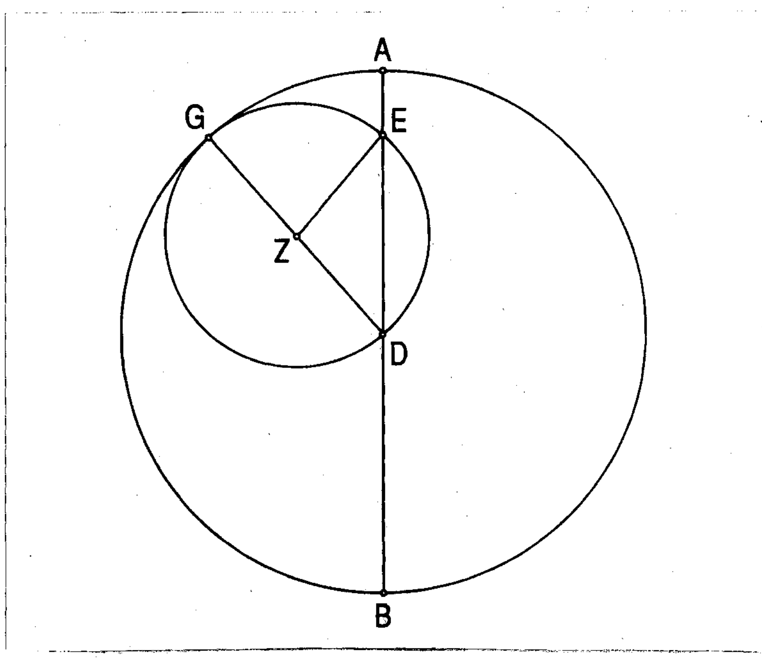


Fig. C10

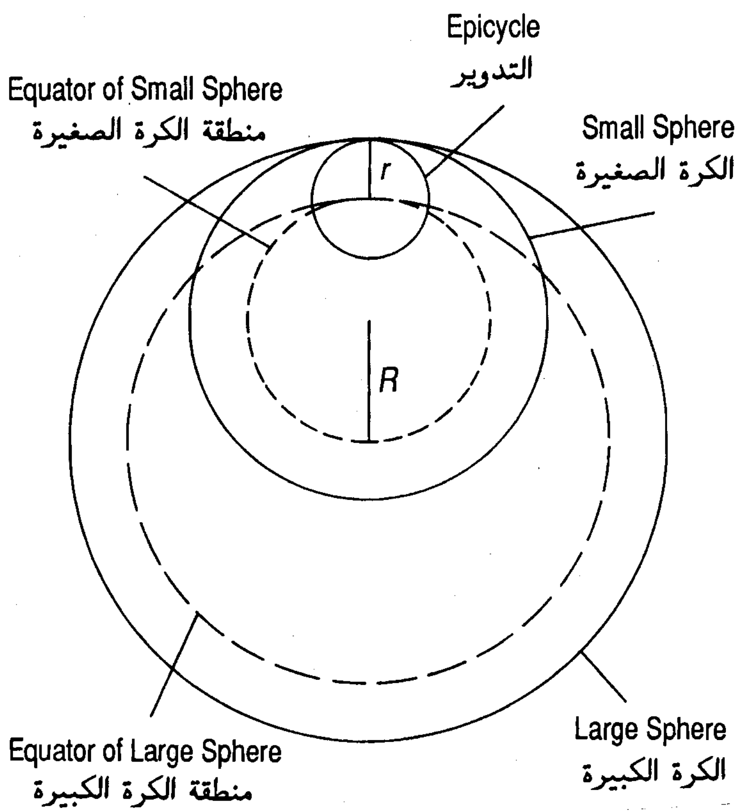

Epicycle
التدوير
Equator of Small Sphere
منطقة الكرة الصغيرة
Small Sphere
الكرة الصغيرة
r
R
Equator of Large Sphere
منطقة الكرة الكبيرة
Large Sphere
الكرة الكبيرة

Fig. C11

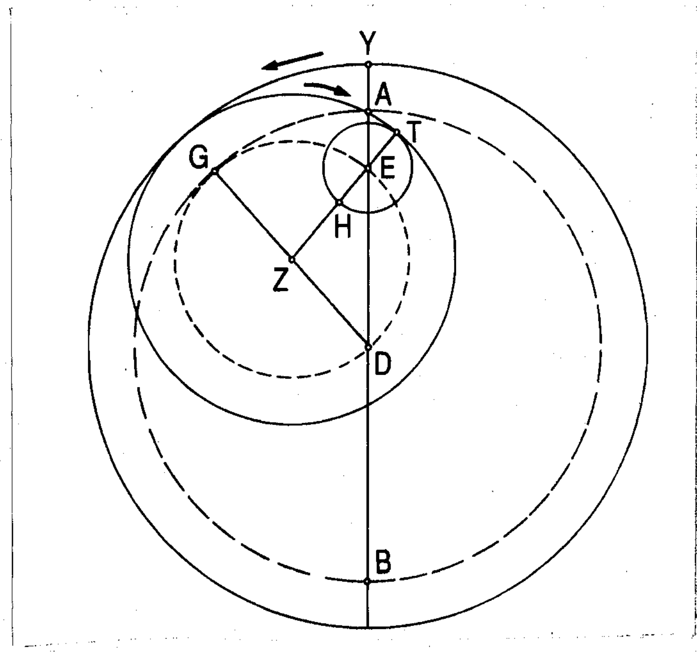


Fig. C12

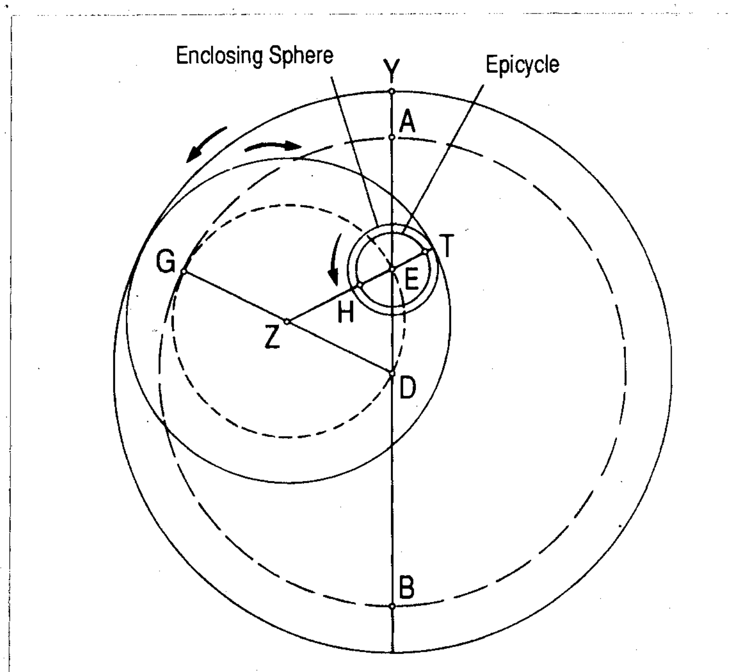


Fig. C13

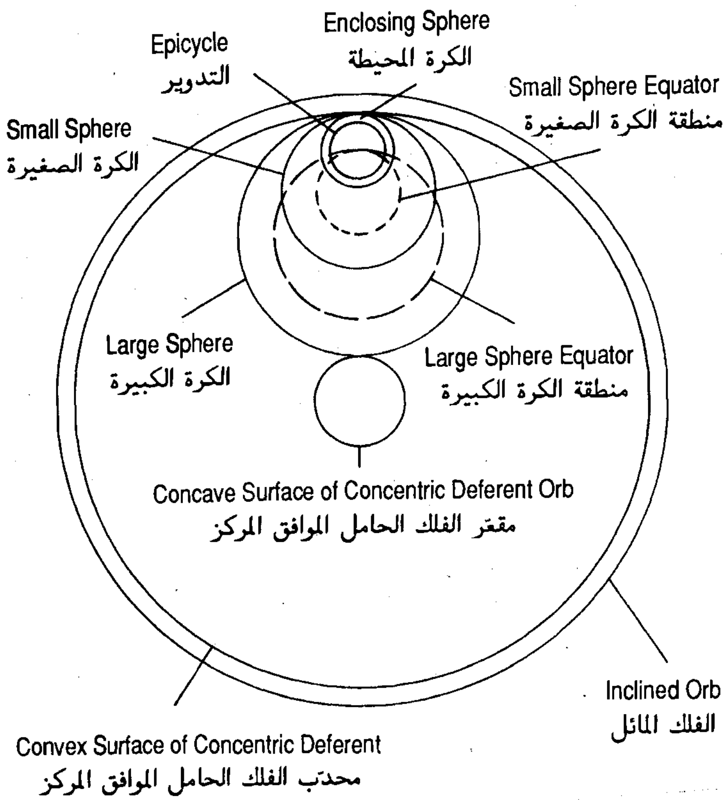

Enclosing Sphere
الكرة المحيطة
Epicycle
التدوير
Small Sphere Equator
منطقة الكرة الصغيرة
Small Sphere
الكرة الصغيرة
Large Sphere
الكرة الكبيرة
Large Sphere Equator
منطقة الكرة الكبيرة
Concave Surface of Concentric Deferent Orb
مقعّر الفلك الحامل الموافق المركز
Inclined Orb
الفلك المائل
Convex Surface of Concentric Deferent
محدّب الفلك الحامل الموافق المركز

Fig. C14

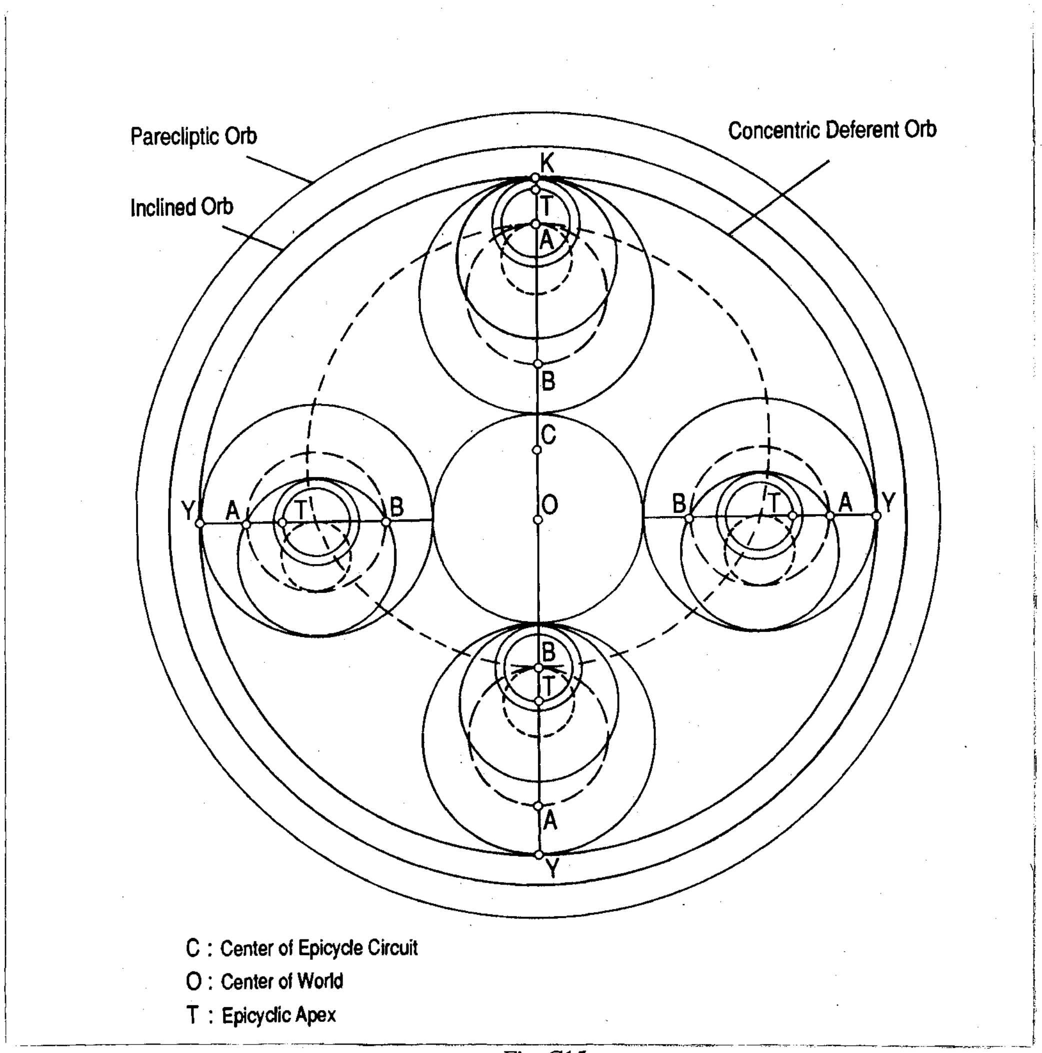

Pareccliptic Orb
Concentric Deferent Orb
Inclined Orb
K
T
A
B
C
O
Y
A
T
B
B
T
A
Y
B
T
A
Y
C : Center of Epicycle Circuit
O : Center of World
T : Epicyclic Apex

Fig. C15

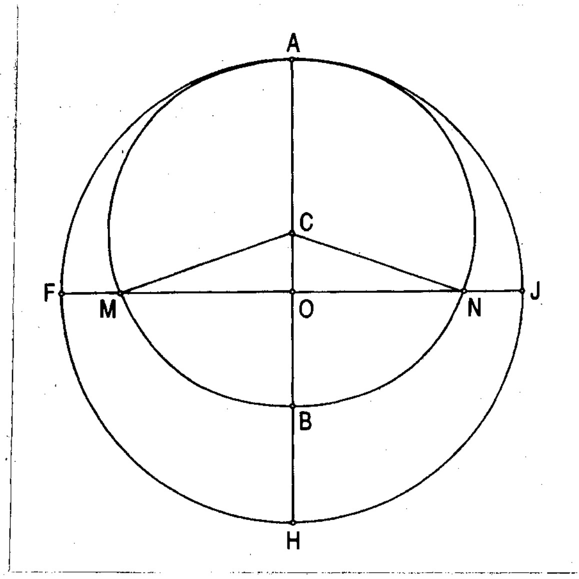


Fig. C16

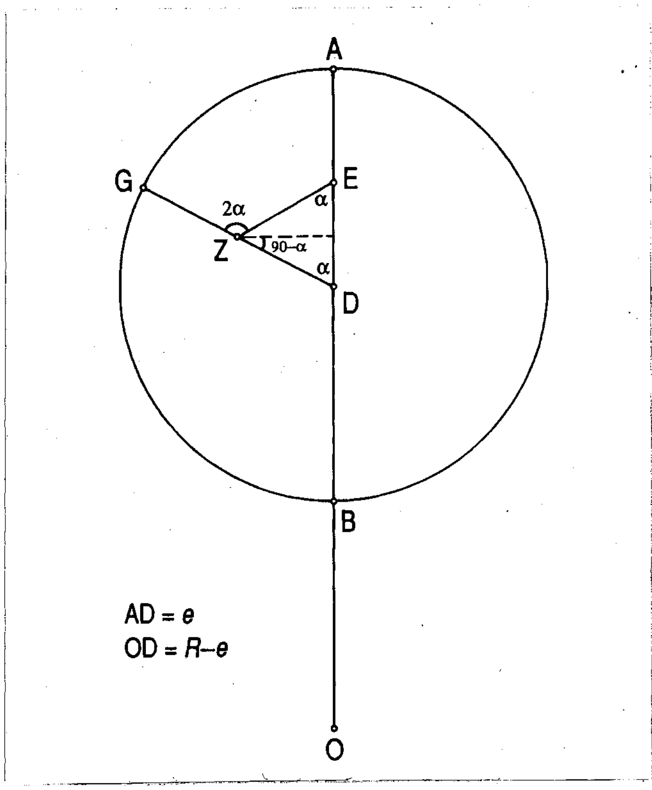


Fig. C17

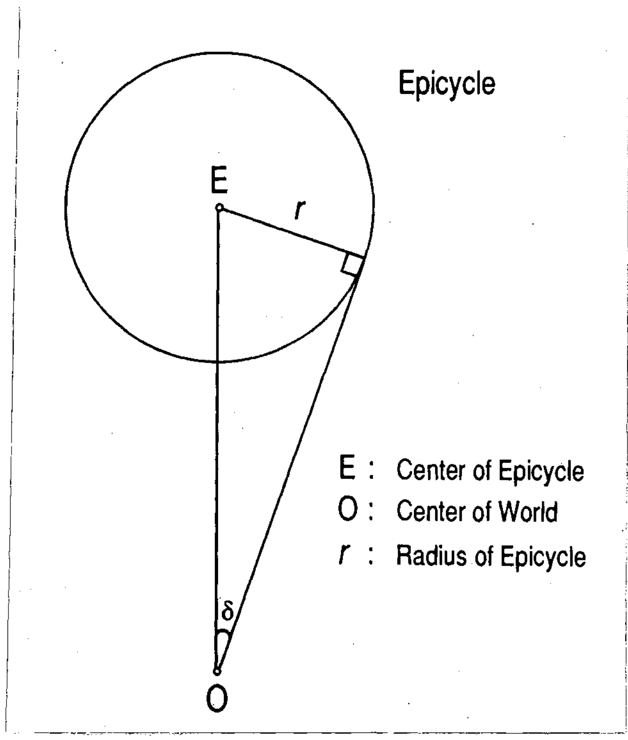

Epicycle
E
r
δ
O
E : Center of Epicycle
O : Center of World
r : Radius of Epicycle

Fig. C18

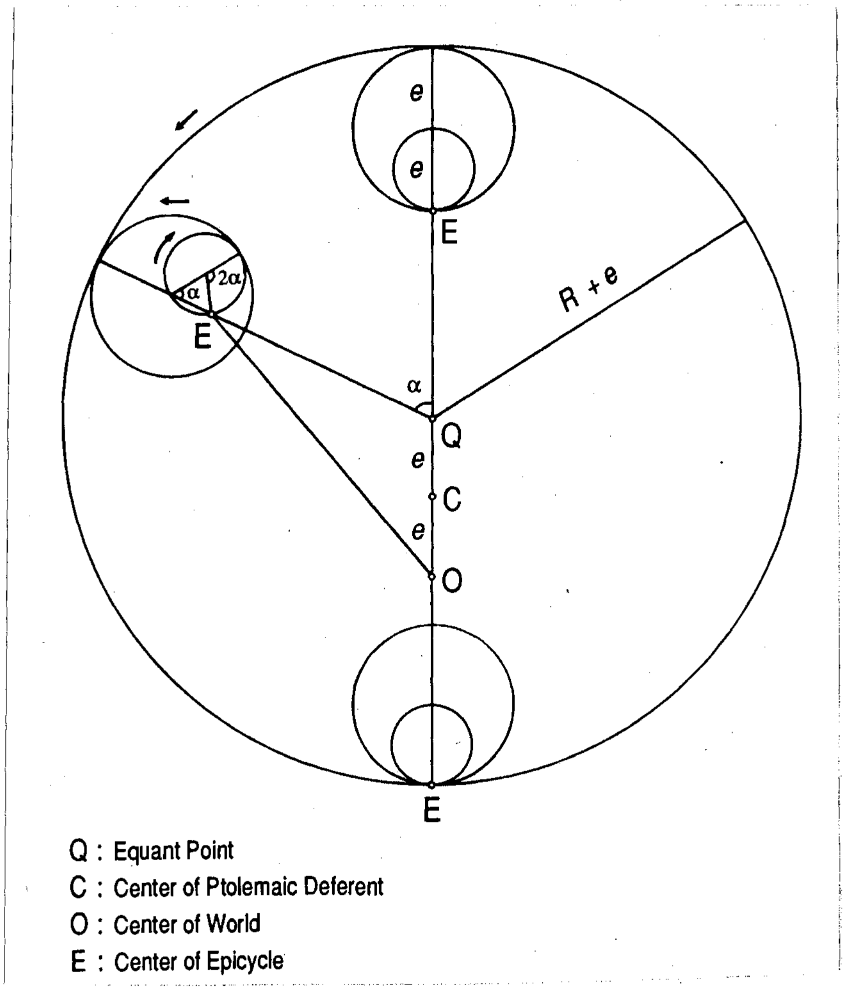

e
e
E
2α
α
E
R + e
α
Q
e
C
e
O
E
Q : Equant Point
C : Center of Ptolemaic Deferent
O : Center of World
E : Center of Epicycle

Fig. C19

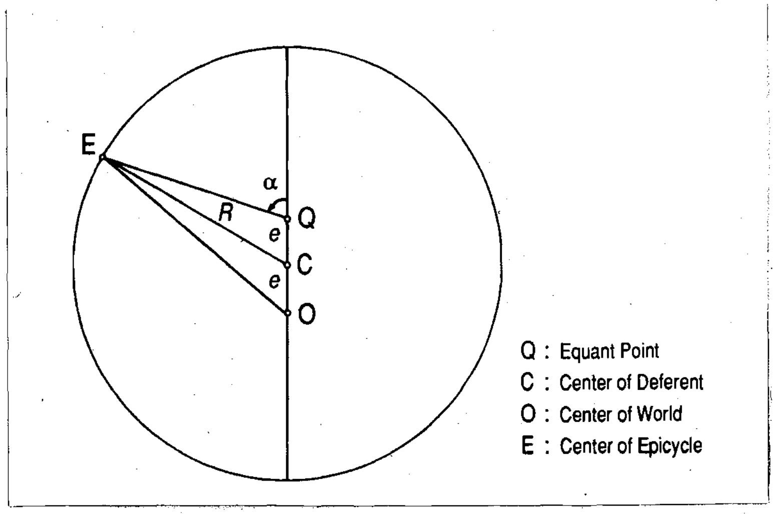


Fig. C20

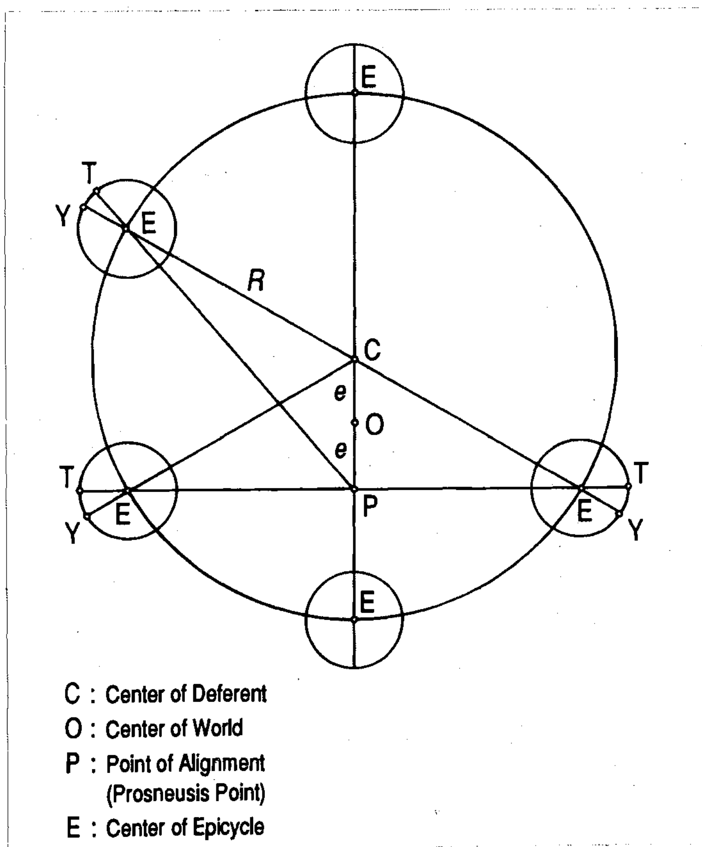


Fig. C21

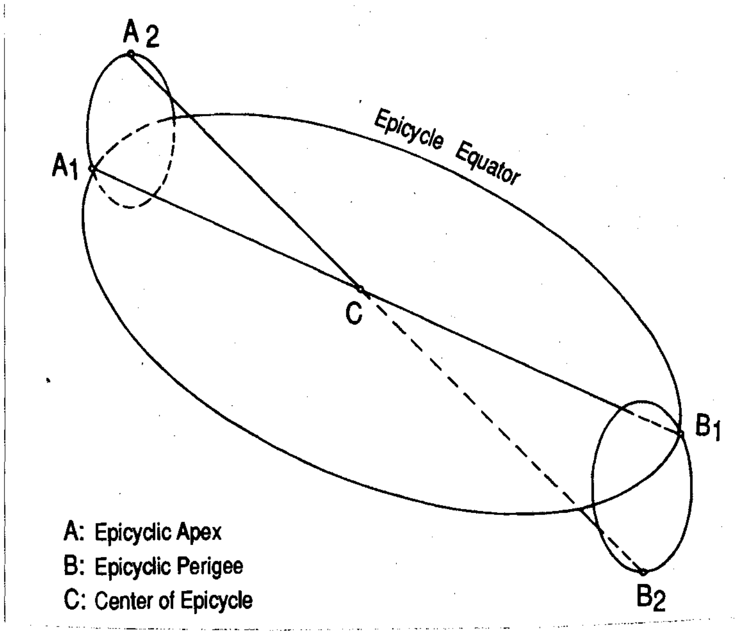


Fig. C22

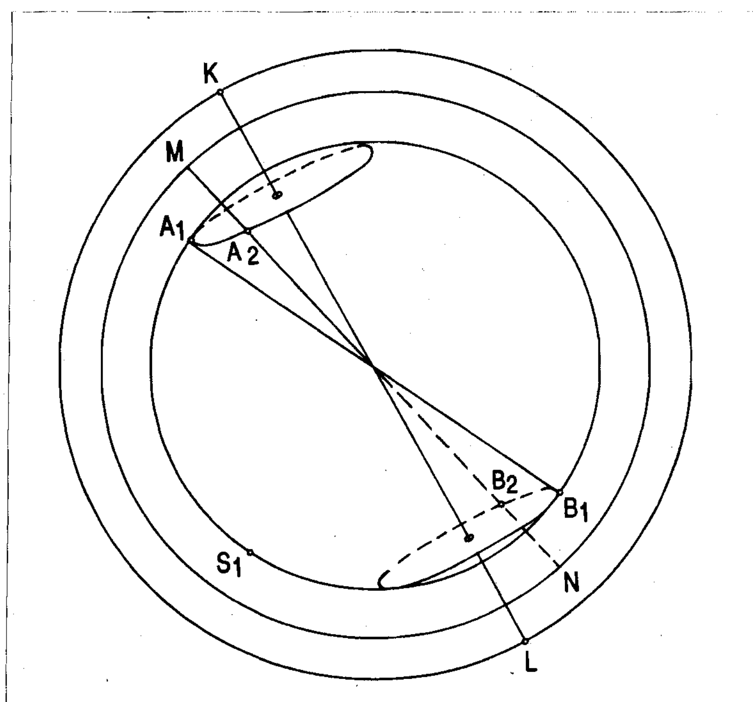


Fig. C23

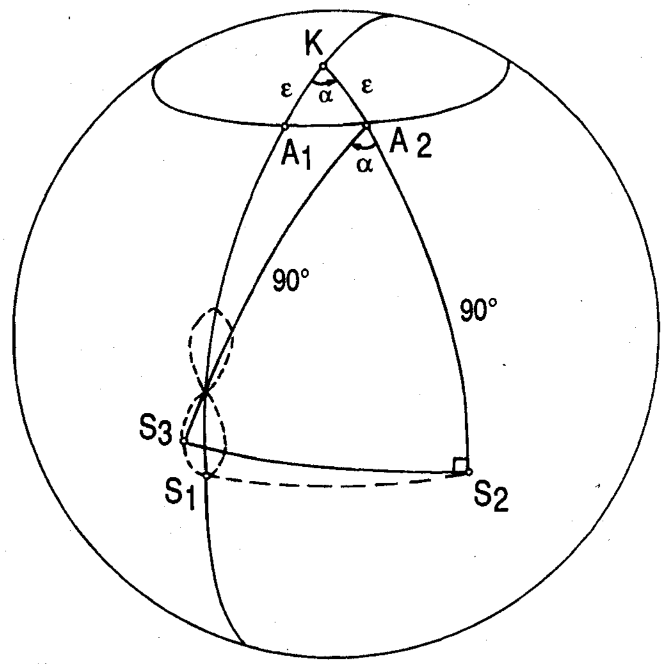


Fig. C24

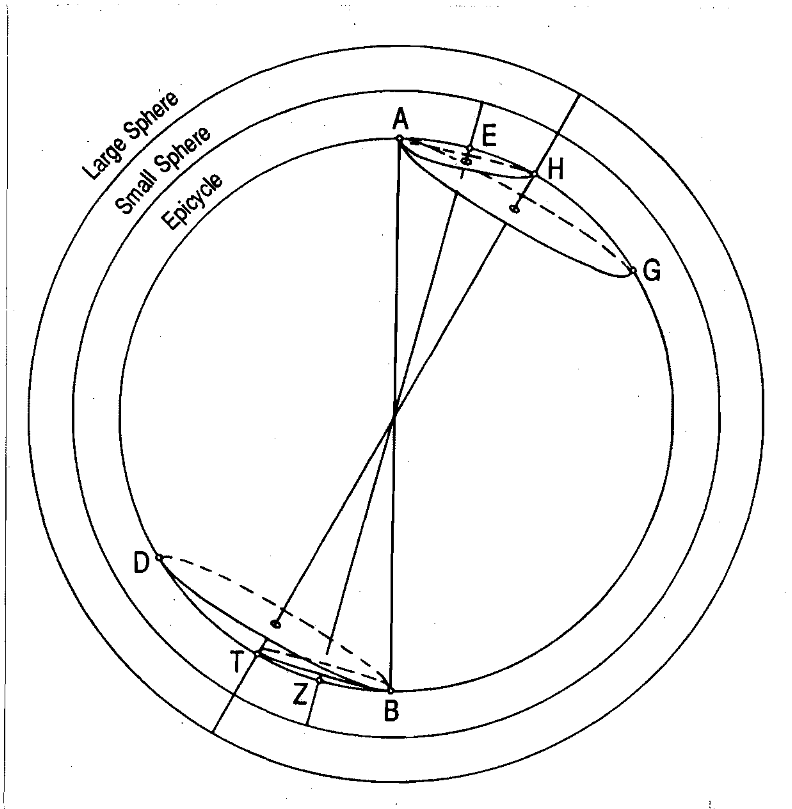


Fig. C25

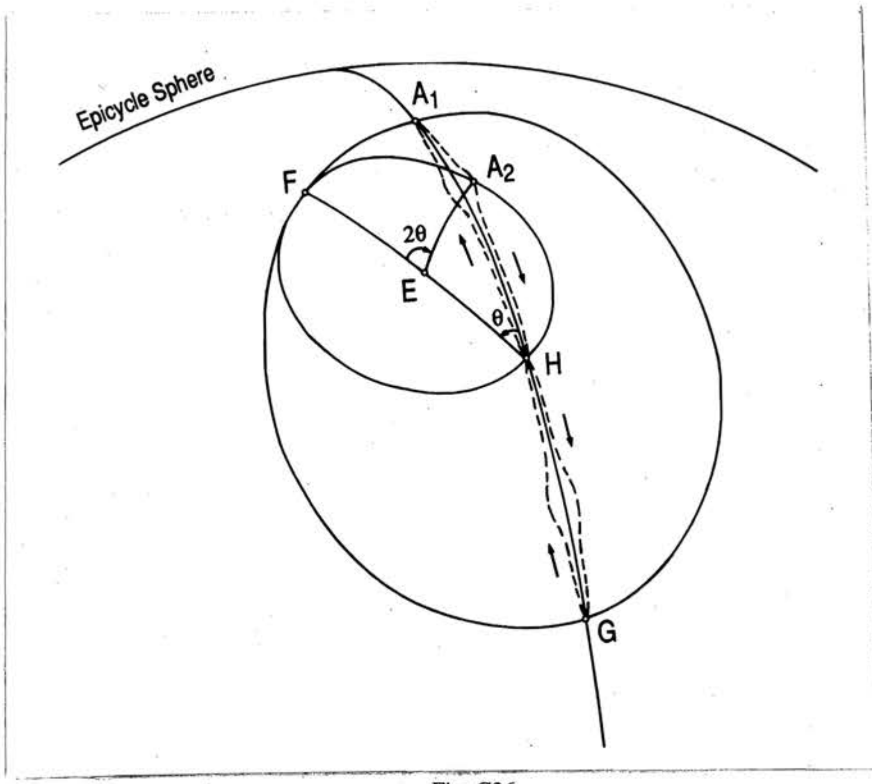


Fig. C26

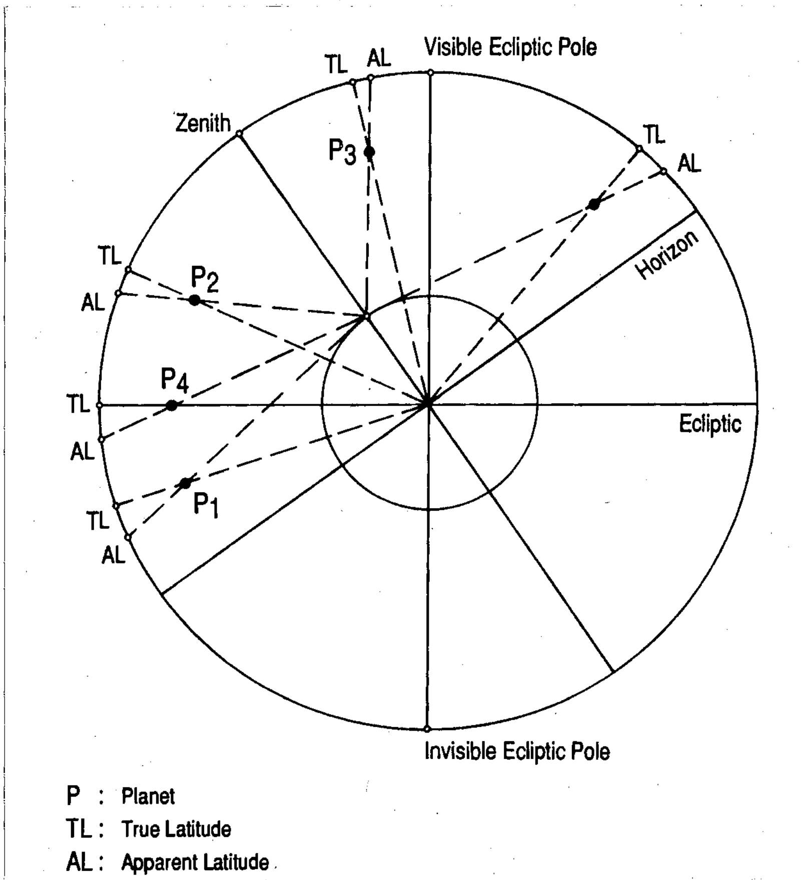

Visible Ecliptic Pole
TL
AL
Zenith
P3
TL
AL
Horizon
TL
P2
AL
P4
TL
Ecliptic
AL
P1
TL
AL
Invisible Ecliptic Pole
P : Planet
TL : True Latitude
AL : Apparent Latitude

Fig. C27

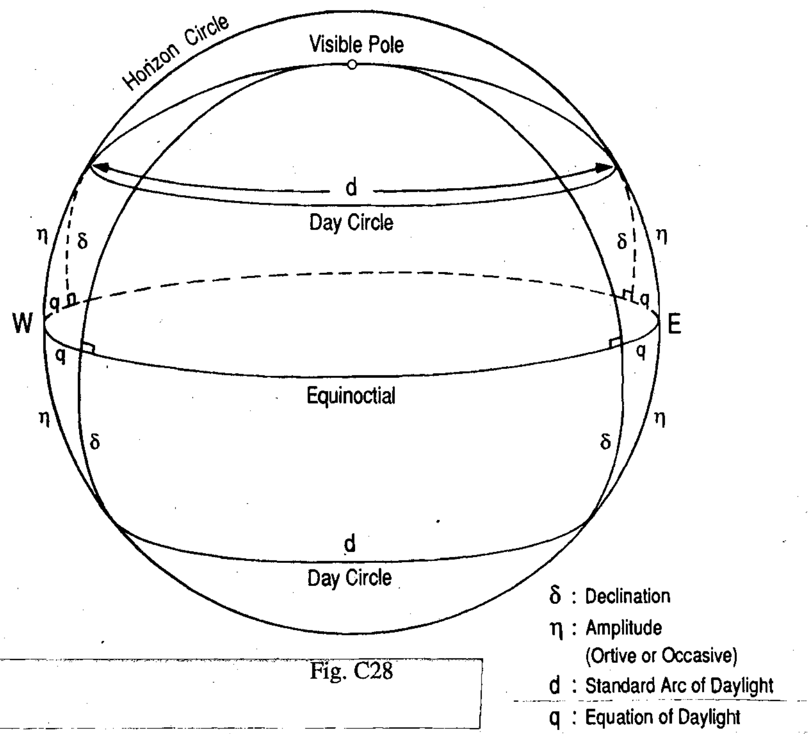


Fig. C28

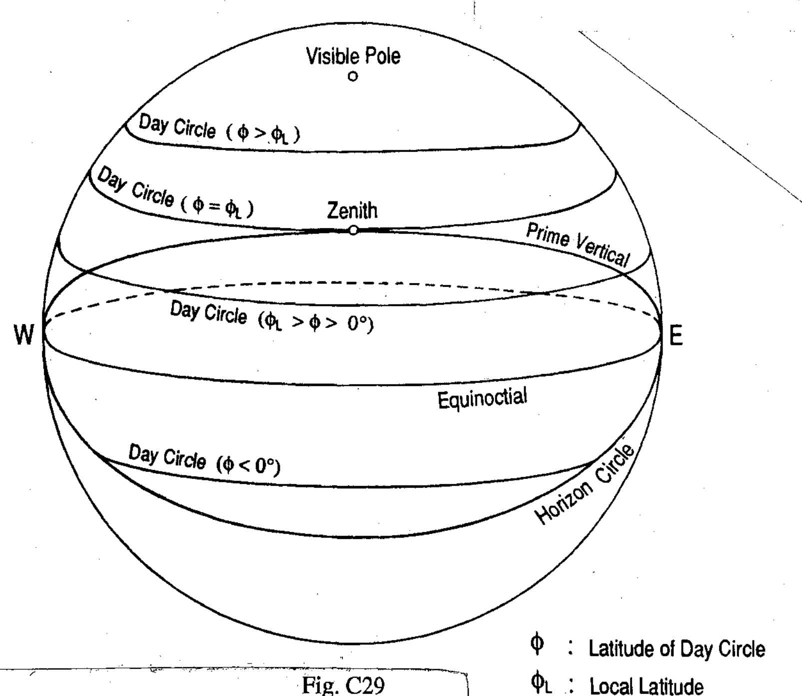


Fig. C29

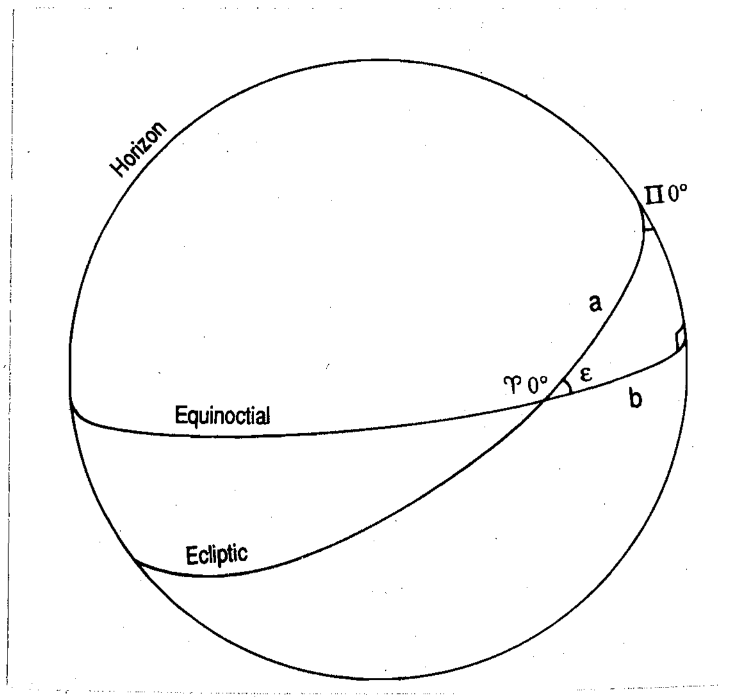


Fig. C30

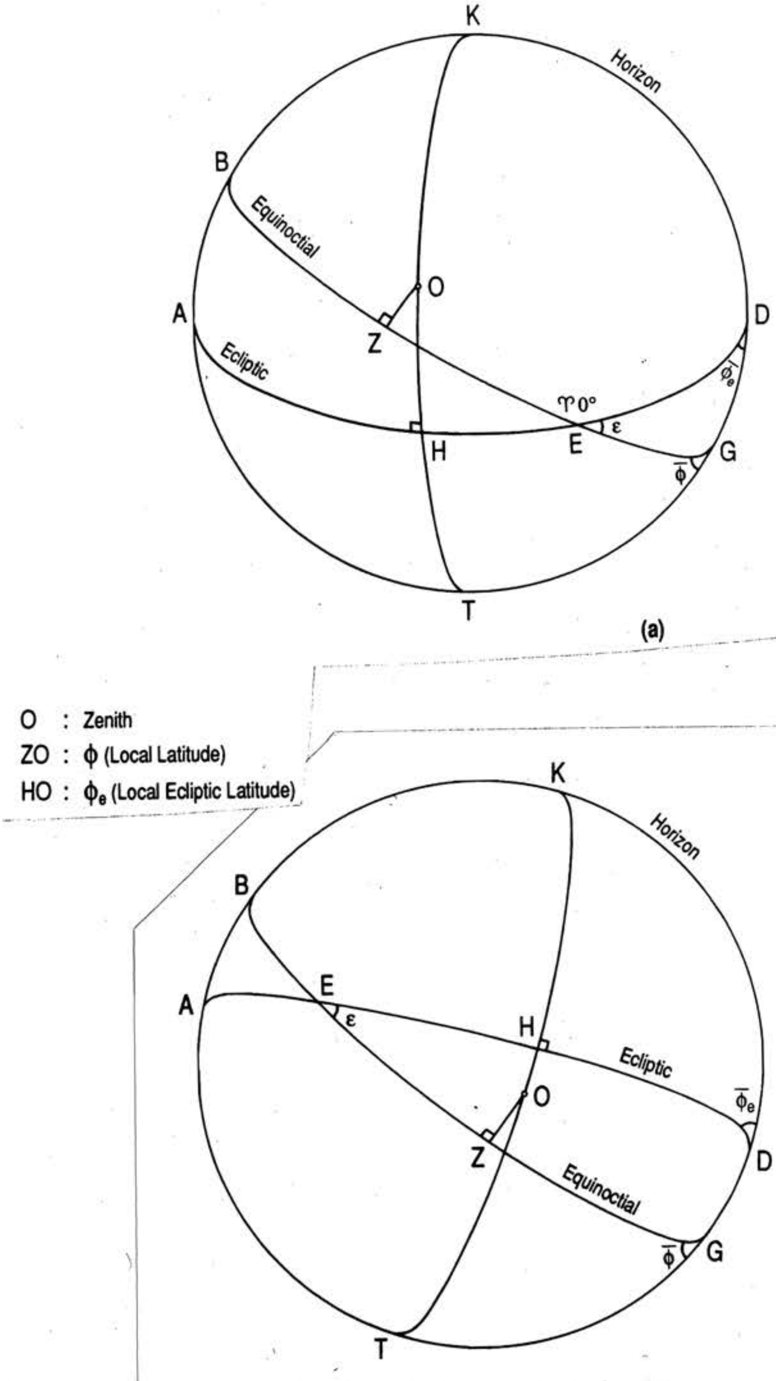
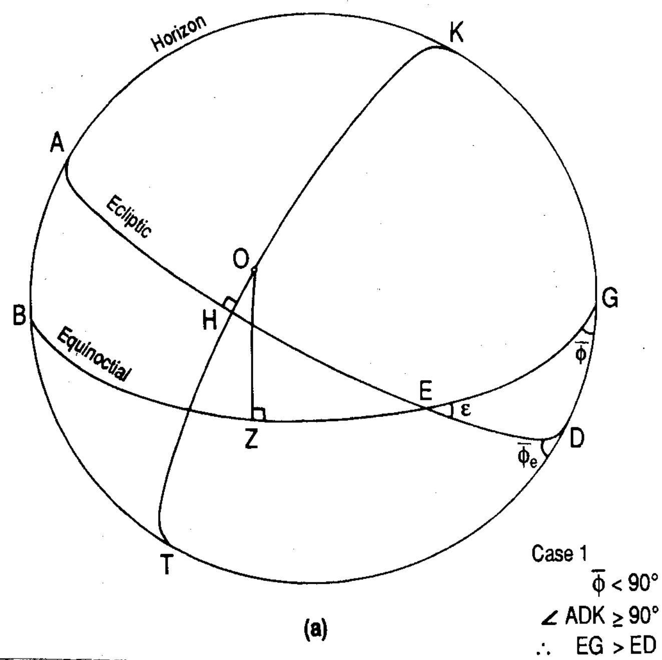
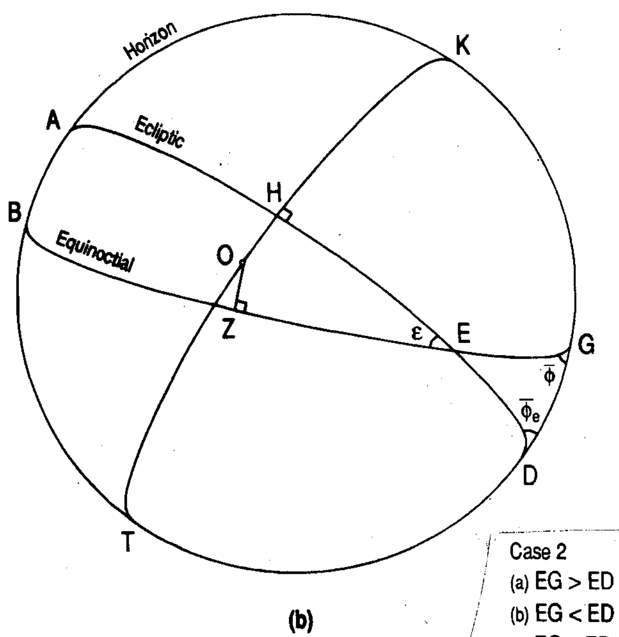

K
Horizon
B
Equinoctial
O
A
D
Z
Ecliptic
♈0°
ε
E
H
G
$\bar{\phi}_e$
$\bar{\phi}$
T
(a)
O : Zenith
ZO : ϕ (Local Latitude)
HO : ϕ_e (Local Ecliptic Latitude)
K
Horizon
B
E
ε
A
H
Ecliptic
O
$\bar{\phi}_e$
Z
D
Equinoctial
$\bar{\phi}$
G
T
(b)

Fig. C31


Horizon
K
A
Ecliptic
O
G
B
H
Equinoctial
$\bar{\phi}$
E
ε
Z
$\bar{\phi}_e$
D
T
Case 1
$\bar{\phi} < 90°$
∠ ADK ≥ 90°
∴ EG > ED

(a)


Horizon
K
A
Ecliptic
H
B
Equinoctial
O
Z
ε
E
G
$\bar{\phi}$
$\bar{\phi}_e$
D
T
Case 2
(a) EG > ED
(b) EG < ED
(c) EG = ED

(b)

Fig. C32

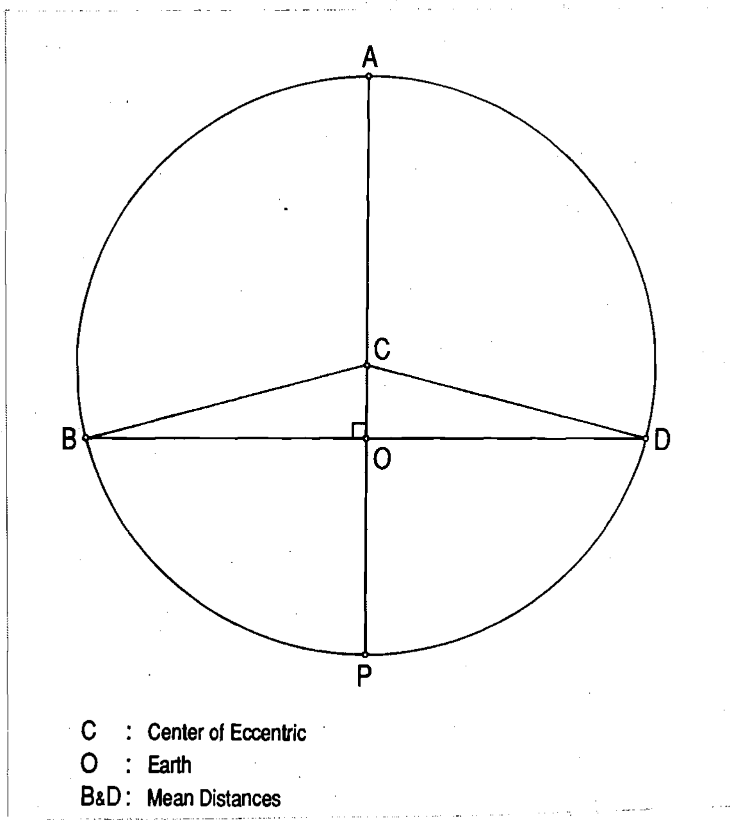


Fig. C33

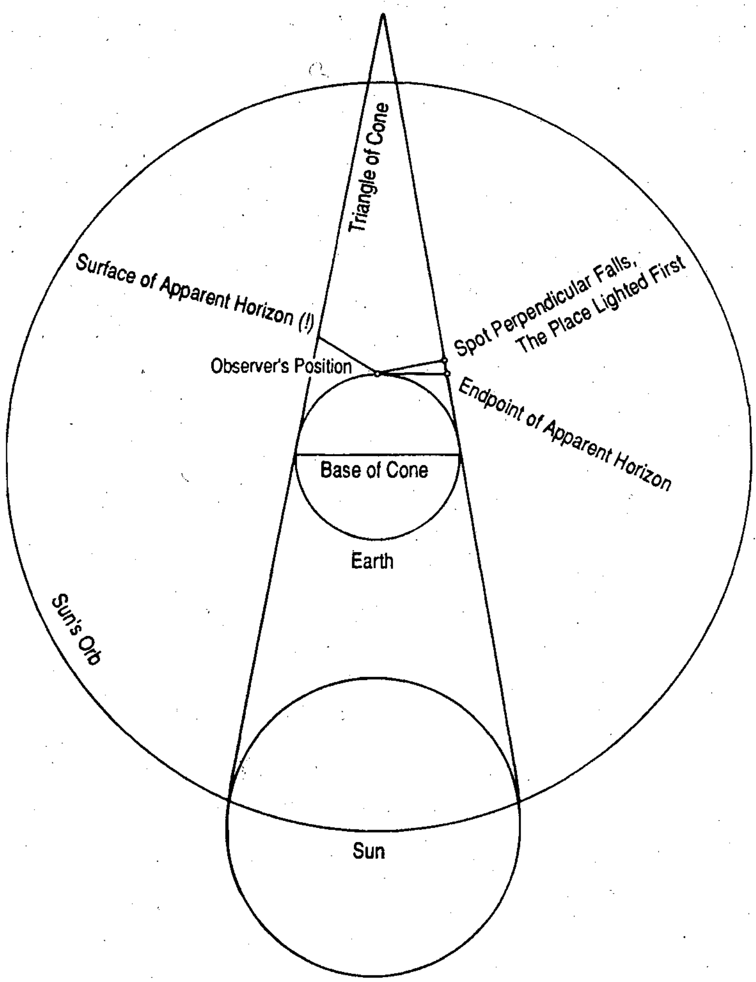

Triangle of Cone
Surface of Apparent Horizon (!)
Spot Perpendicular Falls,
The Place Lighted First
Observer's Position
Endpoint of Apparent Horizon
Base of Cone
Earth
Sun's Orb
Sun

Fig. C34a

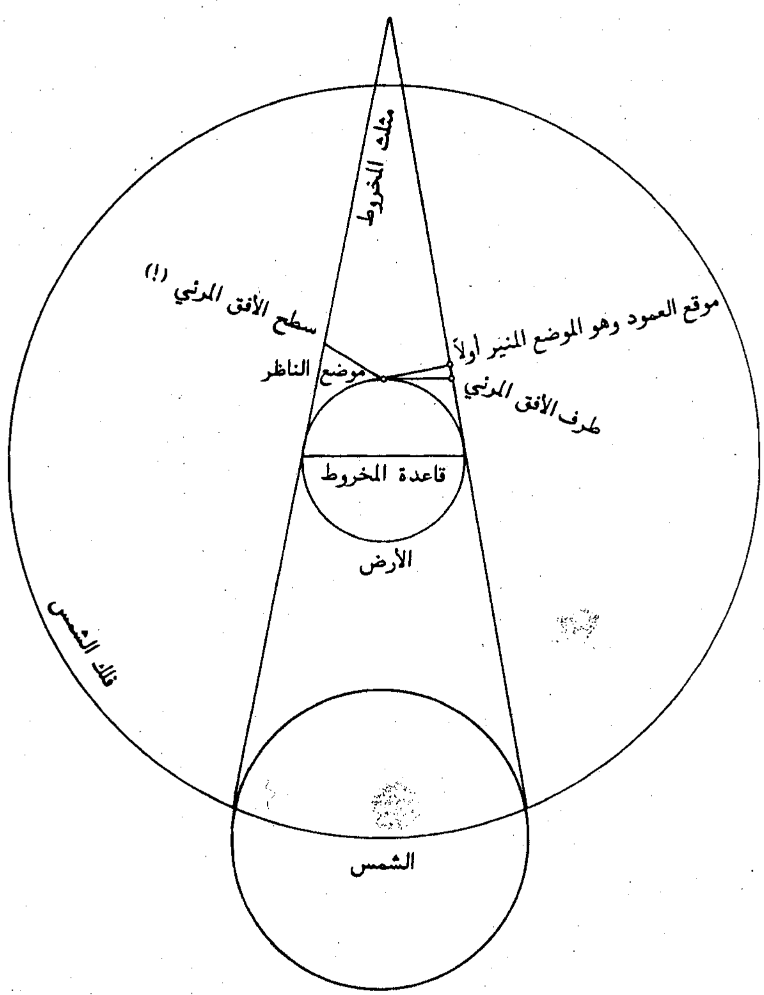

مثلث المخروط
سطح الأفق المرئي (!)
موقع العمود وهو الموضع المنير أولاً
موضع الناظر
طرف الأفق المرئي
قاعدة المخروط
الأرض
فلك الشمس
الشمس

Fig. C34b

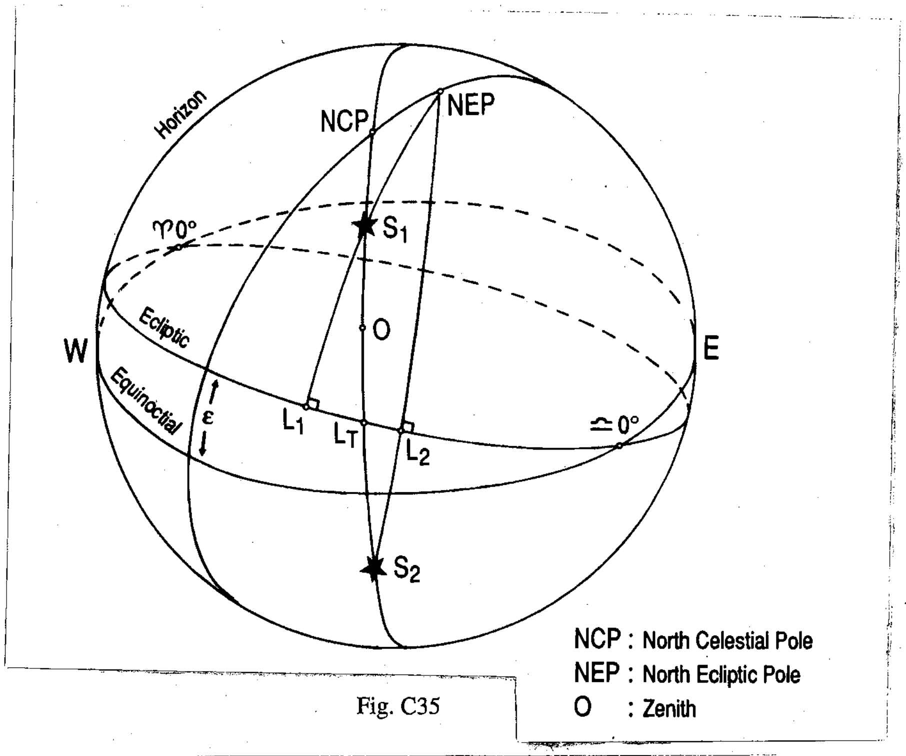


Fig. C35

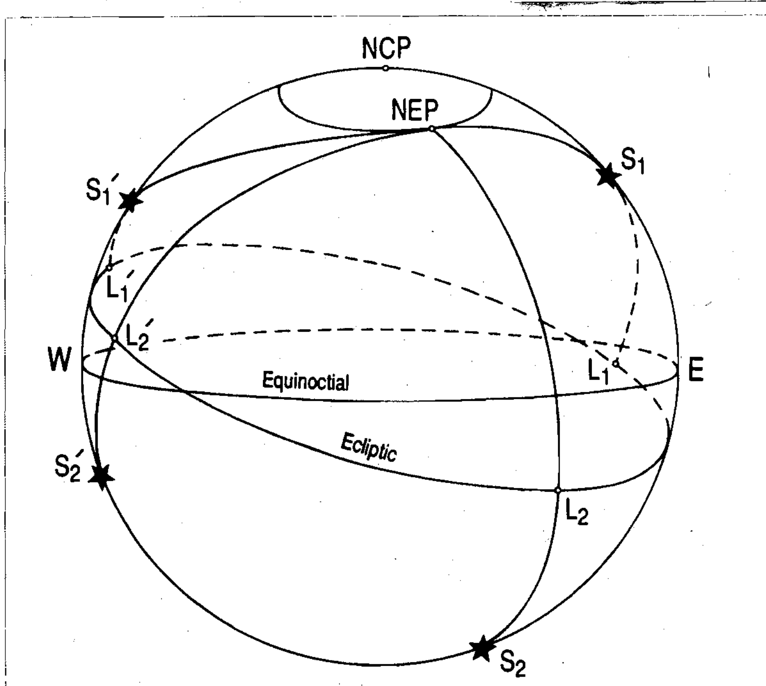


Fig. C36

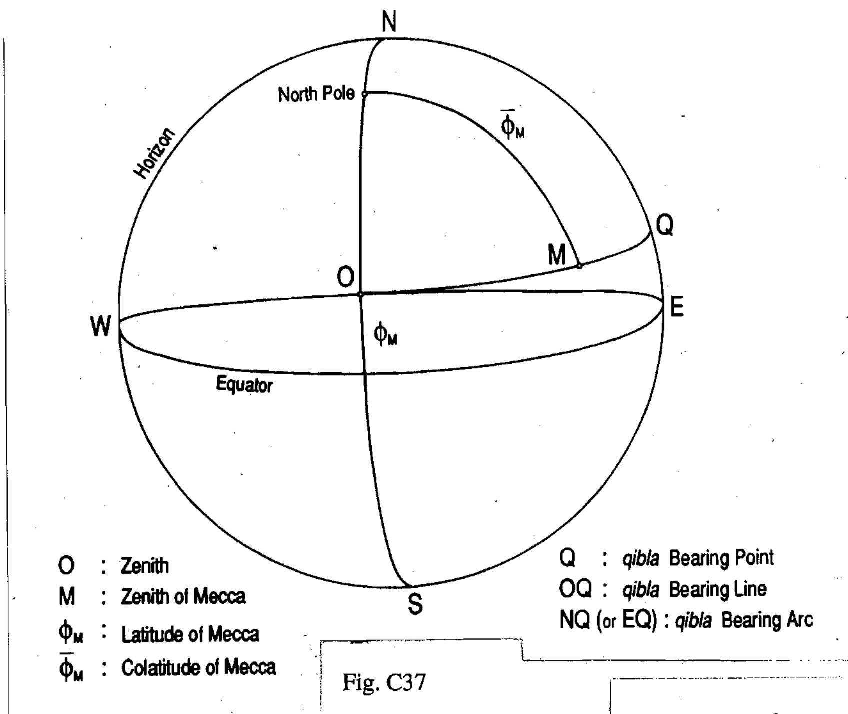


Fig. C37

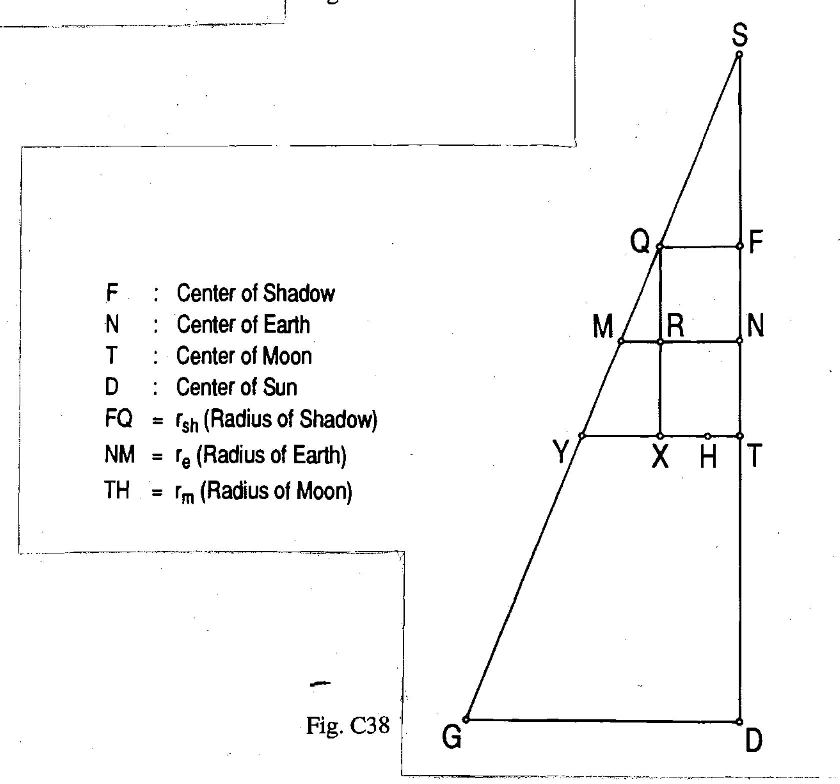


Fig. C38

Volume Two

Part IV

Commentary

Part V

Critical Apparatus

Part VI

Appendices and Indices

xv e

Part IV

Commentary

373 e

BOOK I

Preface

I.Pref. [2]5. *jumal^{an} min ^{c}ilm al-hay'a* (a summary of astronomy): As Ṭūsī repeats many times throughout the text, this work is to be considered a summary rather than a complete exposition as is the case with the *Almagest.* The development of this type of astronomical summary in the Islamic tradition is of more than passing significance as we have discussed in the introduction, §2.C–D. Compare Ptolemy's remarks at the beginning of the *Planetary Hypotheses*, where he states that his aim is to set forth the results of the *Almagest* in "a summary fashion" (κεφαλαιωδῶς) (p. 70, line 12); this was translated into Arabic as *jumal* (p. 13, line 10).

I.Pref. [2]5. *tadhkira li-ba^{c}ḍ al-aḥbāb* (a memento for one of our dear friends): There is a play here on the word *tadhkira*, which has approximately the meaning of "memoir" as used in the title but here has the import of "memento." The friend referred to is, according to Ibn al-Fuwaṭī, ^{c}Izz al-Dīn al-Zinjānī, whose *tadhkira* on grammar was written for Naṣīr al-Dīn. (See p. 71 of Volume One.)

Book I, Introduction

I.Intr. [2]13–14. *wa-mawḍū^{c} al-hay'a...wa-ḥarakātihā al-lāzima lahā* (The subject of astronomy...and intrinsic motions): In his general remarks on astronomy, Tahānawī, the 18th-century Indian compiler of a dictionary of technical terms, notes that the restriction to the quantities, qualities, positions, and intrinsic motions of the simple bodies is made so as to distinguish astronomy (*hay'a*) from the tradition of Aristotle's *De caelo* (*al-samā' wa-'l-^{c}ālam*), which also studies the simple bodies but from the point of view of their natures (*ṭabā'i^{c}*).[1] The latter science, for example, does not study the simple bodies in terms of the actual quantity of their motions as does *hay'a*. (See the introduction, pp. 35, 38–41 and the commentary to II.1 [8].)

[1] Tahānawī, *Dictionary*, 1: 47. For the most part he depends on late astronomers for his information and interpretations, especially commentators on the *Tadhkira*.

I.Intr. [2]16. *bi-a^c^yānihā* (in and of themselves): Khafrī explains this phrase as referring to the question of the number of simple bodies, the size of each individual body, and similar questions relating to the bodies *qua* bodies (*bi-dhawātihā*).

I.Intr. [3]2,4. *^c^alā sabīl al-ḥikāya* (in narrative form); *ḥikāya mā ^c^ammā thabata fīhi* (a report of what is established therein): Ṭūsī wishes to indicate by the use of *ḥikāya* that his work is for the most part an account of the astronomical system of the *Almagest*. As such, he cautions against taking it as a substitute for the *Almagest* or a similar exposition in which a full development with proofs is given. This is a fundamental aspect of the type of astronomical literature known as *al-hay'a al-basīṭa* (simplified *hay'a*). (See the introduction, pp. 24–25, 35–38.)

Book I, Chapter One

I.1 [1]. Compare Euclid, *Elements*, Bk. I, Defs. 1–3, 5–6, 13 and Bk. XI, Defs. 1–2.

I.1 [2]18–19. *wa-'l-mustaqīm...tufraḍ ^c^alayhi* (A straight line...one another): This definition is one of the many that were advanced as alternatives to that of Euclid. It somewhat resembles one found in Hero of Alexandria, Def. 4 (4: 18), which was known to Nayrīzī (fl. ca. 897 in Baghdad). (Compare Heath, *Euclid's Elements*, 1: 168, no. 3 and Proclus, *Commentary*, p. 89.)

I.1 [2]19–20. *wa-'l-mustawī...al-jihāt mumkin^an^* (A plane surface...is possible): This definition is significant not only because it is non-Euclidian (cf. Euclid, *Elements*, Bk. I, Def. 7), but also because it is the first of several passages that allows us to establish that the *Tadhkira* underwent multiple revisions. (See the introduction, §2.J, pp. 71–75.) There are three versions of the definition in the textual tradition (see the apparatus), and Jurjānī gives a lucid account of the relation between them:

> *The plane surface is the one whereby the lines assumed on it in all directions are straight.* This has been nullified because the plane in which arcs were assumed on it in all directions would be excluded from [this definition]. Therefore this phrase has been changed in some of the copies that were read under [the supervision] of the author to his statement: *it is the one in which it is possible to produce straight lines in all directions.* In some of the copies it is thus: *it is the one on which the assumption of straight lines in all directions is possible.* Each of the two modifications has the same meaning.

It is interesting to note that Ṭūsī was still modifying his definition of a plane some 10 to 15 years after writing his recension of the *Elements* (in 1248 A.D.). The older and incorrect definition is found in MSS DGT, and is the only one mentioned by Nīsābūrī; this represents the "Marāgha" version. The first modification is found in M, which was prepared by Shīrāzī under Ṭūsī's supervision. The second modification is found in MSS FL and in the margin of T; this represents the final, or "Baghdad," version of the text. As is often the case, Khafrī copies Jurjānī's remarks without attribution; Bīrjandī is aware of them but in contrast does credit his sources. The implications of this passage for establishing the text of the *Tadhkira* are discussed more fully in the introduction (§2.J, pp. 71–75 and §2.M, pp. 85–88). It may be significant that Ṭūsī's definitions do not resemble any of those in Proclus, but there may be a connection to the one mentioned by Nayrīzī in his remark that "others defined the plane surface as that in which it is possible to draw a straight line from any point to any other" (cf. Heath, *Elements*, 1: 171–172).

I.1 [3]. Compare Euclid, *Elements*, Bk. I, Def. 8 and Bk. XI, Def. 11; note that Euclid speaks of an angle as an "inclination" (κλίσις), whereas Ṭūsī defines it as a "surface" or a "solid"; Heath discusses other alternatives to Euclid's definition that were proposed (1: 176–181 and 3: 267–268).

I.1 [5]. Compare Euclid, *Elements*, Bk. I, Defs. 10–12.

I.1 [6]. Compare Euclid, *Elements*, Bk. XI, Defs. 3–4.

I.1 [7]. Compare Euclid, *Elements*, Bk. I, Def. 23 and Bk. XI, Def. 8.

I.1 [8]. Compare Euclid, *Elements*, Bk. I, Defs. 15–17.

I.1 [9]. The sine function was imported from India at an early stage in the history of Islamic astronomy; it effectively displaced the rather cumbersome chord function that had been used by Ptolemy and other Hellenistic astronomers.

I.1 [9]2. *wa-niṣf al-watar li-niṣf al-qaws jayb* (Half a chord is the sine of half the arc): Jurjānī, who not surprisingly found the Arabic inelegant, supplies the following, rather clearer, formulation: "The sine of every arc is half the chord of twice that arc."

I.1 [10]. Ṭūsī's formulation is essentially that occurring in Theodosius's *Sphaerics*, Bk. I, Defs. 1–3 (Ṭūsī's recension, p. 2) as well as in Hero, Defs. 76–77 (4: 52); it differs markedly from that of Euclid, Bk. XI, Def. 14, who gives the means for generation rather than a definition. See Heath, *Elements*, 3: 269–270.

I.1 [11]. Compare Theodosius, *Sphaerics*, Bk. I, Props. 1, 6 (Ṭūsī's recension, pp. 3–5).

I.1 [12–13]. Compare Autolycus, *On a Moving Sphere*, Prop. 1 (Ṭūsī's recension, pp. 2–3).

I.1 [13]. Compare Theodosius, *Sphaerics*, Bk. I, Props. 1, 6 (Ṭūsī's recension, pp. 3–5) and Bk. II, Props. 1–2 (Ṭūsī's recension, p. 13).

I.1 [14]. Compare Theodosius, *Sphaerics*, Bk. I, Props. 11–12 (Ṭūsī's recension, Props. 12–13(?), p. 8).

I.1 [15]. *falak* (orb): Hartner has detailed the history and etymology of *falak* in *EI*², 2: 761–763; however, there are several points that should be emphasized in order to avoid potential misunderstandings. The definition of *falak* contained in this paragraph is completely explicit and unequivocal. It is a spherical body but not necessarily a sphere except in the limiting case where the concave surface degenerates to a point. It is defined strictly geometrically, there being no reference to motion at all. We should add here that this meaning of *falak* is the one usually encountered in *hay'a* texts, especially in the later centuries (*i.e.* post-11th century A.D.).

A problem arises because *falak* has various other significations (presumably inherited from the early period of translations from Greek sources) that were not easy to eradicate by later writers on account of their widespread usage. Thus despite the unambiguous meaning of *falak* in this paragraph, Ṭūsī must admit in II.3 [2] that the celestial equator may be called either the equinoctial circle (*dā'ira*) or the equinoctial orb (*falak*). The practice of designating this equator by the word orb is, he tells us, "permissible" (*tajawwuzᵃⁿ*), but it is clear that he wishes to imply that such a usage is secondary and not entirely satisfactory.

On the other hand, Bīrūnī does not seem as concerned as Ṭūsī to limit the meaning of *falak*:

> Circle (*dā'ira*) and orb (*falak*) are two names that may be used in turn for the same thing and are thus interchangeable. "Orb" may sometimes refer to the entire sphere (*kura*), especially when it is moving since "orb" is not applicable to something at rest. The name orb (*falak*) is used simply by way of comparison to the whorl (*falaka*) of a rotating spindle.[2]

Ibn al-Haytham, a contemporary of Bīrūnī, also retains this dual usage for *falak*.[3] I believe that it is safe to conclude that the primary meaning of *falak* became delimited between the time of Bīrūnī and Ṭūsī.

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[2] Bīrūnī, *Qānūn*, 1: 54–55; cf. idem, *Tafhīm*, p. 43.

[3] See Langermann [1982], pp. 112–113. One should note that Langermann's very careful remarks deal with terminological issues. It is inappropriate—indeed dangerous—to infer from conventional usages of *falak* anything about the cosmological status of the solid orbs. This unhappy line of reasoning occurs in Rosen [1983], p. 168.

We should make one final point. That "orb" should not be used for a non-rotating spherical body would seem confirmed by Ṭūsī himself despite his lack of any explicit mention of such a restriction in this paragraph. Thus in II.2 [5], Ṭūsī states that "the orbs terminate with the orb of the moon." Below the moon, one finds the elements, which are divided not into orbs, even though they occur as bodies with the shapes of orbs, but into levels (*ṭabaqāt*). Presumably this is because the elements do not move with a circular motion. Our interpretation is made explicit by Nīsābūrī in his commentary on II.3 [2] and is echoed by the other commentators:

> ...This is called the equinoctial circle and it is also called its orb (*falak*) because they have applied the name "orb" to some of the [great] circles, namely those that occur in terms of (*bi-'ᶜtibār*) motion. Thus they do not say "the horizon orb" or "the altitude orb." This is also something that indicates that "orb" takes into account motion in its meaning (*uᶜtubira fī mafhūmihi al-ḥaraka*) as we have indicated previously. [added by Jurjānī: ...by likening it to the whorl of a moving spindle. It is therefore necessary to add the restriction of motion to its well-known description so that it does not encompass spheres (*al-kurāt*) that do not move, such as the elements (*al-ᶜanāṣir*) and the planets (*al-kawākib*).] This application [of orb] is permissible (*ᶜalā sabīl al-tajawwuz*), something in the way of using a location (*maḥall*) to designate some state (*ḥāl*), such as their saying "the river bed (*al-wādī*) flowed."

I have tried to adhere to translating *falak* by "orb" whether a circle or a solid body is meant in order to retain the ambiguity of the Arabic. (See the commentary to II.3 [2]). When *falak* refers to the heavens as a whole, however, I have translated it as "celestial sphere." (See, for example, II.1 [6] and II.3 [12] and [13].)

I.1 [16–17]. Compare Euclid, Bk. XI, Defs. 18–23. As in the case of the sphere, Ṭūsī gives definitions for the cylinder and cone, while Euclid describes how they are generated. See Heath, *Elements*, 3: 270.

I.1 [16]8–9. *wa-yakūnu al-khaṭṭ al-wāṣil...kānat al-usṭuwāna qā'ima* (The line that joins...the cylinder is a right one); **I.1 [17]11–13.** *wa-'l-khaṭṭ al-wāṣil...kāna al-makhrūṭ qā'im*ᵃⁿ (The line that joins...the cone is a right one): Jurjānī, Khafrī, and Bīrjandī tell us that these two sentences were revisions of two passages that originally read: "the line that joins the two centers is perpendicular to the planes of the two circles, and it is the axis of the cylinder" and "the line that joins the point and the center of the base is perpendicular to the base and it is its axis." These original passages occur in the Marāgha version. Khafrī attributes both revisions to the author himself and a marginal note in MS D states that the new text in par. 16 is due to the author. This is important evidence that the Baghdad version, which has the revisions, is due to Ṭūsī rather than, say, a student. (See the introduction, §2.J, pp. 71–75.)

Clearly the new versions were intended to allow for the possibility of an oblique circular cylinder and an oblique circular cone. On the whole the changes produce a more correct formulation (as one finds stated by the copyist of MS T in a marginal note), but I am baffled by the change from definiteness to indefiniteness of "axis" in par. 16 ("is *the* axis of the cylinder" vs. "is *an* axis of the cylinder"). Note that the revision in par. 17 retains the definiteness of "axis."

Book I, Chapter Two

For a general discussion of the physical principles, see the introduction, §2.E, pp. 41–46.

I.2 [1]21; I.2 [2]7. *ᶜalā nahj wāḥid* (monoformly): This phrase should not be translated as "uniform" since it includes the accelerating motion of falling bodies (see above, pp. 44–46). The uniform motion of the celestial bodies is referred to as *mutashābiha*, a term introduced later in this chapter (I.2 [4]26).

I.2 [2]. The use of *ṭabᶜ* (a nature) in this paragraph seems at first sight confusing and even contradictory since it is the principle of motion for both the "natural" (*ṭabīᶜiyya*) motion of the four elements as well as the "voluntary" (*irādiyya*) motion of the orbs. On the other hand, "soul" (*nafs*) as a principle of motion is reserved for vegetative and animal movement. The crucial distinction, as we have noted in our section on principles (p. 45), is that the motion of the orbs, even though they may move through the medium of soul, is regular, whereas that which is vegetative and animal is not. In his commentary on *Al-Ishārāt*, Ṭūsī makes an explicit contrast between *ṭibāᶜ* (a variant or perhaps plural of *ṭabᶜ*) and *ṭabīᶜa*:

> *al-ṭibāᶜ*...is an innate [or essential] quality (*al-ṣifa al-dhātiyya*) of anything [whereas] *al-ṭabīᶜa* may designate (*qad takhuṣṣu*) that from which motion and rest issue forth (*yaṣduru*) involuntarily in whatever it occurs innately [essentially] (*awwal*ᵃⁿ *bi-'l-dhāt*).[1]

Thus both the motions of the orbs and the elements may be said to be due to a *ṭabᶜ* since this merely implies that they are endowed with a certain innate characteristic that results in their motion in a single manner (*i.e.* monoformly). On the other hand, only the motion of the elements may be called *ṭabīᶜī* since in their case the resulting motion is involuntary.

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[1] Ibn Sīnā, *Ishārāt*, 2: 56 (Tehran edition).

I.2 [2]10–11. *wa-'l-mutaḥarrik bi-ghayrih...fa-'l-ḥaraka ᶜarḍiyya* (When a mobile is moved by something other than itself...for it naturally to be): The standard examples of accidental motion in which the mobile is part of the mover are a planet embedded within an epicycle and a ring on the finger; as for the case where the mover is the natural place for the mobile, one finds the examples of an orb that contains another (such as the "enclosing orb" of II.11 [4]) or of the ship with respect to its passengers.

I.2 [3]14. *wa-humā ayniyyatān mustaqīmatān* (These two [motions] are displacing and rectilinear): The use of a derived form of *ayna* (where) to designate a motion that is from one place to another (and thus "displacing") rather than "in place" (*waḍᶜiyya*) is comprehensible but nevertheless curious. Among other alternatives one encounters are *ḥaraka intiqāliyya* (translocation) (Ibn Sīnā, *Al-Samā' wa-'l-ᶜālam*, Ch. 6, p. 45 in *Shifā'*) and *al-ḥaraka fī al-makān* (motion through space) (Ghazālī, *Maqāṣid*, Bk. III, Ch. 1, p. 308).

I.2. [4]. These characteristics of the celestial bodies are not simply a priori assumptions; each is a consequence of Physical Principle 5, namely that "a simple body has a single nature and what issues forth from that nature does so monoformly" (see I.2 [1]20–21 and pp. 44–46). Since this means that they are restricted to circular motion and cannot undergo rectilinear motion, they cannot experience those aspects of the sublunar world dependent on rectilinear motion. This is made explicit by al-Nīsābūrī in his commentary on this passage:

> The orbs, which have a principle of circular motion, do not tear or mend because this would demand a straight-line motion of the parts. They also do not grow or diminish because each of these does not occur except after a straight-line motion of the parts...They do not expand or contract[2]...as these require the separation of the body from its place or the evacuation of part of it by means of a straight-line motion. And because their motion occurs in a single way, this motion neither intensifies, weakens...nor undergoes retrogradation, turning or stopping...

Compare Aristotle, *De caelo*, Bk. I, Ch. 3, where he emphasizes the lack of contraries to explain the unchanging nature of the heavens but without explicitly relating this to the absence of rectilinear motion.

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[2] Khafrī and Bīrjandī make it clear that growth and diminishment (*numūw* and *dhabl*) refer to a gain or loss in the number parts of a body, whereas expansion and contraction (*takhalkhul* and *takāthuf*) refer to an increase or decrease in the same number of parts such as occurs in the expansion or compression of a cotton ball.

BOOK II

Book II, Chapter One

This chapter, in the main, corresponds to the *Almagest*, Bk. I, Chs. 3–7. However, in contrast to Ptolemy, Ṭūsī relies, as he tells us in the last paragraph, on *innī* proofs, *i.e.* ones that use observational evidence to establish certain facts, rather than *limmī* proofs that seek to show why something must occur in a certain way and not another. Thus Ṭūsī, for example, does not resort to the physical arguments of the *Almagest* for the sphericity of the heavens, namely that since the aether has the finest and most homogeneous parts it follows that its surface must be spherical, since the sphere is the only solid whose surface is composed of homogeneous parts (Bk. I, Ch. 3). (There is an interesting exception to this avoidance of physical arguments that occurs with regard to the proof of the Earth's state of rest; see the commentary to II.1 [6].) For a discussion of this important topic and its implications for the Arabic *hay'a* tradition, see the introduction, §2.D, pp. 38–41.

II.1 [title]5&6. *al-samā'* (the Sky): The translation here is dictated by the analogy with circumference; under other circumstances "heavens" is equally if not more appropriate.

II.1 [1]21–1. *fa-inna tarākum...wa-bi-'l-ḍidd* ([which can be explained by] the accumulation...and the opposite holds): The problem here referred to, which has come to be known as the "moon illusion," has a remarkable history in the Middle Ages, which has recently been dealt with by Sabra [1987b]. What is significant is that Ṭūsī has simply paraphrased Ptolemy's erroneous view, which is based on a misapplication of the latter's theory of refraction inasmuch as the eye is here in the denser medium, whereas the viewing of an object in water presents one with the opposite case—this despite the fact that a much more sophisticated treatment, based on a psychological explanation, had been presented by Ibn al-Haytham in his *Optics* some two centuries before. Ṭūsī's uneven knowledge of the works of his predecessors will be noted again, in particular in our commentary on the next paragraph and on II.11. Part of any history of Islamic science must take into account the irregularity of the transmission of texts and scientific knowledge over time and between the various regions of Islam. In the case of Ibn al-Haytham's *Optics*, the work was not well known in the Islamic world from the time it was written until it was revived at the beginning of the

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14th century by Kamāl al-Dīn al-Fārisī, who wrote a critical commentary on the *Optics*, the *Tanqīḥ al-Manāẓir*, at the suggestion of his teacher Quṭb al-Dīn al-Shīrāzī, who seems not to have been familiar with the work. Shīrāzī, of course, was one of Ṭūsī's students, and this, along with the persuasive evidence of Ṭūsī's own writing on optics, make it abundantly clear that Ṭūsī had not studied the *Optics*.[1]

II.1 [2]9–13. *wa-taḍārīsuhā...ᶜalā misāḥat al-arḍ* (The [Earth's] undulations...the area of the Earth): See the commentary to IV.1 [5].

II.1 [4]21–22. *aw ᶜind kawnihā...bi-sayrihā al-khāṣṣ bihā* (or when it is at opposite parts...proper motion): By proper motion (*sayrihā al-khāṣṣ*), Ṭūsī means the sun's motion around the ecliptic. This "evidence" is in addition to what one finds in the *Almagest*, p. 42 (H19).

II.1 [5]4. *wa-sa-nubayyinu dhālik fī mawḍiᶜih* (we shall make this clear in the appropriate place): Parallax is treated in II.12.

II.1 [6]. Ṭūsī here rejects those proofs against a daily rotational motion for the Earth that one may find, among other places, in the *Almagest*, Bk. I, Ch. 7. These arguments, based as they were on the alleged dire consequences that would occur if the Earth were in motion, were empirical inasmuch as they depended upon counterexperience. Although Ṭūsī is not completely explicit on the point, Shīrāzī as well as all the commentators interpret Ṭūsī's stance as indicating a claim that there are no observational tests to determine whether or not the Earth is in motion, or at least that the proposed ones are inadequate. As Nīsābūrī states, Ṭūsī has used observation and testing (*al-raṣd wa-'l-iᶜtibār*) as the basis for the *innī* proofs that confirm the circularity of the Earth and the heavens, the Earth's centrality, and so on; here, however he has had to resort to a *limmī* proof from natural philosophy, namely that the Earth cannot move naturally with a circular motion due to its rectilinear inclination, to establish this fundamental aspect of Ptolemaic cosmology. (See the commentary for par. [8], pp. 386ff for a discussion of *innī* and *limmī* proofs.) Note, however, that Ṭūsī does not say anything about whether a circular motion could be accomplished by compulsion.

To bolster his position, Ṭūsī suggests the possibility that if the Earth did indeed rotate, then all of Ptolemy's alleged horrors would not occur if the air were able to rotate with the Earth; anything in the air could then be carried along in a way such that an observer on the Earth would not be able to determine from them whether or not he were in motion. This argument leaves a great many questions unanswered (for example, would objects be carried by the air accidentally or by constraint?), and Ṭūsī attempts to strengthen it by appealing to what

[1] Sabra [1972], p. 196.

he considers to be the analogous situation of comets. According to Aristotle, the comets are a phenomenon of the sublunar region and not of the unchanging celestial realm. But in order to explain their apparent participation in the daily rotation of the heavens, one must assume that the level of fire, the level of the sublunar region closest to the orbs, moves with the diurnal east-west motion (*Meteorology*, 344a5–20). If one believes this widely accepted view, Ṭūsī is saying, then one should not find disturbing the possibility of the air's conforming motion with the Earth's rotation.

Many objections were raised against this passage by later Muslim astronomers. Indeed Ṭūsī's own student Quṭb al-Dīn al-Shīrāzī claimed that if the air were in motion with the Earth, then a large and a small rock thrown straight up in the air would return to Earth at different locations since each would be moved differently by the air (*Nihāya*, maqāla II, bāb 1, faṣl 4; *Tuḥfa*, bāb II, faṣl 4). In general he seems to agree with Ptolemy that observation can determine the question of the Earth's motion. Shīrāzī also denied the relevance of the use of comets. In the *Tuḥfa* he disputed the Aristotelian claim that the comets moved with the daily motion of the orbs. Later commentators were stimulated by this dispute between teacher and student to analyze the question of the Earth's motion with a certain amount of care. Both Nīsābūrī and Jurjānī, as well as most of the other commentators, criticized Shīrāzī on the matter of the two rocks; they held that they would in fact have the same quantity of motion as that of the rotating Earth. If there were a difference, it would be such that an experimenter would not be able to detect it. On the other hand, many of the commentators agreed with Shīrāzī that the comets did not follow the daily motion of the orbs.

This late medieval tradition of discussing the question of the Earth's motion is, of course, interesting in its own right and deserves a much more extensive treatment than is possible here. It also, for better or for worse, cannot but bring to mind Copernicus. What makes this more than a case of free association is the remarkable similarity between this passage in the *Tadhkira* and one in *De revolutionibus* (Bk. I, Ch. 8, f. 6r). Like Ṭūsī before him, Copernicus evokes the possibility that the air and what is in it could move with the Earth's rotation. But, after all, this idea is already in the *Almagest* where it is promptly rejected (H25). What is much more decisive is Copernicus's appeal to the analogous motion of comets to legitimize the possibility. One could hypothesize that both Ṭūsī and Copernicus are relying on a common source, but what seems to me much more likely is that Copernicus has been influenced, either directly or indirectly, by the late medieval *hay'a* tradition. (We should note here that two commentators, Khafrī and Bīrjandī, are contemporaries of Copernicus.) The significance of this case of influence, if that is indeed what this is, is that we are not dealing with models that could be transmitted simply by diagrams, but instead we are faced with a rather subtle argument that would seem to require either a textual or scholastic mode of transmission.

It is important to keep in mind what is at stake here. Neither Ṭūsī nor any of his commentators, as far as I have been able to determine, defended a rotating

Earth. What was at question was whether or not a stationary Earth could be established by strictly observational data as would befit a mathematical science such as astronomy. Ṭūsī seems to conclude that it is not possible and must then resort to an argument from natural philosophy about the nature of the element earth. Since it can only move in a straight line, it cannot rotate. But this left open the possibility that a different natural philosophy, for example one that asserted that the whole of an element might have a different motion than its parts, would lead to a different conclusion. With Copernicus, this is precisely what has occurred (see *De rev.*, f. 6r–6v).

II.1 [6]11. *bimā yattaṣilu bihā* (along with whatever is joined to it): MSS BCDFGLMST, ᶜUbaydī's commentary, and the *Nihāya* (Ahmet III MS 3333, f. 47a) have *bihā*, while MSS HN and the commentaries of Nīsābūrī, Jurjānī, Shīrwānī, Khafrī, and Bīrjandī have *bih*. The latter explain the phrase as meaning that whatever is joined to the air, such as a rock or a bird, will participate in the air's conforming motion to the Earth's rotation. The former choice, *bihā*, would indicate that the referent is the Earth, which does not make much sense in context and, even if it did, would be redundant. Though I agree with the consensus of the commentators, I have followed my standard editorial procedures and retained *bihā* (cf. p. 87).

II.1 [6]11. *kamā yushāyiᶜu al-athīr al-falak* (just as the aether conforms to the orb): All the commentators are in agreement that *al-athīr* (aether) here designates the level of fire in the upper atmosphere; this is a somewhat unusual usage and, as noted by Shīrwānī, *al-athīr* is much more commonly used with reference to the orbs.[2] According to Bīrjandī, the orb referred to is that of the moon. Presumably what is meant is that the concave surface of the moon's inclined orb is able to impart its motion to the level of fire in the upper atmosphere. This would be consistent with the view that the daily motion is transmitted downward from the highest orb. The diurnal motion of the level of fire, however, would be difficult to reconcile with the alternative view that the heavens as a whole rotate as a single unit, a view held by Ptolemy in the *Planetary Hypotheses* and by Ibn Rushd in his *Talkhīṣ mā baᶜd al-ṭabīᶜa* (see our commentary to II.4 [6]).

II.1 [7]19–22. *wa-'l-inā'...hādhihi al-masā'il* (The amount of water...these matters find strange): Livingston has called these remarks about a filled vessel containing more water when it is closer to the center of the Earth a "proof for the roundness of the Earth and the Heavens."[3] As should be clear from the context, this argument is simply one of several consequences, rather than proofs, arising from the sphericity of the Earth that Ṭūsī wishes to note. Compare Aristotle, *De caelo*, Bk. II, Ch. 4, 287b4–14.

[2] Note, though, that Ḥabash uses it in the sense of fire in his *Book of Bodies and Distances*; see Langermann [1985], p. 111.
[3] Livingston [1973], p. 274.

II.1 [8]. *Inniyya* derives from the word *inna* (that), while *limmiyya* comes from *lima* (why);[4] thus, literally, we can translate these as the "proof of the that" and "proof of the why." Ultimately they can be traced to the distinction that Aristotle makes in the *Posterior Analytics*, Bk. I, Ch. 13 between what he calls the fact (τὸ ὅτι) and the reasoned fact (διότι).[5] Ṭūsī, however, is using this dichotomy in a way different from that of Aristotle, and it will be worthwhile to pursue this difference.

Aristotle first considers a single science in which there are those proofs that provide the fact and those that provide the reasoned fact. An example of a proof of the fact is the following syllogism:

(1) The planets do not twinkle.
(2) That which does not twinkle is near.
∴ (3) The planets are near.

We have thus deduced that the planets are near based on an observation for (1) and "through induction or through perception" for (2).[6] The conclusion, however, does not follow from a premise that is its cause since it is not the case that the planets are near because they do not twinkle. But inasmuch as one may reverse this syllogism, one may provide a proof of the reasoned fact as follows:

(1) The planets are near.
(2) That which is near does not twinkle.
∴ (3) The planets do not twinkle.

Here the conclusion is a reasoned fact since the cause of the planets not twinkling is the premise that they are near.

In this example, the proofs of the fact and the reasoned fact occur in the same science. But Aristotle also takes up the case that is of immediate concern to our own inquiry, namely where the fact and the reasoned fact are investigated by different sciences. Observational astronomers, for example, gather the facts, whereas the mathematical astronomers seek to know the reasoned facts since they "have the demonstrations of the explanations."[7] It is interesting that Aristotle accepts as a matter of course that the mathematicians "often...do not

[4] Technically speaking, the derived forms should be *innayya* (as one finds in MS M) and *limayya*, but it is hard to imagine such things flowing from the lips of Arabic speakers (as distinct from Orientalists whose insistence on "purity" is legendary). Goichon in her *Lexique* does give the latter but for some reason that escapes me has *iniyya* for the former (p. 22). I use the adjectival forms *innī* and *limmī* mostly for their euphonic quality, but these forms are also explicitly given in Harvard's copy of Bīrjandī's commentary (Houghton MS Arabic 4285) and *limmī* can also be found in MS D.

[5] This is the *quia / propter quid* distinction of the Latin scholastics.

[6] *Posterior Analytics*, Bk. I, Ch. 13, 78a34–35.

[7] Ibid., 79a3–4.

know the fact";[8] this does not bother him since "mathematics is about forms, for its objects are not said of any underlying subject."[9] The example from astronomy is fairly typical of the others given by Aristotle in this context. The science in which the fact occurs is empirical and subordinate, whereas that in which the reasoned fact is demonstrated is mathematical and superior.

The situation of the fact/reasoned fact distinction in the Islamic philosophical tradition as well as in Islamic astronomy is rather different. In general, the emphasis is on the proof itself rather than on its individual elements (*e.g.* an observational fact). Thus a *limmī* proof provides the reason for the occurrence of the conclusion (*ᶜilla wujūd al-natīja*), whereas an *innī* proof only gives the reason for asserting that occurrence (*ᶜillat al-taṣdīq bi-'l-wujūd*).[10] An *innī* proof will then give the fact, while the *limmī* proof will give the reasoned fact. But as we have seen, Aristotle uses a distinction based on proof only in the case where the fact and the reasoned fact occur in the same science. When they are found in different sciences, the fact is an observational fact, not the conclusion of a proof; these observational facts from one science are provided with causes by the reasoned facts of another, usually mathematical, science.

In contrast with Aristotle, Arabic mathematical astronomers do not see themselves as giving proofs of the reasoned fact but rather of the fact. These "facts," however, are not observations but rather the configuration (*hay'a*) of the simple bodies; this is arrived at by *innī* proofs that are based on observations. On the other hand, this sort of proof does not indicate the cause for such a configuration in the sense that it does not furnish the answer to the question of why such a configuration occurs and not some other. This question is answered by a *limmī* proof which is given in the *al-samā' wa-'l-ᶜālam* (*De caelo*) literature.

Ṭūsī here, for example, gives what he calls *innī* proofs for the circularity of the heavens, the Earth and water, for the centrality of the Earth, and for the imperceptible size of the Earth with respect to the orb of Mars and those orbs beyond. All of these proofs (for which Ṭūsī uses the term *dalīl* instead of the stronger *burhān*, demonstration) are based upon observations.[11] For example, the earlier rising and setting of celestial objects as one travels East and the increasing altitude of the pole as one goes North are indications of the circularity of the Earth. But this proof does not, as Ṭūsī states, "demonstrate the necessity" (*tufīd wujūb*) for its so being; for this he refers the reader to *De caelo*. Al-Khafrī in his commentary on this passage further elucidates the situation:

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[8] Ibid., 79a3–4.

[9] Ibid., 79a7–8.

[10] This is the formulation in Ghazālī's *Maqāṣid al-falāsifa*, Bk. I (*Al-Manṭiq*), Ch. 5 (*Fī Lawāḥiq al-qiyās*), Part 2, pp. 120–121; his source is Ibn Sīnā's *Al-Ishārāt wa-'l-tanbīhāt*, Bk. I, Ch. 9, Sec. 5 (1: 534–538 of the S. Dunyā edition). Note that Ṭūsī's wording in this paragraph is a bit different; *innī* proofs "assert existence" (*tufīd al-wuqūᶜ*), while *limmī* proofs "demonstrate the necessity of existence" (*tufīd wujūb al-wuqūᶜ*).

[11] This is also *generally* the case in the *Almagest*, Bk. I, Chs. 3–6.

> These proofs (*adilla*), *i.e.* those which he adhered to (*tamassaka bihā*) in the determinations (*aḥkām*) of this chapter, are *inniyya* demonstrations (*barāhīn*), which convey existence, *i.e.* necessitate the judgment (*taṣdīq*) that those bodies occur according to the well-known configuration (*hay'a*) and the previously mentioned circumstances, without including concurrently the cause (*ᶜilla*) for their so being. The proofs that convey the necessity of that existence are *limiyyāt* and include the causes (*ᶜilal*) for those determinations according to both the mind (*dhihn*) and the external world (*al-khārij*). The latter [proofs] are given in Natural Philosophy in the book *De caelo*. For example: the orbs are simple (*basīṭa*), but then the simple requires (*yaqtaḍī*) a circular form; hence, it is the cause for the judgment (*taṣdīq*) as well as for the establishment (*thubūt*) of the necessity of that determination as long as the subject itself exists. To make this discussion clear: it is the case that the problems of this chapter are common to both Natural Philosophy and this science. The difference is rather due to the [type of] proof as has been shown.

There are several points that should be emphasized. First, in contrast to what we found to be the case with Aristotle, the fact and the reasoned fact are here the same; they are only distinguished by the proof that is used to establish them. Thus it is possible for two different sciences to deal with the same subject matter and to prove the same things. One science would establish the fact that something occurs in a certain way whereas the other would seek to give an ultimate reason for that occurrence. Second, it should be clear that *hay'a* is here being portrayed as a science based on observations. On the other hand, *al-samā' wa-'l-ᶜālam* has taken on the character of a discipline whose premises are rational and a priori. As such, an important demarcation has occurred whose result—if not intention—is to make astronomy an undertaking that could be seen as autonomous and generally independent of the philosophical literature concerned with the heavenly bodies.

Compare pp. 38–41 and 45–46 of Volume One.

II.1 [8]24. *kitāb al-samā' wa-'l-ᶜālam* (the book *De caelo*): It is not clear whether Ṭūsī is here referring specifically to Aristotle's *De caelo* or generally to the entire tradition of these types of works. For example, he may have also had in mind the appropriate part of Ibn Sīnā's *Shifā'*.

Book II, Chapter Two

In this chapter we encounter for the first time a procedure that Ṭūsī will use again and again in subsequent chapters for establishing the configuration (*hay'a*) of the World. First he lists the observations that are to be explained. He then describes the orbs that are meant to account for the observations. In later chapters he will also give the individual motions of each of the orbs as well as the so-called anomalies that result from these motions for each planet.

II.2 [3]19–21. *wa-lammā lam takun li-bāqī al-kawākib...jā'iz*^{an} (Since the rest of the stars...conceivable): The idea that there is no necessary reason to place all the fixed stars on a single orb can be found in the *Shifā'* of Ibn Sīnā,[1] who had Geminus and Proclus as classical antecedents.[2] Fakhr al-Dīn al-Rāzī quotes Ibn Sīnā approvingly and adds his own comments in his exegesis on the *Qur'ān*.[3] Maimonides as well is open to the possibility that the number of orbs may equal the number of stars.[4]

II.2 [3]21–23. *wa-ayḍ*^{an} *isnād...li-wujūdihā* (In addition, the attribution...its existence): In his inimitable way, Shīrāzī informs us in *Fa*^{c}*alta* that the additions to this sentence (occurring between slashes in our text and translation) are completely meaningless (*lā ma*^{c}*nā lahā aṣl*^{an}); he is quick to add that these were made after he had left the service (*khidma*) of the author (f. 27a; repeated by Nīsābūrī). A marginal note in MS M that gives the variant concurs in this judgment: "There is no need for the qualification—the correct [version] is what occurs in the [original] text." The reason for the opposition to the revision is rather abstruse; a bit of the historical context may serve to clear up, at least partially, the confusion.

Among the guidelines underlying *hay'a*—axioms being rather too strong a word for the complexities of the situation—are two that are of relevance here: (1) that there should be nothing superfluous in the heavens and (2) that each independent motion in the heavens should be effected by a separate orb. Earlier in this paragraph, for example, one can see the results of these underlying guidelines—all the fixed stars, which share the same motions, are placed on a single orb in the interests of simplicity, and the nine primary motions are assigned to nine separate orbs. The question arises, though, whether one may dispense with the ninth orb, which brings about the daily motion, for the sake of simplicity, thus promoting one guideline at the expense of another. What Shīrāzī and other commentators object to is that Ṭūsī's revision would seem to imply

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1 *Al-Samā' wa-'l-*^{c}*ālam*, Ch. 6, p. 46.

2 *HAMA*, 2: 584.

3 Nallino [1911], pp. 257–259.

4 Maimonides, *Guide*, Bk. II, Ch. 11 (p. 274 of Pines's translation, p. 167 of Friedländer's translation).

that one cannot do this because of the existence of another primary motion, namely precession. But why could not one place all the fixed stars on the eighth orb and then have all the eight orbs move as a whole with the daily motion? (See the commentary to II.4 [6].) Indeed this proposal had already been made, Shīrāzī tells us, by Muḥammad b. Mūsā, one of the Banū Mūsā, in a treatise reproduced in *Fa^{c}alta* (ff. 27b–31b).[5] Shīrāzī claims for himself the proposal to do away with the eighth orb as well, thereby placing the fixed stars on the convex surface of Saturn's parecliptic (f. 32a). He goes on to state that he mentioned this to Ṭūsī who "found it pleasing and commended me" (*istaḥsanahu wa-athnā ^{c}alayya*). (This may be why Shīrāzī is so miffed by the revision, which seems to disallow this possibility.)

Bīrjandī, with several centuries of mulling time behind him and without Shīrāzī's predilection for making tempests in teapots, offers some plausible suggestions that help to disentangle some of these disparate strands. First of all, he holds that one cannot eliminate both the eighth and ninth orbs since this would involve having two different souls with conflicting motions attached to the same entity, *i.e.* the orbs taken as a whole. He has no objection to dispensing with the ninth orb alone; however he feels that this would lead to the problem of deciding which of the two primary motions, both of which encompass all the orbs and both of which are about the center of the World, is to be that of the eighth orb and which is to be that of the eight orbs taken as a whole. Since there is no reason to prefer one over the other, Bīrjandī assumes that Ṭūsī did not mean to imply that the existence of the two primary motions made dispensing with the ninth orb impossible as such but rather that he wished to point out that this would be inappropriate on aesthetic grounds (*yakūnu al-murād bi-'l-imtinā^c huwa ^{c}adam al-istiḥsān lā ^{c}adam al-imkān*). This would certainly make sense if Ṭūsī's intention was to make an addition with as little change to his basic text as possible, as seems to be the case; in doing so, however, he failed to realize that the retention of "precluded" (*mumtani^c*) would lead to misinterpretations. Another consideration for Ṭūsī may have been that a ninth orb provided a starless orb that could be used as a reference system upon which to place the fixed zodiacal signs (not to be confused with the constellations of the same name; cf. II.3 [5]).

II.2 [4]2. *al-falak al-aṭlas* (the *aṭlas* orb): That the name of this orb has something to do with the mythic figure of Atlas holding up the World is certainly a possibility, but I have no explicit evidence to substantiate this. The com-

[5] In all probability, this is the work attributed to Muḥammad's brother Aḥmad by Ibn al-Nadīm and entitled *Kitāb Buyyina fīhi bi-ṭarīq ta^{c}līmī wa-madhhab handasī annahu laysa fī-khārij kurat al-kawākib al-thābita kura tāsi^{c}a* (book in which it is proven by mathematical means and a geometrical approach that there is no ninth sphere beyond the sphere of the fixed stars) (Flügel's edition, p. 271; note that Sezgin, *GAS* VI, p. 148 has *bi-khārij*). This work, which seems otherwise to be nonextant, deserves careful study inasmuch as it indicates a very early interest in cosmological issues.

mentators claim the name *aṭlas*, which is the elative of an Arabic root whose base meaning is efface or obliterate, was given to this orb because it is devoid of stars. Giving Arabic etymologies to Greek words was not uncommon; see, for example, the entries for *athīr* (aether) and *asṭurlāb* (astrolabe) in Lane's *Lexicon.*

II.2 [4]7–10. *wa-ja*ᶜ*alū al-shams...ghayrihimā* (They placed the sun...in yet another): This notion of the sun's centrality is, of course, not new with Ṭūsī; in fact, it can already be found in the *Almagest*, Bk. IX, Ch. 1 (H207), and it is treated in a separate chapter in the epistle on *hay'a* by the Ikhwān al-Ṣafā'. For a discussion of this issue in the Latin West, see Grant [1978], p. 279. Grant draws an interesting contrast between those who held that the sun occupied the crucial "middle" position in the universe and Averroes who equated nobility with distance from the Earth.

II.2 [4]10–11. *wa-kāna ayḍ*ᵃⁿ *bu*ᶜ*duhā...al-waḍ*ᶜ (In addition the known distance...positioning): In other words, the ancient value of the Earth-sun distance was compatible with placing the orbs of Mercury, Venus, and the moon between the Earth and the sun. See IV.5 [5]; Pedersen [1974], pp. 393–394; and Goldstein [1967], pp. 7–11.

II.2 [4]11–13. *wa-qad qīla inna al-zuhara...fī ṣafḥatihā* (There are also reports that Venus...on its surface): There were a number of "reports" of Venus transits in the medieval literature; see Goldstein [1969] and [1972]. To these we may add the claimed observation by Ibn Bājja (12th c. Spain) of a simultaneous transit of Mercury and Venus, which is cited by Shīrāzī (*Nihāya*, II.2, f. 49b; cf. Sayili [1960], pp. 184–185). Shīrāzī also notes two transit observations made twenty-some years apart during the first of which Venus was at the apex of its epicycle, while during the second it was at the perigee; he remarks that a single transit would not be decisive for determining whether Venus's orb was below or above the sun since it would not contradict the proposal that "Venus and Mercury, along with the sun, were in a single sphere, the center of their epicycles being the center of the [sun]," whereas these two observations, if correct, would.

II.2 [5]17–18. *wa-bi-falak al-qamar...ṭabaqāt* (The orbs terminate...levels): Note that a work on *hay'a* would not be complete without a listing of all the parts of the universe, both celestial and sublunar. Though the divisions of the sublunar region have the shape of the orbs, each is called a *ṭabaqa* (level) rather than a *falak* (orb) since the latter is usually considered to have a circular motion (see commentary to I.1 [15]).

II.2 [5]20–21. *wa-rubbamā tūjadu mutaḥarrika...lahu* (they are sometimes found to move...orb): For a discussion of the participation of the comets and meteors in the diurnal motion of the celestial sphere, see the commentary to II.1 [6]. Note that Ṭūsī here says this conforming motion occurs "sometimes," a position somewhat closer to that of Shīrāzī.

II.2 [5]20&22. *al-nayāzik* (meteors); *al-shuhub* (shooting stars): I am at a loss as to how to distinguish between these two phenomena. Compounding the difficulty is that here both are apparently substantial, whereas Ibn Sīnā only uses *shuhub*, which he also states is constituted of the rising smoke (*dukhān*), as such. (Comets and lightning bolts are for him likewise made from smoke.) On the other hand, he employs *nayzak* to designate an image occurring in the atmosphere upon the vapors (*bukhār*), thus putting it in the same class as a rainbow (*al-Maᶜādin*, p. 39). Bīrjandī notes that some unnamed persons hold that *shuhub* are formed in the same place as comets, which is at odds with the description given by Ṭūsī. It may be that *nayzak*, which is from a Persian word meaning short spear, has a shorter streak than a *shihāb*, but this is far from clear.

II.2 [5]24–1. *wa-baᶜḍ hādhihi al-ṭabaqa munkashifa ᶜan al-arḍ* (part of which has been drawn aside, uncovering the earth): This uncovering of the Earth was generally considered to be providential, allowing for a habitat for terrestrial plants and animals, not the least of which is humankind. (This interpretation is confirmed in Khafrī's commentary.)

Book II, Chapter Three

II.3 [2]13. *wa-qad yuṭliqūna ism al-falak...tajawwuzᵃⁿ* (their use of the name "orb"...being permissible): Although Ṭūsī seems to indicate some hesitation over the use of *falak* (orb) to designate an equator, he does employ it quite frequently in this way. I have opted to translate *falak* as "orb" even when it really means an equator. This has seemed to me the only way to preserve the ambiguous Arabic usage and thus allow the English reader to judge whether or not the context is sufficient for distinguishing the strictly physical from the more mathematical usage. For an extended discussion of *falak*, see the commentary to I.1 [15].

II.3 [3]3. *al-mayl al-kullī* (the total obliquity): *Mayl* may in general mean any type of inclination, and thus its use for the obliquity of the ecliptic may seem straightforward enough. However, I think that Ṭūsī has in mind here the technical use of *mayl* to mean declination as defined in II.3 [6], pp. 114–115. The obliquity of the ecliptic is thus the "total" declination to distinguish it from a "particular (or partial) declination" (*mayl juz'ī*).

II.3 [5]13. *burj* (zodiacal sign): It is important to note that the *burūj* (zodiacal signs) do not refer to the constellations that bear the same names but rather to the 12 equal divisions of the ecliptic equator that begin with the vernal equinox. Clearly any alignment of the two will over time be disrupted due to precession. Whether Ṭūsī is serious about renaming the zodiacal signs, or is simply indulging his humor, is not clear.

II.3 [5]14. *min al-thawābit* (from among those fixed): This phrase modifies *kawākib* (stars).

II.3 [8]10. *taqwīm*: This word, which literally means setting upright or rectification, may appropriately be translated as "true position" when it is used to mean the longitude of a star or planet.

II.3 [16]20. *dā'ira wasaṭ samā' al-ru'ya* (ecliptic meridian circle): This circle is analogous to the meridian in that it divides the visible and the invisible halves of the ecliptic orb just as the meridian does for the equinoctial. Similarly one can define local ecliptic latitude with reference to the ecliptic pole on the model of the more standard local latitude, which is with reference to the equinoctial pole. Clearly, however, the local ecliptic latitude is of rather less utility since it is constantly changing due to the motion of the ecliptic pole. Nīsābūrī notes that the importance of this circle is that the ascendent is one of its poles.[6]

The Arabic terms used to designate this circle are somewhat unusual and need explanation. *Samā' al-ru'ya* literally means "the sky of appearances." This, according to both Nīsābūrī and Khafrī, is another name for the ecliptic orb. Khafrī further explains that it has this name because of the great number of stars (hence, "appearances") on it. Since this circle goes through the apparent middle (*wasaṭ*) of the ecliptic, the translation "ecliptic meridian" seems to me to bring out its appropriate parallel with the celestial meridian. As can be seen from the phrase *ᶜarḍ iqlīm al-ru'ya* (local ecliptic latitude), *ru'ya* without *samā'* may alone refer to the ecliptic.[7]

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[6] For a discussion of these terms and Kāshī's use of local ecliptic latitude in his parallax theory, see Kennedy [1956], pp. 37–38.

[7] Note: Kennedy, *Survey*, finds the latter to be *ᶜarḍ iqlīm li'r-rū'ya* [*sic*], which he calls the "latitude of the visible climate" (p. 145). I cannot, frankly, understand this translation and, in view of the above commentators, I think that it should be rejected.

Book II, Chapter Four

II.4 [1]. Ptolemy's value for the obliquity of the ecliptic is 23;51,20°, which is probably the value referred to here as being less than 24°.[1] An obliquity of 23;33° is found in the "purported" *Al-Zīj al-mumtaḥan*; it is also given by Ḥabash as the value used by Thābit.[2] Ṭūsī is certainly correct when he notes that most astronomers (*al-jamhūr*) have adopted 23;35°; it is found in the *zīj*es of Ḥabash, Battānī, Kūshyār, Bīrūnī, and Khāzinī.[3] Ibn Sīnā, in the appendix to his summary of the *Almagest*, gives the value found at the time of Ma'mūn as 23;35°. After the time of Ma'mūn, he continues, the value was found to have decreased one minute (to 23;34°), whereas he personally has subsequently observed a further decrease of approximately one-half degree (to a value, we may presume, of 23;33,30°).[4]

Khafrī states that after writing the *Tadhkira* (presumably the original version in 1261 A.D.), Ṭūsī himself found the obliquity of the ecliptic to be 23;30°, a value less than the minimum that he reports in this paragraph. This is the value adopted in the *Īlkhānī Zīj*, a work written at least four years after the *Tadhkira*; it also appears in *Al-Zīj al-jadīd* of Ibn al-Shāṭir (who also uses 23;31°) and in the *Zīj-i Khāqānī* of al-Kāshī.[5] Finally Khafrī notes that in the most recent observations, which were undertaken under the auspices of Ulugh-beg at Samarqand, the obliquity of the ecliptic was found to be 23;30,17°.

II.4 [2]21. *za*ᶜ*ama ba*ᶜ*ḍuhum* (some have maintained): *ba*ᶜ*ḍ* may have the meaning of "some" or "one." Since the commentators imply that this is a view held by a number of persons, I have opted for the more indefinite "some."

Khafrī identifies Ibn Sīnā as someone who was inclined toward (*māla ilā hādhā*) such a motion for the ecliptic in the *Shifā'*. The most likely passage being referred to, part of which is quoted directly by Shīrwānī, merely states that "it is conceivable that what some [or someone] has maintained is true" (*fa-yushabbihu*[6] *an yakūna mā qāla ba*ᶜ*ḍuhum ḥaqq*ᵃⁿ).[7] What Ibn Sīnā describes turns out not to be a model for bringing about the change in obliquity alone but rather the trepidation model of Ibrāhīm ibn Sinān (who is not explicitly named), whose purpose is to explain a supposedly variable rate of precession as well as a changing value for the obliquity of the ecliptic (see the commentary to II.4 [5]). Whether Ṭūsī himself has anyone specifically in mind here other than Ibrāhīm ibn Sinān is not clear.

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1 *Almagest*, p. 63 (H67–68).
2 Kennedy, *Survey*, pp. 145, 151.
3 Kennedy, *Survey*, pp. 151, 153, 154, 156, 158, 159.
4 Ibn Sīnā, *ᶜIlm al-hay'a* (s.v. *Shifā'*), p. 652
5 Kennedy, *Survey*, pp. 161, 163, 164.
6 Following Shīrwānī and not the editor who has the incomprehensible *fa-nisbatuhu*.
7 Ibn Sīnā, *ᶜIlm al hay'a* (s.v. *Shifā'*), p. 652.

II.4 [2]23–[3]16. *thumma al-minṭaqa...fī buqᶜa bi-ᶜaynihā* (Now the [ecliptic] equator...in a given locality): This version of the passage, which presents eight possibilities, comes from what is referred to as the new text (*al-nuskha al-jadīda*), which was completed in Baghdad probably in 672/1274. The old text (*al-nuskha al-qadīma*), which was completed in 659/1261 in Marāgha, has four possibilities, but there are several significant variations between them. Below, I give a translation of the variants, which are edited in the apparatus, followed by a few notes and some speculation on the relation between them. (I have also discussed this passage and its variants, which are of crucial importance for determining the dating and relationship of the various versions of the *Tadhkira*, in the introduction, §2.J3 and §2.M2.)

I. Passage from the "old version" as found in MSS DMT and Nīsābūrī's commentary, and as the alternative given by Bīrjandī:

> Now the equator, if it moves, may complete a revolution or it may not complete it but instead move to a certain limit then return. This limit may be after it has coincided with the equinoctial and departed from it; it may be at its coincidence; or, it may be before its coincidence. On the first assumption, the halves of the ecliptic orb, *i.e.* the northern and the southern, could completely interchange. On the second assumption, this could partially occur. On the third assumption, this could not occur; however, day and night would become equal during the coincidence under all circumstances and the seasons of the year would cease to occur. On the fourth assumption, this would not be the case; however, altitudes [of stars] and the extent of days and nights would increase and decrease in a given locality.

II. Passage in MS G:

> Now the equator, if it moves, may complete a revolution or it may not complete it but instead move to a certain limit then return. This limit may be at its first coincidence and it may be before its first coincidence. On the first assumption and the second, if it coincides twice, the halves of the ecliptic orb, *i.e.* the northern and the southern, could completely interchange in terms of its surface and its equator. On the remaining assumptions, that would be possible in part [reading *bi-'l-baᶜḍ* for *bi-'l-ᶜarḍ*] for the surface. As for the case on assumption two whereby it does not coincide with it except a single time, the interchange would be for the equator only. On the third assumption, this could not occur; however, day and night would become equal during the coincidence under all circumstances and the seasons of the year would cease to occur. On the fourth assumption, this would not be the case; however, altitudes [of stars] and the extent of days and nights would increase and decrease in a given locality.

III. Fragment identified by Khafrī as the older text:

> Now the equator, if it moves, will either complete [reading *tutammimu* for *tatimmu*] a revolution or will not but instead move to a certain limit then return. This limit is either after it has coincided with the equinoctial and departed from it once or twice, or else at one of the two coincidences, or else before it.

In MSS DMT, the context makes it clear that it is the first coincidence of the ecliptic and the equinoctial that is being referred to. In MS G, this is made explicit and there is some attempt to differentiate between the cases of the first and second coincidences of the ecliptic with the equinoctial. It would seem reasonable to assume that this represents an intermediate version between the old text of four assumptions and the new text with eight. (Note: In MS G, assumption two, in which the limit is after the first coincidence, has been left out, no doubt due to scribal error.) Khafrī's version is very close to that of MSS DMT; for the most part, the variations seem to be his own attempt to clarify the text by adding some words and phrases, but in the process he has extended the number of assumptions beyond the four of MSS DMT.

With the exception of Nīsābūrī, who adopts it, the commentators either ignore completely the older version or else quote it with disapproval. There is no question that the newer version is much clearer from a didactic standpoint.

II.4 [4]17–1. *wa-ayḍ*$^{\mathrm{an}}$ *waqa*^{c}*a al-ikhtilāf...sab*^{c}*īn sana* (Furthermore there was a divergence...70 years): One degree per 100 years is the value for precession adopted by Ptolemy and the lower limit set by Hipparchus.[8] The early Muslim astronomers, whom Khafrī identifies as those who worked during the reign of the Caliph al-Ma'mūn, found precession to be at the somewhat faster rate of one degree per 66 years.[9] This rate is also adopted by al-Battānī in his *Al-Zīj al-Ṣābi*^{c}, apparently on the basis of independent observations.[10] Khafrī attributes the value of 1°/70 years (cf. modern value of approx. 1°/71.6 years) to Ibn al-A^{c}lam (ca. 960 A.D.) and "others" (possibly Ibn Yūnus [d. 399/1009]). This rate was confirmed by Ṭūsī himself at Marāgha and appears in the *Īlkhānī Zīj*; it is also found in the *zīj*es of Ibn al-Shāṭir and al-Kāshī.[11] Finally Khafrī notes that 1°/70 years was also the rate found by the observers at the Samarqand observatory, and thus it had become the accepted value by his time (10th/16th century).

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[8] *Almagest*, p. 328 (H15–16).

[9] This seems to be the rate given in the "purported" *Al-Zīj al-mumtaḥan* and in the Istanbul copy of the *Zīj* of Ḥabash; see Kennedy, *Survey*, pp. 146, 153.

[10] Battānī, *Zīj*, 3: 187–188.

[11] Kennedy, *Survey*, pp. 161, 163, 165.

II.4 [4]1. *ahl al-ṭalismāt* (the practitioners of [the art of] talismans) (Note: I have followed the voweling given by Bīrjandī, who also gives *ṭillismāt* as a possible alternative.): A talisman in its usual meaning is a magical charm inscribed with some combination of mysterious words or symbols. Nīsābūrī in his commentary would seem to indicate that the group referred to here are indeed magicians of a sort since they "transfuse (*yumāzijūna*) the faculties (*quwā*) of the celestial bodies to terrestrial recipients (*qawābil*) so that extraordinary effects (*āthār*) might come to pass."[12] It is somewhat odd that the concocters of such stratagems would come to be linked with trepidation. But we also find this association explicitly made by Battānī in his *Zīj* and by Bīrūnī in his *Al-Tafhīm li-awā'il ṣināᶜat al-tanjīm.*[13] But Bīrūnī, unlike later Muslim astronomers, explains *aṣḥāb al-ṭalismāt* as simply astrologers (*aṣḥāb al-aḥkām*). The source for connecting the *ahl al-ṭalismāt* with trepidation would seem to be Theon of Alexandria (4th c. A.D.) whom Bīrūnī and Ṣāᶜid al-Andalusī refer to in this connection.[14] And indeed Theon in his *Small Commentary to the Handy Tables* of Ptolemy attributes trepidation in its simple linear oscillatory version to οἱ παλαιοὶ τῶν ἀποτελεσματικῶν.[15] This phrase, however, has usually been rendered, just as we have noted in Bīrūnī's *Tafhīm*, simply as "ancient astrologers" without any reference to "talismans."[16] Accepting this, one could then view accounts such as we find in Nīsābūrī simply as later embellishments with no historical basis.[17]

Such a neat explanation has its problems, however. Both Battānī and Bīrūnī, this time in his earlier *Al-Āthār al-bāqiya ᶜan al-qurūn al-khāliya* (The Chronology of Ancient Nations), cite Ptolemy, not Theon, for their information concerning trepidation.[18] One might, as Neugebauer does, simply see this as a

[12] Nīsābūrī, *Tawḍīḥ*, Najaf MS 649, f. 26b, lines 17–18.

[13] Battānī, *Zīj*, 3: 190; Bīrūnī, *Tafhīm*, p. 101.

[14] Ibid.; Ṣāᶜid, *Ṭabaqāt*, pp. 40, 54 (= Fr. trans., pp. 86, 110).

[15] Tihon [1978], p. 236.

[16] See, for example, Neugebauer, *HAMA*, 2: 632 (n.7) and Dreyer [1953], p. 204.

[17] One should, however, note the obvious similarity of *ahl al-ṭalismāt* and ἀποτελεσματικοί, which could lead to such confusion. But this leaves open the question of when ἀποτελεσματικά, which means "astrological influences," (and is, not incidentally, another name for Ptolemy's *Tetrabiblos*) went from being used for prognostication to having the connotation of astral magic. For some interesting insight and speculation on this problem, see Pingree [1980].

[18] Battānī, *Zīj*, p. 190 (Latin trans., 1: 126–127); Bīrūnī, *Āthār*, pp. 325–326 (Engl. trans. p. 322). One might contend that the report from Ptolemy in the *Āthār* concerns only the information regarding the Greeks; the following statement about the "Chaldeans" in which the account of trepidation occurs would then be a separate report. Indeed, Sachau vowels the crucial word *ḥukiya* (it is reported) rather than *ḥakā* (he, *i.e.* Ptolemy, reported). But in the table on page 327 (Engl. trans., p. 323), the matter would seem settled since the information concerning the "Chaldeans" is attributed to Ptolemy.

misconstruction;[19] for example, Battānī as well as Bīrūnī in his earlier writing may have mistakenly attributed the commentary on the *Handy Tables* to Ptolemy rather than Theon. But against this is the fact that Bīrūnī cites a specific work, purportedly by Ptolemy, called *Al-Madkhal ilā al-ṣināᶜa al-kuriyya* (Introduction to Spherical Construction) that is clearly not the *Handy Tables*.[20] Furthermore Bīrūnī makes the additional connection between the *ahl al-ṭalismāt* and the "Chaldeans" (in the *Āthār*) or "Babylonians" (according to the *Tafhīm*). We should make the point that contrary to the statement of Neugebauer,[21] the linking here of trepidation with the "Chaldeans" or "Babylonians" is not an inference but rather a direct report. As this identification does not occur in Theon, one might suspect an additional source though any evidence at this point is indirect and rather insubstantial.

Whoever were the originators of trepidation, the basic idea as described here is quite simple.[22] The ecliptic orb accedes (*aqbala*) in the sequence of the signs at a rate of 1°/80 years for 640 years thus traversing 8° and then reverses direction and recedes (*adbara*) at the same rate and for the same length of time. Note that in this account of the ancient theory of trepidation, the vernal point is the fixed reference point, just as it is with a straightforward precession, and the so-called fixed stars perform the actual motion.[23] This reverses the relationship that seems to be indicated by Theon, whereby it is the stars that are fixed, while the solstitial points are claimed to move.[24] What Ṭūsī has evidently done is modify the theory as reported by Theon in order that it be compatible with his general cosmological perspective in which it is the vernal equinox that defines the reference system.

II.4 [4]3–6. *fa-samiᶜa dhālika baᶜd...ilā al-tawālī* (Some of the practitioners of this discipline...sequentially from its position): Since the period between Hipparchus and the early period of Islamic science did not witness a reversal in direction of the fixed stars,[25] one would have thought that the theory of trepida-

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[19] *HAMA*, 2: 631 (n. 4).

[20] *Āthār*, p. 325, line 23 (Engl. trans., p. 322, line 16); cf. Sezgin, *GAS*, 6: 96. The *Handy Tables* were known in Arabic as *Kitāb al-Qānūn fī ᶜilm al-nujūm wa-ḥisābihā wa-qismat ajzā'ihā wa-taᶜdīlihā*; cf. Sezgin, *GAS*, 6: 95. As for the actual identity of this work, I am at a loss. Ptolemy's *Planisphaerium*, usually called *Risāla fī Tasṭīḥ al-kura*, does not seem a likely candidate. Theon's purported work on the armillary sphere, extant only in Arabic translation, is a possibility; in any event it would be well worth investigating for a mention of trepidation; cf. Sezgin, *GAS*, 6: 101–102. On the attribution in Arabic sources of Theon's works to Ptolemy, see Neugebauer [1949], pp. 242–243.

[21] *HAMA*, 2: 598, 633 (n. 11).

[22] For details concerning the early history of trepidation, see *HAMA*, 1: 297–298, 2: 631–634. The account as presented in our commentaries is, of course, from the point of view of a cosmology of solid orbs.

[23] Cf. the commentary to II.3 [5]13.

[24] Cf. Neugebauer, *HAMA*, 2: 632–633.

[25] This should have taken place in 483 A.D.

tion would have been allowed to die an unmourned death. This was not to be, however. The theory was revived in several new guises whose purpose was to account for the differences found in the rates of precession, in particular between that of Ptolemy and those of the early Islamic observations. Here we find described one of these attempts whose basic approach is to superimpose a trepidation function upon a constant term of precession thus resulting in a variable rate of motion for the fixed stars but maintaining a constant direction for that motion. An example should help clarify the situation. If we take the classical rates for accession and recession, namely 1°/80 years, clearly we must add a precessional rate that is greater than 1°/80 years in order to guarantee that during recession the stars will have a motion that is in the direction of the signs. In particular, if we wish to have the stars move at a rate of 1°/100 years during the recessional period, one must postulate a precessional rate of 9°/400 years. Unfortunately this will give the stars a motion during the accessional period of about 1°/28.5 years, a rate considerably faster than any observed value. One could, of course, arrive at more acceptable values by simply changing the rate of accession and recession. For example, one could take the average rate of 1°/70 years and 1°/100 years as one's precessional value and then add or subtract an appropriate trepidation value. One could even retain the 640-year period from the classical theory since Ptolemy's value could fall between, let us say, 0 A.D. and 640 A.D. while the early Islamic value would be valid until 1280 A.D. Whether anyone followed such a procedure I do not know. Nīsābūrī, who expounds upon the brief remarks made by Ṭūsī, seems to think that the follower(s) of this theory retained the ancient value of trepidation. Finally we should note that Ṭūsī's statement that the vernal equinox "moves from its position" is rather confusing since the vernal equinox is a stationary reference point. What he probably wants to say is that the point on the ecliptic previously aligned with the vernal point has now moved from that position. This type of misstatement by Ṭūsī concerning reference points is quite widespread and is often remarked upon by the commentators.[26]

Who introduced this synthesis of precession and trepidation? Battānī (ca. 244–317/858–929) seems to refer to just such an idea in his *Zīj* (3: 190–191), so I am led to believe that we are dealing with a theory dating from the 9th century. Hāshimī (ca. 890 A.D.) in his *Kitāb fī ᶜilal al-zījāt* (Book of Explanations of the *zīj*es) mentions several persons who did work on what he calls *irtifāᶜ al-falak wa-inkhifāḍuhu* (the elevation and depression of the orb), which Kennedy and Pingree take to be trepidation.[27] Indeed one of these is Theon, lending credence to this interpretation. The others are al-Fazārī (ca. 770 A.D.), Yaḥyā ibn Abī Manṣūr (d. ca. 830 A.D.), Abū Maᶜshar (787–886 A.D.), and Ḥabash al-Ḥāsib (ca. 850 A.D.).[28] Now Hāshimī states explicitly that Abū Maᶜshar does not

[26] In this case by Bīrjandī. See also the commentary to II.11 [3].
[27] Hāshimī, p. 225.
[28] Hāshimī, f. 97r, lines 1–3.

agree with Theon and the others, so we are left with Yaḥyā, al-Fazārī, and Ḥabash as viable alternatives. I have been unable to find any other reference to Yaḥyā and al-Fazārī in connection with trepidation,[29] but we learn from Ṣāᶜid al-Andalusī that Ḥabash used "the motion of accession and recession of the ecliptic orb" in his first *zīj* (*i.e.* the one that followed the *Sindhind*) "according to the theory of Theon in order to correct the position of the stars in longitude."[30] He thus becomes the most likely originator of the theory mentioned by Ṭūsī.

II.4 [4]6–7. *wa-dhālika ayḍ*^{an}...*mā marra* (This would...those already mentioned): Here again we can see Ṭūsī's concern with providing physical models (cf. II.11 [21]).

II.4 [5]. This innocent looking paragraph concerns one of the most complex and, for the modern historian, most vexing problems in the history of Arabic astronomy. A full treatment of the theory referred to here and its development is beyond the scope of our commentary. What I propose to do instead is give a summary account of some of the results of my research thus far; at a later date, I hope to return to the very interesting problems posed by this paragraph and fill in the many gaps and details missing below.

In the modern period, it has generally been accepted since Delambre that the originator of the theory proposed here (or some approximation thereof) is Thābit ibn Qurra (d. 901 A.D.).[31] But the reasons for such an attribution are far from compelling. The treatise *On the Motion of the Eighth Sphere*, a work that does not seem to be extant in Arabic, exists in Latin translation[32] where, among other variations, the attribution of authorship is made as *Tractatus patris Ascen Thebit filii Chorat de motu octave spere*.[33] Though this may appear conclusive, it should be remembered that the Latin tradition is not free from errors as concerns the identification of authors of Arabic works.[34] What seems to me to carry more weight is the lack of any association of this theory with Thābit in the Arabic tradition despite a rather extensive literature dealing with similar models and tech-

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[29] It is significant that Ibrāhīm ibn Sinān (908–46 A.D.), who has a great deal to say about Yaḥyā's *Zīj al-Mumtaḥan* in his *Kitāb fī Ḥarakāt al-shams*, does not allude to any trepidation theory contained therein even though he sets forth his own theory of trepidation. Also there is apparently no mention of trepidation in the extant parts of the *Zīj al-Mumtaḥan* (see Kennedy, *Survey*, pp. 145–147).

[30] *Ṭabaqāt*, p. 54 (Fr. trans., pp. 109–110); virtually the same report occurs in Qifṭī, *Ta'rīkh*, p. 170. I do not know if any of the extant sections from the "*zīj*es" of Ḥabash correspond to this early one; cf. Kennedy, *Survey*, pp. 126–127.

[31] Delambre [1819], pp. 73ff and pp. 264ff; however, cf. p. 175.

[32] The text has been published several times, in particular by Millás Vallicrosa [1950], pp. 496–509 and by Carmody [1960], pp. 102–113 (2 versions).

[33] See Carmody [1960], p. 102 and the variants in MSS CSU, p. 107.

[34] One important example was the misattribution to Naṣīr al-Dīn of the commentary on Euclid's *Elements* that was printed in Rome in 1594; see Sabra [1969], p. 18.

niques.[35] In addition, there is some rather strong internal evidence for doubting Thābit's composition of this treatise. The author of *On the Motion of the Eighth Sphere* quotes al-Battānī's remarks concerning trepidation as follows (Neugebauer's translation):

> I do not see this variation proceed in a (fixed) proportion of velocity and slowness; if there existed any motion which we do not know and which we do not understand, those who come after us will observe it and verify it, as we did (with our predecessors).[36]

This is a fair translation of two excerpts from Chapter 52 of Battānī's *Zīj* (3: 191, lines 12 and 17–19). (The correspondence is surprisingly good considering that we are comparing the English translation of a Latin version of an Arabic paraphrase with the original Arabic.) Now Battānī completed his *Zīj*, at least in the form that we now have it, sometime after August 901 A.D., the date of one of his fundamental observations.[37] Since Thābit died in February 901 A.D., it would seem reasonable to conclude that he could not have written *On the Motion of the Eighth Sphere*. Nallino, however, accepting his authorship, assumes that Thābit must be referring to an earlier version of Battānī's *Zīj*, presumably written before 901.[38] But this leads to a number of untoward consequences. First we would need to conclude that Battānī, in revising his first version, did not deal with, or even mention, the treatise of his celebrated fellow Ḥarrānian despite being cited in it. He would have continued to treat trepidation in the rather unsophisticated version that we have discussed above and completely ignored the vastly more refined model of Thābit. When we combine this with the fact that no medieval Arabic author has yet been found who associates this model with Thābit, not even Thābit's grandson Ibrāhīm ibn Sinān who, as we shall see, treats of similar matters,[39] the claim that Thābit wrote *On the Motion* becomes

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[35] A similar argument was made by Duhem, *Système*, 2: 246ff.

[36] Neugebauer [1962a], p. 294; Carmody [1960], p. 104 (no. 21 of Version M) and p. 111 (no. 21 of Version N). Cf. Carmody's translation, p. 89.

[37] Nallino, "Praefatio" to Battānī, *Zīj*, 1: xxxii.

[38] Ibid. Hartner ("al-Battānī," *DSB*, 1: 508) follows Nallino. The evidence for two versions (*nusakh*) comes from Ibn al-Nadīm's *Fihrist*, p. 279 (repeated by Qifṭī, p. 281).

[39] If Thābit did write *On the Motion*, Ibrāhīm's ignorance or neglect of it would be especially surprising. Neither in the *Ḥarakāt al-shams* (Motions of the Sun) nor in his description of it in his account of his works written at age 25 (ca. 933 A.D.) (*Fī waṣf al-ma*^{c}*ānī…*) does he mention his grandfather. We should contrast this with his citation of Thābit's work on the mensuration of the parabola in his own *On the Area of the Parabola* (p. 57); indeed, he betrays a bit of family chauvinism when in *Fī waṣf al-ma*^{c}*ānī* he writes regarding this treatise that he "would not like al-Māhānī having written a work superior to that of my grandfather, and there not being among us [*i.e.* the Ibn Qurra family] someone who could surpass what he had done" (p. 29).

tenuous if not untenable.[40]

Who then wrote *On the Motion of the Eighth Sphere*? Before we can answer this rather difficult question, we must first deal with the related but nevertheless distinct problem of identifying the originator of the theory described by Ṭūsī in this paragraph. We should note that the proposed model consists of a single additional mover that will bring about both divergences mentioned previously, namely the variations in the obliquity and the precessional rate that seemingly had occurred between the time of Ptolemy and the Islamic period. Thus far, Ṭūsī's account is consistent with the model in *On the Motion of the Eighth Sphere* inasmuch as one does indeed find there a "moving ecliptic" that is in addition to a "fixed ecliptic" and the equator. The rest of the paragraph, however, can only be described as misleading at best, utter nonsense at worst. It is clearly impossible to move a sphere in such a way that every point on it describes a small circle; and even if such a feat of contortion were possible, it would be totally irreconcilable with the cosmological doctrines of medieval Islamic astronomy. It thus becomes a hopeless task to try to understand this paragraph as it stands.

Fortunately the commentators once again come to our rescue. Although all of them explain this very perplexing passage to some degree of adequacy, Khafrī, who has at his disposal the explanations of the earlier commentators as well as that of Shīrāzī, gives the most extensive and, in my opinion, the most intelligent account. We may summarize his exposition with the aid of Figure C1, which is adapted from his commentary.[41] Consider an orb with pole Y and equator AKGL at a fixed inclination to the equinoctial equator ABGD whose pole is E. Inside this orb we place the ecliptic (*i.e.* the orb containing the fixed stars) concentrically and such that its pole M is at a given distance from Y. Now when orb AKGL rotates, it will cause the ecliptic to move so that pole M will trace a small circle about Y. At the same time, every other point on the ecliptic, with the exception, of course, of the two points opposite the poles of orb AKGL, will also describe circular paths. This will be the motion of precession. We should note, however, that the reference points by which one may measure precession are no longer fixed as they are when the obliquity of the ecliptic is a constant value. This is most easily seen in the figure by examining the changing position of the solstice point Q. As the ecliptic pole moves along the circular path MNTS, the summer solstice will describe an oval path FQZR. Thus a given

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[40] Note that Duhem [1914], pp. 257–258, had already pointed to this problem in dating and used it to question Thābit's authorship and to advocate al-Zarqāllu's. Millás [1943–1950], pp. 487–494, countered Duhem by adducing Ṣāᶜid's testimony, which we shall discuss presently. While showing that Duhem was clearly wrong in attributing *On the Motion* to al-Zarqāllu, Millás did not really deal with Duhem's more substantial arguments against Thābit's authorship.

[41] Though it is rather schematic and not in perspective or to scale, it will serve our purposes here; it also gives an idea how complex spherical diagrams were presented in the manuscripts.

star, though moving at a constant rate, will appear to go faster or slower with respect to the solstices (and also, of course, with respect to the equinoxes). If the motion of the ecliptic pole is in the direction SMN, then accession will occur during the period the pole travels on arc SN, while recession will take place on arc NS, where N and S are the points of tangency between the solstitial colure and the circular path of the ecliptic pole. The obliquity will also vary since the distance of the ecliptic pole from the equinoctial pole increases during the motion of the ecliptic pole from M to T and decreases from T to M.

We can now return to Ṭūsī's description to determine how closely it is in accord with this account. Taking MNTS to be the "small circle," we can see that he is approximately correct in stating that accession will occur from the motion in one half of the circle, while recession will result from motion in the other half. But it should be clear that SN is not exactly equal to NS, a fact that the commentators, at least, are aware of. Furthermore the obliquity will increase from M, the midpoint of arc SMN, to T, the midpoint of arc NTS, while it will decrease from T to M. Ignoring the fact that SMN and NTS are not half circles, we again find that Ṭūsī's statements conform to this model. What remains puzzling is Ṭūsī's assertion that every point on the ecliptic moves about a small circle. According to the explanation of Khafrī, every point will move on a circle parallel to the path of the ecliptic pole (with the exception, of course, of the two points opposite the poles of the orb that moves the ecliptic). But it is unlikely that Ṭūsī would refer to these complete revolutions as "small circles." Khafrī seems to believe that Ṭūsī is referring not to actual points on the ecliptic but to the reference points such as the solstices. But these will describe, as Khafrī proves, ovals rather than circular paths. (The equinoxes are exceptions to this; they will oscillate between two extreme points with a straight-line motion along the equinoctial equator.)

Because of this ambiguity, it will be worthwhile to examine briefly the question of whether Ṭūsī could be referring to a model other than the one presented by the commentators. One possibility would be the one proposed in *On the Motion of the Eighth Sphere*. Here we find that the sidereally fixed "first of Aries" ♈ and "first of Libra" (not shown) do indeed undergo a circular motion (see Figure C2).[42] And furthermore the vernal equinox, indicated in the figure by the intersection of the "movable ecliptic" ♈ P and the equinoctial equator AB, will accede or recede according to the motion of "the first of Aries" in one half or the other of its circular path.[43] But Ṭūsī's statement concerning the continual increase or decrease of the obliquity according to the motion in a certain half of the circle or the other is not applicable to this model. For as we can see from Figure C3, the ecliptic pole M will describe a hippopede; hence, each of the maximum and minimum obliquities will be reached twice during the

[42] Figure C2 is adapted from Goldstein [1964], p. 233.

[43] Neugebauer [1962a], pp. 292–293.

revolution of "the first of Aries." Thus we must conclude, as do the commentators, that the previous model is the one that Ṭūsī is attempting to describe.

Let us now return to the problem of origination. Khafrī specifically identifies a certain Ibrāhīm b. Naṣr b. Sinān as the original proponent of the model illustrated by Figure C1.[44] It is reasonable to assume that the person in question is Ibrāhīm b. Sinān (908–946 A.D.), the grandson of Thābit b. Qurra. Further corroboration for this hypothesis comes from Bīrūnī, who in his *Al-Āthār al-bāqiya* mentions Ibrāhīm b. Sinān as one of two people (the other being al-Khāzin) who offered an explanation (*bayān*) for trepidation. He further states that this occurs in his *Kitāb Ḥarakāt al-shams*.[45] One of Ibrāhīm's main concerns in this treatise, which was completed sometime before 933 A.D.,[46] is to understand the reason for the different lengths of the year given by Ptolemy and by the astronomers of the Islamic period. He is reluctant to dismiss the ancient observations out of hand even though he does criticize Ptolemy's observational method at one point.[47] He must therefore accept the reality of a variable tropical year. If we recall that Arabic astronomers, unlike Ptolemy, considered the solar apogee to move with the motion of the ecliptic orb for cosmological reasons (this motion being in addition to any other proper motion of its own),[48] we can then see that a variable motion of the ecliptic could have the necessary effect upon the tropical year. Ibrāhīm remarks that the slowness of the motion of the solar apogee during Ptolemy's time may have been why he did not discover it; only later, with its increased speed due to the faster motion of the ecliptic (and thus also the rate of precession), did it become more apparent.[49]

The model he proposes is conceptually the same as that described by Ṭūsī and the commentators. It is not fully worked out, however, and no parameters are presented. This is not surprising in view of the circumstances under which Ibn Sinān tells us he has been forced to write this treatise. In his introduction, he laments the lack of access to his books (because of unspecified political reasons) and is thus reduced to approximating the observational results of Ma'mūn's astronomers as well as those of even his own father Sinān.[50] But here we face a puzzle. How are the commentators able to quote specific parameters for this model, in particular the 4° radius for the path of the ecliptic pole? It is not inconceivable that what we have in the extant text is a preliminary work that was

[44] Khafrī, *Takmila*, Damascus, Ẓāhiriyya MS 6727, f. 90b, lines 8–9. "Ibrāhīm b. Sayyār b. Thābit b. Qurra" occurs in Anon., Istanbul, Ahmet III MS 3316, f. 29a and in Shīrwānī, Ahmet III MS 3314, f. 102a.

[45] Bīrūnī, *Āthār*, p. 326. Cf. idem, *Taḥdīd*, p. 101 (= Engl. trans. p. 70).

[46] The new edition, due to A.S. Saidan, supplants the incomplete, corrupt, and scattered Hyderabad printing.

[47] Ibrāhīm ibn Sinān, *Ḥarakāt*, pp. 282–283.

[48] The motion of the solar apogee in Islamic astronomy has been dealt with by Toomer [1969].

[49] Ibrāhīm ibn Sinān, *Ḥarakāt*, pp. 282–284.

[50] Ibid., pp. 275–276.

later revised and refined. Indeed Ibrāhīm states that he wishes here to present only the basic ideas; later, he tells us, he may be able to add further calculations and corroborative observations to the text.[51]

Is *On the Motion of the Eighth Sphere* this promised revision? Before presenting the strictly historical evidence, we should examine the relationship of Ibn Sinān's model with that of pseudo-Thābit's. One fundamental difference between the two models concerns their *hay'a*, or physical configuration. The model presented in *Ḥarakāt al-shams*, at least in the interpretation of later commentators (which is not inconsistent with the text itself), adheres to the physical requirements laid down by the Islamic *hay'a* tradition. Thus the additional mover AKGL with pole Y moves the ecliptic in a physically unobjectionable way so that each point on the ecliptic must travel in a complete revolution that parallels the motion of the ecliptic pole (see Figure C1). But this places a great restriction, in particular, on the ability of the model to deal with the discrepancy between the value for the obliquity given by Ptolemy and that found by the early Islamic observers. In order to attain a range for precession between 1°/100 years and 1°/60 years, one would need to postulate a mean motion of the ecliptic pole of 1°/80 years and about a 4° radius for its path about point Y. But this would mean that the obliquity could have an 8° range of variation (from, say, 16° to 24°), an amount far in excess of what could reasonably be expected based on the modest change from the time of Ptolemy to the 9th century A.D. (from 23;51,20° to 23;33°). This basic flaw in the model would become obvious to any competent mathematician as soon as he attempted to use it with actual parameters; indeed, virtually all the commentators on the *Tadhkira*, as well as Shīrāzī, recognize this defect and dismiss the model for this reason, among others.[52]

Let us assume for the moment that the author of *On the Motion of the Eighth Sphere* knew Ibn Sinān's model. In experimenting with it, he would presumably have realized rather quickly that the problem was to find a means of achieving a relatively large variation in the precessional rate that would occur simultaneously with a relatively small change in the obliquity. The small circle would therefore need to be transferred from the vicinity of the pole of the fixed ecliptic to the fixed ecliptic itself. But the requirement of *hay'a* that the complete revolution of a point on the small circle be paralleled by complete revolutions of all the other points on the ecliptic would need to be abandoned; otherwise, the complete revolution of the ecliptic pole on a great circle would cause an even more drastic variation in the obliquity than that of the previous model. To someone well-versed in the *Almagest*, an alternative that would have presented itself would have been Ptolemy's latitude theory for the epicycles of the planets.[53] The epicyclic poles likewise describe circular paths with minimal disturbance to the rest of the epicycle.[54] Thus in the model adopted by the author of *On the*

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[51] Ibid., p. 276.

[52] Shīrāzī, *Nihāya* (maqāla II, bāb 4); *Tuḥfa* (bāb II, faṣl 7).

[53] *Almagest*, Bk. XIII, Ch. 2.

[54] See II.10 [4] and the commentary to II.11 [14].

Motion of the Eighth Sphere, there is no "mover" in the sense used by Ṭūsī; rather, the two points, "the first of Aries" and "the first of Libra," move along circular paths without the other points on the ecliptic participating in a parallel motion. This approach to astronomical modeling is, to say the least, contrary to the thrust of eastern Islamic astronomy from at least the late 10th or early 11th century A.D. (Ibn al-Haytham [d. ca. 1040 A.D.] criticizes just this type of model in his *Maqāla fī ḥarakat al-iltifāf*;[55] Ṭūsī, as we shall see, continues the work of his predecessor and uses his couple to help give a physical structure [*hay'a*] to Ptolemy's models.[56]) The point I wish to make here is that the author of this treatise has been willing to sacrifice at least some of the principles of *hay'a* in order that his model be able to explain the observations available to an Arabic astronomer of the 9th or 10th century A.D., namely the values of precession and obliquity reported by Hipparchus, Ptolemy, and the astronomers of 9th-century Iraq. Considering the complexities of the model, in particular the changing obliquity based on the figure 8 motion of the ecliptic pole and the difficulty of coordinating this with instantaneous rates of precession resulting from a sine function, this represents no mean feat.[57]

Having briefly set forth our analysis of the mathematical relationship between the two models, we should now turn to the historical evidence to determine the actual connection between them. First it is important to note that the model of *On the Motion of the Eighth Sphere* is virtually unknown in the Islamic East. Except for a brief and vague mention by Bīrjandī (who, of course, is quite late) of Zarqāllu's model for trepidation, which is similar in crucial

[55] Even though this work is not extant, we know a fair amount about its contents; see Sabra [1979] as well as the commentary below to II.11 [16]. It is interesting that even as early as Battānī the cosmological issue plays a role in evaluating models. With regard to the ancient model of trepidation described by Theon, Battānī not only criticizes it as being contrary to observation but also as lacking a physical structure (*lā yatahayya'u*) (*Zīj*, 3: 190).

[56] For more details, see our commentary to II.11 [14–19].

[57] Admittedly, this dogmatic presentation of my investigation of the model must here be taken on faith since a full exposition would take us far afield. Neugebauer [1962a], Goldstein [1964], and Mercier [1976–1977] have been the most recent commentators on trepidation, and they all make valuable contributions. However, their ignorance of the sources mentioned above as well as their somewhat disparaging attitude toward this theory, especially in the case of Mercier, have blinded them to the truly ingenious ability of the author of *On the Motion of the Eighth Sphere* to construct a model that fits the data at hand to a remarkable degree. In the case of Neugebauer, his disdain of anything to do with cosmology forces him to dismiss the very critical relationship between trepidation and the variability of the tropical year (see especially pp. 293–294) and thus miss the important observational data that is crucial for evaluating the theory. We should recall here that such an understanding of the relationship of the tropical year and trepidation is taken as a given in the Arabic sources; for example, this is one of the main points made by Battānī in his discussion of trepidation.

respects to that of pseudo-Thābit,[58] there does not seem to be any knowledge of this trepidation theory outside of Spain and the Maghrib. The impression that this model is a western Islamic, and in particular a Spanish, phenomenon is further strengthened by the fact that *On the Motion of the Eighth Sphere* was available to Gerard of Cremona for translation in Spain. Even stronger evidence is provided by the *Toledan Tables*, which contain a section on trepidation whose tables are identical in all details with those of *On the Motion of the Eighth Sphere*.[59] But an important passage by Ṣā^{c}id al-Andalusī (1029–70 A.D.) casts doubt on a Spanish origin:

> Among [the Arab astronomers] is al-Ḥusayn b. Ḥamīd, known as Ibn al-Ādamī, the author of the great *zīj* (handbook) that was completed after his death by his student al-Qāsim b. Muḥammad b. Hishām al-Madā'inī known as al-^{c}Alawī. He called it *Kitāb Naẓm al-^{c}iqd* and published it in the year 338 [H. = 949 or 950 A.D.]. It is a book comprehending the making of [planetary] equations and including the principles of the cosmography of the orbs and the calculation of the motions of the stars according to the doctrine of the *Sindhind*. Concerning the motion of accession of the orb and its recession, he mentions [in the *zīj*] what no one had mentioned previously. Before this book had reached us, what we had heard about this motion was incomprehensible and did not incorporate an established principle (*qānūn*). Then this book came to us and we understood the manner (*ṣūra*) of this motion. This was the reason [following the variant, p. 110, and Qifṭī, p. 282] for working with [or investigating] it [the motion] for some time until it became clear to us what we do not believe was clear to anyone else. We pursued various things about it that I have explained in my compilation (*kitābī al-mu'allaf*) "On correcting the motions of the stars" (*fī iṣlāḥ ḥarakāt al-nujūm*).[60]

Because of Ṣā^{c}id's very close association with the *Toledan Tables*,[61] I think that it is reasonable to assume that the trepidation theory that is found there is

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[58] See Goldstein [1964], pp. 238–244.

[59] See Toomer [1968], pp. 118–122.

[60] *Ṭabaqāt*, pp. 57–58 (French trans., p. 114).

[61] On this question, see now the masterful treatment by Richter-Bernburg [1987], pp. 385–390. I regret not having had the benefit of this article before embarking on my own, much more limited, investigations of Ṣā^{c}id. Although I am in substantial agreement with Richter-Bernburg, I am uneasy with his hypothesis that Ṣā^{c}id began compiling the *Toledan Tables* in the last two years of his life. I would like to leave open the possibility that the work referred to by Ṣā^{c}id as *Fī iṣlāḥ ḥarakāt al-nujūm*, which he calls *kitābī al-mu'allaf* (my compilation?), is substantially similar to the *Toledan Tables* themselves.

very similar, if not identical, to the one being discussed in this passage.[62] But since the trepidation theory of the *Toledan Tables* is identical to that of *On the Motion of the Eighth Sphere*, we are led to the inevitable conclusion that the model of the latter treatise, if not the work itself, was contained in Ibn al-Ādamī's *zīj*. Does this mean that Ibn al-Ādamī, or less likely his student al-ᶜAlawī, is pseudo-Thābit? I very much doubt that this is the case; my own inclination is to believe that Ibn al-Ādamī incorporated the work of his contemporary and fellow Baghdādī Ibrāhīm ibn Sinān, who, after gaining access to his books and the reports of observations, modified his original theory in the manner we have outlined above. But for some reason, this revised version, unlike *Ḥarakāt al-shams*, was unknown in eastern Islam, perhaps because it was only to be found in Ibn al-Ādamī's *zīj*, a presently nonextant work that does not seem to have had a wide circulation in the east.[63] On the other hand, it seems, on Ṣāᶜid's testimony, to have been well-known and appreciated in the west. If *On the Motion of the Eighth Sphere* were indeed by Ibrāhīm ibn Sinān ibn *Thābit ibn Qurra*, this would offer us an explanation of how it came to be ascribed to Ibrāhīm's much more well-known grandfather by the Latin translator, who either dropped or misread part of the name.[64] What remains puzzling is why Ṣāᶜid did not know the author of the trepidation theory that he adopted, whereas the Latin translator did, though imperfectly;[65] one possible explanation is that the attribution only occurred in some copies of Ibn al-Ādamī's *zīj*. Admittedly my proposal as to the authorship of *On the Motion of the Eighth Sphere* remains speculative, but it does have the virtue of accounting for most of the known facts.

II.4 [6–7]. The question of the dynamics of celestial motion was an issue that was raised, as one might expect, along with the other physical questions associated with *hay'a*. For the eccentric orbs and epicycles, there was no problem since they were simply carried inside other orbs with different centers. One might perhaps worry about the interface of the encompassing and the contained orbs in such a situation; there had to be complete freedom of movement for the contained orb to rotate with its own proper motion. But since

[62] This was the opinion of Millás [1943–1950], pp. 27, 492–494, who originally pointed out the significance of this passage for understanding the history of trepidation.

[63] It was known to Ibn Yūnus (pp. 126–128), but it is not mentioned in the *Fihrist* (p. 280). Qifṭī (pp. 270–271, 282) derives his information from Ṣāᶜid. Cf. Sezgin, *GAS*, 6: 179–180 and Kennedy, *Survey*, p. 127.

[64] An explanation whereby someone dropped "Ibrāhīm ibn Sinān" leaving "Thābit ibn Qurra" is appealing but clearly incomplete given the numerous Latin manuscripts that not only give Thābit's name but additionally his correct filionymic of Abū al-Ḥasan (patris Asen) (see p. 400 and footnote 33). The latter, we should note, would have been reasonably well-known in Spain (for example, Ṣāᶜid gives it); it does not overly strain our credulity to imagine a process whereby the filionymic is added along the way.

[65] Ṣāᶜid does not mention Ibrāhīm ibn Sinān in his *Ṭabaqāt* even though he is aware of other members of the Ibn Qurra family (p. 37=Fr. trans., pp. 81–82).

the aether was by nature a perfect substance and not subject to friction, I doubt if this really constituted much of a problem.

On the other hand, there *was* a problem in understanding how one orb could move another orb concentric to it. Part of the reason for the difficulty arose because of a somewhat different conception of the orbs in the Arabic *hay'a* literature from what one might find, say, in Ptolemy's *Planetary Hypotheses* and in the less-specialized Arabic literature such as Ibn Rushd's *Talkhīṣ mā ba^{c}d al-ṭabī^{c}a* or the *Rasā'il* of the Ikhwān al-Ṣafā'. There the heavens are stated to be a single, living being. Thus the daily motion is simply the motion of the whole, and the other orbs are then seen as parts of this whole.[66] But in the *hay'a* literature, the daily motion is caused by the ninth orb, which is a discrete orb as defined in I.1 [15], namely a solid body contained between two spherical surfaces. As such, there must be some way for it to move the orbs below it. This interpretation is confirmed by the first sentence of paragraph [7] in which Ṭūsī states that the eighth orb must be moved by the ninth. One way for this to occur would be by means of the classical solution, such as that proposed by Eudoxus, whereby the poles of the eighth are attached to the ninth orb. But the commentators reject this solution on the grounds that this implies that the aether would somehow be different at the point of attachment. There are even greater difficulties involved in the case of one orb moving another concentric to it when the axes of the two orbs are collinear. One instance of this involves the movement of the apogees of the planets and the luminaries. The ecliptic orb must move all the pareclipitic orbs, which are concentric and share the same poles, so that the apogees will share in the motion of precession. But in addition, the parecliptics of the sun and moon must also have a proper motion as dictated by the motions of the solar apogee and the lunar nodes.[67] Another example is brought about by Ṭūsī's lunar model. He surrounds the lunar epicycle with another orb, called the *muḥīṭa*, that is concentric and coaxial with it. But nevertheless it is expected to move the epicycle with a circular motion.[68] It is clear that in such a case mere attachment would be useless for moving the enclosed orb; nor can we resort to dragging, friction or some similar means since all are precluded in the heavens.

The manner chosen by the commentators to explain the moving of one orb by another in such cases is an interesting one. Basically they reject the approach represented by physical attachment and instead resort to what one may call action at a distance. What this amounts to is that the soul of the encompassing orb may have a sufficient moving faculty to cause the enclosed orb to move as well. The succinct statement of Jurjānī is fairly typical of what we find in the other commentators:

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[66] For Ibn Rushd, see *Talkhīṣ*, p. 133. For Ptolemy, see his *Planetary Hypotheses*, British Museum MS Arab. 426, f. 93a, lines 22–23 (reproduced in Goldstein [1967], p. 36); German translation by L. Nix in Heiberg [1907], p. 112. Cf. Plato, *Timaeus*, 32d.

[67] See II.6 [3] and II.7 [8].

[68] See the commentary to II.11 [4].

From antiquity, it has been well-known that in the celestial region an enclosing [orb] (*al-ḥāwī*) will move [another] contained within it (*al-maḥwī*) by necessity (*bi-'l-ḍarūra*) if their two centers are different and if the axis of the enclosing [orb] does not pass through the center of the contained [orb]. Otherwise, if the enclosing [orb] moves but the contained [orb], whether it be an epicycle or an eccentric, does not, consequences will follow that will violate the principles, specifically tearing and mending or expansion and contraction. [The moving of one orb by another] might also occur by attachment (*bi-'l-tashabbuth*) when the poles are different [but the centers are the same]. The poles of the contained [orb] are attached to two points on the concave surface of the enclosing [orb]... The poles do not depart from the two points; they revolve with them thus causing the contained [orb] to move with the revolution of its poles. This has been rejected since the postulated points on the concave surface of the enclosing [orb] are of the same essence (*māhiyya*) on account of its being simple; thus the attachment of the two poles of the contained [orb] to two designated points on the enclosing [orb] to the exclusion of any other points is implausible.

It is clear from the statement in the text, as illustrated by the motion of the occupant of a ship, that the motion of the contained [orb] results from the motion of its place since that which is in place (*al-mutamakkin*) has the same status as a part of that place. So just as a part moves with the motion of the whole, that which is in a place will also move with the motion of the place. This always holds true for motion of displacement (*al-ḥaraka al-ayniyya*). As for motion in place (*al-ḥaraka al-waḍᶜiyya*), it will necessarily hold for the first case that we have described and will possibly hold for the other cases, namely when the orbs are concentric (whether or not the axes are the same) and when the orbs have different centers but the axis of the enclosing [orb] passes through the center of the contained [orb]. [In these latter cases], the moving soul (*al-nafs al-muḥarrika*) of the enclosing [orb] may have a sufficient faculty (*al-quwwa*) to move the contained [orb], and hence will move it, inasmuch as every action is not contingent upon a corporeal instrument (*āla jusmāniyya*), or it may not have [a sufficient faculty] whereupon it will not move [the enclosed orb].

The contained [orb] may then, in addition to its motion due to the enclosing [orb], have a proper motion since it is not impossible for a single body to have two motions—one accidental, the other essential. These two motions may or may not be in the same direction...[69]

[69] Jurjānī, *Sharḥ*, Damascus, Ẓāhiriyya MS 3117, ff. 36b-37a.

II.4 [7]5–7. *wa-kull kawkab yusāwī...wāḥida* (Every star whose latitude is equal...latitude): This sentence is badly worded. The solstice point is, of course, not on the equinoctial equator; the point of tangency in question will be at the intersection point of the solstitial colure and the equinoctial. I am unable to offer any suggestion as to how to rectify the text.

II.4 [9]. My voweling and pointing of constellation names follows a rough consensus of the manuscripts; variants may be found in the apparatus. Note that "corrupt" forms may result (*e.g. qayqāwus* for Cepheus). Whether what we have in the manuscripts indicates standard late medieval pronunciation is difficult to judge based on such a small sample. Compare Kunitzsch [1974], pp. 172–203.

II.4 [10]. Aristotle held that the Milky Way was a sublunar phenomenon (*Meteorology*, Bk. I, Ch. 8), but this was not universally accepted in antiquity. By the Islamic period, such writers as Ibn al-Haytham and Bīrūnī had made convincing arguments for its occurrence in the celestial sphere, a position that Ṭūsī here follows without hesitation. See Kunitzsch for an elaboration and bibliographic details (*EI*², 5: 1024–1025).

II.4 [11]. For a listing of the lunar mansions and an extensive bibliography, see Kunitzsch, *EI*², 6: 374–376.

II.4 [12]. Ṭūsī's remarks here tend to confirm some of the arguments we have made in Volume One (pp. 37–38) that the *Tadhkira* should be considered a summary and that each branch of astronomy had its own specialized treatises.

Book II, Chapter Five

Ṭūsī here introduces and explains the Ptolemaic "models" (*uṣūl*, the Arabic translation of the Greek *hypotheses*) that he will use in developing his cosmography, namely concentrics, eccentrics, and epicycles. We should note that he gives both plane and solid versions. The latter is necessary for those who wish "to understand the principles of motion" (II.5 [10]11). It is interesting that part of this introduction consists of giving the proportions by which one may achieve retrogradation, stations, and variations in direct motion. This seems somewhat out of place in an "elementary" text; indeed, Ptolemy deals with this difficult problem in a separate chapter.[1]

II.5 [3]21. *al-buᶜdayn al-awsaṭayn* (the two mean distances): See Fig. T1; for the epicycle's mean distance, see the next paragraph. Note that this "mean

[1] *Alm.*, Bk. XII, Ch. 1; for extended discussions of this aspect of Ptolemaic astronomy, see Pedersen [1974], pp. 329–349, *HAMA*, 1: 190–206, and Neugebauer [1959].

distance" is the position at which mean motion occurs; cf. II.5 [9]1–2_2. For an alternative "mean distance" that is actually based on distance, see II.6 [5]1–3.

II.5 [5]. For this equivalence of the eccentric and epicyclic hypotheses, see *Alm.*, pp. 148–150 (H225–227).

II.5 [6]8–9. *fa-li-dhālik ḥakama Baṭlamyūs...min al-tadwīr* (Therefore Ptolemy considered...than the epicycle): See *Alm.*, III.4, p. 153 (H232).

II.5 [8]6–8. *nisbat al-khaṭṭ al-wāṣil...niṣf quṭr al-tadwīr* (the ratio of the line connecting...radius of the epicycle): This version of the proportion is given in MSS FL and in the margins of MSS DT. (MS L substitutes "nearest distance" for both occurrences of "perigee".) A note in MS D states that this is from "the new, emended version" (*al-iṣlāḥ al-jadīd*), which confirms the evidence from the manuscripts that this variant comes from the "Baghdad version." In the original "Marāgha" version of the text we find the following:

> Let us make the ratio of the eccentric radius to the [distance] between the two centers the same as the ratio of the [epicycle's] deferent radius to the radius of the epicycle.

What makes the Baghdad emendation interesting is that it gives a proportion that is clearly useless since it does not preserve the relation of the extremal distances in the two models.[2] For this to occur, one would need to change the text so that "eccentric radius" reads "eccentricity" or else change "radius of the epicycle" to "radius of the deferent of the epicycle"; the latter change is the most easily made to the Arabic text since it only involves the addition (or perhaps substitution) of one word. I do not believe, however, that we are dealing with a copyist error. Most of the commentators, with the exception of ᶜUbaydī, simply ignore the "improvement" and use the older passage. Kamāl al-Dīn al-Fārisī in his gloss on this paragraph and the next refers to this variant as "off the mark" (*baᶜīd*).[3] Bīrjandī shows the two versions are equivalent by substituting "radius of the deferent" for "radius of the epicycle," but my suspicion is that this is his own emendation since it is attested nowhere else.

Why would Ṭūsī have made this change, one that would have been inconsequential even if he had gotten it right? There seem to be two related possibilities. He may have wished to give this proportion in the same terms (*i.e.* of nearest distances) as those of the proportion stated later in this paragraph (lines 11–15) that establishes the limiting conditions for the occurrence of stations and retrogradation. Or perhaps he may have been attempting to correct the mistake

[2] On my justification for following the Baghdad version even when it is "wrong," see the introduction, §2.M2, p. 87.
[3] *Ḥāshiya*, Najaf MS 649, f. 111a.

in this paragraph in which he states that the motions of the two concentrics (those of the eccentric and the epicycle) should be equal so that the two models be equivalent. In order to delve into this question a bit more deeply, we will need to refer to Figure C4, adapted from the *Almagest*,[4] in which circle ABGD with center E represents both the epicycle and the eccentric, Z is the ecliptic center in the epicyclic model, and K is the ecliptic center in the eccentric model.[5]

As shown in the *Almagest* (pp. 561–562 [H463–464]) retrogradation will occur when

$$EG : GZ > \text{speed of epicycle} : \text{speed of planet.} \qquad (1)$$

Remembering that for Ṭūsī the motion of the epicycle center is brought about by a concentric deferent and that for him the "motion of the epicycle" is what Ptolemy calls "the motion of the planet," we obtain Ṭūsī's proportion corresponding to (1):

$$\text{motion of epicycle} : \text{motion of its concentric deferent} > GZ : EG. \qquad (2)$$

In order that the two models be equivalent, the motion of the deferent in the eccentric model must be equal to the sum of the deferent in the epicyclic model plus the motion of the planet on the epicycle; furthermore, the motion of the planet on the epicycle must be equal to its motion on the eccentric.[6] Thus one may obtain the following from (2):

$$\text{motion of eccentric} : \text{motion of its concentric deferent} > GZ : EZ. \qquad (3)$$

Using $GZ : EZ = KG : EG$ (following Bīrjandī's emendation of the Baghdad (β) version), Ṭūsī's proportion for retrogradation in the eccentric model follows immediately from (3), namely

$$\text{motion of eccentric} : \text{motion of its concentric deferent} > KG : EG. \qquad (4)$$

The problem is, however, that Ṭūsī states that the motion of the two concentric deferents are equal (*mutashābihatayn*),[7] which would preclude (3) following from (2). But if one were to take the incorrect proportion of the Baghdad (β) version, then $GZ : EG = KG : EG$. Applying this otherwise worthless proportion would allow one to use the incorrect relation of the motions of the

[4] *Alm.*, p. 556 (H453).
[5] On the significance of using a single figure to illustrate the possibility of transforming one model into the other, see Neugebauer [1959], pp. 8–9.
[6] *Alm.*, p. 558 (H455). For a reasonably clear explanation of this, see Pedersen [1974], pp. 339–340 (especially Fig. 11.5). Cf. Neugebauer [1959], pp. 6–8.
[7] Its use here to mean equal rather than uniform is confirmed by the commentators.

concentric deferents so as to obtain (4) directly from (2) showing that sometimes two wrongs do make a right.

All the commentators[8] recognize Ṭūsī's mistake concerning the ratio of the concentric motions and correct it. What makes it even more puzzling is that Ṭūsī himself, as Bīrjandī points out, got it right in his recension of the *Almagest* written many years before the *Tadhkira*.[9] It is reasonable to assume that Ṭūsī, or someone else who pointed it out to him, recognized that something was amiss in the Marāgha (α) version. But instead of correcting the relationship of the concentrics, he instead changed the underlying geometrical relationship between the two models in order that the motions of the deferents, just as those of the eccentric and the epicycle, could be equal. Whether this was an absent-minded mistake or a misguided attempt at elegance is not clear.

II.5 [8]24. *wuqūfayn* (two stations): The technical term for station is *maqām*.

II.5 [9]. Referring to Figure C4, we obtain the following proportions:[10]

motion of eccentric : motion of its concentric deferent = KH : ½DH

motion of epicycle : motion of its concentric deferent = HZ : ½BH

(As we noted above, the motions of the eccentric and the epicycle are what Ptolemy calls the "speed of the planet," while the motions of the concentrics correspond to his "speeds of the eccentric and the epicycle.")

II.5 [9]3$_2$. *fī al-tadwīr* (in the case of the epicycle): This phrase, which occurs in MSS GL and in the margins of MSS DF, would thus appear to be a late insertion. It was added to indicate that the mean speed was the motion of the concentric in the case of the epicycle but not the eccentric model.

II.5 [10]21–24. *wa-minṭaqatuhu madār markaz al-tadwīr...li-l-buᶜd al-abᶜad* (Its [inner] equator is the circuit of the epicycle's center...the farthest distance): For a discussion of the curious use of *minṭaqa* (equator) to mean the circuit of the epicycle center, see the commentary to II.11 [4]. In order to distinguish this meaning of *minṭaqa* from its standard usage, I add "inner" in brackets in my translation since this circuit occurs inside an orb. Figure C5 indicates the two possibilities for the concentric inner equator referred to in the text. Note that *muḥādhiya* (facing) in the phrase "a point facing the farthest distance" has the import here of "adjacent" rather than "opposite" (cf. I.1 [2]).

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[8] This includes Shīrāzī in the *Tuḥfa* (II.8, Mosul MS 287, f. 57b) but curiously not in the *Nihāya* (see II.5, Istanbul, Ahmet III MS 3333, f. 62b).

[9] Ṭūsī, *Taḥrīr al-Majisṭī*, XII.1, Damascus, Ẓāhiriyya MS 7790, f. 150b.

[10] Cf. *Alm.*, p. 559 (H458).

Book II, Chapter Six

II.6 [1]11–12. *qarīb*[an] *min intiqālāt al-thawābit* (that is approximately the movement of the fixed stars): Taken in isolation, this phrase seems to imply that Ṭūsī recognizes that the solar apogee may have a proper motion of its own that is distinct from precession. I do not know if he is aware of the work of Zarqāllu though one may assume that he knows Bīrūnī's discussion.[1] The setting of the solar apogee motion equal to precession had been made in the early period of Islamic science by (pseudo-) Thābit, Battānī, and others. On the solar apogee problem in Islam, see Toomer [1969], Hartner and Schramm [1963], Hartner, "al-Battānī," *DSB*, 1: 510–511, Neugebauer [1962a], p. 267, and Morelon [1987], pp. LIII–LVII, LXIV–LXXV and 189–215.

II.6 [2]15–16. *idhā naqaṣa...yaqūlu bihā* (the motion of the apogee being subtracted from the mean motion among those who propound it): The motion of the sun resulting from the eccentric orb moving it in the sequence of the signs combined with the motion of the apogee, which moves the eccentric orb itself sequentially, yields the mean motion. Thus the motion of the eccentric alone is obtained by subtracting the motion of the apogee from the mean motion.

II.6 [2]22–23. *wa-Baṭlamyūs...absaṭ* (Ptolemy...simpler): See *Alm.*, III.4, p. 153 (H232) and compare II.5 [6].

II.6 [3]2–3. *wa-ammā ᶜalā aṣl al-tadwīr...huwa al-mumaththil* (For the epicyclic model...the parecliptic): If I understand this phrase, Ṭūsī seems to be saying that the eighth orb suffices for moving the orbs in the epicyclic but not in the eccentric model, a position I find puzzling. For if the eighth orb moves everything below it, should it not move the eccentric parecliptic as easily as the epicyclic deferent?[2] There seem to be several requirements at work, not all of which are entirely compatible with one another. On the one hand, one *must* have a concentric parecliptic in the eccentric model in order to fill what would otherwise be a void between the sun's eccentric and the orbs of Mars above and Venus below. But an orb usually moves with its own motion,[3] and this may be why Ṭūsī has it move with its own motion rather than be moved by the eighth orb. On the face of it, however, this would seem to contradict the statement in this paragraph that "[the eighth orb] moves everything below it." The epicyclic deferent, on the other hand, already has a motion of its own. For consistency, one could surround this deferent with a parecliptic that would then move the apogee, but this may have appeared to Ṭūsī to be a gratuitous multiplication of

[1] *Qānūn*, Bk. VI, Chs. 7–8 (2: 650–685).
[2] Cf. the commentary to II.4 [6–7].
[3] See the commentary to I.1 [15].

Book II, Chapter Six

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[1] *Qānūn*, Bk. VI, Chs. 7–8 (2: 650–685).
[2] Cf. the commentary to II.4 [6–7].
[3] See the commentary to I.1 [15].

orbs.[4] Khafrī, following Jurjānī, wishes to have the parecliptics in all cases have their own motion rather than depending on the eighth orb; he even suggests that the motion of the deferent in the solar epicyclic model should be a combination of its own motion and the motion of the apogee to avoid having it be moved by the eighth orb, which is one way of getting around Ṭūsī's difficulty here.[5] In conclusion, I think that it is safe to say that there is considerable confusion and ambiguity concerning some aspects of the dynamical problem despite a brave front in II.4 [6–7].

II.6 [4]11–13. *wa-yakūnu bi-qadr...sittīn* (it depends...eccentric being 60): Battānī's value for the eccentricity was 2;04,45,[6] which was also given by Abū Ja^c^far Muḥammad ibn al-Ṭabarī (second half of the 11th century).[7] Bīrūnī used[8] 2;05 as did, according to Kennedy, Ḥabash, Abū al-Wafā' al-Būzjānī, and the Banū Mūsā.[9]

II.6 [4]12. *aṣḥāb al-arṣād* (observational [astronomers]): This phrase conveys the sense that those who conduct observations form a distinct, specialized subgroup within the astronomical profession. But since Ṭūsī himself was both an observer and a theoretician, it would be unwise to interpret this dichotomy too strictly. Compare *ahl al-raṣad* in II.7 [18]3.

Book II, Chapter Seven

II.7 [1]. Ṭūsī here follows his normal procedure of listing the fundamental observations that provide the basis for the orbs, motions, and anomalies that will be listed later in this chapter. In paragraph [14], he explains how most of these observations can be explained by the Ptolemaic orbs and motions.

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[4] One should compare this with the moon in which a concentric parecliptic surrounds a concentric inclined orb. But the lunar and solar cases may have not been considered strictly comparable since the lunar parecliptic must account for the motion of the nodes, which is a combination of a proper motion and the motion of the fixed stars; see II.7 [8].
[5] Khafrī suggests a similar solution to the problem of the "enclosing sphere" in II.11 [5].
[6] Battānī, *Zīj*, Ch. 28, 3: 73 (Latin trans., 1: 48).
[7] Kennedy and Hamadanizadeh [1965], p. 444.
[8] Kennedy and Muruwwa [1958], p. 115.
[9] Kennedy [1960], p. 170; as-Saleh [1970], p. 143.

II.7 [1]18–21. *wa-ḥarakatuhu ᶜalā dhālik al-madār...bi-zamān qalīl* (Its motion on that circuit...to what was comparable to it): The manifold significations of *ikhtilāf* (*lit.*, difference) and its derivatives make a consistent translation virtually impossible.[1] Here I have translated *ikhtilāf* as anomaly, but this should not be understood in the technical sense explained in detail in II.7 [16–18], pp. 156–159. Nor should it be understood as the proper anomaly (*khāṣṣa*) defined in paragraph [29] even though the return to the same proper anomaly, which is the return of the planet to the same location on the epicycle, is the explanation for the return to a comparable speed.[2] In the present context, Ṭūsī is reporting observations to be explained, not the models that are meant to explain them. Thus an *ikhtilāf* is simply an irregular, or anomalous, speed, *i.e.* one that differs from the mean. One should be careful not to think of a return of an anomalous speed in terms of a fixed rate. As he mentions here, the return is to "what is comparable," meaning that it is relative. Thus a return to a slowest or fastest value would be to the relatively slowest or fastest speed during a revolution. Clearly, though, one could not easily observe the return of intermediate rates of speed without some model to depend on.

II.7 [1]21–22. *fī al-buṭ'...fī al-surᶜa* (during the slower motion...during the faster motion): The slower motion of a planet occurs when its direction on the epicycle is opposite that of the deferent, while its faster motion is when the two directions are the same. The dividing points between the slower and faster motions are the two mean distances as defined in II.5 [3–4]. (Compare II.5 [5] and [7]).

II.7 [7]17. *al-jawzahr*: Hartner ("Djawzahar," *EI*[2], 2: 501–502) details the history and etymology of the word; he states unequivocally that it derives from the Avestan *gao-čit͟hra*, an epithet of the moon. The commentators, however, propose other etymologies. Bīrjandī gives us two choices. The first is *jawz-chihr*, which means the "image of the *jawz* [nut?]." He claims this is on the analogy of *jawz-girih*, which is a Persian word for one kind of knot. (Steingass gives "coat-button" as the meaning for the latter.) The other possibility, which is also reported by Khafrī, is *kū-zahr*, which means the "location of the poison." This would, of course, refer to the head and tail of the snake or dragon that were imagined to be at the nodes of the moon.

Since *jawzahr* is an Arabized word, the voweling could vary (*e.g. jawzahar*, etc.). It occasionally is found in the dual form (*jawzahrān*), but the Persian encompasses both nodes in its meaning. Ṭūsī would seem to have preferred the singular form as evidenced by the manuscripts.

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[1] Compare Toomer's remarks concerning the analogous problem of translating ἀνωμαλία in the *Alm.*, p. 21.

[2] The return is not to the exact same speed since the epicycle center will be at a different distance from the Earth after a return in anomaly; see commentary to II.7 [14]15, p. 418.

II.7 [14]15. *al-ᶜawd ilā ikhtilāf bi-ᶜaynihi* (the return to the same anomaly): Here again there is a problem in interpreting what exactly is meant by *ikhtilāf*. As noted elsewhere in this chapter (II.7 [1]18–21; II.7 [14]17–21), the moon does not return sometime after a complete revolution to the exact same speed but rather to what is "comparable to it" since the epicycle center will be either closer or farther away from the center of the World when the moon has returned to the exact same position on the epicycle. In the comment on II.7 [1]18–21, I interpreted a similar use of *ikhtilāf* as "anomalous speed," but this seems inappropriate here since a certain relative speed (*e.g.* the fastest speed) will not be the same but vary from one revolution to the next. Unless Ṭūsī has simply made a misstatement, which would not be that extraordinary, he may mean by the return to the exact same *ikhtilāf* the return of the moon to the same position on the epicycle.

II.7 [14]17–21. *wa-li-kawn niṣf quṭr al-tadwīr...wa-ghayruhumā min al-ikhtilāfāt* (Because the radius of the epicycle...as well as the other anomalous speeds): There are several ways of reading various parts of this passage that lead to fairly divergent interpretations by the commentators. Bīrjandī understands *fī al-falakayn* (in the two orbs) as referring to the eccentric and the epicycle. In this reading, Ṭūsī is not thinking simply about the varying distance of the epicycle from the Earth due to the motion of the eccentric; he also has in mind the different positions of the epicycle radius, which will result in the radius appearing to vary in size from the Earth. Thus the different rates of speed that are being referred to are due not only to the varying distance of the epicycle center from the Earth but also to the planet's motion on the epicycle. In this broad interpretation of Ṭūsī's intention, the phrase *aqdār al-buṭ' wa-'l-surᶜa*, which I have translated "the rates of the slowest and fastest speeds," could refer not only to particular rates of speed but also to the varying lengths of the arcs on the epicycle that determine the slower and faster motions. In keeping with this view, Bīrjandī takes *ikhtilāfāt* in line 21 to refer not to other particular anomalous speeds but to other anomalies (such as the equation of the proper anomaly, the anomaly of the nearest distance and so on).

My own understanding of this passage as reflected in the translation is that Ṭūsī is trying to explain why a particular speed, say the fastest or slowest, varies from one revolution to another. We should recall that this was one of the observations that he reported at the beginning of the chapter (see II.7 [1]18–21). The only variation in distance needed to explain this would then be that of the epicycle center from the Earth. There would be no need to bring in the motion on the epicycle since we are comparing one particular speed from one revolution to the next. There are some problems with this interpretation, however. What does Ṭūsī mean by the "varying distance...in the two orbs" if only the eccentric is needed to resolve the observation in question? Jurjānī and Khafrī, who basically confirm my interpretation, suggest deleting *fī al-falakayn* and indicate that in some copies of the *Tadhkira* this was done (see, for example, Leipzig, University Library MS 261 (K.203), f. 11b). Another possible way to resolve this is to take "the two orbs" to mean the eccentric and the inclined since both acting together bring the epicycle toward and away from the Earth. Yet another problem with our interpretation is that one would need to take *buṭ'* and *sur*^{c}*a* not in their usual senses as the faster and slower motions that occur on the two segments of the epicycle (see the comment to II.7 [1]21–22) but in the sense of particular speeds, namely the fastest and the slowest. But this seems to me the only interpretation that would allow one to understand *ikhtilāfāt* as "anomalous speeds," which is the most natural reading and one that conforms to its use in II.7 [1]20. This permits us to avoid Bīrjandī's strained resort to "anomaly," which distorts the flow of the sentence.

Let me conclude with a more general point. Ṭūsī here and elsewhere (see, for example, II.11 [13]) discusses instantaneous and maximum speeds. One of the reasons for belaboring the problems of understanding this passage, problems that apply to the medieval commentators no less than ourselves, is in order to see the difficulties involved in dealing with a concept such as speed without a well-developed vocabulary. I would suggest that an understanding of these linguistic difficulties and the attempts to resolve them should form an integral part of the story of the development of the calculus.

II.7 [18]17. ^{c}*alā tasdīs al-shams aw tathlīthihā* (at the sextiles or trines with respect to the sun): The maximum of this anomaly is sometimes stated to be at the octants, perhaps based on a less than precise statement by Ptolemy (*Alm.*, pp. 226–227 [H367]). The actual maximum occurs at 57° and 123°, as one can easily ascertain from the appropriate table in the *Almagest* (p. 238, col. 3; cf. Pedersen [1974], p. 192). This is no doubt the basis for Ṭūsī's statement here.

II.7 [18]1–2. *nuqṭat al-muḥādhāt* (point of alignment): For this aspect of his lunar theory, Ptolemy uses the word *prosneusis*, which, according to Toomer, is not being employed by him in this context as a technical term (*Alm.*, p. 227, n. 19). It is used more generally simply to mean direction. At least in some instances, it is rendered in Arabic by *mayl*, which I have usually translated as "inclination." For example, in *Almagest* I.7, p. 43, Toomer translates *prosneusis* as direction (*sc.* of the motion of a heavy body), whereas Ṭūsī uses *mayl* for this concept (II.1 [7]). On the other hand, *muḥādhāt* has more the sense of facing or being aligned with rather than inclining toward (cf. I.1 [2]).

II.7 [18]3. *ahl al-raṣad* (observational [astronomers]): See commentary to II.6 [4]12.

II.7 [22]. The shapes of the moon are discussed in II.13 [1].

II.7 [23]. The theory enunciated here concerning the existence of other bodies in the lunar epicycle also occurs in Ibn Sīnā's *Shifā'* (*Al-Samā' wa-'l-*^{c}*ālam*, pp.

37ff). This explanation is rather different from the one offered by Ibn al-Haytham.[3]

II.7 [25]. Here, as well as in II.8 [19], II.9 [15], and II.10 [2] and [6], one finds an enumeration of Ṭūsī's criticisms of the Ptolemaic models. His counter-proposals are found in II.11. See also the introduction, §2.F2, pp. 48–51.

II.7 [26]. Parallax is discussed in II.12.

II.7 [28]4. *law lā ḥarakat al-shams* (If the motion of the sun is disregarded): In other words, if the sun were stationary, the epicycle center would perform the oval circuit indicated in Figure T7. But since the sun moves an appreciable distance in a month, the actual circuit will be rather more complex.

II.7 [29]1. *khāṣṣa* (proper anomaly): This is usually rendered simply as anomaly;[4] however, I feel that anomaly should be reserved as the general term to translate *ikhtilāf*. Since *khāṣṣa* has to do with the proper motion, *i.e.* that of the planet itself on the epicycle, I have added the qualifier as a way to distinguish the general meaning of anomaly from this specific one.

II.7 [29]2. *ᶜalā al-tawālī al-mafrūḍ fīhi* (sequentially as defined for the [epicycle]): As defined in II.7 [13], the epicycle moves in the counter-sequence of the signs in its upper half; thus this is the direction that is sequential when measuring arcs on the epicycle.

Book II, Chapter Eight

II.8 [1]18–21. *wa-idhā qīsa...wa-fī baᶜḍihā akthar* (If one compares...while in other parts they will be greater): Slower and faster motions (*buṭ'*; *surᶜa*) refer to the motions on the two segments of the epicycle discussed in II.5 [7].

Bīrjandī states that this passage is true only for retrograde motion and for the faster motion. In both cases, as the epicycle distance from the Earth increases, the arcs will decrease in size for two reasons: first the actual arcs on the epicycle will decrease, which thus will decrease the time period of these motions, and second the arcs themselves will appear smaller. For the direct motion and for the slower motion, however, he notes that the time and size are not necessarily coordinated since with an increase in the distance between the epicycle center and the Earth, the actual arcs will increase in size, which will increase the time peri-

[3] See Sabra [1977].

[4] *E.g.* Bīrūnī, *Tafhīm*, p. 94 and Kennedy [1960], p. 91.

od involved; this contradicts Ṭūsī's general statement that it will decrease. The apparent size, however, will indeed decrease with increasing distance. Admitting that it is difficult to resolve how the two effects will interact in a general way, Bīrjandī says that he has consulted a *zīj* and found that the decrease in apparent size outweighs the increase of the arc on the epicycle.

II.8 [1]21–22. *wa-'l-juz'...al-thawābit* (That part...fixed stars): The part being referred to here is the apogee, where apparent motion will be at its slowest. I am not quite sure what is meant by *zamān* (time period) being at its shortest there since this is true of some categories of motion but not others as we saw in the previous comment. At first reading, Ṭūsī seems to be saying in this sentence that the slower motion will take place in the least time when it is in the vicinity of the apogee, but this is not the case. What he is probably thinking of is the retrograde motion, which in fact will take place in the shortest time near the apogee since the retrograde arc on the epicycle will there have its smallest size.

II.8 [1]1–2. *wa-fī muqābila dhālika...tilk al-ghāya* (Directly facing that...maximum extent): As happens all too frequently, Ṭūsī has made things more complicated than need be by an excessive use of pronouns. The first in "directly facing that" I take to refer to the apogee, which would indicate that Ṭūsī wishes to describe the situation 180° from the apogee. What he finds is similar to "this part," which I have concluded is the trine; in other words, the situation at 180° is similar to that at 120° though the maximum effect is at the latter. Though this interpretation is not obvious given the structure of the sentence, it does correspond reasonably well with the analysis of Mercury's epicycle center distance from the Earth and its effect on the anomaly made by Pedersen ([1974], pp. 326–328, especially p. 328) and it is furthermore confirmed by Nīsābūrī. Bīrjandī, however, has a different reading. He takes the first "that" as referring to the trine from which it follows that "this part" would be the sextile, which is directly opposite the trine. Though neither my interpretation nor Bīrjandī's is without its problems, mine does correspond somewhat better with a close analysis of the situation; whether this corresponds with Ṭūsī's own understanding is another matter.

II.8 [4]9. *wa-sa-tajī'u ṣifatuhā* (its description will be forthcoming): See II.10 [1–2].

II.8 [7]20. *ᶜalā mā sa-yajī'u bayānuh* (as will be explained below): See II.10 [4–5].

II.8 [10]5. *ḥawl nuqṭa sa-nadhkuruhā* (about a point we will discuss below): See II.8 [14].

II.8 [11]19–21. *wa-lā fī al-tarbīᶜayn...laysā bi-mutasāwiyayn* (nor is [this distance] at the quadratures...are not equal): Ṭūsī seems to be thinking here of the situation of the moon where the nearest distance does occur at the quadratures as a result of the "two opposite distances," *i.e.* those at conjunction and opposition, being equidistant from the Earth. This is, of course, a consequence of the motion of apogee being about the center of the World. In the case of Mercury, however, the corresponding distances will not be the same since the motion of the apogee is with respect to the dirigent center and not the center of the World.

II.8 [15]5. *al-ta*ᶜ*dīl al-thānī* (the second equation): It is a bit confusing, to say the least, to call the first anomaly the second equation. Bīrjandī notes in his commentary to II.7 [16] that this terminology is that of the practical astronomers (*ahl al-*ᶜ*amal*), who call it second because it is computed after the equation of center, which is called by them the first equation. On the other hand, the theoretical astronomers (*ahl al-hay'a*) call this the first equation and the independent equation as we saw in II.7 [16]. We should note that *al-thānī* (second) occurs only in MS L and in the margin of MS F, which indicates that it is a late addition of the Baghdad (β) version. (Compare Kennedy, *Survey*, p. 142).

Book II, Chapter Nine

II.9 [1]9. *aw ba*ᶜ*dahu bi-qalīl* (or is a little beyond it): Since this reading occurs in MSS F and M, which usually witness a later version, it seems reasonable to assume that it is meant to replace the redundant *qablahu* (before it), which is found in MSS DGLT. *ba*ᶜ*dahu* (beyond it) allows for the possibility that a station could occur after the sun has reached the second trine. Indeed Bīrjandī, who admittedly simplifies the situation by calculating only the mean positions, finds that the stations will be before the first trine and after the second for Saturn. (For Jupiter and Mars they are after and before.) Interestingly enough, the newer version also occurs in both the *Nihāya* (II.8, f. 79a) and the *Tuḥfa* (II.11, f. 105b). This may indicate that this change is due to Shīrāzī, a view suggested by both Jurjānī and Bīrjandī. This is consistent with what we know about the status of MS M.[1]

II.9 [6]12. *wa-taḥduthu fī al-mumaththil* (and there occurs on the parecliptic): In MS G and in the margins of MSS MT, one finds immediately afterward the phrase *bi-ḥaraka markaz al-tadwīr* (due to the motion of the epicycle center). It is absent in MSS DL and is marked for deletion in MS F. At some point in the text tradition, Ṭūsī, or perhaps an associate, wished to indicate that the plane of the inclined orb is determined by the motion of the epicycle center. However, the epicycle center obviously does not produce a great circle on the parecliptic, and the phrase was subsequently excised to avoid any ambiguity.

[1] See the introduction, §2.J3, pp. 71–75.

II.9 [8]21. *ḥaraka markaz al-tadwīr* (the epicycle's motion of center): A variant reading is *ḥaraka markaz al-kawkab* (the planet's motion of center) found in MSS DGM. Bīrjandī prefers this variant since he claims it is the correct technical terminology. He does admit that "the epicycle's motion of center" does seem appropriate in context here.

II.9 [14]. See the text and commentary to IV.6 [3].

II.9 [15]18. *dūn alladhī bi-sabab al-muḥādhāt* (but without there being the [difficulty] due to the alignment): The difficulty alluded to here is the one mentioned in connection with the moon, where the epicycle diameter is aligned with a point other than its center of motion (see II.7 [18]). For Mercury and for these four planets, however, this difficulty does not arise inasmuch as the epicycle diameter passing through the mean apex and perigee is always aligned with the center of motion, *i.e.* the equant center (cf. II.8 [19]).

Book II, Chapter Ten

II.10 [2]7–8. *wa taḥtāj...al-mutaqaddimūn* (These two motions...[our] predecessors): Ṭūsī's solution is in II.11 [19].

II.10 [4]25. *fī ghāyat al-buᶜd al-shamālī* (at the northern limit [*lit.*, at the maximum northern distance]): For the upper planets, the northern (or southern) limit refers to the point on the inclined deferent at which the epicycle center is at its maximum northern (or southern) inclination from the ecliptic. Since the apogee is in the northern part of the deferent for all the upper planets, the apparent deviation of the epicyclic apices and perigees will be less at the northern limit than at the southern limit (see Table 3).

II.10 [4]15. *fī ghāyatay al-buᶜdayn* (at both maximum distances): In contrast with II.10 [4]25, here the maximum distance refers to the maximum deviation in either direction of the apex (or epicyclic perigee) from the inclined deferent. Since for both lower planets this will occur at nodes that are equidistant from the Earth, there is no need to take into account the northern or southern limit of the deferent as was the case for the upper planets; consequently there is only one maximum deviation for each of the apex and epicyclic perigee.

II.10 [4]18$_2$. *al-mayl* (deviation): *Mayl* is used in various contexts to indicate an inclination of one sort or another; thus translating it here as "deviation" may seem to be assigning it a technical meaning it does not have. *Mayl*, for example, is also used in II.10 [1]3 to mean the inclination between the inclined and parecliptic equators. On the other hand, Ṭūsī does seem to intend this aspect of the latitude theory to have a specific designation; this is confirmed by Bīrjandī who calls the inclination of the inclined equator "the latitude of the epicycle center" (*ᶜarḍ markaz al-tadwīr*) in order to distinguish it from this *mayl*. The third component of the latitude theory, the slant, goes under a number of different names, happily none of which is *mayl* (cf. II.10 [5]9).

II.10 [4]18–19$_2$. *ghayr hādhayn al-ᶜarḍayn* (only these two latitudes): These two are the inclination of the inclined equator and the deviation.

II.10 [4]&[5]. The following tables list the values given in these paragraphs for the deviation and the slant. See also Figure C6 and compare Pedersen [1974], Fig. 12.5, p. 362 and Toomer, *Alm.*, Fig. U, p. 624 and footnote 42, p. 623. Figure C6a is a cross-sectional view; it is not applicable to the lower planets since their maximum deviation occurs at the nodes. Figure C6b is somewhat "artificial" (quoting Toomer) since when the epicycle center is in the ecliptic, the slant is zero; however, the slant is measured with respect to a plane parallel to the ecliptic and so we can take the figure conveniently to represent, in Ptolemy's phraseology, the "separation of effects." Finally note that Ṭūsī calls the line connecting the morning and evening endpoints a "diameter"; strictly speaking, however, the line connecting the mean distances, as shown in Figure T1, is not a diameter.

Table 3. Deviation.

Upper Planets					
Planet	j_m	a_{mn}	a_{ms}	p_{mn}	p_{ms}
Saturn	†4½°	†26′	*28′	33′(†34′)	*35′
Jupiter	†2½°	†24′	*25′	35′(†36′)	*38′
Mars	†2¼°	22′(*52′)	27′(*56′)	3;22°(†3⅓°)	6¹⁄₁₀° (†6°)

Lower Planets			
Planet	j_m	a_m	p_m
Venus	†2½°	†1;2°	6;23°(†6;22°)
Mercury	†6¼°	1¾°(†1;46°)	4;4°(†4;5°)

Table 4. Slant.

Planet	k_m	m_{ma}	m_{mp}
Venus	†3½°	2½°(†2;27°)	2½°(†2;34°)
Mercury	†7°	2¼°(†2;17°)	2¾°(†2;46°)

Key: j_m = maximum deviation measured from epicycle center; a_m = apparent maximum deviation of epicyclic apex; p_m = apparent maximum deviation of epicyclic perigee; a_{mn} (a_{ms}) = apparent maximum deviation of epicyclic apex at northern (southern) limit on inclined deferent; p_{mn} (p_{ms}) = apparent maximum deviation of epicyclic perigee at northern (southern) limit on inclined deferent; k_m = maximum slant measured from epicycle center; m = morning endpoint; m_{ma} = apparent maximum slant at deferent apogee; m_{mp} = apparent maximum slant at deferent "perigee"; † = value stated explicitly in text of *Almagest*; * = value derivable from latitude tables in *Almagest* XIII.5; divergent values from *Almagest* are in parentheses.[1]

[1] Cf. *Alm.*, pp. 602 (H536–7), 604–605 (H540, H542), 626–627 (H573, H575), 629–630 (H577–9), and 632–634 (H582–6).

I do not know the reasons for the small discrepancies between Ṭūsī's values and those of Ptolemy. In some cases (*e.g.* $p_{ms} = 6\frac{1}{10}°$ for Mars), Ṭūsī has probably obtained the value from Ptolemy's latitude table (*Alm.*, p. 633) by subtracting the inclination i from the tabulated value (in this case, $7;7°-1° = 6;7° \approx 6\frac{1}{10}°$). In general, Ṭūsī's values are correct to within a few minutes irrespective of how one calculates them; in the case of Mars, however, the amounts given for a_{mn} and a_{ms} are incorrect. The values should be[2] (using *Alm.*, XI.11, cols. 5–7, p. 551)

$$a_{mn} \approx (2;24° - 0;8°)\ 2;15/6 \approx 51'$$

$$a_{ms} \approx (2;24° + 0;9°)\ 2;15/6 \approx 57'$$

It is a mystery to me how Ṭūsī could be so far off; if one were inadvertently to drop 2;15 from the above calculation (or, what amounts to the same thing, if one were to confuse the declination with the inclination of the inclined equator, which is 1° for Mars), one could conceivably arrive at Ṭūsī's figures with some fudging. This is only conjecture, however, and not a particularly strong one at that.

II.10 [5]9. *wa-hādhā al-ᶜarḍ...wa-'l-iltifāf* (This latitude...[*iltifāf*]): On the Arabic terminology for this aspect of the Ptolemaic latitude theory, see Sabra [1979], pp. 389–390, especially note 5.

II.10 [6]10–11. *wa-kull...al-qudamā'* (Each...Ancients): This problem is dealt with in II.11 [13–15] and [18].

II.10 [6]11. *al-muta'akhkhirīn* (recent [astronomers]): The person (singular, not plural) referred to is Ibn al-Haytham; see II.11 [16].

II.10 [6]13. *ᶜalā mā dhukira fī al-majisṭī* (based on what is stated in the *Almagest*): Here again we see that the basic purpose of the *Tadhkira* is to summarize the *Almagest*; as Bīrjandī points out, Ṭūsī gives the Ptolemaic values despite having access to more recent Islamic values including those that came out of the Marāgha observations.

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[2] For the justification of this method, see Pedersen [1974], pp. 362–363. From the table (*Alm.*, p. 633), one can derive 52′ for a_{mn} and 56′ for a_{ms} ($1°-0;8°$ and $1°-0;4°$, respectively).

Book II, Chapter Eleven

For an overview of this chapter, see Ragep [1987].

II.11 [title]15. *al-ishāra* (An indication): Both in its presentation and in its historical role, this chapter should be seen as a program for research. By using *al-ishāra* in the title, Ṭūsī is clearly making this explicit. He is, in effect, telling his medieval audience that he does not claim a definitive solution to the problems of astronomy but only an approach. In various places throughout the chapter, Ṭūsī reiterates this point and challenges his contemporaries and successors to continue in this endeavor. This attempt to institutionalize a certain program was obviously quite successful. One is reminded here of Plato's proposal to the astronomers to "save the phenomena" with uniform circular motions.[1] But in contrast to our somewhat limited knowledge of the immediate response to the Platonic challenge, we have with Ṭūsī and his successors a well-documented record of an on-going—even systematic—response to the unresolved problems (dare we say anomalies?) of astronomy. As such, one has a very important case study with which to test the conflicting claims that have been proposed in recent years of how scientific change occurs.

II.11 [title]17. *allatī sabaqat al-ishāra ilayhā* (Referred to Previously): The referent here is *al-ishkālāt* (the difficulties), which Khafrī finds to be sixteen—two for the moon (the irregular motion of its deferent and the oscillation of its epicycle as a result of the alignment of its diameter with the prosneusis point), four each for the two lower planets (the irregular motions of their deferents, of their epicyclic apices and of the endpoints of their mean epicyclic diameters, and the oscillations of the equators of their deferent orbs), and two each for the three upper planets (the irregular motions of their deferents and of their epicyclic apices).[2] These have been cited in II.7 [25], II.8 [19], II.9 [15], and II.10 [2] and [6]. Note that the related problems associated with trepidation and the motion in latitude of the ecliptic do not figure in this list since they were usually not considered, at least in late eastern Islamic *hay'a*, to have been observationally established (see II.4 [2–5] and II.11 [21]).

II.11 [1]. Because of the intriguing historical questions raised by this paragraph, it is important to examine it closely. The first difficulty (*ishkāl*) referred to concerns the irregular motion of the center of the moon's epicycle about its deferent center. Ṭūsī's statement that nothing concerning this problem has reached him from his predecessors reaffirms his statement in II.7 [25] that no one in the profession has shown how this motion is composed nor has anyone even

[1] This is reported by Simplicius in his commentary on Aristotle's *De caelo*; see Duhem [1908], p. 3 (Engl. trans. [1969], pp. 5–6).

[2] Khafrī, *Takmila*, f. 189a–b. See also the introduction, §2.F2, pp. 48–51.

ventured (*ta*ᶜ*arraḍū*) to deal with this difficulty nor the other difficulties associated with the moon. Both Ibn al-Haytham and Jūzjānī, however, should at least be credited with discussing some of the difficulties even though they may not have reached a solution acceptable to Ṭūsī.[3] But we should emphasize again that he is explicit: "No statement (*kalām*) has reached me concerning [this difficulty] (*fīhi*)... ."[4]

How should we interpret this sentence? At first glance Ṭūsī would seem to claim originality not only for the solution but also for the statement of the problem. One would then be forced to conclude that he was unaware of *Al-Shukūk* ᶜ*alā Baṭlamyūs* (Doubts Concerning Ptolemy) by Ibn al-Haytham, especially in view of his willingness to acknowledge him in connection with *iltifāf* motion later in this chapter.[5] But we should not be too hasty in drawing sweeping conclusions based on a lack of evidence. It strains one's credulity to assume that Ṭūsī rediscovered single-handedly the entire body of *ishkālāt* (difficulties) associated with the *Almagest*. My own sense is that the difficulties themselves were fairly common knowledge and were possibly discussed even before Ibn al-Haytham.[6] Thus the fact that Ṭūsī did not have the *Shukūk* does not in itself preclude his knowledge of the arguments contained therein. Tentatively then I would be inclined to say that we should regard Ṭūsī's statements here and in Chapter Seven as referring to the lack of an acceptable solution rather than to the statement of the problem.

Jūzjānī's work confronts us with an even more difficult situation to resolve. There is conclusive evidence that Quṭb al-Dīn al-Shīrāzī knew of Jūzjānī's work. In the former's *Fa*ᶜ*alta fa-lā talum*, a strongly worded attack on a commentator of the *Tadhkira*, there is extensive reference to Jūzjānī, especially in the part of the work corresponding to the chapter on the moon (Book II, Chapter 7). We are here faced with a dilemma. If we assume that Ṭūsī did not know of Jūzjānī's work at the time he wrote the *Tadhkira*, we have the problem of explaining how it became widely known—and even championed by Ḥimādhī, the above-mentioned commentator—shortly after Ṭūsī's death. If we assume that he did know it, then why did he state that he knew of no previous work in this area? One is tempted to say that Ṭūsī, as Shīrāzī after him, did not take Jūzjānī seriously;[7] he therefore simply ignored him. I do not think that this is the case, however. Ṭūsī evidently did not think much of Ibn al-Haytham's efforts to

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[3] Cf. the introduction, §2.F2. Ṭūsī himself mentions in II.11 [12] an anonymous writer who had dealt with the moon's prosneusis.

[4] This statement and the statement in II.7 [25] closely echoes his assertions in the *Mu*ᶜ*īniyya*; see the introduction, §2.J1, especially p. 68.

[5] There seem to be few citations of the *Shukūk* in the astronomical literature; interestingly, the Spanish philosopher Ibn Bājja (d. 1138–39) does know—and even criticizes—the work. See Langermann [1990], p. 32.

[6] See the introduction, §2.F2.

[7] See Shīrāzī's statement in Book II, Chapter Seven of *Fa*ᶜ*alta*: "The model of Abū ᶜUbayd is unsound (*fāsid*)...and nothing can be resolved with it at all" (ff. 145b–146a).

resolve the problem of *iltifāf* motion with techniques very similar to those of Jūzjānī; nevertheless, he at least mentioned and criticized Ibn al-Haytham's models.

Finally, we should turn to Mu'ayyad al-Dīn al-ᶜUrḍī, a contemporary of Ṭūsī who did indeed work on the *ishkālāt*. Unless one wishes to claim that Ṭūsī was being deceptive, an unlikely possibility in view of the close association of the principals at Marāgha, the only reasonable conclusion is that he here in the *Tadhkira* is simply repeating his claim of priority and the lack of knowledge of any previous work on the *ishkālāt* that he had made some 25 years earlier in the *Risālah-i Muᶜīniyya*. Even if he had become aware of ᶜUrḍī's *Kitāb fī al-Hay'a*, or of Jūzjānī's work, in the intervening period, he no doubt saw little reason to modify this claim of priority whose point of reference would be 1235 and not 1261.[8]

II.11 [1]18. *al-ishkāl al-awwal al-madhkūr fī hay'at aflāk al-qamar* (the first difficulty, which was cited [in connection] with the configuration of the moon's orbs): The "first difficulty" here referred to concerns the motion of the center of the epicycle about a point other than its deferent center. See II.7 [25].

II.11 [1]19. *istanbaṭtu* (I have devised): On the use of this term to indicate innovation, see Sabra [1984], p. 133.

II.11 [2]. Ṭūsī here states the famous lemma that has recently come to be associated with his name. Since a question of priority has been raised concerning this lemma, it would be wise to analyze this conflicting claim.

First we should ask whether Ṭūsī himself claims priority in this matter. Virtually all modern commentators have agreed that he has[9] but the situation is not as clear-cut as may at first appear. Ṭūsī claims priority, as we have stated above, for his lunar model. He at no place *explicitly* states that he has invented this lemma. What he literally says is as follows: "Let us set forth for that [purpose] a lemma, which is thus... ." The "that" clearly refers to the previous statement in which he has declared that he will now present what he has invented, *i.e.* his model. It remains moot, however, whether the lemma itself is his own discovery since he apparently, if one takes this passage completely literally, only claims priority for his model, *i.e.* the use to which he puts the lemma. On the other hand, one must admit that the "Ṭūsī couple" cannot merely be called a preliminary theorem. It is, as we shall see, at the heart of every model he presents in this chapter. In fact, after his lemma is presented and proved, the

[8] For a discussion of the relationship of the *Tadhkira* to its original Persian version (consisting of the *Muᶜīniyya* and the appendix to it, the *Ḥall-i mushkilāt-i Muᶜīniyya*), see the introduction, §2.J1, pp. 65–70. For another view of this matter, one which does not take into account Ṭūsī's earlier work on the *ishkālāt*, see Saliba [1979], pp. 571–576.
[9] Hartner [1971], p. 631; Veselovsky [1973], p. 129.

actual modeling becomes more or less trivial. In conclusion, we should acknowledge that Ṭūsī is claiming the lemma as his own, at least in the form presented.

But does this passage mark the first appearance of the lemma? I. N. Veselovsky, for one, wishes us to see Proclus (or someone Proclus follows) as the originator of this theorem.[10] The relevant remarks occur in Proclus's *Commentary on the First Book of Euclid's Elements*, a work apparently unknown in the Arabic tradition.[11] In a passage devoted to the classification of lines, Proclus examines the question of whether there are only two simple lines (straight and circular) or more.[12] In particular, Proclus is worried that a simple line produced by a combination of simple motions may not be so simple, a situation that might undermine the cherished Platonic distinction. He then sets forth several examples, presumably from Geminus, of what he means. One of these involves the resulting circular motion described by the midpoint of a line whose endpoints move along the two sides of a right angle (see Figure C7, which has been adapted from Neugebauer, *HAMA*, 3: 1431). This is the converse of the Ṭūsī couple, whose purpose is to produce straight-line from circular motion. Furthermore, one must considerably expand and modify what amounts to a passing comment in Proclus to reach Ṭūsī's couple. Nevertheless Veselovsky states: "...it is much more plausible to suppose that Copernicus took the argument he needed from Proclus (5th century A.D.) and not from Naṣīr al-Dīn; at all events, the priority of the latter is undermined."[13] And the normally precise Neugebauer remarks that "In its astronomical applications, Proclus's theorem appears in a slightly [*sic*] modified form."[14] Perhaps to a modern mathematician this transformation from Proclus to Ṭūsī represents a "slight modification." However as an historian, I must admit of a great deal of unease in accepting such a characterization. I dare say that no one—including Neugebauer—would have made the connection between Proclus and Ṭūsī had not Copernicus explicitly mentioned Proclus when he used the couple for his Mercury model:

> This [motion along a straight line] can be the result also of uniform circular motions, as was shown above in connection with the precession of the equinoxes [III,4]. There is nothing surprising in this, since Proclus too in his *Commentary on Euclid's Elements* declares that a straight line can also be produced by multiple motions.[15]

[10] Veselovsky [1973], pp. 129–130.
[11] This statement is based on the absence of any listing by Sezgin, *GAS*, 5: 104.
[12] Proclus, *Commentary*, p. 86.
[13] Veselovsky [1973], pp. 129–130.
[14] Neugebauer, *HAMA*, 2: 1035.
[15] Copernicus, *De rev.*, V.25, p. 164v (Rosen trans., p. 279).

Far from being the source for Copernicus, this passage has all the earmarks of an afterthought—a kind of classical padding that is hardly unknown in the work of Copernicus or other Renaissance scholars. This is further borne out by the observation originally made by Prowe and later repeated by both Swerdlow and Rosen that Copernicus's knowledge of this commentary could not have occurred before he was given the work as a gift by Rheticus in 1539.[16] It should be noted here that Copernicus already makes use of the couple in the *Commentariolus*, which was written before May 1514.[17] Furthermore, Copernicus himself implicitly indicates in the famous deleted passage in Book III, Chapter 4 of *On the Revolutions* that other persons before him knew and had used this theorem.[18] But, as Rosen notes, he does not identify the "some people" who "call this the 'motion along the width of a circle.' "[19] It should be clear, though, that he does *not* mean Proclus since the latter neither uses the term "motion along the width of a circle" nor does he state the theorem in a way in which such a terminology would make sense.

Veselovsky also brings up the point that the figure used by Copernicus has an extra circle *cef* that, he alleges, corresponds to the motion of the midpoint of the given line in Proclus's formulation. Neugebauer as well draws our attention to this correspondence in his Fig. 137b (*HAMA*, 3: 1431). Copernicus uses this additional circle to facilitate his proof, which is rather shorter than Ṭūsī's. By indicating that arc FC is twice arc GH, he eliminates the need to deal with the exterior angle step in Ṭūsī's proof. Whether Copernicus saw his proof as simpler than the other(s) he knew or whether he was only acquainted with a diagram and had to supply a proof is, of course, unknown. At any rate, it does not seem plausible to assume that he was motivated into adding this circle by Proclus since he admits knowing the Ṭūsī version as we have seen.

Another point should be made regarding the Proclus connection. In the deleted passage of III.4, Copernicus mentions that if one varies the sizes of the circles, one will obtain not a straight-line motion but an ellipse. This is somewhat reminiscent of Proclus who notes that points other than the midpoint will describe ellipses. But Copernicus makes no explicit mention here of Proclus as he does later in his chapter on Mercury. And this oblique reference to ellipses hardly constitutes conclusive evidence.

It is ironic that the mathematical aspect of the passage in Proclus has received most of the attention, whereas the physical content of his remarks have been generally ignored. The close relation of physics to geometry was an

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16 Rosen, Commentary to *On the Revolutions* by Nicholas Copernicus, pp. 369, 429; Swerdlow [1975], pp. 146 (n. 5), 155 (n. 8). Swerdlow's arguments, including his observation that the marginal reference to Proclus was penned later than the other marginal notes on that page, are quite compelling.

17 Swerdlow [1973], p. 431.

18 Rosen, "Commentary," pp. 384–385. Swerdlow [1972] deals with Giovanni Battista Amico's use of the Ṭūsī couple in a treatise published in 1536.

19 *On the Revolutions* (Rosen trans.), p. 126.

important theme for some Hellenistic and Islamic mathematicians and philosophers. It should therefore not surprise us to find Proclus interspersing his discussion concerning the classification of simple lines with comments about simple motions. For, as he tells us, "Aristotle's opinion is the same as Plato's; for every line, he says, is either straight, or circular, or a mixture of the two. For this reason there are three species of motion—motion in a straight line, motion in a circle, and mixed motions."[20] And further along he continues: "...and Geminus has rightly declared that, although a simple line can be produced by a plurality of motions, not every such line is mixed, but only one that arises from dissimilar motions."[21] Proclus then sets forth several examples to show that even though one may produce what appears to be a simple line (or motion) by a mixture, that line will not actually be simple but presumably mixed.[22] Thus the circular path resulting from the midpoint of the moving line EK in Figure C7 is not a simple circle but a mixed line. Why does Proclus insist on what is a not very significant mathematical distinction?[23] Clearly the need to preserve such a classification of lines has its origin in physics, not mathematics. If one could somehow produce circular from straight-line motion or vice versa, would this not undermine the distinction between sub-lunar and supra-lunar motion? Proclus himself states that "a circular line is generated as a result of nonuniform motion of the middle point, under the condition given that the line is moving not naturally, but with its extremities on the sides of a right angle."[24] The fact that the line is not moving naturally would seem then to be the operative principle that allows a circular line to be generated but not to occur simply and naturally.

Hartner has called attention to this very point when he remarks that "by proving...that combined circular motion may produce rectilinear motion, the Aristotelian distinction between terrestrial and celestial natural motion becomes seriously weakened though of course not invalidated completely because the lemma concerns only cases of alternating and limited rectilinear motion in space."[25] Whatever one may think of Hartner's argument, such considerations do seem to be at the root of Proclus's attempt to save his overstretched categories. It is not without interest in this connection that Shīrāzī in his *Al-Tuḥfa al-shāhiyya* applies Ṭūsī's lemma to a problem of *terrestrial* physics. "It is possible to use this [lemma] to show the impossibility (*imtināᶜ*) of rest between a rising and falling motion on the line (*samt*) of a terrestrial diameter."[26] Khafrī, a

[20] Proclus, *Commentary*, p. 85.
[21] Proclus, *Commentary*, p. 86.
[22] The argument as presented by Proclus is not very clear; my conclusion, which follows Morrow, p. 86 (n. 40), is a suggested interpretation.
[23] Cf. Morrow's note to Proclus, *Commentary*, p. 90 (n. 54); Tannery [1912–1950], 2: 37.
[24] Proclus, *Commentary*, p. 86.
[25] Hartner [1969], p. 289; cf. Rosen, Commentary to *On the Revolutions*, p. 385.
[26] Shīrāzī, *Tuḥfa*, Bk. II, Ch. Eight, f. 66b, lines 12–14. Khafrī additionally quotes Shīrāzī as saying: "Thus Aristotle's statement that there must be rest between the two straight motions of rising and falling is invalidated" (Khafrī, *Takmila*, f. 191a).

16th-century commentator on the *Tadhkira*, disputes Shīrāzī on this point in a way that is vaguely reminiscent of Proclus:

> There is nothing in this because there results from this lemma only rising and falling by means of motions that are in themselves circular (*bi-'l-ḥarakāt al-mustadīra fī nafs al-amr*); straight motion will occur visually (*bi-ḥasab al-ru'ya*) but will not be the straight motion brought about by a straight-line inclination (*al-mayl bi-'l-istiqāma*).[27]

We must conclude that the problem posed by the Ṭūsī couple in confounding straight-line and circular motions was an issue—though probably not a significant one—raised in the Islamic Middle Ages.

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Turning our attention to the lemma itself, we find that the conditions are all straightforward except for those involving the motions of the two circles. These conditions are as follows: (1) the two motions must be simple; (2) they are in opposite directions; (3) the motion of the small circle is twice that of the larger one. In point of fact, Ṭūsī does not need two motions to achieve the oscillation of his given point along the diameter of the larger circle; he merely needs to allow the smaller circle to "roll" inside the larger one, which would remain stationary. To see this we will refer to Figure C8. Circle Z rolls inside circle D. At the starting point, A and E coincide; after the smaller circle has rolled along arc AG, point E will be at a distance GE from the point of tangency G. It is clear that GE = AG; therefore, $\angle$ GZE = 2$\angle$ GDA since the radius of the smaller circle is half that of the larger. Thus mathematically, this rolling is equivalent to having the smaller circle rotate twice as fast as the larger one in the opposite direction. We may also, being anachronistic, find the locus of the point E by noting that the parametric equations of $\overrightarrow{DZ} + \overrightarrow{ZE}$ are

$$x = r\cos\alpha + r\cos(-\alpha) = 2r\cos\alpha$$

$$y = r\sin\alpha + r\sin(-\alpha) = 0$$

which indicate that point E will oscillate on a straight line. (Note that it is unnecessary to make any assumptions about the whereabouts of point E or that $\angle$ GDE = $\angle$ GDA.)

I have belabored this rather obvious point in order to underscore the fact that Ṭūsī does *not* proceed in this manner. The reason is that this lemma is not simply a mathematical theorem; it is meant to have a physical application and

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[27] Khafrī, *Takmila*, f. 191a, lines 14–18.

therefore the added stipulations are necessary. For there can be no rolling in the heavens; only rotation in place is allowed since there is no void. It is unfortunate that Kennedy—and others—have persisted in calling the Ṭūsī couple a "rolling device," which it emphatically is not.[28] This is only one of many cases where modern commentators have taken Ṭūsī's strict adherence to physical principles less than seriously.

II.11 [3]. Ṭūsī here presents his proof for the lemma. He remarks that he had not intended to present any proofs in this compendium whose purpose is, as stated in the introduction, to provide a summary of astronomy. This again reinforces our view that the primary purpose of the *Tadhkira* was not to make new departures in astronomy but to give a useful and easily comprehensible summary of the *Almagest.* The prototype for this was Ṭūsī's earlier Persian work *Risālah-i Muᶜīniyya*, which contained none of the material found in this chapter. But ironically it was precisely this sort of cosmographical summary that provided the occasion for new models.

The proof itself is imprecise in several places. Usually the ambiguity is trivial but in at least one case the problem is major. The reason for this is Ṭūsī's unwillingness to distinguish between a fixed point on one of the circles and a point defined by a relation between the two circles. Some examples will, I hope, clarify this (see Figure C9).

First, we should note that point G is a fixed point on the larger circle and it also designates the point of tangency of the two circles. It is not, however, a fixed point on the smaller circle but rather a reference point by which to measure the rotation of the smaller circle. Likewise, point A is not fixed on the circumference of the larger circle but is the reference point for measuring its rotation. Point E, the given point, is fixed on the smaller circle. Therefore, when the larger circle rotates, point G will move with respect to line AB, which is fixed in space, so to speak, but is not a fixed diameter of the larger circle. At the same time, the smaller circle will be carried along due to this rotation, and it is also free to have its own proper motion, which is twice that of the larger circle. Thus point E will move along an arc measured from G that is twice arc AG. This is implicit in what Ṭūsī has stated but may not be completely clear on first reading. Any imprecision that may arise from Ṭūsī's language is, thus far, not serious. Anyone who reads the text carefully can easily see that the smaller circle does not roll but is carried by the larger circle; the smaller circle is thus always tangent to the larger circle at the same point G, which is fixed on the circumference of the larger circle.

A serious problem does arise, however, when we reach the part of the proof that begins: "Then angle GZE is twice angle GDA on account of the two motions." This, at least, is clear. However Ṭūsī then states: "It [*i.e.* angle GZE] is also twice [angle GDA] because it is an exterior [angle] of triangle EZD and

[28] Kennedy [1966], pp. 368–370.

equal to the two interior [angles] ZED and ZDE... ." The pronoun *hā* in the word *ḍiᶜfuhā* (twice it) can only refer to angle GDA but this will make the proof invalid since Ṭūsī would then have assumed what he is trying to prove, namely that E is on line ADB. Clearly what he means to say is that angle GZE is also twice angle GDE and then his result will follow. What seems to have happened is that Ṭūsī has absentmindedly confused the given point E with the intersection of the smaller circle with diameter AB. The diagrams in all copies of the *Tadhkira* indicate them to be the same but, of course, this is what is to be proved (see Figure C10). This error would seem to be from Ṭūsī himself since only in MS B (Paris 2509, f. 38b) do we find "angle ZDE" but the copyist (or perhaps annotator) of this MS has a penchant for correcting mistakes, both perceived and real. In his commentary, Khafrī, who is usually reluctant to tamper with the received text, has changed "angle GDA" to "angle GDE" so that *ḍiᶜfuhā* would also refer to "GDE." But this expedient not only fails to resolve the difficulty, it compounds the problem since one now has no way of equating $\angle$ GDA to $\angle$ GDE.

II.11 [4]. Ṭūsī, who has just completed the explanation and proof of his couple in a plane, now wishes to show how to apply his results in the case of solid bodies. To do this, he will make the two circles from II.11 [2–3] *minṭaqatay* (two equators) of two solid orbs. In general, *minṭaqa* is equivalent to "equator" as we can see from Ṭūsī's definition in I.1 [13]: "The great circle that is equidistant from the two poles is the sphere's equator." Here, however, the *minṭaqa* of the small sphere is the circuit (*madār*) of the epicycle center within the sphere. Since the epicycle must have a certain thickness, it is clear that the circuit of the epicycle must be *inside* the orb, not on the surface. Thus the *minṭaqa* is concentric and coplanar with the equator of the orb but does not have the same radius. In the case of the larger sphere, its *minṭaqa* is stated to be a circle whose radius is equal to the diameter of the *minṭaqa* of the smaller sphere. If we refer to Figure C11, we can see that the two equators form a Ṭūsī couple, that is, they are tangent at a point and the radius of the smaller is half that of the larger. However, the ratio of the radii of the two *spheres* is not $\frac{1}{2}$ but rather $(R + r)/(2R + r)$, where R is the radius of the smaller equator and r is the radius of the epicycle. This interpretation, which is crucial for understanding the arrangement of the orbs in the lunar and planetary models, is confirmed by the commentators. In particular, Nīsābūrī in his *Tawḍīḥ al-Tadhkira* elaborates this passage as follows:

> If we take an epicycle in the interior (*fī jawf*) of another small sphere such that the convex [surface of the former] is tangent to the convex [surface of the latter] at a common point between them and their centers are not the same, then if the small sphere moves through one rotation, the epicycle center will no doubt traverse a circuit (*madār*) about its center, *i.e.* the small sphere, which will be its equator (*minṭaqatuhā*). And if we take another sphere, a larger one, that encloses (*yuḥīṭ*) the small sphere just as the small sphere enclosed the epicycle so as to be tangent [to it] and without having the same centers, then if the large sphere moves through one rotation, the center of the small sphere will no doubt traverse a circuit about the center of the large sphere which will be, strictly speaking (*bi-'l-ḥaqīqa*), its equator; however, the equator of the large sphere is instead used to designate (*yuqāl li-*) an imaginary circle whose center is that of the large sphere and whose diameter is twice the diameter of the equator of the small sphere so that the distance of the center of the large sphere from the epicycle center at the initial position is twice the distance of the center of the small sphere from the epicycle center. But then it is called the equator of the large sphere because if not for the small sphere then it would be the circuit of the epicycle center.[29]

Rather surprisingly, the source of much of the misunderstanding concerning this chapter has occurred, I believe, because of a confusion about this quite simple but nevertheless fundamental meaning of *minṭaqa*. Ṭūsī himself is partially to blame since he begins this paragraph by saying that he will now simply make the two circles equators of solid bodies; however, in his next sentence he has gratuitously added an epicycle into his scheme, which, of course, changes the commonly understood meaning of equator. Only further on do we find that this "epicycle" is in fact a "given sphere" that has replaced the "given point" of the planar case. This "given sphere," of course, will eventually become the epicycle of the planetary and lunar models.

But Ṭūsī is here faced with a problem. As can be seen in Figure C12, the "given sphere" TH will move up and down a straight line as was the case with the "given point." But as an epicycle, the "given sphere" must have a reference point from which to measure the uniform circular motion of the planet on the epicycle. Thus in addition to having the sphere oscillate on a given line, the apex (*dhirwa*) of the epicycle must also remain on that line. But herein lies the difficulty: the apex, point T, will clearly not be on line AB except when G is at A or B. And to compound the difficulties, when G is at B, T will no longer be the apex but instead the epicyclic perigee (*ḥaḍīḍ*). To solve this unfortunate state of affairs, Ṭūsī proposes another sphere, which he calls "enclosing" (*muḥīṭa*), to

[29] Nīsābūrī, *Tawḍīḥ*, Najaf MS 649, 57a (BM MS Add. 7472, f. 65b).

keep the diameter containing the apex and perigee of the given sphere coincident with the "diameter" AB. (We should remember that AB is a reference line and not a fixed diameter of the equator AGB.) To do this, it must return the given sphere back in the direction of the motion of AGB in the amount of angle TEA which is precisely the motion of the large sphere (see Figure C13). Furthermore, it is obvious that the enclosing sphere must be *concentric* with the given sphere; otherwise, the center E of the given sphere will deviate from line AB.[30]

The "enclosing sphere" is itself problematical because it must move another sphere with the same poles and center. Unfortunately, Ṭūsī does not seek to justify this departure from the traditional view of celestial "dynamics." But as I have discussed in the commentary to II.4 [6], the problem of how one sphere moves another was discussed extensively by the commentators who concluded that an enclosing sphere may move an enclosed sphere with the same poles and center.

II.11 [4]. *minṭaqa* (equator): This has been replaced consistently, after the first usage, by *al-dā'ira* (the circle) in MSS FHL (the Baghdad (β) version) and inconsistently in MSS BN. As we have seen there is some confusion involved in using *minṭaqa* to mean a circle inside a sphere. However, the use of *minṭaqatay* in the first sentence of this paragraph, which makes the circles of the Ṭūsī couple into two equators of spheres, is attested to by all MSS, and the commentators have adequately explained this extended usage of *minṭaqa*.

There are grammatical considerations that must also be dealt with. In the phrase *al-murād min minṭaqat al-ṣaghīra* (what is intended by the [inner] equator of the small [sphere]) of version α, it is not immediately clear what *al-ṣaghīra* (the small) refers to. The only apparent possibility is *falak* (orb); however, *falak* is masculine, while *ṣaghīra* is feminine, and we find no variants in any MS. Replacing *minṭaqa* with *al-dā'ira* (circle) solves the grammatical difficulty; however, after *minṭaqat al-ṣaghīra* comes the phrase *madār markaz al-tadwīr fīhā* (the circuit of the epicycle center in it). One would not expect the pronoun of *fīhā* (in it) to refer to a circle but rather to a solid body. This has obviously been felt by those copyists (or redactors?) who have used *al-dā'ira* instead of *minṭaqa*; for we find that two have written *fīhā* above the line (MSS H and N), one has crossed out the word (MS F), and two have left it out entirely (MSS B and L). Tentatively, I would conclude that version β was meant to correct the problem of *al-ṣaghīra* by changing *minṭaqa* to *al-dā'ira*; this, however, left *fīhā* as a rather awkward additional word that was subsequently dealt with accordingly.

Khafrī in his detailed commentary has followed the α version but added *al-kura* (the sphere) between *minṭaqa* and *al-ṣaghīra* to indicate why the latter is

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30 Unfortunately, this was not so obvious to Carra de Vaux, who was misled by a poor diagram in MS B to assume that the given and the enclosing spheres were tangent at a point (Carra de Vaux [1893], p. 361).

feminine. *Al-ṣaghīra* and *al-kabīra* are then the actual spheres that we have drawn in Figure C11. This would resolve the difficulties of version α and is in accordance with Ṭūsī's own usage of *al-kura al-kabīra* later in this paragraph; it was not, however, an expedient of which Ṭūsī availed himself.

II.11 [4]6. *ghayr zā'il ᶜan waḍᶜihā* (not deviate from its position): Although a majority of manuscripts have *waḍᶜihā*, which apparently refers to *al-kura al-mafrūḍa* (the given sphere) (see the indication to this effect in MS F), it is more precise to speak of the given sphere's diameter not deviating from the diameter (*quṭr*) of the large sphere, in which case one would have liked to find *waḍᶜihi*. This latter reading is preferred by Khafrī, who, however, takes *waḍᶜihi* abstractly to mean the *state* of the given sphere's diameter being coincident with the diameter of the large sphere rather than simply referring to the diameter of the large sphere.

II.11 [4]9. *wa-yushtaraṭ fīhā* (It is [also] stipulated for it [them?]): This one has even the commentators stymied. *Fīhā* could conceivably refer to *al-kura al-mafrūḍa* (the given sphere) or perhaps generally to *al-ukar* (the spheres). If we take it to be *fīhi*, with MSS S and M, then, as Khafrī tells us, it could refer to *al-farḍ* (the premise), *i.e.* the premise concerning the given sphere.

We should note here the somewhat odd repetition of the dimensions of the two equators, which have already been given a few lines earlier.

II.11 [4]10–12. *wa-ḥīna'idh*$_{\text{in}}$*...dhālik al-inṭibāq* (Thereupon...that coincidence): MSS H and L have a slightly different version of this passage. The additional *quṭruhā* (its diameter) after *munṭabiq* would refer to the given sphere, while the second *quṭruhā* would refer to the large sphere. I fail to see, however, how the masculine *mutaradd*$^{\text{an}}$ (oscillating) of MSS HL could replace the feminine *mutaraddat*$^{\text{an}}$ since it can only refer to *al-kura al-mafrūḍa*. On the other hand, *zā'ila* (deviating), which modifies *al-kura al-mafrūḍa*, could conceivably be replaced by *zā'il* of MSS HL in which case the modified word would be the added *quṭrihā* (its diameter).

II.11 [5]. Ṭūsī now presents his lunar model. Basically it is the physical adaptation of his couple, which was described in the previous paragraph, nested within a concentric deferent. The place of the "given sphere" is now taken over by the moon's epicycle; in turn, it is placed within the enclosing sphere. These two spheres are then nested within the "small sphere" which is itself nested within the "large sphere." The "large sphere" is embedded within a concentric deferent that is enclosed by the inclined orb. Although Ṭūsī does not so indicate here, the inclined orb is itself within the moon's parecliptic orb (see Figure C14).

II.11 [5]14–15. *markazuhu...al-qamar* (Its center is the point E and its circumference has the same dimension as the moon's epicycle): This phrase occurs in MSS FHL, which indicates that it belongs to the Baghdad (β) version of the *Tadhkira*. (See the introduction, §2.J3 and §2.M2.)

II.11 [5]15. *kura ukhrā muḥīṭa bihi* (another sphere...that encloses it): We have already discussed the role of the enclosing sphere; now that we have reached the specific case of the moon, we must confront the question of why it is necessary at all. This, of course, is different from asking whether one sphere may move a sphere within it that is also concentric and coaxial to it; this problem has been dealt with in the commentary to II.4 [6–7]. But even if we allow its possibility, why would one need such an additional sphere?[31] For the epicycle itself would be sufficient if its motion were made the sum of the motion of the Ptolemaic epicycle and the enclosing sphere. The epicyclic apex would then remain on the fixed diameter and the planet would move with its characteristic motion on the epicycle. The answer, of course, is that strictly speaking it is not necessary, and Khafrī, for one, proposes doing away with it by letting the epicycle take on both motions. We are then faced with explaining why Ṭūsī retains the enclosing sphere when he could dispense with it in his final model. One likely reason is that he wishes to remain as close as possible to the Ptolemaic model; as such, he preserves the individual elements from the *Almagest* in his version. By supplementing without displacing the lunar epicycle, Ṭūsī has reaffirmed his aim to correct but not overthrow the Ptolemaic system.

II.11 [5]16–17. *wa-yanbaghī...kathīr*^an^ (it should not be large lest it occupy too much space): The exact size of the enclosing sphere is not as big an issue as Hartner has claimed.[32] Since the size of the two equators does not depend on the size of the small and large spheres, the effect of the couple is not dependent on the radius of the enclosing sphere. In theory, one is restricted by the nearest distance of the moon predicted by the Ptolemaic model (33;33 earth radii).[33] The orb of fire should begin at that point. However, this boundary between the upper and lower bodies could not, of course, be verified independently; as such, the fact that the space occupied by the lunar shell might be larger than predicted by Ptolemy would not have any earth-shattering effects. In the other direction, it was well-known that some empty space existed between the orbs of the sun and Venus as predicted by Ptolemy. The enclosing sphere could, if nothing else, help fill in this gap.[34]

II.11 [5]17–18. *iḥdāhumā...mā bayn al-markazayn* (one of which, the deferent for these two, takes the place of the small sphere and its diameter is the same size as the eccentricity): "Its diameter" seems grammatically to refer to the small sphere. It should, however, be understood to refer to the equator of the small sphere. Similarly, the next occurrence of *quṭruhā* must refer to *dā'ira* and not *al-kabīra*. The reason this must be the case is that the center of the epicycle,

[31] I am indebted to A. I. Sabra for having posed these important questions.
[32] Hartner [1969], pp. 288–293 passim and p. 302.
[33] Ibid., p. 291.
[34] See further the interesting remarks of Pedersen [1974], p. 394.

whose distance varies from the center of the World by twice the eccentricity, travels along a straight line equal to the diameter of the *equator* of the large sphere (see the commentary for II.11 [4]). As such, *quṭruhā* (its diameter) must refer to an understood *minṭaqa* (equator) and not *kura* (sphere).

II.11 [5]19–20. *markazuhā markaz...fa-yakūn* (its center is that of a circle that the epicycle center touches at its farthest and nearest distances): This supposedly clarifying sentence, which is present in MSS FL, absent from MSS DGMT and Khafrī's commentary, and erased in MS B, is from the new but not necessarily improved version if this sentence is any indication. In order to make sense of it, we must assume the "touching" that the epicycle center does to this circle only occurs at the endpoints of its straight-line path. This would exclude the possibility that the specified circle might have a different center or diameter but still be traversed—and thus "touched"—by the epicycle center. *bu^{c}dayhā* (the two distances) then refers to the two points that are farthest and nearest to the Earth on the circle that is the equator of the large sphere.

II.11 [5]21. *al-kabīra fī thikhan ḥāmil...al-mā'il* (the large [sphere] is within the thickness of a concentric deferent that is enclosed by the inclined [orb]): Unlike Ptolemy who must resort to an eccentric deferent and the so-called "crank mechanism" to achieve the necessary variation in distance of the epicycle center from the Earth, Ṭūsī, thanks to his couple, can employ a concentric deferent. The variation in distance, as we will detail below, occurs because of the back and forth motion of the epicycle center on its straight-line path. This concentric deferent is then placed within the inclined orb. But because the deferent and inclined orb in Ṭūsī's model have the same center and poles, we are faced with exactly the same problem that we discussed earlier in the case of the epicycle and its enclosing sphere. As with the enclosing sphere, we must conclude that the inclined orb is not, strictly speaking, necessary; its motion could be assumed by the deferent. Ṭūsī, however, wishes to keep the individual Ptolemaic parameters.

Although it is not mentioned here, the inclined orb is contained within the moon's pareclipic orb.

II.11 [5]22. *al-muḥīṭ bi-'l-tadwīr alladhī fīhi* (the enclosing [orb] of the epicycle, which is inside it): According to Khafrī, *alladhī* (which) refers to *al-muḥīṭ* (the enclosing [orb]), while the "it" of *fīhi* refers to *ḥāmil* (deferent). There are several other possible readings, all of which are inconsequential. An indication in MS F, for example, directs the reader from *fīhi* back to *al-muḥīṭ*.

II.11 [5]22–23. *^{c}ind al-dhirwa* (at the [epicyclic] apex): The Marāgha (α) version has *bi-qurb min al-dhirwa* (near the [epicyclic] apex). This formulation would serve to indicate that the tangent point of the enclosing sphere and deferent is not exactly at the epicyclic apex since the enclosing sphere intervenes between the epicycle and the deferent. The Baghdad (β) version has *^{c}ind* (at) instead of *bi-qurb min* (near) but this would only be true if one dispensed with the enclosing sphere by having the epicycle assume its motion (see commentary to II.11 [5]15). Khafrī also notes that if we assume that the center of the moon is on the *equator* of the epicycle, then there would also be space between the apex (also considered to be on the equator of the epicycle) and the deferent.

II.11 [5]23. *wa-l-yutawahham...thābit*an (Let the diameter of the deferent that passes through the point of tangency be considered fixed): This diameter is for reference and hence does not move with the deferent.

II.11 [6]. For a listing of the parameters in this paragraph, see Table 5 on p. 457.

II.11 [7]. It is important to keep in mind that, for the most part, this paragraph concerns only the behavior of the epicycle center taken in isolation from the rotation of the inclined orb (motion of the apogee). Thus the diagram accompanying this paragraph (Fig. T13; cf. Fig. C15) does not represent the circuit of the epicycle center during a month but rather during half a synodic month. This circuit, as we shall see, is intended to replace the eccentric deferent of the Ptolemaic model.

Once again, we must be careful to note our reference points. Point Y, which is the reference point for the motion of the large circle, is no longer fixed in space. Being the point of tangency of the large sphere and the concentric deferent, it is now carried along by the deferent. The diameter AB of the equator of the large sphere, upon which the epicycle center moves, remains in all circumstances aligned with the center of the universe. The apex T of the epicycle will also, thanks to the enclosing sphere, remain on AB and will always be the closest point on the epicycle to Y. (Conversely, it is the farthest point on the epicycle from the center of the universe.) Point K, from which the motion of the deferent is measured, has become the fixed point of reference in this diagram. We may define it as the point of intersection of the line OY with the deferent orb when the epicycle center is at its farthest distance from the center of the universe. (This cumbersome definition is necessary because the enclosing sphere intervenes between the epicycle and the deferent orb.) Thus point K corresponds to the lunar apogee in the Ptolemaic model. Now as seen from O (the center of the World), the path of the epicycle center, due to the motions of the deferent orb and the large and small spheres, will be similar to (but not exactly—see II.11 [9]) a circle. The center of this "pseudo-circle" will be at point C, which is on line OK at distance e from O, and the radius will be $R - e$ ($R = 60^p$). These, of course, are the parameters of the Ptolemaic deferent. At $\alpha = 0°$ (where α measures the rotation of the deferent), the epicycle center will be at its farthest distance R from O, while at $\alpha = 180°$ it is at its nearest distance $R - 2e$. The mean distance $R - e$ occurs at $\alpha = 90°, 270°$.

II.11 [7]6. *nazala...mulāzim*an (the diameter of the epicycle will descend as it adheres): Although this phrase is awkward, it is attested to by a majority of manuscripts. The alternative *lam yazal* (it continues...) for *nazala* is preferred by a few copyists as well as by Khafrī. Of some relevance in choosing between these alternatives is the use of *al-taṣā*^{c}*ud* (ascending) later in the paragraph to indicate the opposite motion of the epicycle from the one occurring here.

II.11 [7]7–8. *quṭr al-ḥāmil al-mārr bi-nuqṭat al-tamāss al-madhkūra* (the diameter of the deferent that passes through the tangent point cited above): This point is what we have labeled K. It is the fixed point of reference cited at the end of II.11 [5] from which the motion of the concentric deferent is measured.

II.11 [7]10. *madār shabīh bi-muḥīṭ dā'ira* (a circuit resembling the circumference of a circle): See the commentary to II.11 [9].

II.11 [7]12. *min quṭr al-kura al-kabīra wa-'nṭabaqa quṭruhā thāniy*^{an} (of the large sphere's diameter, which will once more coincide): The diameter referred to here is that of the large sphere passing through point Y.

II.11 [7]13–14. *wa-tumāssu al-muḥīṭa...ḥaḍīḍ al-tadwīr* (the epicycle's enclosing [sphere] touches the concave [surface] of the deferent near the epicyclic perigee): The epicycle's perigee at $\alpha = 180°$ reaches its nearest distance to the Earth. It does not, however, touch the concave surface of the deferent because of the intervening enclosing sphere.

II.11 [7]15. *wa-kāna dhālik al-quṭr...wa-'l-aqrab* (that diameter will pass through the farthest and nearest distances): The diameter of the deferent passing through point K also passes through the lunar apogee and perigee.

II.11 [7]16. *al-taṣā*^{c}*ud* ^{c}*alā al-quṭr al-madhkūr* (will begin to ascend on the above diameter): This is the diameter of the large sphere.

II.11 [7]20–21. *wa-yakūnu al-faḍl...al-markazayn* (The difference between the farthest and nearest distances is in the amount of twice the eccentricity): The nearest distance of the epicycle center is $R - 2e$, while the farthest distance is R; obviously, the difference is $2e$.

II.11 [7]21–1. *wa-takūnu ma*^{c} *dhālik...mutashābiha* (Despite this, the motion [of the epicycle center] about the center of the World is uniform): "The motion" refers to that of the center of the epicycle, which indeed moves uniformly about the center of the World since it is always on a given diameter of the uniformly moving concentric deferent. The "despite this" calls attention to the fact that the epicycle center moves uniformly even though it also, due to another component of its motion, travels alternatively closer and farther away from the center of the World.

II.11 [7]1–2. *wa-yastaqbiluhu...awwal*^an (As was the case previously, the apogee will meet it due to the motion of the inclined [orb]): Our "fixed" point K, the lunar apogee, moves as a result of the motion of the inclined orb (as well as the parecliptic, which is not mentioned here; in order to simplify the argument that follows, we also shall ignore its small effect.) This motion, in conformity with Ptolemy's attempt to account for the so-called evection of the moon's motion, brings the epicycle center to its nearest distance from the center of the World at the moon's quadratures from the sun. Thus at $\alpha = 180°$, the center of the epicycle will be at quadrature and not opposition to the sun since the apogee is moving 11;9°/day in counter-sequence of the signs, whereas the deferent moves 24;23°/day in the opposite direction. (The sun, moving at 0;59°/day in the sequence of the signs, will always be halfway between the apogee K and point Y of the deferent.) At opposition ($\alpha = 360°$), the epicycle center will be "met" by the apogee as was the case for the Ptolemaic model described in II.7 [11].

II.11 [8]. Once again Ṭūsī affirms that this model is his own invention. The three additional orbs he refers to are the large and small spheres of the Ṭūsī couple and the *muḥīṭa* (enclosing orb) of the epicycle. The concentric deferent is not an additional orb since it has taken the place of the eccentric deferent of the Ptolemaic model.

II.11 [9]. The proof that the eccentric path of the epicycle center is not a circle is fairly straightforward. To avoid confusion, however, we should first note that here Ṭūsī is not referring to the actual circuit of the epicycle center during the course of a month but rather, as we have discussed in our commentary to II.11 [7], to the motion of the epicycle center taken in isolation from the rotation of the inclined orb. As such, we are here dealing with the circuit of the epicycle center as shown in Figure C15. In the Ptolemaic lunar theory, the corresponding circuit of the epicycle center is obviously a circle since the epicycle is carried by an eccentric deferent whose motion, though not uniform, does rotate in place about its center. The situation here, however, is rather more complex.

Referring to Figure C16 (which is simply a schematized version of Figure C15),[35] we may proceed with the proof. AFHJ is the concentric deferent;[36] A and B are the farthest and nearest distances of the epicycle center, respectively; C is the midpoint of AB; M and N are the mean distances; and O is the center of the World. As described in II.11 [7], when the deferent has rotated 90°, the center of the epicycle will have traveled half-way along its straight-line path in an

[35] Cf. Fig. T14. Note that its lettering, which only occurs in a few manuscripts, has not been used for Fig. C16 in order to avoid potential confusion. Following the manuscripts, with the possible exception of MS D, we have made ATYH a circle in Fig. T14. On the other hand, we have attempted to give the corresponding circuit AMBN in Fig. C16 an appropriate bulge.

[36] Actually, this is an "inner equator" of the solid concentric deferent.

amount equal to the eccentricity e in the Ptolemaic model. In Figure C16, the position of the epicycle center will at that point be at M. OM will therefore be equal to $R - e$, where R = OF = the radius of the concentric deferent. But since AC = CB = $R - e$, CM would also have to be equal to $R - e$ for AMBN, the circuit of the epicycle center, to be a circle. But clearly CM > OM and so it follows that the circuit is not a true circle.

It is notable that Ṭūsī is concerned with comparing his model with that of Ptolemy. So far, as we have seen, he has noted the difference in the number of orbs and in the shape of the circuit of the epicycle center. What is of rather more interest, though, is his calculation of the maximum difference in the predicted positions of the two models as well as the point at which this maximum difference will occur. Presumably Ṭūsī has arrived at his value of $\frac{1}{6}°$ at the octants based on arithmetical methods, which may indicate that he at some point prepared tables for his model that were never published. This seems implied, to a certain degree, in Khafrī's commentary.[37] In order to test Ṭūsī's claim, however, we shall not prepare tables, of course, but instead resort to analytical methods. (In the following, I have relied on Hartner [1969], pp. 298–299.)

We first must determine the radius vector for the actual path of the epicycle center for each model. For that of Ṭūsī, we may refer to Figure C17 where AGB is the equator of the "large sphere" with D as its center, DZ and ZE are radii of the equator of the "small sphere," and E is the center of the epicycle. Thus for the radius vector $\overrightarrow{OE}$ we have

$$\begin{aligned}\overrightarrow{OE} &= \overrightarrow{OD} + \overrightarrow{DZ} + \overrightarrow{ZE}\\ &= (R - e) + 2(ZD)\sin(90 - \alpha)\\ &= (R - e) + 2(e/2)\cos\alpha\\ &= R - e + e\cos\alpha\end{aligned}$$

where α, the double elongation of the center of the epicycle from the mean sun, is also the motion of the "large sphere" and the concentric deferent orb. For the Ptolemaic model, the radius vector is

$$p = \sqrt{(R - e)^2 - e^2\sin^2\alpha} + e\cos\alpha$$

where $R = 60^p$ and is the distance from the Earth to the apogee of the deferent.[38] The difference between these vectors is then

$$\begin{aligned}\overrightarrow{OE} - p &= R - e + e\cos\alpha - (\sqrt{(R - e)^2 - e^2\sin^2\alpha} + e\cos\alpha)\\ &= R - e - \sqrt{(R - e)^2 - e^2\sin^2\alpha}\end{aligned}$$

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[37] *Takmila*, f. 197b.

[38] In Pedersen [1974], p. 194, R is the radius of the deferent, which accounts for the difference in the formulas; cf. Hartner [1969], p. 298.

By inspection, one can see without too much difficulty that the maximum difference occurs when $\alpha = 90°, 270°$. Remembering that α is the double elongation from the sun, we may conclude that the greatest variation between the Ptolemaic and Ṭūsī models will occur at the octants, exactly as Naṣīr al-Dīn has stated. To find the maximum difference in the predicted values for the true position of the moon, we need to take into account, of course, the position of the moon on the epicycle. Since the maximum prosthaphairesis occurs when the moon is at the mean distance on the epicycle, *i.e.* at the point at which a line drawn from the center of the World is tangent to the epicycle, the maximum value at the octants for the Ṭūsī model will be (see Figure C18)

$$\begin{aligned} \sin \delta_T &= r/(R - e + e \cos \alpha) \\ &= 5;15^p/(60^p - 10;19^p + e \cos 90°) \\ \delta_T &= 6;4° \end{aligned}$$

while for the Ptolemaic model we have

$$\begin{aligned} \sin \delta_P &= r/(\sqrt{(R-e)^2 - e^2 \sin^2\alpha} + e \cos \alpha) \\ &= 5;15^p/(\sqrt{(60^p - 10;19^p)^2 - (10;19^p)^2 \sin^2 90°} + e \cos 90°) \\ \delta_P &= 6;12° \end{aligned}$$

Thus the maximum difference is 0;8°, which again is in agreement with Ṭūsī's statement.[39]

There are several points that should be made regarding the final sentence of this paragraph. In it Ṭūsī remarks that 10 minutes of arc is imperceptible when the moon is at the octants. Since this statement comes from the director of an observatory, it, of course, carries considerable weight and should be taken into account in any future studies on medieval observations. We should note that Ṭūsī is not saying that one cannot in general observe the moon to within 10 minutes of arc; one could, presumably, make finer judgments during eclipses. This is why he restricts his statement by adding *hunāk* (there, *i.e.* at the octants) to the end of the sentence. It turns out that the maximum difference between the predictions of Ṭūsī's planetary models and those of Ptolemy may exceed ⅙ of a degree (see commentary to II.11 [10]). Future research may indicate whether later astronomers were influenced in any way by such considerations.

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39 Though the effect of the prosneusis point reaches a maximum near the octants, it is clear that Ṭūsī wishes us to ignore it for the time being. As we shall see in II.11 [21], he will propose a method to replace the prosneusis point that will approximate its effect upon the position of the moon. Thus up until now, we have been dealing with what has been designated as Ptolemy's "Second Lunar Model" rather than his "Third" and final version.

II.11 [9]1–2$_2$. *wa-mā bayn al-bu^{c}dayn al-awsaṭayn...bayn al-bu^{c}dayn al-ākharayn* (the [distance] between the two mean distances...between the other two distances): As it stands, this statement is at best misleading, at worst simply wrong. If one reads it straightforwardly, it is certainly incorrect since the distance between the mean distances measured as MON is the same, not longer, than AOB (see Fig. C16). Being charitable, one might take the first part of the phrase, *lit.*, "what is between the two mean distances," as referring to MCN rather than MON. Clearly this ambiguity was felt by Ṭūsī's medieval audience since in MS M the phrase has been replaced by the correct, if inelegant, "the [distance] between the two mean distances on it and the midpoint of the other two distances is longer than half the [distance] between the other two distances." Not surprisingly, all the commentators follow this reading; in fact, with the exception of ^{c}Ubaydī, they completely ignore the variant. It is therefore puzzling that the correction did not make it into the Baghdad (β) version. Its occurrence in MS M could indicate that it is one of Shīrāzī's corrections that was picked up by the commentators but not necessarily by the manuscript tradition; for another example where Shīrāzī's suggestion to Ṭūsī was ignored, see the commentary to II.2 [3]21–23.

II.11 [10]. Ṭūsī here proposes his model for the superior planets and for Venus. His description is rather abbreviated, but the missing details may be easily supplied by analogy with the lunar model. The deferent in this case is an eccentric orb, embedded within the parecliptic, whose center Q is that of the equant in the Ptolemaic model, *i.e.* it is at a distance $2e$ from the center of the World O and at a distance e from the center C of the Ptolemaic deferent (see Figure C19; note that for simplicity this diagram shows only the "inner equators" of the solid orbs. The reader should refer to the diagrams for the moon for the elaborated solid version of the model.). Embedded within the deferent of this model is a Ṭūsī couple, whose small circle has a diameter of e and whose large circle has a diameter of $2e$. Now in order for the distance OE from the center of the World to the epicycle center to be $R + e$ at apogee and $R - e$ at perigee as in the Ptolemaic model (see Figure C20), the epicycle center E at apogee must be at its closest position to Q, while at perigee it must be at its farthest distance. It then easily follows that the deferent in this model has a radius of $R + e$ and the starting position of the epicycle center will not be on the deferent, as was the case for the moon, but 180° from that position on the equator of the large sphere. As he states, Ṭūsī will need three additional spheres for his model over what is used in the Ptolemaic planetary configuration: the large and small spheres of his couple and an "enclosing sphere" for the epicycle. This latter, as is the case for the lunar epicycle, is needed to keep the epicyclic apex and perigee aligned with the point about which uniform motion occurs, in this case the equant center Q.

Ṭūsī declares that the distances OE in both his model and that of Ptolemy will be essentially the same; at least, his model will not disturb the circumstances (*aḥwāl*) of the Ptolemaic system. Be this as it may, it must have been clear to Ṭūsī that the distances OE that result from his model are not precisely the same as those of the classical theory. The reason is exactly the same as given by Ṭūsī for the moon; the epicycle center will not describe a perfectly circular path about the Ptolemaic deferent center C but will instead "bulge out" as can most easily be seen at the quadratures (see above commentary to II.11 [9] and Figure C16). Thus my interpretation of this sentence, as reflected in my translation, is that the difference between the models is insignificant and not that the models are completely equivalent.

This brings us to an interesting question. As we found in the case of the moon, Ṭūsī was willing to tolerate a 0;10° maximum difference in the *true* positions as predicted by the two models. But is he willing to overlook a larger variation in the case of the planets? Although he does not deal with this problem directly beyond the short statement mentioned above, it seems unlikely that having found the maximum variation for the lunar model he would have completely ignored this issue in the case of the planets. At any rate, it will be useful to find the maximum variation on the assumption that Ṭūsī did know it; we will then be in a position to see what quantity he considered insignificant. In addition, it will be useful to have such information if future research should turn up other discussions of this important subject.

First let us find OE, the distance between the Earth and the center of the epicycle in the Ṭūsī model. From Figure C19, it is clear that

$$(\mathrm{OE})_{\mathrm{T}}{}^2 = (\mathrm{QE})^2 + (2e)^2 - 2(\mathrm{QE})\,(2e)\cos(180° - \alpha)$$

Now

$$\mathrm{QE} = R - e\cos\alpha$$

as we found for the moon (cf. Figure C17; note that the change in sign is due to the different starting position of the couple). Thus we have

$$(\mathrm{OE})_{\mathrm{T}} = \sqrt{R^2 + 2eR\cos\alpha - 3e^2\cos^2\alpha + 4e^2}$$

For the Ptolemaic model (see Figure C20), one may obtain the following:[40]

$$(\mathrm{OE})_{\mathrm{P}} = \sqrt{[\sqrt{R^2 - (e\sin\alpha)^2} + e\cos\alpha]^2 + (2e\sin\alpha)^2}$$

The maximum difference $|(\mathrm{OE})_{\mathrm{T}} - (\mathrm{OE})_{\mathrm{P}}|_{\mathrm{Max}}$ occurs near the quadratures when $\alpha \approx 90°, 270°$.[41] At those positions, $(\mathrm{OE})_{\mathrm{T}} = \sqrt{R^2 + 4e^2}$, while $(\mathrm{OE})_{\mathrm{P}} = \sqrt{R^2 + 3e^2}$ and the maximum prosthaphairesis for each model will be given by $\delta = \arcsin(r/\mathrm{OE})$ (see Figure C18, where r is the radius of the epicycle). We will then obtain the following results:

[40] Pedersen [1974], p. 280.

[41] The actual values near the first quadrature, which are dependent on the eccentricity, are: Venus, 90°; Mars, 89;53°; Jupiter, 89;59°; and Saturn, 89;59°. Rounding off to 90° will have an insignificant effect on the accuracy of δ to the nearest minute.

	δ_P (Max. Prosthaphairesis for Ptolemaic Model)	δ_T (Max. Prosthaphairesis for Ṭūsī Model)	$\delta_P - \delta_T$
Venus	45;58°	45;57°	0;1°
Mars	40;26°	40;12°	0;14°
Jupiter	11;1°	11;°	0;1°
Saturn	6;11°	6;11°	—

As can be seen, Ṭūsī is essentially correct in his assessment of the two models except in the case of Mars. Whether or not 14 minutes of arc was considered equally negligible by later Islamic astronomers will, one hopes, be the subject of further research.

II.11 [11]. This straightforward paragraph was misread by Hartner, who thought that Naṣīr al-Dīn was saying that he had "invented a theory based on the same principle but too complicated to be explained here, which he hopes to bring as an appendix."[42] Clearly, Naṣīr al-Dīn is saying no such thing; he is, at the time of writing, unable to devise a Mercury model because of the complicated motion of that planet. If he should discover a solution, he promises to append it to the *Tadhkira*, presumably in the manner in which he appended the *Ḥall* to his *Risālah-i Muᶜīniyya.* I very much doubt that Naṣīr al-Dīn ever solved the problem of Mercury; if he did, there is certainly no trace of it to be found.

II.11 [12]. Ṭūsī now turns his attention to the moon's prosneusis point, a problem he has thus far neglected in the lunar model presented above. He begins by outlining a previous attempt to solve this difficulty; an additional orb is postulated, presumably containing the epicycle, that would have the prosneusis point as its center. Ṭūsī notes that it is not clear how one could add such an orb to the existing configuration of lunar orbs without disrupting their motions. Note that whatever Ṭūsī's feelings are about this proposal, it calls into question his earlier statement in II.7 [25] that his predecessors had "not ventured any explanation" for the difficulties of the moon.

II.11 [13]. Part of Ṭūsī's genius lies in his ability to deal with a wide range of seemingly disparate problems with a single, unified approach. Here he indicates that the problem of the moon's prosneusis point may be viewed as an oscillatory motion of the lunar epicycle and thus basically similar to the problem of producing latitudinal variations by means of the motions of the epicycles of the planets. As the reader may surmise, the solution to this problem, which will occupy the rest of the chapter, will consist of producing an oscillatory motion along a single arc of a sphere. Obviously, this is analogous to the rectilinear oscillation in the

[42] Hartner [1969], p. 299; he was probably misled by Carra de Vaux's translation, p. 353.

plane produced by the Ṭūsī couple. As we shall see, Naṣīr al-Dīn proposes using this "curvilinear" Ṭūsī couple to solve a number of the difficulties with the Ptolemaic system that he has previously pointed out.

Regarding the specifics of this paragraph, we should point out that a somewhat similar discussion of the problem of the prosneusis point occurs in Ibn al-Haytham, *Shukūk*, pp. 15–20.[43] The mean apex, from which the mean argument of the epicycle is measured, is on a diameter of the epicycle that Ptolemy had found to be always aligned with a point P at a distance $2e$ from the center of the deferent.[44] As the epicycle center E moves along the deferent, the apex will oscillate on the epicycle between two maximum points that are reached when the epicycle center is on a line perpendicular to the line of apsides and passing through the prosneusis point (see Figure C21). To check this, we note that the inclination of the apex of the epicycle is given by

$$\angle \mathrm{CEP} = \arcsin\,[(2e/R) \cdot \sin \angle \mathrm{CPE}]$$

which clearly reaches its maximum absolute value when $\angle$ CPE = 90°, 270° as stated by Ṭūsī. Taking the derivative of the right-hand side of the equation, we find

$$f' = \left[1/\sqrt{1 - [(2e/R) \cdot \sin \angle \mathrm{CPE}]^2}\,\right] \cdot (2e/R) \cdot (\cos \angle \mathrm{CPE})$$

which reaches its maximum absolute value when $\angle$ CPE = 0°, 180°. This confirms Ṭūsī's statement that the maximum speed (*ghāyat al-surʿa*) of the apex will occur at the apogee and perigee of the deferent.

It is important to keep in mind here that for Ṭūsī the motion of the apex is due to the motion of the epicycle as a whole. He must therefore find a way to move this entire sphere with this oscillatory motion.

II.11 [14]&[15]. Here Ṭūsī gives an account of Ptolemy's explanation of the latitudinal variations due to the motions of the diameters of the planetary epicycles. In the cases of all five planets, the endpoints of the diameters of the epicycles that are aligned with the equant will perform a revolution upon a small circle thus producing what is known as the deviation (*mayl*) (see Figure C22). For the two inner planets, the endpoints of a second diameter at right angles to the first and in the same plane will perform a similar revolution upon small circles (see II.10 [5]). This latter variation is known as the slant (*inḥirāf*). The apparent quote at the end of [14] seems to be a paraphrase of a passage in Bk. XIII, Ch. 2 of the *Almagest*.[45]

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[43] This has been translated by Sabra [1978], pp. 124–127.
[44] *Alm.*, V.5.
[45] *Alm.*, pp. 599–600 (H530–531).

Ṭūsī's objections are quite straightforward. Objection 1: one needs to account for these motions by a physical body. A point or line in a medieval cosmological system cannot simply move by itself. The fact that the endpoints travel along circular paths is completely irrelevant for the requirements of *hay'a*. Objection 2: because the motion of the endpoint along the small circle has the same movement as the motion of the epicycle center on its deferent, one will encounter the same difficulty in the former as in the latter, namely the motion will be nonuniform. Objection 3: the endpoints, as a result of their circular paths, cannot help but cause a displacement in longitude as well as latitude. Because the centers of the small circles occupy the former positions of the apex and perigee in the simpler model that is meant to account for longitude, it is clear that the displacement of the apex and perigee to the circumferences of the small circles will cause a *longitudinal* difference to result. Furthermore, the line joining these new positions of the apex and perigee will no longer be aligned with the equant point as they had been in the simpler model. It should be clear that this objection is not, as are the previous two, a problem of physics (or, as some would have it, "philosophy"). It concerns the disruption of the predicted positions in longitude due to the latitude theory.

The latitude theory was problematic from its inception. In fact, Ptolemy himself, in his famous aside in Bk. XIII, Ch. 2 of the *Almagest*, acknowledged the difficulties with his theory.

We should note in passing that the Ptolemaic approach to the latitudinal variations of the epicycles is virtually identical to the approach of pseudo-Thābit to trepidation (see the commentary to II.4 [5] and the accompanying Figures C2 and C3). This has, I believe, important implications for the history of the Ṭūsī couple, a subject we shall explore in the comments to the following paragraphs.

II.11 [16]. The subject of this paragraph, Ibn al-Haytham's *Maqāla fī ḥarakat al-iltifāf* (Treatise on the Motion of *iltifāf*), is no longer extant. However, his reply to a critic of his work does exist and has been recently edited by A. I. Sabra [1979]. The original *Maqāla* was known to ᶜUmar al-Khayyām who, as we learn from Shīrāzī's *Nihāyat al-idrāk*, appended his own remarks on latitude theory to the text.[46] Ṭūsī himself also discusses the treatise at some length in Ch. 5 of his *Ḥall*.

There is some ambiguity in the Arabic concerning the identity of the critic of truncated orbs (*manāshīr*) in lines 4–5 at the end of the paragraph. It is possible to interpret the phrase to mean that Ṭūsī is criticizing Ibn al-Haytham for suggesting the use of truncated orbs. But in view of Ibn al-Haytham's attempts to disassociate himself from *manāshīr* in his extant reply to his contemporary critic, the most reasonable interpretation is that the phrase is meant to be Ibn al-Haytham's own refusal to accept them.[47]

[46] This appendix, and the judgment on it, occurs in the *Nihāya*, Istanbul, Ahmet III MS 3333 (2), ff. 94b–95a; cf. Kennedy [1966], pp. 377–378 and Sezgin, *GAS*, 6: 34.
[47] Ibn al-Haytham, *Iltifāf*, p. 410 (= p. 195 Arabic numeration), especially lines 6–7.

Ibn al-Haytham's proposal is fairly simple. He first encloses the epicycle with a concentric sphere whose poles K and L are located at a distance equal to the maximum inclination (which may be either the deviation or the slant) from the apex A and the perigee B of the epicycle (see Figure C23). (To avoid potential confusion, we should recall that the plane of the paper is that of the equator of the epicycle.) The enclosing sphere rotates with the same motion as that of points A and B on the small circles of the Ptolemaic model; thus A and B will trace out the same path at the same rate in Ibn al-Haytham's system. (Note that the enclosing sphere will not rotate uniformly.) However, it is clear that every other point on the epicycle, with the exception of the two points on axis KL, will also perform a complete revolution thereby disrupting the Ptolemaic planetary model. For this reason, Ibn al-Haytham postulates another enclosing sphere between the first and the epicycle whose motion is the same as the first but in the opposite direction. Its poles M and N are directly opposite points A and B; obviously, M and N will also describe circular paths due to the motion of sphere KL. What will be the effect of such a combination of motions on the epicycle? The answer is greatly simplified by noting that these two spheres form what may be called a "Eudoxan couple," *i.e.* two spheres corresponding to the third and fourth spheres of Eudoxus's planetary model that move in the above manner thereby producing a hippopedal motion. In Figure C24, we see that point S, an endpoint of the so-called mean diameter of the epicycle lying midway between A and B on the epicycle equator AB in Figure C23 will, due to the motion of sphere KL, move from S_1 to S_2 on a path parallel to that of point A from A_1 to A_2. Now the motion of sphere MN will, of course, leave A stationary at position A_2; it will, however, move S from S_2 to S_3. This will result in S performing a hippopedal motion of narrow dimension with respect to the great circle A_1S_1.[48] Ibn al-Haytham's physical model will therefore very nearly duplicate Ptolemy's mathematical model. (We cannot say exactly duplicate since the motion of point S is not precisely defined by Ptolemy; presumably, it will remain on A_1S_1 in the way that the point P (♈+ 90°) remains on the fixed ecliptic in the pseudo-Thābit trepidation model (see the commentary to II.4 [5] and Figures C2 and C3).

The purpose of Ibn al-Haytham's proposal is not to "correct" Ptolemy in the manner of the Ṭūsī couple, but rather to provide a physical mechanism, *i.e.* solid spheres, for this aspect of his latitude theory. This is clear since Ibn al-Haytham attempts to reproduce the Ptolemaic model as closely as possible, thus retaining the flaws noted by Ṭūsī in his second and third objections. He has in no way dealt with these concerns of late medieval astronomy, namely the nonuniformity of motion on the small circle and the disruption by this model of the longitude theory. (Note that in the *Ḥall,* he does not mention the third objection, that pertaining to the disruption in longitude; see the commentary to II.11 [18].)

[48] The classic discussion of Eudoxus's planetary models and the hippopede is Schiaparelli II, 5–112. Lucid accounts in English are in Heath [1913], pp. 190–211, Dreyer [1906], pp. 87–107 and Neugebauer [1953].

Ibn al-Haytham's model is one of several attempts previous to the time of Ṭūsī to modify the Ptolemaic models. One dominant feature of these proposals, such as that of Ibrāhīm b. Sinān for trepidation, al-Jūzjānī's for the equant, the above model of Ibn al-Haytham and the alternative system of Biṭrūjī, is the use of concentric spheres with different poles. Whether one calls such models Eudoxan or not, the fact remains that the basic idea is the same as that found reported in Bk. XII, Ch. 8 of Aristotle's *Metaphysics*. There has been considerable debate about whether Biṭrūjī, for one, might have been influenced by Eudoxus.[49] My own feeling is that such questions are beside the point. As should be abundantly clear from this chapter and the commentary to II.4 [5], the Eudoxan *technique* was clearly understood and utilized at least as early as Ibrāhīm b. Sinān (d. 335/946) in his *Kitāb fī ḥarakāt al-shams* and, as we have seen, formed the basis for Ibn al-Haytham's only known attempt to modify the Ptolemaic system. There hardly seems any reason, therefore, to question the use by Biṭrūjī of such a method as if it were somehow an isolated event. There is a certain irony, though, in the use of Eudoxus's technique especially as it pertains to Ibn al-Haytham. Ptolemy, in an attempt no doubt to simplify his models, dispensed with additional spheres (or circles) and had his desired motion of the epicycle in latitude occur on its surface. But here we have Ibn al-Haytham rejecting such "simplicity" and returning to a much earlier methodology that Ptolemy had specifically attacked in his *Planetary Hypotheses*.[50] We hardly need reiterate that the history of science is not a linear affair.

II.11 [17]. Ṭūsī here proposes a modification of Ibn al-Haytham's configuration. He suggests placing the poles of the first assumed sphere at a distance from the poles of the epicycle (rather than from the apex and perigee as before) equal to the radius of the small circle. Khafrī and Bīrjandī both take this to be equivalent to Ibrāhīm ibn Sinān's trepidation model described in II.4 [5] by which one mover with poles near the ecliptic poles will supposedly be able to cause every point on the sphere to describe a circle. This, of course, is utter nonsense as the commentators realize. It would be ironic, to say the least, for Ṭūsī, who is hardly reticent in complaining about the failure of others to adhere to the principles of *hay'a*, to advocate such a model. I am therefore inclined to believe that he means this alternative model to have a second sphere, just as was the case for Ibn al-Haytham, to prevent a dislocation of the entire epicycle.

But what would be the effect of Ṭūsī's alternative model on the motion of the apex and perigee? I am tempted to see this modification as a realization by Ṭūsī that by placing the poles of the Eudoxan couple in the vicinity of the epicyclic pole, the apex and perigee will perform hippopedal motions that will lessen the longitudinal dislocation. His further modification in the next paragraph, the curvilinear Ṭūsī couple, would then be a natural outgrowth of this sort

[49] See, for example, Goldstein [1964] and Kennedy [1973].

[50] Goldstein [1967], pp. 38–39 (BM 94a–94b) [German trans., pp. 115–117].

of experimentation. But against this is the fact that all the commentators, beginning with Shīrāzī, assume that Ṭūsī believed that the motion of a small circle near the pole would somehow result in the motion of a small circle near the equator. (Presumably this is because Ṭūsī seemed to accept this in Ibrāhīm ibn Sinān's trepidation model.) Thus placing the Eudoxan couple of Ibn al-Haytham's model near the apex or near the pole would, for Ṭūsī, produce the same effect, *viz.* a circular motion of the apex and perigee.

II.11 [18]. Here Ṭūsī introduces the curvilinear version of his couple. Because he presents it as an outgrowth of Ibn al-Haytham's model, it will be useful to see how well this presentation conforms to the historical record. Ṭūsī had announced the existence of the rectilinear version of his couple in the *Risālah-i Muᶜīniyya*, written in 632/1235, and exhibited full control of it in the appendix to that work, the *Ḥall-i mushkilāt-i Muᶜīniyya*, written shortly thereafter.[51] In the latter, he used it for the longitudinal motions of the moon and planets (*Ḥall*, Ch. 3). He did not, however, have the curvilinear version that he details here and in the next paragraph; at least he made no mention of it. What he did instead was simply provide a summary of Ibn al-Haytham's treatise (*Ḥall*, Ch. 5). The obvious conclusion is that at the time of writing the *Ḥall* Ṭūsī had not yet realized that he might modify his couple in order to produce a curvilinear oscillation. In fact, he does not mention the disruption of the longitude by the latitude theory (Objection 3) that the curvilinear version is meant to resolve. He was, however, intrigued by Ibn al-Haytham's approach and was using it as a starting point with which to deal with the difficulties of Ptolemy's latitude theory.[52] By the time of writing his *Taḥrīr al-Majisṭī* (Recension of the *Almagest*) in 644/1247, Ṭūsī could suggest using a mathematical version of his rectilinear couple to produce an oscillation in latitude so as to avoid a disruption in longitude (*Taḥrīr*, Bk. XIII, Ch. 2, f. 161a–b). He had also realized that the difficulty brought on by the prosneusis point was fundamentally the same as that of the latitude difficulty since he stated as much in the *Taḥrīr* (Bk. V, Ch. 5, f. 66a).[53]

[51] See the introduction, pp. 67–70 of Volume One.

[52] We should note, though, that even as early as the *Muᶜīniyya* Ṭūsī was not satisfied with Ibn al-Haytham's "solution" and refers to several remaining "defects"; see p. 68. But he had solution of his own to offer, which may account for his lack of criticism of Ibn al-Haytham in the *Ḥall*.

[53] To his credit, Saliba has brought attention to these passages in his [1987a]. Unfortunately his attempt to use them to advocate the chronological priority of the *Taḥrīr* over the *Ḥall* is misguided; cf. p. 69, footnote 15. To repeat, the *Ḥall* has no trace of the second (curvilinear) version of the Ṭūsī couple. Furthermore, the connection Ṭūsī draws in the *Taḥrīr* between the problem of the moon's prosneusis point and that of the latitude theory, as well as Objection 3 to the latitude theory, are all missing in the *Ḥall* but, of course, are repeated in the *Tadhkira*. It would be highly unlikely that Naṣīr al-Dīn would have omitted these crucial aspects of his planetary theory from the *Ḥall* if he had written it after discussing them in the *Taḥrīr*. Saliba insists, however, that "the identity of the solution for the prosneusis point and that of latitude, and the possibility of explaining such a motion, was proposed in the *Ḥall*" (p. 11) but significantly gives no reference.

Thus the need for a model that could produce a curvilinear oscillation had become manifest, and Ṭūsī would have then seen the applicability of his already available couple. We should remember, however, that the couple of the *Ḥall* was used to produce rectilinear oscillation, whereas now he needed curvilinear oscillation. Because the device in the *Taḥrīr* is so schematic (unlike a normal Ṭūsī couple, he uses two intersecting circles of the same size, one of which carries the center of the other), it is difficult to determine whether he also possessed the full physical model of the *Tadhkira's* curvilinear couple. It may well be that he only had the general idea of using his couple for the latitude theory at the time of writing the *Taḥrīr*; it was only later in writing the *Tadhkira* that he felt that a *hay'a* work required him to give the full physical model. It is here that he could also draw the connection between his curvilinear couple and Ibn al-Haytham's model. But in attempting to understand the historical situation of the mathematical device in the *Taḥrīr*, it is worth noting that Ṭūsī drew a clear distinction between an astronomical presentation limited to circles and one in which the "principles of motion" (*i.e.* solid orbs) were invoked (see the *Tadhkira*, II.5 [10]). In the *Ḥall*, for example, he criticized Khiraqī for accepting circles for the latitude theory in the *Muntahā al-idrāk* while insisting on solid orbs elsewhere in that work (pp. 15–16). Ptolemy, on the other hand, had consistently limited himself to circles throughout the *Almagest*, at least according to Ṭūsī. Thus the fact that Ṭūsī did not use the full physical model of his curvilinear couple in the *Taḥrīr* should not be used to prove that he did not have it at that time; he may simply have felt constrained by the *Almagest* format to restrict himself to circles.

II.11 [18]&[19]. Ṭūsī here presents the curvilinear version of his couple in order to deal with the third objection. We should again emphasize the importance of such a device in allowing the medieval astronomer to confine the effects of his assumed motions to a single parameter without disrupting any other.

As we have stated, Ṭūsī presents his lemma as an outgrowth of Ibn al-Haytham's model. We again have a sphere, which Ṭūsī calls the large sphere, whose axis intersects the epicycle at points H and T, which are at a distance equal to the maximum inclination from apex A and perigee B, respectively (see Figure C25; cf. Figure T15 in the edition and translation, which represents a medieval attempt at drawing in perspective). We should note, however, that great circle AHBT is not the same as AS_1B in Figure C23. In Ibn al-Haytham's model, AS_1B is the equator of the epicycle; here, the great circle shown in the diagram passes through the poles of the equator. We next bisect arc AH and arc BT at E and Z and assume a "small sphere" with axis EZ that is located between the large sphere and the epicycle. The large sphere is then given a certain motion (in this case equal to the mean motion [presumably] of the epicycle center on its deferent) in a given direction, while the small sphere rotates with twice this motion in the opposite direction. Ṭūsī concludes (incorrectly as we shall see later) that the apex will oscillate between points A and G on a great circle arc. This will account for the latitudinal deviation without a corresponding disruption of the apex in longitude (see Figure C26; note that circles E and H are on the surface of the epicycle sphere and are not in the same plane.) But as a result of the motion of the two spheres, other points on the epicycle will be displaced. As with Ibn al-Haytham's model, one will need a third sphere (not shown in Figure C25) located between the small sphere and the epicycle with poles that are always aligned with the oscillating apex and perigee. As should be clear, this sphere must rotate at the same rate and in the same direction as the large sphere in order to bring the rest of the epicycle to its proper position. (This orb is analogous to the enclosing orb [*muḥīṭa*] of the rectilinear version; cf. Figures C12 and C13.) The net result for the epicycle as a whole will be an oscillation on an axis coincident with the mean diameter, *i.e.* the diameter in the plane of the equator of the epicycle perpendicular to AB.

There are problems with the curvilinear version, some of which Ṭūsī acknowledges, some of which he does not. One that seems to have escaped him is the failure of the couple to work as advertised. The resultant locus will not, in fact, be an arc but rather a stretched out figure 8 on the surface of a sphere (see Figure C26).[54] To see this we need only note that in spherical triangle EA_2H the exterior angle FEA_2 must be less than the sum of interior angles EHA_2 and EA_2H;[55] the endpoint of radius vector EA_2 must therefore always extend beyond arc A_1G except when $\theta = n \cdot 90°$, n any integer, in which case A_2 will fall on it. Nevertheless, because of the small size of the arcs of oscillation, divergence will be slight.[56]

Another remaining difficulty, one that Ṭūsī must regretfully admit he is incapable of resolving, is related to Objection 2,[57] namely that concerning the uniformity of the motion about a point other than the center. Because the motion of epicyclic apex A is approximately given by $A_1H - A_1H \cos \theta$,[58] it is clear that its inclination in either direction from H will be exactly equivalent in amount and duration; the Ptolemaic theory, however, requires that this inclination be of longer duration in one half than the other since the motion of point A on the small circle is coordinated with the irregular motion of the epicycle center on the deferent.

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[54] This was originally pointed out to me by E. S. Kennedy. Otto Neugebauer further noted that the figure could not be a hippopede since it has a vertical rather than horizontal tangent at $\theta = n \cdot 90°$ inasmuch as the area of triangle EA_2H will approach 0 at these points. Thus the figure will not be a smooth curve but rather one with pinched cusps.

[55] This was recognized by Khafrī in his commentary, ff. 214b–215a, who cites Proposition 11, Book I of Menelaus's *Spherics*.

[56] Even in the case of the greatest oscillation, the 24.538° in either direction resulting from the moon's prosneusis, the maximum deviation from arc AG will only be 0.214°, which is about 0.87%. This will approximately occur when $\theta = n \cdot 180° \pm 35°$, n any integer. For smaller arcs of oscillation, the percentage deviation will be even smaller.

[57] See paragraph [15].

[58] We here ignore the deviation from the great circle arc.

II.11 [20–22]. Here Ṭūsī mentions the situations in which the curvilinear version of his couple may be applied. In all cases the basic approach is the same. For the motion of the epicycle due to the prosneusis point, Ṭūsī notes a problem that his model is unable to solve, namely that his construction will result in a motion of inclination that is symmetrical with respect to the line joining the centers of the epicycle and the deferent, whereas the Ptolemaic theory results in an asymmetrical motion of inclination (see II.11 [13] and Figure C21). This drawback of the Ṭūsī model is somewhat analogous to its inability to deal with Objection 2.

The suggested use of the Ṭūsī couple for trepidation and the variability of the obliquity is of great historical interest since Copernicus introduces it for just this purpose in *De revolutionibus*, III.4; this should obviously be the subject of further research.

II.11 [23]. These sorts of comments give us some understanding of the attitude of Ṭūsī toward what we might call the progress of science. Provided that we do not make too much of them, they are well worth noting. It is clear that he is more than aware of the remaining flaws in his models and seems to have no hesitation in accepting that they may be superseded. Ṭūsī perhaps hoped that his work would be the basis for a continuing program of research; if so, he would have been well pleased with subsequent developments.

Table 5. Parameters of Ṭūsī's Models for "Difficulties" 1–6.†

	Moon	Venus	Mars	Jupiter	Saturn
Epicycle Radius (r) Motion(γ)	 5;15^{P} 13;4°/day cs	 43;10^{P} 0;37°/day s	 39;30^{P} 0;28°/day s	 11;30^{P} 0;54°/day s	 6;30^{P} 0;57°/day s
Enclosing Sphere Thickness (ε) Radius (r+ε) Motion (α)	 unspecified 5;15^{P}+ε 24;23°/day s	 unspecified 43;10^{P}+ε 0;59°/day s	 unspecified 39;30^{P}+ε 0;31°/day s	 unspecified 11;30^{P}+ε 0;5°/day s	 unspecified 6;30^{P}+ε 0;2°/day s
Small Sphere Radius ($\frac{1}{2}e$+r+ε) Motion (2α)	 10;24½^{P}+ε 48;46°/day cs	 43;47½^{P}+ε 1;58°/day cs	 42;30^{P}+ε 1;2°/day cs	 12;52½^{P}+ε 0;10°/day cs	 8;12½^{P}+ε 0;4°/day cs
Small Sphere **Inner Equator** Diameter (= e)	 10;19^{P}	 1;15^{P}	 6;00^{P}	 2;45^{P}	 3;25^{P}
Large Sphere Radius (= e+r+ε) Motion (α)	 15;34^{P} + ε 24;23°/day s	 44;25^{P} + ε 0;59°/day s	 45;30^{P} + ε 0;31°/day s	 14;15^{P} + ε 0;5°/day s	 9;55^{P} + ε 0;2°/day s
Large Sphere **Inner Equator** Diameter (= 2e)	 20;38^{P}	 2;30^{P}	 12;00^{P}	 5;30^{P}	 6;50^{P}
Deferent Orb Radius Moon: R+r+ε Planets: R+e+r+ε Motion (α)	 65;15^{P} + ε 24;23°/day s	 104;25^{P} + ε 0;59°/day s	 105;30^{P} + ε 0;31°/day s	 74;15^{P} + ε 0;5°/day s	 69;55^{P} + ε 0;2°/day s
Deferent **Inner Equator** Radius Moon: R Planets: R + e	 60;^{P}	 61;15^{P}	 66;^{P}	 62;45^{P}	 63;25^{P}
Inclined Orb Thickness (κ) Radius (R+r+ε+κ) Motion	 unspecified 65;15^{P}+ε+κ 11;9°/day cs	 unspecified NA NA	 unspecified NA NA	 unspecified NA NA	 unspecified NA NA
Parecliptic Thickness (μ) Radius Moon:R+r+ε+κ+μ Plan.:R+e+r+ε+μ Motion	 unspecified 65;15^{P}+ε+κ +μ 0;3°+/day cs	 unspecified 104;25^{P}+ε+μ 1°/70 years s	 unspecified 105;30^{P}+ε+μ 1°/70 years s	 unspecified 74;15^{P}+ε+μ 1°/70 years s	 unspecified 69;55^{P}+ε+μ 1°/70 years s

† Motion in the sequence (s)/counter-sequence (cs) of the signs is determined by the motion of the orb's apogee point; R = radius of Ptolemaic deferent; e = eccentricity; NA = not applicable.

Book II, Chapter Twelve

For this chapter, see *Alm.*, pp. 243–244 (H401–402), 258–273 (H427–459); *HAMA*, 1: 100–101, 112–117; and Pedersen [1974], pp. 203–204, 213–219; see also Kennedy [1956]. Ṭūsī's attempt to be brief in this chapter results in some less than clear formulations. Bīrjandī is particularly upset and accuses him of being obscure and overgeneralizing, complaints that lead Bīrjandī in his commentary to rewrite a good part of the chapter in what he considers a clearer style.

II.12 [4]4&6; II.12 [5]10; II.12 [6]22. *falak al-burūj* (ecliptic orb): *Falak* (orb) is here being used in its secondary sense of equator; see commentary to II.3 [2]13. Compare II.12 [1]21–22 where *falak* is used in its more customary sense of solid orb.

II.12 [5]9–11. *wa-idhā kāna al-kawkab...ikhtilāf al-ᶜarḍ bi-ᶜaynih* (If the planet...precisely the difference in latitude): To understand this passage, it is helpful to keep in mind that in this particular case the ecliptic meridian circle and the altitude circle are one and the same; cf. II.3 [17] 6–9. On the ecliptic meridian circle, see the commentary to II.3 [16]20.

II.12 [6]. Figure C27 (admittedly a grossly oversimplified diagram since it assumes the planet to be on the ecliptic meridian circle) should help give an indication of Ṭūsī's intention in this paragraph. When the planet P has the same direction in latitude as the invisible ecliptic pole (either north or south), the absolute value of the difference in latitude is added to the absolute value of the true latitude to arrive at the apparent latitude (position P_1). When the planet has the same direction as the visible ecliptic pole, one subtracts the absolute value of the difference in latitude from the absolute value of the true latitude (position P_2). An exception to the latter rule occurs when the planet is between the zenith and the visible ecliptic pole whereupon one adds the difference in latitude (position P_3). In the case where the planet has no true latitude (position P_4), the apparent latitude will have the same direction as the invisible ecliptic pole.

The "same reason as before" at the end of the paragraph refers to the rule that the apparent position will always be closer to the horizon.

II.12 [7]. The moon's distances are dealt with in IV.2.

II.12 [8]. For solar parallax, see *Alm.*, pp. 258–260 (H428–432), 265 (H442–443).

Book II, Chapter Thirteen

For this chapter, see *Alm.*, Bk. VI, pp. 275–313 (H461–535); Pedersen [1974], pp. 220–235; *HAMA*, 1: 118–141.

II.13 [1]16. *mā lā yaṣilu ilayh nūr al-baṣar* (that which the light of the eye does not reach): The theory of vision implied here is that of the Greek mathematical tradition (*viz.* that of Euclid and Ptolemy) whereby vision was brought about by visual rays emanating from the eye. The Peripatetic tradition opposed this so-called extramission theory of vision with an intromission theory based on forms arising from an object and then traveling to the eye. Although Ibn al-Haytham had two centuries prior to Ṭūsī brought about a brilliant synthesis of these two traditions—one that maintained an intromission theory that at the same time employed the visual rays of the mathematicians—Ṭūsī here and elsewhere seems totally innocent of any knowledge of the optical work of his great predecessor.[1] Again we have the curious historical situation of major breaks in the transmission of science within the Islamic tradition (cf. commentary to II.1 [1] on the moon illusion problem; note though that Ibn al-Haytham's work in astronomy was generally known to Ṭūsī—see II.11 [16]).

II.13 [4]11. *ᶜalā minṭaqat al-burūj* (on the [plane of the] ecliptic equator): Ṭūsī is rather confusing here since the center of the shadow cone is obviously in the plane of the ecliptic rather than on the ecliptic equator itself. The same will apply to *ᶜalā al-minṭaqa* in II.13 [4]14–15.

II.13 [5]. For the derivation of the eclipse limit of 12° (12;12° in the *Almagest*), see *Alm.*, pp. 282–287 (H476–484). The value 2⅗ for the ratio of the shadow circle's diameter to the lunar disk is from *Alm.*, pp. 254, 285 (H421, 480). Compare IV.3 [1].

II.13 [5]2. *al-aṣābiᶜ* (digits): On linear digits (for diameters) and area digits (for bodies), see Kennedy, *Survey*, p. 143 and *Alm.*, pp. 302–305 (H512–518), p. 308.

II.13 [6]. On the intervals between lunar eclipses, see *Alm.*, pp. 287–290 (H485–489).

II.13 [7]. On the need and method of adjusting for parallax in the case of solar eclipses, see *Alm.*, pp. 173–174 (H267), 310–313 (H527–535).

II.13 [8]2–4. *wa-quṭr al-shams…ilā sitt wa-thalāthīn* (The solar diameter…to 36 minutes): The following table is a sampling of some of the better known values for the apparent diameters of the sun (d_s) and moon at syzygy (d_m):

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[1] Sabra [1972], pp. 190–197, especially 190–192, 196; see also Lindberg [1976], pp. 11–17, 58–67, 71–80.

Table 6. Solar and Lunar Apparent Diameters.

Astronomer	d_s Apogee	d_s Perigee	d_m Apogee	d_m Perigee	Source
Ptolemy (2nd c. A.D.)	0;31,20°	0;31,20°	0;31,20°	0;35,20°	*Alm.*, pp. 252, 254, 284, 285
Khwārizmī (9th c.)	0;31,20°	0;33,48°	0;29,16°	0;34,34°	*Khw.*, pp. 175,180; Neugebauer [1962b], pp. 105–6
Battānī (10th c.)	0;31,20°	0;33,40°	0;29½°	0;35⅓°	*Batt.*, 1: 58, 236; 3: 87–8
"Moderns"	0;31,03°	0;33,33° *			Nīsābūrī
Kūshyār (*Zīj al-jāmi*ᶜ) (10th c.)			0;29⅔°	0;35⅓°	Bīrjandī
Ṭūsī (13th c.)	0;31°	0;34°	0;29°	0;36°	*Tadhkira*, II.13 [8]
Shīrāzī (13th c.)	0;30,02°†	0;32,38°†			*Tuḥfa*, IV.1 (f. 270a)
Kāshī (*Zīj-i Khāqānī*) (15th c.)	0;30°	0;32,12°	0;31,4° (lunar)‡ 0;31,24° (solar)‡	0;37,2° (lunar)‡ 0;37,46° (solar)‡	Bīrjandī

Notes:

* This value is also given by Bīrūnī, *Qānūn*, 2: 869.

† These values are derived by assuming that Ptolemy's value of 0;31,20° occurs at the sun's mean distance; elsewhere (*Tuḥfa*, II.15 [Mabḥath 3], f. 195b) Shīrāzī adopts Ṭūsī's figures.

‡ Kāshī makes a distinction between the moon's diameter during lunar eclipses, which he measures from the center of the World, and its diameter during solar eclipses, which he measures from the observer. (Note that I am depending on Bīrjandī's account; I do not have access to the actual *zīj*.)

II.13 [8]2–3_2. *wa-in kāna quṭr al-shams...ḥalqat al-nūr* (If the sun's diameter...ring of light): One of the consequences of Ṭūsī's values for the apparent solar and lunar diameters (as well as those of other Islamic astronomers) is that the moon might be smaller than the sun during conjunction, a situation that could lead to an annular eclipse. This possibility is clearly foreclosed by Ptolemy's numbers; it is interesting that certain of Ptolemy's "predecessors," among whom, as Swerdlow has shown, we should include Hipparchus, allowed for annular eclipses—at least inasmuch as they took the moon's diameter to be equal to the sun's at the moon's mean distance rather than at apogee (*Alm.*, p. 252, n. 53 [H417]; Swerdlow [1969], pp. 291–298).

There were a number of reports of annular eclipses that postdate the *Almagest*. In antiquity there is one observed by Sosigenes, the teacher of Alexander of Aphrodisias, probably that of 4 September 164 A.D.[2] Bīrūnī (*Qānūn*, 2: 632) cites the observation of an annular eclipse made on 28 July 873 by Abū al-ᶜAbbās al-Īrānshahrī in Nīsābūr (Schramm [1963], pp. 26–27 and 27, n. 1; Oppolzer, p. 200 and Plate 100).

II.13 [9]11–14. *fa-fī al-iqlīm al-rābiᶜ...ilā sabᶜ darajāt mumkin*ᵃⁿ (In the fourth clime...after the node of the tail): Ptolemy, who calculates the solar eclipse limits by considering the boundaries of the seven climes, namely Meroe with latitude 16;27° and Borysthenes with latitude 48;32°, finds them to be 17;41° when the moon is northerly and 8;22° when it is southerly.[3] On the other hand, Ṭūsī here gives the eclipse limits for the fourth clime, which we learn from III.1 [8] is bounded by latitudes 33 ⅝° and 39 ⁹⁄₁₀°. In order to obtain the maximum limit, at least to a good level of approximation, one considers two cases:[4]

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[2] See Schramm [1963], pp. 24–27 for a discussion of Sosigenes and the significance of his observation for establishing the impossibility of a homocentric cosmology.
[3] Note that these are true not mean values.
[4] For an explanation and justification of what follows, see *HAMA*, 1: 126–129.

Case 1. The moon is north of the ecliptic (or in Ṭūsī's terms when it is "after the node of the head or before the node of the tail"). Since in this case one wishes to maximize the effect of parallax so as to obtain the maximum limit, one assumes: a) the observer at latitude 39 $\frac{9}{10}$°; b) the moon at epicyclic perigee; c) the sun at perigee; d) conjunction occurring at Aries 0° 6 hours before noon with Capricorn 0° culminating. One then finds using Tables II.13 and V.18 from the *Almagest* that the latitude component for the adjusted lunar parallax[5] (p_β) is about 0;54° and the longitude component (p_λ) is 0;28°. Then using Neugebauer's formula[6]

$$\text{solar eclipse limit} = (p_\beta + 0;35) \cdot 11;30 + p_\lambda$$

one obtains a limit of 17;32°, which compares favorably with Ṭūsī's 18°.

Case 2. The moon is south of the ecliptic (or in Ṭūsī's terms it is "before the node of the head or after the node of the tail"). One should note here that since the ecliptic is always to the south for an observer in the fourth clime, one would wish to minimize the effect of the latitude component of the adjusted parallax while maximizing the longitude component in order to obtain the maximum limit. One therefore assumes: a) the observer at latitude 33$\frac{5}{8}$°; b) the moon at epicyclic perigee;[7] c) the sun at perigee; d) conjunction occurring at Libra 0° 6 hours before noon with Cancer 0° culminating. Using the same procedure as before, one finds the latitude component for the adjusted lunar parallax to be about 0;10½° and the longitude component to be 1;04°. The resulting eclipse limit is just under 6°, whereas Ṭūsī reports 7°.

One should not take the above numbers too seriously. For one thing, they are based upon Ptolemy's tables, which have been modified by Islamic astronomers. Furthermore, it is not clear to what extent Ṭūsī, or any other Islamic astronomer, has improved upon Ptolemy's, or Pappus's, rather defective techniques for finding the maximum solar eclipse limits.[8] That my numbers correspond reasonably well with those of Ṭūsī should not disguise the fact that any coincidences in final results may be fortuitous. It is salutary to look at Bīrjandī's comments. For the case of the moon north of the ecliptic, basing himself on a maximum latitude component of 44 minutes and a maximum longitude component of 11 minutes, he finds the eclipse limit to be 15;34°. In the case of the moon south of the

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[5] Adjusted lunar parallax is the difference between the lunar and solar parallaxes. See Kennedy [1956], p. 35.

[6] I have substituted 0;35° for the radii of the two luminaries in place of Ptolemy's 0;33,20°.

[7] Though at first glance it may seem contradictory to place the moon at the epicyclic perigee when we wish to minimize the effect of the latitude component, this small factor is more than compensated for by the maximization of the limit that results from the additional amount of the radii of the two luminaries at perigee and of the longitude component of the parallax.

[8] See *HAMA*, 1: 127–129.

ecliptic, he arrives at an eclipse limit of 7;26°, the latitude and longitude components in this case being 4 minutes and 50 minutes, respectively. But I have been unable to find the basis for his values of parallax, which are considerably at variance with those of Ptolemy and presumably those of Ṭūsī as can be inferred from the divergence in their eclipse limits. In the second case, Bīrjandī's value of 4 minutes for the latitude component seems particularly suspect, but the source of variance could range from a different boundary for the fourth clime to computational error. That Bīrjandī is not above the latter is attested by his finding 0;30°/sin 5° to be 6;36° (correct value: 5;44°). In short, one would need to examine a number of discussions of solar eclipse limits in the *zījes* in order to arrive at some conclusion concerning the assumptions and techniques under which these numbers were arrived at and whether there were any improvements over Ptolemy.

II.13 [9]14–19. *wa-li-dhālik yumkinu kusūfān...jihat al-ᶜarḍ* (It is therefore possible for two solar eclipses...different directions in latitude): On the intervals between solar eclipses, see *Alm.*, pp. 290–294 (H489–498).

II.13 [9]19–21. *wa-li-kawn al-qamar...awwal*an (And because the moon...to reappear): These are rather trivial consequences of the west to east motion of the moon and sun, and of the moon's greater speed.

Book II, Chapter Fourteen

II.14 [1]. A rather more extensive discussion of sectors occurs in Bīrūnī, *Tafhīm*, 201 (pp. 107–110). See also Kennedy, *Survey*, p. 143; idem, [1958], pp. 247–253; and idem, [1960], pp. 218–222. According to Kennedy, the sectors do not appear until Islamic times and seem to be motivated by astrological considerations.

II.14 [2]. On the visibilities of the five planets and the "phases," see *Alm.*, pp. 636–647; *HAMA*, 1: 230–261; and Pedersen [1974], pp. 386–390.

II.14 [2]18. *wa-thālith*an *bi-ḥasab ikhtilāf al-āfāq* (and third, according to different horizons): As we shall see in III.4 [2]4, *ufuq* (horizon) may mean locality, which it probably does here as well. As such, Ṭūsī is not completely accurate since in addition to locality the inclination of the ecliptic to the horizon can affect visibility as stated in the *Almagest* (p. 636 [H590]); this, of course, will vary at a given location. Shīrāzī and the commentators have noted the problem and corrected it.

II.14 [2]19–20. *wa-'l-zahara lā takhtafī...bukra wa-ᶜashiyya* (Venus does not become invisible...while retrograding): Ptolemy states that the interval from evening setting to morning rising is about two days (*Alm.*, p. 641 [H597]).

II.14 [2]9–10. *wa-aqall mā yakhtafī laylatān* (The minimum that it is invisible is two nights): Shīrāzī in the *Nihāya* claims that it would be possible for the moon to disappear for only one night; although unusual this would not be impossible (cf. Ilyas [1984], pp. 100–101).

II.14 [2]10–14. *wa-qad umtuḥina...mar'iyya lahu faqaṭ* ([The matter] has been subjected to testing...in its case only, is apparent): Whether or not Ṭūsī subjected the matter to testing, he ends up with exactly the same values as Ptolemy (*Alm.*, pp. 639 [H595], 640 [H597]). Note, though, that whereas for Ptolemy these values are of the sun's depression at the moment the planet is on the horizon, for Ṭūsī these amounts are the altitude of the planet when the sun is at the horizon.[1] Bīrjandī states that this change was made because a depression is a calculated value, whereas an altitude could be observed.

It is of some interest that Ptolemy ignores the problem of the moon's visibility, this in spite of its probable importance for the Babylonians. Ṭūsī's 8° is a bit low compared with the values deduced by Hogendijk [1988] for Khwārizmī's *zīj* (8;54°), and for the *Mufrad zīj* of al-Ṭabarī and for a table attributed to al-Khāzin (both 9;30°). But 8° is given as a rule of thumb by the Royal Greenwich Observatory (Ilyas [1984], p. 100). (8° is also an implied minimum for Battānī; see Bruin [1977], p. 355.)

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[1] On the constant depression criterion, see Hogendijk [1988], pp. 29–30; cf. Bruin's [1977] "arc of descent," p. 333.

BOOK III

Book III, Chapter One

III.1 [1]7. *fī awwal al-kitāb* (in the first part of the book): See II.1 [2–3] and [7].

III.1 [3]8–11. *wa-innamā ḥukima bi-anna al-ma^{c}mūr...niṣf dawr al-falak* (The inhabited world has been determined...one-half revolution of the orb): The ultimate source for this is probably *Alm.*, p. 75 (H88); Toomer (n. 3) discounts what seems to be implied, namely that Ptolemy had records of simultaneously observed eclipses at widely separated locations.

III.1 [4]21. *al-jibāl...al-manṣūba ilā al-qamar* (mountains...named for the moon): The "mountains of the moon" are mentioned by Ptolemy in the *Geography* where he states they lie between 57° and 67° longitude (measured from the Fortunate [Canary] Islands) and at 12½° south latitude.[1] In view of this there seems little to recommend reading *qamar* (moon) as *qumr* (white [*pl.*]), which is given as a possibility by Bīrjandī and by Wright in a note to his translation of Bīrūnī's *Tafhīm* (p. 143, n. 8). Modern attempts have been made to associate the mountains of the moon with the snow-clad peaks of Ruwenzori located between Lake Edward and Lake Albert on the border between Zaire and Uganda, which do indeed feed into the Nile; but this seems problematical due to their great elusiveness and to the existence of the more prominent Mounts Kenya and Kilimanjaro, the melting snows of which, however, do not drain into the Nile.[2]

III.1 [5]25–26. *ka-'lladhī bayn al-Maghrib...wa-'l-Shām* (such as: that between the Maghrib...and Syria): Note that Ṭūsī divides the Mediterranean into two parts, a not uncommon practice; cf. Dunlop, *EI*^{2}, 1: 934–936.

III.1 [5]27–1. *al-khalīj al-Barbarī wa-huwa aqrabuhā ilā al-Maghrib* (the Gulf of Barbary [Gulf of Aden], which is the nearest of them to the Maghrib): This is the Gulf of Aden, which in actuality is east of the Red Sea. Maps by Bīrūnī (*Tafhīm*, p. 124) and Nīsābūrī (f. 78a) confirm Ṭūsī's error.

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[1] Müller ed., IV, 8, 2 [2: 789].

[2] Cf. Mountjoy and Embleton [1967], pp. 100–101.

III.1 [5]1. *al-khalīj al-akhḍar* (the Green Gulf): Although there is considerable difference of opinion concerning the identity of the "Green Gulf"—indeed Shīrāzī gives two widely divergent accounts—I believe that we may safely place it in East Asia and furthermore make it roughly coincident with the South China Sea. The source of the confusion is that it is often not distinguished from the Green *Sea*, which may be: (a) the Encompassing Sea or "Ocean";[3] (b) the Indian Ocean and Chinese Sea;[4] (c) the Indian Ocean itself;[5] or (d) part of the Indian Ocean.[6] In the *Tuḥfa* Shīrāzī places the Green Gulf in the vicinity of the South China Sea, at least this is a reasonable inference since Khānqū (Canton) and Khānjū (Quanzhou ?)[7] are mentioned as coastal cities. He furthermore seems intent on rectifying what he stated in the *Nihāya* since he says that the Green Gulf is "in the most distant of the land of China, not India (*fī aqṣā bilād al-Ṣīn lā al-Hind*).[8] Shīrāzī's *Tuḥfa* account is followed by Nīsābūrī and Bīrjandī, but Jurjānī, copied by Khafrī, conflates the two accounts and states that it is the "Chinese and Indian Sea" (*baḥr al-Ṣīn wa-'l-Hind*). This confusion leads an exasperated Bīrjandī to declare that "whoever claims that [the Green Gulf] is in the land of India and calls it the Indian Sea is mistaken." Shīrāzī also gives the dimensions and delineation of the Green Gulf: it is basically triangular in shape with the eastern side being about 110 parasangs, while the western side along the Chinese coast is some 500 parasangs. This is a rough but recognizable description of the South China Sea.

III.1 [5]2. *wa-ka-baḥr Warank* (and such as the Sea of Warank): According to Minorsky ([1942], pp. 115–116), *Warank* was first used by Bīrūnī to indicate the Baltic, but he seems not to have distinguished between it and the Beloye More (White Sea).[9] The word itself derives from the Varangians, one of the Scandinavian roving bands that invaded Russia in the 9th and 10th centuries and even managed to reach Constantinople.

III.1 [5]6. *al-masālik* (the geographical placebooks): On this genre of geographical works, see Pellat, "*al-Masālik wa 'l-Mamālik*," *EI*^{2}, 6: 639–640; cf. Maqbul-Ahmad, "*Djughrāfiyā*," *EI*^{2}, 2: 575.

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[3] Ibn Khaldūn, *Muqaddimah*, 1: 96.
[4] Bīrūnī, *Qānūn*, 2: 547–549, 552.
[5] Abū al-Fidā', p. 22 (Arabic text), p. 27 (French trans.).
[6] See Khwārizmī, *Ṣūra*, p. 75 and cf. Nallino, *Racc.*, 5: 478, n. 3, who suggests that it be identified with the Erythraean Sea of the Ancients. See also Shīrāzī, *Nihāya*, III.1, f. 120a (German trans. by Wiedemann [1970], p. 802 and French trans. by Ferrand [1913–14], 2: 613).
[7] Cf. Kennedy and Kennedy [1987], p. 180.
[8] *Tuḥfa*, III.1 (ff. 210a, 211b).
[9] Cf. Bīrūnī's *Taḥdīd*, p. 142 (Engl. trans. p. 107); Kennedy, *Taḥdīd Comm.*, p. 81.

III.1 [6]7–15. *wa-qad qāla ba*ᶜ*ḍ...ma*ᶜ*mūr*ᵃⁿ (Some practitioners of this science have stated...would be inhabited): Ṭūsī is probably referring to either Bīrūnī[10] or Khiraqī,[11] or perhaps to both. Each emphasizes that the sun is at perigee near winter solstice, which makes the southern region beneath the maximum declination of the sun hotter and hence uninhabitable. Bīrūnī further claims that southern latitudes beyond the maximum obliquity would have unbearably cold winters since at summer solstice the sun is near its apogee. Note that underlying Bīrūnī's argument is the assumption that the effects of actual solar distance are of the same order as zenith distance. On the other hand, Ṭūsī seeks to minimize the effect of the variability in distance by noting that the resulting size difference is imperceptible (31 to 34 minutes in the diameter of the solar disk; see II.13 [8]). Shīrāzī, however, strongly rejects Ṭūsī's reasoning and insists that perceptible size is irrelevant to whatever effects may result from actual distance; since the difference between the distance of the sun at apogee and perigee is approximately 100 terrestrial radii (see IV.5 [1]), Shīrāzī concludes that this must have a substantial effect.[12] (The ratio of the nearest to farthest distance of the sun in medieval cosmology was about 0.92, whereas the modern value is closer to 0.97.) Shīrāzī is generally followed by the commentators.

III.1 [6]15–20. *wa-dhakara ayḍ*ᵃⁿ *ba*ᶜ*ḍuhum...yunāfī dhālik al-ḥukm* (In addition some have stated...contradicts this judgment): Bīrūnī gives a similar report in the *Taḥdīd* of what "some people have argued" (pp. 56–58; trans. pp. 26–29). He also tends to discount the possibility that the southern seas and northern lands would exchange positions due to the motion of the apogee; he gives several reasons the first of which is virtually the same as Ṭūsī's.[13]

III.1 [6]22. *al-ṭarīqa al-muḥtariqa* (the combust way): The "combust way" was defined in at least two ways. Here it is a narrow latitudinal band around the Earth (corresponding to a band on the celestial sphere) demarcated by the "fall" (*hubūt*) of the sun and of the moon, which are Libra 19° and Scorpius 3°, respectively.[14] This then corresponds to a region between about 7½° and 13° south latitude. The commentators cite Ptolemy's *Geography* as evidence against the notion that this region is uninhabited.[15] Another definition is given by Khiraqī for whom the "combust way" is a small circle (rather than band) of local latitude

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[10] *Taḥdīd*, pp. 59–60 (trans. pp. 29–31); cf. Kennedy, *Taḥdīd Comm.*, pp. 11–15.

[11] *Muntahā*, II.4, f. 10a–b.

[12] *Nihāya*, III.1, ff. 120b–121b.

[13] Cf. Kennedy, *Taḥdīd Comm.*, p. 12.

[14] "Fall" (also known as "dejection" or "depression") is an astrological concept that is the opposite of "exaltation" (*sharaf*); see Bīrūnī, *Tafhīm*, p. 258 and Ptolemy, *Tetrabiblos*, I.19.

[15] Cf. *Geog.*, I.23 (p. 56) and IV.7–8, passim; cf. Bīrūnī, *Tafhīm*, p. 317 and Pingree, *Dorotheus*, V, 5, 8 and V, 20, 6.

that is directly below the sun at perigee.[16] Since he gives the solar perigee as 4° before winter solstice, this will result in a southern latitude circle of about 23½° (assuming an obliquity of 23;35°). This is evidently an attempt to give a scientific basis to an astrological concept, thus being comparable to Ibrāhīm ibn Sinān's approach to trepidation (see commentary to II.4 [5]).

Ṭūsī's disdain for some astrological doctrines is clear but any final judgment of his attitude should take into account his role as a practicing astrologer at the time he wrote these words.

III.1 [6]25. *al-ᶜināya al-ilāhiyya* (divine providence): On the question of the reason for the lack of habitation in the southern region, both Bīrūnī and Ṭūsī invoke "divine providence" but they use it in curiously divergent ways. For Ṭūsī, the fact that the inhabited quarter is in the north is a chance occurrence unconnected with a purposeful cause; in this sense it is similar to our modern "act of God." On this issue, Bīrūnī subscribes to a more intentional universe that is closer to Aristotle's teleological approach:

> In the south, however, the extreme of zenith culmination is added to extreme proximity to Earth, and no balanced effect is reached. All this is designed by the All-Wise. It is not fortuitous or haphazard, for He placed the water where civilization is not possible because of unfavorable climatic conditions, and made the land emerge where habitation and civilization can flourish (*Taḥdīd*, p. 60 [trans., p. 31]).

One man's providence is another man's science.

III.1 [7]. For the climes, see Honigmann [1929], *HAMA*, 1: 333–335, 2: 727–736, and *Alm.*, pp. 19, 82–87 (H101–111), 122–130 (H172–189).

III.1 [7]10. *al-khālidāt* (the Eternals): These islands, also called the Fortunate Islands (*suᶜadā'*), are usually identified with the Canaries; their use as zero meridian goes back to Ptolemy's *Geography*. The designation of the western coast of Africa, taken to be 10° east of the Fortunate Islands, is apparently an Islamic innovation whose use is fairly widespread.[17] Bīrjandī, elaborating on Ṭūsī's remarks about their current lack of habitation, states that they are in fact under water and for this reason are no longer used as the starting point for longitudes. (Is this somehow related to the myth of Atlantis?) Bīrūnī in the *Taḥdīd* explains the different initial meridians by citing a discrepancy of about 10° (13½° for Fazārī) that was found in calculating the meridian of a locality from the east versus from the west.[18]

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[16] *Muntahā*, II.4, f. 10a–b.

[17] Kennedy and Kennedy [1987], p. xi; Nallino, *Racc.*, 5: 490–491.

[18] *Taḥdīd*, pp. 156–157 (trans. pp. 120–121); cf. Kennedy, *Taḥdīd Comm.*, p. 91.

III.1 [7]13. *Kankdiz* (Kangdezh): This is a mythical Iranian castle supposedly built by a legendary king (either Kaykāwus or Jam) on the equator at the farthermost reaches of the inhabited world (180° from the Fortunate Islands).[19] Bīrjandī states that it is reported that Indian scientists have an observatory there; he gives them as a group who use it as the initial meridian. Some Islamic astronomers also seem to have used an eastern starting point for longitudes. Bīrūnī in the *India* (p. 259 [trans., 1: 304]) cites Abū Maᶜshar as having used Kankdiz and in the *Taḥdīd* he implies that there were others (see preceding comment). Hamdānī (d. 334/945) measures from the east and relates his longitudes to those of Ptolemy by the formula $L + L_P = 193.5°$. This seems to be derived from the discrepancy of 13½° between using an eastern and a western starting meridian that was attributed to Fazārī (see commentary to III.1 7[10]); Hamdānī in fact quotes values from Fazārī in his geographical work.[20] Finally we should note that in later sources one often finds reference to a S-lā island that is at the eastern extreme of China, but I do not know if it was ever taken as the eastern boundary. Bīrūnī's stated longitude for it of 170° from the coast of the Western ocean makes this a distinct possibility (*Qānūn*, 2: 549). In the modern literature, S-lā is usually associated with Korea, but the basis for this is rather obscure. Since it is given a latitude of 5° and since Shīrāzī makes it the eastern extreme of the Green Gulf (*i.e.* the South China Sea; see commentary to III.1 [5]1), it does not seem farfetched to identify it with the Sulu archipelago, which was the first part of the Philippines to be Islamized.

III.1 [7]14. *qubbat al-arḍ* (cupola of the Earth): That the cupola is on the equator means that it should not be identified with Ujjain, the so-called Indian Greenwich that was the basis for the computations of the *Sindhind.* Bīrūnī, basing himself of Indian sources, identifies it with the island of Lañkā (longitude 100;50° on the equator in the *Qānūn*, 2: 547). Ujjain itself is on the meridian that joins Lañkā with the alleged mountain Mīrū below the North Pole. As Ṭūsī states, the actual location of the cupola depends on one's starting point to which Bīrjandī adds the comment: "technical terms are not contestable."[21]

III.1 [8]. Table 7 summarizes the reported values and for comparison also gives those of Ptolemy, Bīrūnī, and a modern recomputation.

The differences between Ptolemy's values and those of Islamic astronomers are due to the use of a different value for the obliquity (see commentary to II.4 [1]) and to more precise trigonometric tables. The new values are already found in Battānī, whose numbers do not significantly differ from Bīrūnī's (see Honigmann [1929], p. 163 for a convenient listing).

[19] Nallino [1911], pp. 187–188 (*Racc.*, 5: 234); Kennedy and van der Waerden [1963], p. 319.
[20] Kennedy and Kennedy [1987], pp. xi, xxi.
[21] Cf. Bīrūnī, *Tafhīm*, p. 140; *Taḥdīd*, pp. 206 (trans. p. 172), 293 (trans. p. 263); Kennedy, *Taḥdīd Comm.*, p. 126.

Table 7. Maximum Daylight and Latitudes of Climes.

Clime	Maximum Daylight (Hours)	Latitudes			
		Ptolemy*	Bīrūnī†	Ṭūsī‡	Modern Recomputation#
	12¾	12;30°	12;39,5°	12;40°	12;39,17°
I	13	16;27°	16;38,34°	16;37,30°	16;38,48°
	13¼	20;14°	20;27,29°	20;27°	20;27,47°
II	13½	23;51°	24;4,30°†	24;5°‡	24;4,47°
	13¾	27;12°	27;27,40°	27;30°	27;28,55°
III	14	30;22°	30;39,27°	30;40°	30;39,47°
	14¼	33;18°	33;36,56°	33;37,30°	33;37,21°
IV	14½	36;00°	36;21,29°	36;22°	36;21,55°
	14¾	38;35°	38;53,36°	38;54°	38;54,1°
V	15	40;56°	41;13,52°	41;15°	41;14,19°#
	15¼	43;1°*	43;23,5°	43;22,30°	43;23,32°
VI	15½	45;1°	45;22,8°	45;21°	45;22,29°
	15¾	46;51°	47;11,26°†	47;12°	47;11,54°
VII	16	48;32°	48;52,21°	48;52,30°	48;52,35°
	16¼	50;4°*	50;24,34°†	50;20°	50;25,13°

Notes:

* *Alm.*, pp. 84–87 (H105–111). These values are reported as Ptolemaic by Khiraqī in the *Muntahā* in two places, in the text (II.4, ff. 7a–9a) and in a table (f. 12a), but with two variants: (1) instead of 43;1, one finds 43;15, which derives from the Arabic manuscript tradition of the *Almagest* (see *Alm.*, p. 86, n. 43); (2) one finds 50;15 rather than 50;4, which occurs in the table but neither value is in the text.

† These are Dallal's restored values from the *Qānūn* ([1984], p. 14); the same are in the *Taḥdīd* (p. 141; trans., p. 106) and in the *Tafhīm* (p. 138 though here rounded to minutes; for 24;13 read 24;4—corrected in the Persian edition). They are also given by Khiraqī (*Muntahā*, f. 11a) and attributed to the "moderns" with two exceptions: (1) 47;52,21° is instead of 47;11,26, an obvious scribal error repeating the minutes and seconds of the following entry; (2) 50;24,34° is not given.

‡ These values, with one interesting exception (24;5°; see following discussion), are also found in Shīrāzī's *Tuḥfa* and *Nihāya* and in Jaghmīnī. [The exceptions noted by Honigmann ([1929], p. 163) are due either to misreadings by Rudloff and Hochheim or else the use of a defective manuscript.]

Cf. Kennedy, *Taḥdīd Comm.*, p. 80 and Dallal [1984], p. 14. The value 41;13,19° in Dallal should be corrected.

For the most part, Ṭūsī seems to have adopted a unit fraction approximation of Bīrūnī's figures though there are some discrepancies (*e.g.* 27;30° and 50;20°). These values, which do not occur in the earlier *Muᶜīniyya* (where Ṭūsī simply reports Ptolemy[22]), seem to originate here in the *Tadhkira.* We may confirm this by following the brilliant textual analysis of Bīrjandī. As he notes, the value (24 + ½ + ⅙), or 24⅔, occurs as such in Jaghmīnī's *Mulakhkhaṣ*, Shīrāzī's *Tuḥfa* and *Nihāya*, and in most manuscripts of the *Tadhkira.* But if Ṭūsī had intended 24⅔, he would have used two-thirds as he does in two other places and not unit fractions; on this basis Bīrjandī concludes that the mix-up must be due to scribal error. The correct reading should therefore be one-half *of* one-sixth (*niṣf suds*) rather than one-half *plus* one-sixth (*niṣf wa-suds*), the former being more in conformity with the *zīj*es. (Note that the difference in Arabic script is only one letter.) This error in the *Tadhkira* manuscript tradition must have occurred quite early since virtually all manuscripts have ½ + ⅙; the notable exception is MS D, which purportedly is copied from an autograph.

As a corollary, we may conclude, as does Bīrjandī, that Jaghmīnī's *Mulakhkhaṣ* is dependent on the *Tadhkira* since there we find an unambiguous 24;40 written in alphabetical numerals (II.1; Rudloff and Hochheim, p. 261).[23]

III.1 [8]11–12. *wa-qawm...khaṭṭ al-istiwā'* (One group...equator): Khiraqī in the *Muntahā* (II.4, f. 7a) makes the equator the starting circle for the climes; he gives the circle with 12¾ hours maximum daylight as the choice of "some." In either case, the middle circle of the First Clime corresponds to 13 hours maximum daylight. No doubt the inspiration for starting with the equator goes back to Ptolemy: "We begin with the parallel beneath the equator itself, which forms, approximately, the southern boundary of the [earth's] quarter which comprises our part of the inhabited world" (*Alm.*, p. 82 [H101]).

III.1 [9]. The following table summarizes the reported values and gives those of Ptolemy for comparison (*Alm.*, pp. 87–90 [H112–117]). The recomputation for 17–24 hours has been done using modern trigonometric values and $\varepsilon = 23;35$.[24] For 1–6 months, I have used Bīrūnī's declination table.[25] The two values given by Battānī (69;44° for 2 months and 78;28° for 4 months) agree exactly with the recalculation.[26]

[22] See *Muᶜīniyya*, pp. 61–63; although some of the numbers are garbled, the Ptolemaic character is clear.
[23] Cf. the introduction, p. 35, footnote 11.
[24] Cf. Dallal [1984], pp. 3–4.
[25] *Qānūn*, 1: 373–377; for the method, see Toomer, *Alm.*, p. 649.
[26] *Batt.*, 3: 24–25 (trans. 1: 16).

Table 8. Maximum Daylight for Latitudes > 54°.

Maximum Daylight	Latitude		
	Ptolemy	Ṭūsī	Recomputation
17 hrs.	54;1°	54+°	54;21°
18 hrs.	58°	58°	58;19°
19 hrs.	61°	61°	61;11°
20 hrs.	63°	63°	63;15°
21 hrs.	64½°	64½°	64;43°
22 hrs.	65½°	65+°	65;41°
23 hrs.	66°	66°	66;14°
24 hrs.	66;8,40°	colatitude of obliquity	66;25°
1 mo.	67°	67¼°	67;16°
2 mos.	69½°	69¾°	69;44°
3 mos.	73⅓°	73½°	73;33°
4 mos.	78⅓°	78½°	78;28°
5 mos.	84°	84°	84;3°
6 mos.	90°	90°	90;°

Evidently Ṭūsī has simply reported (or approximated) Ptolemy's values for 17–24 hours but has recalculated (or taken over someone else's recalculation) for 1–6 months using $\varepsilon = 23;35$.

Book III, Chapter Two

III.2 [1]. Compare *Alm.*, pp. 82–83 (H101–104) and *Tafhīm*, pp. 124–125.

III.2 [1]23. *dūlābiyy*^{an} (wheel-like): A *dūlāb* is a water wheel; the analogy is meant to evoke the circular motion of the attached jug coming out of and going back into the water. *Dūlābī*, however, is not simply descriptive; it is used in the literature as a technical term to describe the motion of the orb for an observer at the equator and is contrasted with *raḥawī* ("spinning," the motion at the poles) and *ḥamā'ilī* ("slanted," the motion everywhere else).[1]

III.2 [1]2. *āfāq al-falak al-mustaqīm* (horizons of the right orb): What I translate as "right orb" is, of course, the *sphaera recta* of medieval Latin astronomy.[2] As stated previously, I have consistently used "orb" to render *falak* in order to retain in translation the ambiguity of its usage (see commentary to I.1 [15] and II.3 [2]).

[1] Cf. III.3 [1] and III.6 [1], and Nallino [1911], pp. 261–263 (*Racc.*, pp. 281–282).
[2] Cf. Toomer, *Alm.*, pp. 18–19.

III.2 [1]3&4–5$_1$. *sa^{c}at al-mashriq, sa^{c}at al-maghrib* (ortive amplitude, occasive [setting] amplitude): See III.3 [2].

III.2 [2–4]. The argument concerning the temperateness of the equatorial region occurs in Ibn Sīnā's *Qānūn fī al-ṭibb*;[3] unless the passage occurs elsewhere, Ṭūsī is giving a paraphrase rather than a direct quotation. Fakhr al-Dīn al-Rāzī (543(?)–606/1149–1209) was a famous theologian who did much to bring Hellenistic ideas and concepts into the mainstream of dialectical theology (*kalām*). According to Bīrjandī, Rāzī's criticism of Ibn Sīnā comes in his commentary on the *Qānūn*, a work that is extant but which I have not seen.[4] Ṭūsī, who generally tends to defend Ibn Sīnā from Rāzī's criticisms when commenting on Rāzī's *Muḥaṣṣal* and on his commentary to Ibn Sīnā's *Kitāb al-Ishārāt wa-'l-tanbīhāt*, here reverses roles and sides with Rāzī. The commentators tell us that Rāzī only rejected the "first argument" (that the equator is the most temperate locality); he agreed with Ibn Sīnā that the hottest localities would be those whose latitude is equal to the obliquity.

Bīrūnī in the *Tafhīm* (p. 125) seems to have heard of Ibn Sīnā's position (though he does not name him); his rejection of it, based on the supposedly intemperate characteristics of the inhabitants along the equator, is quite similar to Ṭūsī's. That the climate (or air) can account for human variation is an idea going back to antiquity (*e.g.* in *Airs, Waters, Places* of the Hippocratic corpus).

Book III, Chapter Three

This chapter deals generally with latitudes greater than 0° and less than 90° (the "oblique horizons"). Chapter III.4 deals specifically with latitudes between the equator and the colatitude of the obliquity, while III.5 is concerned with locations whose latitudes are greater than the colatitude of the obliquity and less than 90°.

III.3 [1]. Compare II.1 [1].

III.3 [1]16. *ḥamā'iliyy^{an}* (slanted): A *ḥamā'il* is a shoulder belt worn at an angle whence it evidently came to describe the motion of the orb for the *sphaera obliqua*. (See Bīrūnī, *Tafhīm*, p. 140; Lane, *Lexicon*, 2: 650; and commentary to III.2 [1]23 for further references.)

III.3 [2]. Compare *Alm.*, pp. 76–77. In Figure C28,[1] the triangles (with sides δ, η, q) occurring toward the invisible pole have sides δ and q in solid lines, those

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[3] Bk. I, Pt. 2, Thesis 2a, Ch. 8 (Rome ed., pp. 43–44; Būlāq ed., p. 88 [Engl. trans., p. 197]); cf. *Alm.*, p. 83 (H103).
[4] *GAL*, 1: 457 and S1: 824.
[1] Cf. Kennedy, *Taḥdīd Comm.*, p. 71 and *Alm.*, Fig. 2.1, p. 76.

toward the visible pole have δ and q in broken lines. In the eastern triangle the amplitude (η) is ortive (or rising), while in the western triangle the amplitude is occasive (or setting).[2] If one measures the day-circles and equinoctial arcs in hours, and D is the amount of daylight, then the equation of daylight (q) is ½(D − 12).[3] For declination (δ), see II.3 [6]. Finally we should note that Ṭūsī does not restrict the concepts of "daylight" (*nahār*) and "amplitude" to the sun but is willing to apply them to stars and planets as well; this extended usage, at least for amplitude, was presaged by Ptolemy.[4] "Daylight" for a star or planet, of course, is simply the period it is above the horizon; the "arc of daylight" is the measure of this in equinoctial degrees (see III.10 [1]9–15).

III.3 [2]Fig. T19. This diagram occurs only in MS L; similar ones can be found in the commentaries. The attempt to show what is on the other side of the sphere by projecting it outward is fairly typical; cf. Fig. T15.

III.3 [3]. For a depiction of the four possibilities, see Fig. C29. Despite some ambiguity, the last three day-circles mentioned are meant to be in the direction of the visible pole. For the prime vertical circle (*dā'irat awwal al-sumūt*), see II.3 [14].

Book III, Chapter Four

III.4 [2]. In the tropics there are two points on the ecliptic that will coincide with the zenith. If the sun is at one of these points, then at noon the ecliptic will be at right angles to the horizon circle and the ecliptic poles will be on the horizon. When the sun is on the arc that is toward the visible equinoctial pole and that connects these two points, then at noon the ecliptic pole adjacent to this pole will be below the horizon; the depression will reach its maximum value of $\phi - \varepsilon$ (ϕ being the local latitude and ε the obliquity) when the sun is at the solstice. When the sun is on the other arc of the ecliptic, this ecliptic pole will be above the horizon at noon and will reach its maximum altitude of $\phi + \varepsilon$ when the sun is at the other solstice.

III.4 [2]19. *fī al-qaws allatī bayn al-nuqṭatayn* (on the arc between the two points): This is the ecliptic arc connecting the two points that pass through the zenith.

[2] On ortive and occasive amplitude, see *HAMA*, 1: 37–39 and Bīrūnī, *Tafhīm*, p. 129.

[3] Not (D − 12) as in Kennedy, *Survey*, p. 141—correct formula in Kennedy, *Taḥdīd Comm.*, p. 70 and Bīrūnī, *Shadows*, 2: 98; cf. Bīrūnī, *Tafhīm*, p. 131.

[4] *HAMA*, 1: 38, 142.

III.4 [2]3–5. *wa-lā takūnu fuṣūl al-sana...lam takun mutashābiha* (The seasons of the year...would still not be uniform): For the case of four seasons, Ṭūsī presumably would take spring to be from spring equinox until the sun reaches the zenith for the first time. Summer would then be appreciably longer than spring since it would last from then through the summer solstice until the fall equinox. Fall and winter would be according to the standard definitions. Four seasons will work reasonably well for a latitude in the tropics close to that of the obliquity; for a latitude nearer 0° one might have recourse to the eight-season possibility mentioned in III.2 [1]. In the latter case one would have two summers of unequal length since half the distance from one zenith coincidence to the summer solstice would not be equal to half the distance from the other zenith coincidence to the winter solstice.

III.4 [2]4. *āfāq* (regions): Note that *āfāq* (horizons) is being used in the sense of localities; cf. II.14 [2]18.

III.4 [4]19. *ka-mā bayyannā* (as we have [already] explained): See III.3 and III.4 [3].

III.4 [4]19–22. *fa-in kāna ᶜarḍ al-balad...wa-marra mā sāwā ᶜarḍuhu al-faḍl* (Where the local latitude...while those whose latitude is equal to the excess [will pass over] once): By "other wandering [planets]" Ṭūsī means other than the sun with which he is here trying to make an analogy. Just as the sun passes over the zenith of places in the tropics, so also may a planet pass overhead if its maximum latitude (measured from the ecliptic; see II.10) is greater than or equal to the excess of the local latitude over the obliquity. (Remember that Ṭūsī is only considering latitudes greater than the obliquity in this paragraph; there is no question that planets may pass overhead in the tropics.)

III.4 [4]22–24. *wa-fī hādhih al-ᶜurūḍ...bi-'zdiyād al-ᶜarḍ* (In these latitudes...with increasing latitude): The rising or setting amplitude is given by $\eta = \arcsin(\sin\delta/\cos\phi)$ where η is the amplitude, δ is the declination and ϕ is local latitude.[1] For any given declination, the amplitude will thus indeed increase with increasing latitude. The equation of daylight q is given by $q = \arcsin(\tan\phi \cdot \tan\delta)$.[2] As is evident, for a given declination the equation of daylight will also increase with increasing latitude. The qualification "in these latitudes" is apparently meant to exclude the arctic regions where the concepts of amplitude and equation of daylight will not be meaningful at certain times of the year.

[1] Kennedy, *Taḥdīd Comm.*, p. 71.
[2] Bīrūnī, *Shadows*, 2: 99–101.

III.4 [5]5–6. *nuqṭa quṭb awwal al-sumūt* (pole of the prime vertical): These poles are the north and south points; see II.3 [14].

III.4 [5]10–11. *al-juz' al-tālī li-'l-munqalab; ᶜalā quṭb awwal al-sumūt* (The point subsequent to the solstice; upon the pole of the prime vertical): "Subsequent" here means following in the sequence of the zodiacal signs. The subsequent points will not in actuality be upon the poles of the prime vertical since only the solstice points can coincide with them.

III.4 [5]23–24. *wa-yakūnu ṭulūᶜ niṣf dawr...lā fī zamān* (The rising of a half revolution...does not [require] time): For a location on the arctic circle, the eastern half of the ecliptic that has the vernal equinox as midpoint will rise all at once, while the western half with the autumnal equinox as midpoint will take a full revolution of the equinoctial to rise (cf. III.4 [5]8–18). Bīrjandī objects to the idea that motion can occur without time. On the other hand, Khafrī states that this example disproves atomism. The two comments are too brief to be fully comprehensible, but they do indicate an issue in late medieval natural philosophy that would be worth pursuing.

Book III, Chapter Five

III.5 [1]12. *maᶜkūsa* (in reverse order); *mustawiya* (in regular order): Regular order is in the sequence of the zodiacal signs, while reverse order is in their counter-sequence.

III.5 [1]15–21. *wa-yakūnu li-'l-munqalab...ᶜalā al-mayl al-kullī* (The visible solstice...over the obliquity): Using modern symbols, we may summarize the passage as follows:

Highest altitude of solstice:	$\bar{\phi} + \varepsilon$ (toward invisible pole)
Lowest altitude of solstice:	$\phi - \bar{\varepsilon}$ (toward visible pole)
Highest altitude of ecliptic pole:	$\bar{\phi} + \bar{\varepsilon}$ (toward invisible pole)
Lowest altitude of ecliptic pole:	$\phi - \varepsilon$ (toward visible pole)

All these altitudes occur on the meridian. (ε = obliquity; ϕ = local latitude; a bar indicates the complement.)

III.5 [1]23. *wa-'l-irtifāᶜayn al-mutabādalayn* (and their altitudes are at opposite [extremes]): In other words, when the solstice is at its lowest altitude the ecliptic pole will be at its highest, whereas the solstice will be at its highest altitude when the pole is at its lowest.

III.5 [2]. Throughout this paragraph, as also in subsequent examples, Ṭūsī is speaking in approximate terms though the chosen latitude of 70° is a good one. From Bīrūnī's declination table one finds that a latitude of 69;43,39,58° will result in precisely 60° of the ecliptic (in other words Gemini and Cancer) being permanently visible.[1]

III.5 [2]4–8. *fa-idhā kāna awwal al-saraṭān...sitt wa-arbaᶜūn daraja wa-rubᶜ wa-suds* (Then when the first of Cancer...(46 + ¼ + ⅙)°): From III.5 [1]15–21, the highest altitude of the solstice is

$$\bar{\phi} + \varepsilon = 20° + 23;35° = 43;35° = (43 + \tfrac{1}{3} + \tfrac{1}{4})°$$

The lowest altitude of the ecliptic pole is

$$\phi - \varepsilon = 70° - 23;35° = 46;25° = (46 + \tfrac{1}{4} + \tfrac{1}{6})°$$

III.5 [3]8. *min al-janūb ilā al-shamāl* ([extending] from south to north): Strictly speaking, the order of the zodiac here is from north to south.

III.5 [4]6–9. *wa-yantahī awwal al-saraṭān...sitt wa-thamānūn daraja wa-rubᶜ wa-suds* (The first of Cancer...(86 + ¼ + ⅙)°): The lowest altitude of the solstice is

$$\phi - \bar{\varepsilon} = 70° - 66;25° = 3;35° = (3 + \tfrac{1}{3} + \tfrac{1}{4})°$$

The highest altitude of the ecliptic pole is

$$\bar{\phi} + \bar{\varepsilon} = 20° + 66;25° = 86;25° = (86 + \tfrac{1}{4} + \tfrac{1}{6})°$$

III.5 [4]11. *ᶜalā tawālī mukhālif li-'l-maᶜhūd* (the directional sequence [of the signs here being] opposite the conventional one): One would normally expect the zodiacal signs to extend from a roughly west to east direction; in this case, the signs go from east to west with the first of Aries at the east point and first of Libra at the west point.

III.5 [7]. This is another Ṭūsī "believe it or not" (cf. II.1 [7], II.9 [14], and III.1 [1]). The basic idea here is that if the horizon were close enough to the equator, then a wandering planet might, by means of its proper (*i.e.* secondary) motion,[2] move enough in a day's time to transfer to a day-circle that would

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[1] *Qānūn*, 1: 376; cf. *Alm.*, p. 72 (H80–81) where the latitude would be 69;29,51°.

[2] The "second motion" (*al-ḥaraka al-thāniya*) usually refers to the motion of the ecliptic orb (the precessional motion), but that would make little sense in this context. Ṭūsī does occasionally use it to mean the proper motion of the wandering stars; see commentary to III.6 [2]13–14.

allow it to set in the east or rise in the west. The best possibility for this would be the moon, which has a mean motion of about 12°/day. It could easily then have a 5° declination shift in a single day.

Book III, Chapter Six

III.6 [1]1. *raḥawiyya* (spinning): A *raḥ*^{an} is a millstone whose spinning motion is here compared to the motion of the orb at the poles. It is used as a technical term, as were *ḥamā'ilī* and *dūlābī*.[1]

III.6 [2]7. *yawm bi-laylatih* (day with its night [nychthemeron]): Bīrjandī complains that for the situation at the pole it is more appropriate to speak of a day *and* a night (*yawm wa-layla*) instead of "a day with its night," the nychthemeron, which should be reserved for a normal solar day. This terminological distinction was apparently introduced by Shīrāzī. On the nychthemeron, see III.8 [1].

III.6 [2]8–10. *fa-yakūnu taḥt al-quṭb al-shamālī...fī awākhir al-qaws* (Beneath the northern pole...at the end part of Sagittarius): Ptolemy gives the period for spring and summer as 187 days and that of fall and winter as 178¼ days, the difference being 8¾ days.[2] If one takes the longitude of the solar apogee to be 90° instead of Ptolemy's 65½°[3] and a solar eccentricity of 2;05 instead of 2;30,[4] one comes up with a difference of a little over 8 days. It is thus a bit mysterious how Ṭūsī arrives at 7 days. Bīrjandī, who along with the other commentators notices the problem, suggests a slip of the pen on Ṭūsī's part since seven and nine are very similar when written out in Arabic script.

III.6 [2]11–12. *wa-takūnu mudda ghurūb al-shafaq...fīmā ba*^{c}*du* (The period for the setting of dusk...later on): In III.9 [2] Ṭūsī gives 18° as the amount of depression of the sun below the horizon needed for first rising of dawn and final setting of dusk. This translates into around a 50° arc of the ecliptic which in turn gives us the 50 days of the text.

III.6 [2]13–14. *wa-yakūnu ṭulū*^{c} *al-shams...min al-ufaq* (Neither the rising nor the setting of the sun...on the horizon): Because he is talking of the rising and setting of the sun and "stars" (*kawākib*), I take it that Ṭūsī intends by the "second motion" (*al-ḥaraka al-thāniya*) the proper motion of the sun, moon, and

[1] See III.2 [1]23 and III.3 [1]16; cf. Bīrūnī, *Tafhīm*, p. 140 and Lane, *Lexicon*, 3: 1057.
[2] *Alm.*, pp. 153–154 (H233–234).
[3] This is not an unreasonable assumption given what Ṭūsī says in this paragraph; cf. Kennedy [1960], p. 168.
[4] See II.6 [4].

planets (*i.e.* their west to east motion through the zodiac). Since in general the periods of these motions are not integral multiples of the daily rotation, these bodies will not rise or set at fixed locations for an observer at the pole. The "second motion" usually refers to the motion of the ecliptic orb (*i.e.* the precessional motion; see II.2 [4], II.3 [3] and [5], and II.4 [4] and [7]).[5] Ṭūsī, however, sometimes uses this phrase to refer to the proper motion of the wandering stars; this is certainly the case in III.2 [1]7 and is probably the case in III.5 [7]15. (For these I use the term "*secondary* motion.")

III.6 [3]. The "stars" (*kawākib*) are here the fixed stars; the "second motion" is that of the ecliptic orb (the precessional motion; see preceding note). A fuller exposition of the information in this paragraph may be found in II.4 [7] to which Ṭūsī is evidently referring.

III.6 [4]22–23. *wa-mā yajrī majrāhā* (and what is comparable to them): Ṭūsī means to indicate the poles, which are degenerate day-circles, and perhaps also the equinoctial, which Bīrjandī tells us was not usually considered to be a day-circle.

Book III, Chapter Seven

For this chapter, see *Alm.*, pp. 90–103 (H118–141); *HAMA*, 1: 34–37; Pedersen [1974], pp. 110–113. Compare Bīrūnī, *Tafhīm*, pp. 145–147.

III.7 [1]4. *maṭāli^c* (co-ascension): This curious word, Bīrjandī tells us, is the plural of *maṭla^c*, a "noun of time" (*ism al-zamān*) that means rising time. This form is used to indicate that the rising time of an ecliptic arc is being measured along the equinoctial whose units are time (*azmān*; see II.3 [2] and cf. *Alm.*, p. 90 [H117]). Bīrjandī rejects any attempt to read *muṭāli^c*, which, if it existed, would mean "corresponding to the rising" arc of the ecliptic.

III.7 [1]5. *al-daraj al-sawā'* (equal degrees): One would prefer to see the more grammatical *daraj al-sawā'*, which one finds in the commentaries, in Bīrūnī, *Tafhīm*, p. 145, and in MSS LT. But the form with the initial article is well-attested in the other manuscripts and analogous forms (*e.g. arḍ sawā'*) are given by Lane, *Lexicon*, 4: 1479, col. 3.

III.7 [2]. Ṭūsī is here giving a general account of the co-ascensions for the equator without attempting to be exhaustive. (Bīrjandī rectifies this.) In Figure C30 we see that two zodiacal signs measured from Aries 0° will be larger than their co-ascensions. (The theorem used in lines 18–19 is from Menelaus,

[5] This is also my interpretation of the usage in the next paragraph.

Spherics, I.24.) The same figure, of course, is applicable to a single sign measured from the equinox. (Note as well that the equal degrees will also be larger than their co-ascensions for signs measured in the other direction from the equinox.) Since a > b, then 90 – a < 90 – b and therefore with the help of the figure one may conclude that Gemini, which adjoins the solstice, will be smaller than its co-ascension.

III.7 [4]13–17. *wa-idhā ṭalaᶜat qaws...bi-ḍidd mā kāna* (When an arc...the opposite of what it was [before]): Bīrjandī gives his usual careful analysis of the situation[6] and finds that Ṭūsī is incorrect in asserting that for the oblique horizons the ecliptic arc in the direction of the visible pole will always be larger than its co-ascension.[7] By referring to Figure C31, we may summarize Bīrjandī's discussion as follows:

[KOT = ecliptic meridian circle; ϕ_e = arc HO = local ecliptic latitude (see II.3 [16]); ϕ = arc ZO = local latitude; ε = obliquity; a bar indicates the complement]

Assume $0 < \phi < \bar{\varepsilon}$. Ecliptic arc ED is in the direction of the visible pole. (For $\phi \geq \bar{\varepsilon}$, see paragraphs [5] and [6].)

(1) For $\phi_e \geq 0$, $\angle$ BGK = $180°-\bar{\phi} > 90°$, and $\angle$ ADT = arc HT = $\bar{\phi}_e \leq 90°$; thus arc ED > arc EG (Fig. C31a).

(2) For $\phi_e < 0$, $\angle$ ADT = arc HT > 90°. There are three cases (Fig. C31b):

(a) if $|\phi_e| < |\phi|$, then $|\bar{\phi}_e|$ = arc HK = $\angle$ ADK > $|\bar{\phi}|$ = $\angle$ BGT; therefore $\angle$ ADT < $\angle$ BGK and thus arc ED > arc EG;

(b) if $|\phi_e| > |\phi|$, then $|\bar{\phi}_e|$ = arc HK = $\angle$ ADK < $|\bar{\phi}|$ = $\angle$ BGT; therefore $\angle$ ADT > $\angle$ BGK and thus arc EG > arc ED;

(c) if $|\phi_e| = |\phi|$, then arc EG = arc ED.

Thus contrary to what Ṭūsī states, $\angle$ ADT will be obtuse for **(2)** and could be right for **(1)**. Furthermore the ecliptic arc will be smaller than its co-ascension for **2(b)** and equal to it for **2(c)**.

An analogous modification should be made for the situation when the ecliptic arc is toward the invisible pole away from the equinoctial. By a simple

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[6] Bīrjandī is aware of his thoroughness; he states self-assuredly that one will hardly find a comparable discussion elsewhere.

[7] One can also see that Ṭūsī is incorrect by examining the accumulated time-degrees for Leo 20° to Virgo 30° in Ptolemy's rising time table for the Avalite Gulf (*Alm.*, II.8, p. 100).

argument from symmetry, it is clear that in this case the opposite set of relationships between arc EG and arc ED will hold (Fig. C32).

III.7 [4]17–19. *wa-yaẓharu min dhālik...takūnu muṭāliᶜuhā mutasāwiya* (It is apparent from the above...their co-ascensions are equal): Ṭūsī engages in a bit of hand waving here; the proof is more involved than he implies. For the details, see *Alm.*, pp. 90–91 (H118–119) and *HAMA*, 1: 35.

III.7 [4]19–23. *wa-'l-falak yanqasimu...wa-'l-ukhrā takūnu aṣghar* (The orb...the other is smaller): For an inhabitant of the northern hemisphere, the first segment of the ecliptic is that bisected by Aries 0°. From III.7 [4]13–17, it is clear that the co-ascension is smaller than the arc bounded by Aries 0° and Cancer 0°. Since the arc bounded by Capricornus 0° and Aries 0° has an equal co-ascension (see III.7 [4]17–19), then this half of the ecliptic has a co-ascension less than 180°. Obviously the other half of the ecliptic has a co-ascension greater than 180°. For someone in the southern hemisphere, the first segment is that which is bisected by Libra 0°, and a similar result will follow.

III.7 [4]23–24. *wa-maṭāliᶜ al-qusiyy al-shamāliyya...wa-kadhālik fī al-janūbiyya* (The co-ascensions of northern arcs...the same holds for the southern [arcs]): For example, the co-ascension for Taurus at some latitude ϕ will be equal to the co-ascension of Scorpius for latitude $-\phi$.

III.7 [4]24–25. *wa-maghārib kull qaws...naẓīr tilk al-qaws* (The co-descension of any arc...of the arc directly opposite): For example, for any latitude the co-descension of Libra is equal to the co-ascension of Aries. The proof is again based on a simple argument from symmetry.

III.7 [5]. Ṭūsī is here referring to locations whose latitude is equal to the complement of the obliquity; see III.4 [5].

III.7 [6]. This paragraph is basically a repetition of what was stated in III.5. What Ṭūsī has left out here is how this relates to co-ascension, which is, after all, the subject of the chapter. Bīrjandī, as usual, fills in the details. From Figs. T20–T23 of III.5, it should be reasonably clear that when Pisces and Aquarius rise (from Fig. T22 to Fig. T23), the ecliptic pole will have traveled less than 90° on its circuit; therefore Aries 0° will have traversed less than 90° and the co-ascension of Pisces and Aquarius will consequently be less than 90°. On the other hand, when Libra and Scorpius rise (from Fig. T20 to Fig. T21), the ecliptic pole will have traveled more than 90°; therefore Libra 0° will have traversed more than 90° and the co-ascension of Libra and Scorpius will consequently be more than 90°. It then immediately follows[8] that the co-ascension of Aquarius,

[8] From III.7 [4]17–19.

Pisces, Aries, and Taurus (these signs rising in reverse order) is less than 180°, whereas the co-ascension of Leo, Virgo, Libra, and Scorpius (these rising in regular order) is greater than 180°. The opposite will hold for the co-descensions.[9]

The "remaining regions" in line 12 refers to locations with latitudes other than 70° but greater than the complement of the obliquity.

Book III, Chapter Eight

For this chapter, see *Alm.*, III.9, pp. 169–172 (H258–263); *HAMA*, 1: 61–68; and Pedersen [1974], pp. 154–158.

III.8 [3]. In Figure C33, the sun will experience its slower speed in the apogeal segment DAB and its faster speed in the perigeal segment BPD; the mean speed will occur at D and B. At D, the position of the mean sun is behind that of the true sun by the amount of the maximum solar anomaly, namely CDO. In going from D to A, the mean sun will steadily gain on the true sun, eventually coinciding with it at the apogee A. From A to B, the mean sun will overtake the true sun until it is ahead of it by the maximum solar anomaly CBO at mean distance B. The reverse process takes place in segment BPD. One may therefore say, as Ṭūsī does, that in the far segment the mean sun will increase its position with respect to the true sun in the amount of twice the anomaly, whereas in the near segment it is the true sun that will increase its position with respect to the mean sun by this amount. In terms of the variability of the nychthemerons, a mean day will be longer than a true day in the far segment and shorter in the near segment;[1] this is because the sun's observed speed is slower than the mean speed in the apogeal half and faster in the perigeal half.

III.8 [4]20–21. *bi-ḥasab al-tafāwut bayn daraj al-sawā' wa-maṭāli*[c] *naẓīrihā* (according to the difference between the equal degrees and the co-ascension of the [arc] directly opposite them): Ṭūsī could also have said "according to the difference between the equal degrees and the co-descension of the arc," but he probably wished to be consistent and state the relationship in terms of the co-ascension; cf. III.7 [4]24–25.

III.8 [4]21–23. *wa-in ju*[c]*ila mabādi' al-ayyām...dūn al-wajh al-awwal* (But if the beginning of the day is made...rather than the first alternative): Since each meridian circle is a horizon of the equator, one can eliminate the variability due to local latitude by having the day begin at noon; cf. III.7 [3].

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[9] Recall from III.5 [2] that Gemini and Cancer are here permanently visible, while Sagittarius and Capricornus are permanently invisible.

[1] Discounting, of course, variability due to co-ascension.

III.8 [5]. This paragraph expands on the statements of III.7 [2]3–6. The reason that Ṭūsī brings up these four segments here is that the maximum difference between the equal degrees and the co-ascension for *sphaera recta*, namely 5°, will occur in going from the beginning to the end of any one of these quarter segments. The way this works for the equation of time is indicated in Fig. T24.[2] In the quarter segments bisected by the equinoxes, the co-ascensions will be smaller than the equal degrees; hence this will act to make the true days shorter than the mean days. In the quarter segments bisected by the solstices, the co-ascensions will be larger than the equal degrees; this will therefore act to make the true days longer than the mean days.

On the other hand, Ptolemy states that "the greatest [accumulated] difference will occur between two points enclosing two signs which are on either side of either a solstitial or an equinoctial point"[3] and finds it to be 4½°. But Ptolemy's own rising-time tables (p. 100) show that Ṭūsī is more correct in considering the maximum difference to occur in a 90° rather than 60° segment of the ecliptic; cf. *HAMA*, 1: 67 (Table 4).

III.8 [5]26, 2, 3; [7]13, 15, 16. *awāsiṭ, awā'il, awākhir* (middle part, beginning part, end part): Ṭūsī uses these plurals of *awsaṭ* (middle), *awwal* (beginning), and *ākhir* (end) to indicate that the point in question is not exactly at the beginning, middle or end. I usually add "part" to indicate this approximation. In the case of the quarter segments of the ecliptic, the dividing points are slightly before the midpoints of Aquarius and Leo, and slightly after the midpoints of Taurus and Scorpius.

III.8 [6–7]. We may express the equation of time as follows:

$$\Delta t - \Delta \bar{t} = \Delta E$$

where t is elapsed true days, $\bar{t}$ is elapsed mean days and E is the equation of the nychthemeron (or time).[4] As Ṭūsī has stated, ΔE is the combination of two "differences," that resulting from the variability of the solar motion through the ecliptic and that resulting from the co-ascension. Now both differences are sinusoidal; combining them yields a curve whose maximum is near the first of

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[2] Cf. *HAMA*, 3: 1222 (Fig. 57).

[3] *Alm.*, p. 170 (H260).

[4] *HAMA*, 1: 66. Note that Neugebauer [1962b], p. 94, has instead what amounts to $\Delta t - \Delta \bar{t} = -\Delta E$. From a modern viewpoint, in which ΔE ranges over positive and negative values, either equation, of course, is appropriate. However in dealing with the ancient and medieval situation, one needs to be careful since ΔE was always taken to be non-negative; thus to avoid confusion, one should consider the first equation to indicate that ΔE is to be subtracted from true days to convert them to mean days, whereas the second indicates it is to be added. For further elaboration, see p. 484 and cf. *HAMA*, 2: 984–985.

Scorpius and whose minimum is near the end of Aquarius.[5] Thus by choosing an epoch near the end of Aquarius, one may ensure that all values of ΔE will be positive.

To see what this will mean in practice, we may go through the steps to convert a given number of true days into mean days. For a certain day, we may calculate the number of true days, hours, and so forth that have elapsed since the epoch. We then enter a table of ΔE (provided in many *zīj*es) with the zodiacal sign and degrees corresponding to our given day to obtain ΔE; we then *subtract* this value from the true days to obtain the mean days that have elapsed during the same period.[6] Note that using this epoch will result in the elapsed true days always being greater than or equal to the elapsed mean days. Obviously by using an epoch near Scorpius 0°, one will have the opposite state of affairs. On the face of it, this seems in direct contradiction to what is in the text since Ṭūsī states that for the epoch in Aquarius "the true days will always be shorter than the mean ones," whereas for the epoch in Scorpius "the true days will always be longer than the mean ones." In order to reconcile these two accounts, one needs to recall how Ṭūsī has set up the problem. As is clear in paragraph [2], he is intent on comparing the length of one or more mean days with the corresponding true day or days.[7] Thus here in paragraph [7], he is comparing the length of a given *whole number* of mean days with the length of the same *whole number* of true days rather than comparing the *amount* of elapsed mean days with the *amount* of elapsed true days in a given *period*. For example, if one were to begin counting days using the end part of Aquarius as the initial point, then 50 true days will elapse in a shorter time than 50 mean days; but in a period of exactly 50 true days measured from the same initial point, there will obviously be fewer than 50 complete mean days. It is in this latter sense that the equation of time quoted at the beginning of this note should be understood and it is in this sense as well that one would normally use a table of ΔE in a *zīj*, namely in order to convert *elapsed* true days into *elapsed* mean days. Ṭūsī's pedagogic intent in [3–6], however, is to show how one obtains ΔE and it is therefore in reference to the former problem, namely of comparing a given *number* of true days with the same *number* of mean days, that he uses "shorter" (*nāqiṣa*) and "longer" (*zā'ida*). The inevitable confusion leads even as competent an astronomer as Nīsābūrī into mistakenly stating that one should *add* ΔE in order to convert true to mean days when the initial point is in Aquarius.

The choice of making the correction for the equation of time always subtractive seems to be fairly standard in the Islamic *zīj* literature; at least this is what one finds in the *zīj*es of Battānī and of Khwarizmī despite the fact that both have

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[5] See the very helpful graph in *HAMA*, 3: 1222 (Fig. 57); the discrepancy with Ptolemy will be dealt with below.

[6] *HAMA*, 1: 67.

[7] This is made quite explicit in Fig. T24; see also above commentary to paragraphs [3] and [5].

epochs whose solar longitudes are not at the end of Aquarius.[8] In the *Handy Tables*, however, the equation of time is always additive and the epoch (–323 November 12) has a solar longitude of Scorpius 17°, which is near the location of the minimum (*i.e.* zero) value for ΔE in the table.[9]

III.8 [7]2. *wa-yataghayyaru...fī mudda ṭawīla* (The difference...over a long period): Because of the motion of the solar apogee,[10] the curve of the solar anomaly (and hence its maximum and minimum) will shift with respect to the ecliptic. (The other component of the equation of time, that of the co-ascension, does not vary over time if one assumes a constant obliquity.) There will therefore also be a slow shift in the curve of ΔE. Thus whereas Ptolemy finds the extremes occurring at approximately Scorpius 0° and Aquarius 15°, Ṭūsī, as we have seen, gives the values as "the beginning part of Scorpius" and "the end part of Aquarius." We find in Khwarizmī's *zīj* that the positions[11] are about Scorpius 10° and Aquarius 22°; from Battānī's *zīj* one extracts around Scorpius 9° and Aquarius 19°.[12]

III.8 [8]4–5. *ta*ᶜ*dīl al-ayyām bi-layālīhā* (the equation of the nychthemeron): Ptolemy talks about ἡ τῶν νυχθημέρων ἀνισότης (the inequality of nychthemerons),[13] but he does not have a word as such for equation, which is apparently a medieval innovation.[14] The Byzantine and Latin terminology is virtually the same as the Arabic, which makes it something of a puzzle as to how the modern transformation to "equation of time" came about.[15]

Book III, Chapter Nine

This chapter does not have an analogue in the *Almagest*. The problem of twilight attained considerably more importance in Islam than in antiquity

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[8] Battānī's epoch is the Seleucid era (–311 March 1) at which the initial solar longitude is Pisces 4°, which is not terribly far from the end of Aquarius. Khwarizmī, however, uses the *hijra* (+622 July 15) at which the solar longitude (Cancer 24°) is quite far from Aquarius. Neugebauer [1962b], p. 65 shows how one can make a correction to Khwarizmī's epoch values for the sun and moon in order to use an always negative equation of time.

[9] See *Handy Tables* , 1: 148–155; cf. *HAMA*, 2: 984–985 and Rome [1939], who provides a full discussion..

[10] See II.6 [1].

[11] Khwārizmī, *Zīj*, Table 68, p. 182.

[12] Battānī, *Zīj*, 2: 61, 64; in his discussion (3: 74 [trans. 1: 49]), he gives "two-thirds of Aquarius" and "the beginning of Scorpius."

[13] *Alm.*, p. 169 (H258, lines 14–15).

[14] Toomer, *Alm.*, pp. 21–22.

[15] *HAMA*, 1: 61 (n. 2).

inasmuch as three of the five Islamic prayer times were set with reference to dawn and dusk. (Evening prayers were to be performed between sunset and nightfall (end of dusk), night prayers between nightfall and daybreak (beginning of dawn), and morning prayers between daybreak and sunrise.)[1] The time for daily fasting to begin during the month of Ramaḍān was also made with reference to daybreak.[2] See Bīrūnī, *Tafhīm*, pp. 51–52; Davidian and Kennedy [1961]; Wiedemann and Frank [1926]; Wiedemann [1912]; and King [1973], pp. 365–368, who provides further references.

III.9 [title]2. *al-ṣubḥ wa-'l-shafaq* (dawn and dusk): I have adopted Goldstein's suggestion that dawn and dusk be used for the intervals from daybreak to sunrise and from sunset to nightfall, respectively, and that twilight denote both phenomena.[3]

III.9 [1]6–8. *wa-l-yamurr saṭḥ...ᶜalā saṭḥ al-makhrūṭ* (Let a plane pass...on the surface of the cone): Ṭūsī is being rather sloppy here. To say that the plane passes through the centers of the sun and Earth as well as through the axis of the cone is to be redundant but without specifying a unique plane. Now Ṭūsī's added stipulation that the resultant triangle will have its base on the horizon (meaning, presumably, in the plane of the horizon) does specify a unique plane for a given moment, but this plane is not very useful for the purpose, namely to understand false dawn. To see this, we may note that at sunrise in the tropics and middle latitudes, the plane of the triangle, whose base would be the intersection of the plane of the horizon circle and the base of the shadow cone, would have a generally north-south orientation, whereas one would certainly need an east-west one. The commentators have noticed the problem and Bīrjandī, for one, proposes that the base of the triangle be the intersection of the base of the shadow cone and the plane of the east-west circle.[4] Bīrjandī also suggests that Ṭūsī added the horizon stipulation on the basis of his illustration, which shows the situation at midnight rather than at sunrise; in this case the base of the triangle is indeed on the horizon if one conceives of the triangle in the plane of the east-west circle. Bīrjandī is probably referring to something similar to Fig. C34, which is what one finds in the earlier Marāgha (α) version of the *Tadhkira*. It is worth noting that MSS FL, which represent the later (β) version of the text, do not have the base of the cone in the plane of the horizon. (See Fig. T25 and the commentary to it.)

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1 Cf. Kennedy [1975].
2 See King [1973], pp. 370–371.
3 Goldstein [1985], p. 117.
4 Actually false dawn is, more or less, in the plane of the ecliptic; see commentary to III.9 [1]13.

III.9 [1]10–11. *awwal mā yurā nūr al-shams* (the first observed light of the sun): The Arabic construction is awkward but comprehensible; Bīrjandī complains that it is unnatural (*fīhi takalluf*).

III.9 [1]13. *al-ṣubḥ al-kādhib* (false dawn): Redhouse [1878] seems to have been the first to identify "false dawn" (also known as the wolf's tail [*dhanab al-sirḥān*] with the zodiacal light.[5] This light is not atmospheric but rather extraterrestrial, its source being mainly the illumination of a disk of cosmic dust surrounding the sun. (Its modern name derives from the fact that this disk is within the zodiacal band, perhaps being a remnant of the early formation of the solar system.) As such, Ṭūsī's explanation is not a particularly happy one since it obviously depends on the illumination of the atmosphere; but even if we accept the premise that the light is atmospheric, the explanation is still rather weak since he has not taken into account the height of the atmosphere.[6] For a modern discussion of the zodiacal light, see Minnaert [1954], pp. 289–295; for some insight into why Ṭūsī's explanation would not work even if we accepted that the light were atmospheric, see ibid., pp. 277–280 as well as the particularly useful discussion and diagrams of Dietze [1957], pp. 207–210.

III.9 [1] Fig. T25. This version of the figure has been drafted using the illustrations in MSS FL. As we stated in our introductory remarks to the edition, these represent a revision of the *Tadhkira*, the so-called Baghdad (β) version of the text.[7] Figure C34 has been drafted using MSS DGMT, which are witnesses of earlier versions of the *Tadhkira*. There seems to have been an attempt to deal with the problems mentioned above (see commentary to III.9 [1]6–8) and indeed Fig. T25 is something of an improvement over Fig. C34.[8] First of all the apex of the cone is placed on the sun's orb in MSS FL, whereas it is beyond it in MSS DGMT. Actually the apex, according to Ptolemaic parameters, should be in Venus's system of orbs (see IV.3 [5]) and thus inside the sun's orb in the figure. Hence MSS FL are somewhat better without being completely accurate.[9] Second, the base of the cone in MSS FL is no longer parallel to the horizon, which may be an attempt to deal with the problem mentioned in the commentary to III.9 [1]6–8. Finally the "plane of the apparent horizon" (*saṭḥ al-ufaq al-mar'ī*) in MSS DMT, which is incomprehensibly drawn obliquely to the horizon, is

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[5] Cf. Wiedemann [1912] and Wiedemann and Frank [1926], pp. 31–32.

[6] Estimates for the height of the atmosphere were made in Islamic astronomy, but I have been unable to ascertain whether anyone related it to the problem of false dawn; cf. Saliba [1987b].

[7] See pp. 74–75 and 87–88 of Volume One.

[8] The β version of the *Tadhkira*, though, is not always an improvement; see for example II.5 [8]6–8 and II.11 [5]19–20.

[9] This may be due simply to problems related to drafting the diagram. Bīrjandī states explicitly on his figure that the apex is in the orbs of Venus and yet it is drawn on the sun's orb by the copyist of Houghton Arabic MS 4285.

replaced in MSS FL by an apparent horizon and by an east-west line, both correctly drawn, that extend across the figure.

III.9 [2]2–4. *wa-qad ᶜurifa bi-'l-tajriba...thamāniya ᶜashr juz'ᵃⁿ* (It has become known by trial and error...18°): The word that I have translated as "trial and error," *tajriba*, has something of our sense of experiment, which it has come to mean in modern Arabic. Without attempting to overgeneralize at this point, one might contrast *tajriba* with *imtiḥān* and *iᶜtibār*, both of which are used in the astronomical literature to refer to the testing of some received value or parameter rather than finding a new one.[10] Bīrjandī gives a number of methods by which one could arrive at a parameter for the solar depression; he wishes to dispel the notion (perhaps by some religious figures?) that one could not obtain a reliable value since the sun could not be observed at the appropriate time. He mentions observations of the fixed stars, the employment of an astrolabe, and the use of clocks. Whatever methods were used, we do know that 18° was only one of several values in use during the Islamic Middle Ages; for some of the different parameters and a discussion, see King [1973], pp. 366–367.

III.9 [2]4–1. *fa-fī al-bilād...fī al-munqalab al-ṣayfī* (Thus in the lands...at the summer solstice): From III.5 [1]15–21, we have

$$\text{lowest altitude of summer solstice} = \phi - \bar{\varepsilon} = 48\tfrac{1}{2}^\circ - 66\tfrac{1}{2}^\circ = -18^\circ$$

which indicates that for $\phi = 48\frac{1}{2}^\circ$ dusk and dawn will be continuous when the sun is at the summer solstice.

III.9 [2]2$_1$. *fī zamān akthar* (for a longer period): There are two ways to interpret this phrase. Shīrāzī understands the increase in time to refer to the continuity—and thus mixing—of dawn and dusk over a single night. At $\phi = 48\frac{1}{2}^\circ$, dawn will begin at just the moment dusk is ending. At higher latitudes where the solar depression at summer solstice decreases, one could say that dawn begins before dusk has ended. Bīrjandī does not think much of this interpretation and instead takes "for a longer period" to refer to the increase in the number of days during which dusk and dawn are continuous for $\phi > 48\frac{1}{2}^\circ$. Although Bīrjandī makes a strong case for his admittedly more sensible interpretation, one must acknowledge that Shīrāzī is closer in time and thinking to Naṣīr al-Dīn; the reader is free to choose between the alternatives.

III.9 [2]3–4. *wa-yatabayyanu mimmā waṣafnā...li-'l-ufaq al-raḥawī* (From what we have described...should be clear): See III.6 [2]11–12, in which Ṭūsī states that the setting of dusk or the rising of dawn will occur in 50 nychthemerons for an observer at the pole; this is a direct consequence of the 18° parameter as explained there in our commentary.

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[10] Cf. Sabra [1968]; also cf. commentary to II.14 [2]10–14.

Book III, Chapter Ten

III.10 [1]. For Ptolemy's discussion of hours and daylight, see *Alm.*, pp. 99, 104 (H142–145); *HAMA*, 1: 36–37, 40–41; and Pedersen [1974], pp. 113–114. Although his methods for determining length of daylight and of converting equal into seasonal hours (and vice versa) are obviously equivalent to those of Islamic astronomers, Ptolemy does not use the terms "arc of daylight" or "equation of daylight" as such; for these see *Tafhīm*, p. 131; Kennedy, *Taḥdīd Comm.*, pp. 70, 113; and III.3 [2].

III.10 [1]9–15. *al-mashhūr...bi-ḥasab dhālik* (It is commonly held...The arc of night is in accordance with the above): One may define the standard arc of daylight (d) as

$$d = 180° + 2q$$

where q is the equation of daylight.[1] But as Ṭūsī notes, the sun will traverse a small arc of the ecliptic during daylight due to its proper motion, and the co-descension[2] of this arc must be added to or subtracted from the standard arc of daylight to obtain the true arc of daylight. Now one would assume at first glance that since the daily motion is from east to west and the sun's proper motion has a generally west to east direction the correction should always be additive, and this is the reading one finds in MSS DGLMT. But in MS F and in the margin of MS T, the words "or less" (*aw anqaṣ*) have been added to indicate a possible subtractive correction. This addition is dismissed by Nīsābūrī and Jurjānī,[3] who take it to be a phrase that has inadvertently (*sahw*^{an}) entered the manuscript tradition. However, an annotator contends in a marginal note to Nīsābūrī's commentary that the additional phrase is not inadvertent but is in fact correct,[4] and this line is supported by Bīrjandī. They point to the Arctic region where the correction will be negative if the sun is on the arc that sets in reverse order.[5] To see how this would work, we can refer to Figs. T20–T22, which represent the situation for $\phi = 70°$. If the sun is at Libra 0° when it rises (Fig. T20), it will move slightly into Libra during the course of the daylight period. But since Libra sets in reverse order, the point the sun has reached in Libra will set before Libra 0° and hence the correction to the standard arc of daylight will be negative (Fig. T22). Bīrjandī further notes that if the latitude is equal to the complement of the obliquity (see III.4 [5]), then when the sun is on the ecliptic arc that sets in one stroke[6] the correction will be zero since the sun thereupon sets simultaneously with its rising point.

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[1] For northern latitudes, q is positive for $180° > \lambda_{Sun} > 0°$ and negative for $360° > \lambda_{Sun} > 180°$; see Fig. C28.

[2] Ṭūsī mistakenly has co-ascension in the text.

[3] Khafrī simply repeats Jurjānī.

[4] Najaf, Āyatallāh al-Ḥakīm MS 649, f. 93a.

[5] See III.5 [1].

[6] In the northern hemisphere, this is the six signs bisected by the autumnal equinox.

There is another anomaly that one might consider in correcting the standard arc of daylight, namely the one due to the shifting of the sun from one day-circle to another in the course of a day. In Fig. C28, this will result in the equation of daylight q in the western triangle differing from that of the eastern triangle. Generally this is a very small correction, but for $\phi > 50°$, it can reach a large enough negative value to result, when combined with the previous positive correction, in a negative correction. But since this second correction is not mentioned by anyone other than Bīrjandī, who dismisses it as insignificant, it seems safe to conclude that the additional phrase "or less" was added to account for the situation in the Arctic region. This would certainly not be inconsistent with what we have previously seen, namely a concern by Ṭūsī and by the commentators to have their statements be as all-inclusive as possible; I have therefore concluded that this addition should be considered part of the β version.

III.10 [1]15–20. *fa-idhā qusima...lā yakhtalifāni* (Now if each of the two arcs is divided by 15...that are invariable): If one divides the day into 24 hours of equal length, which are the kind of hours we are accustomed to using, then in general the number of hours of daylight will not be equal to the hours of darkness. If one has 12 hours of daylight and 12 hours of darkness no matter what the season, then these hours are of variable, or "unequal," length. The former type of hours were referred to by the ancients as equal or equinoctial hours, whereas the latter, which were in general use, were called seasonal or civil hours.[7] In the case of equal hours, the number of degrees/hour is invariable, whereas for seasonal hours it is the hours/daylight or hours/darkness that is invariable.[8]

In order to clarify Ṭūsī's rather tortured phrasing here, let us take an example. Assume the arc of daylight is 150° and the arc of night is 210°. Then since there are 15°/equal hour, one has

$$150° \div 15°/\text{equal hour} = 10 \text{ hours (daylight)}$$

$$210° \div 15°/\text{equal hour} = 14 \text{ hours (night)}.$$

On the other hand, there are 12 unequal hours/daylight or night; thus

$$150° \div 12 \text{ hours} = 12\tfrac{1}{2}°/\text{unequal hour (daylight)}$$

$$210° \div 12 \text{ hours} = 17\tfrac{1}{2}°/\text{unequal hour (night)}$$

Compare *Tafhīm*, pp. 53–54.

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[7] Toomer, *Alm.*, p. 23; *HAMA*, 3: 1069.

[8] In saying that the degrees/equal hour is invariable one ignores the equation of time. One should also note that in assuming 15°/equal hour, one will end up with a solar day longer than 24 hours (but a sidereal day of exactly 24 hours). For simplicity we shall ignore these differences in the sequel.

III.10 [2]. For the synodic month, see *Alm.*, pp. 174–176 (H270–272); *HAMA*, 1: 69–70, 3: 1083–1084; and Pedersen [1974], pp. 160–164. Compare *Tafhīm*, pp. 161–162.

III.10 [2]21–22. *wa-qad tabayyana...min al-shams* (which have been shown...from the sun): This is a reference to II.13 [1].

III.10 [2]2–5. *fa-musta*ᶜ*milūh min ahl al-ẓāhir...yaṣṭaliḥūna* ᶜ*alayh* (Those employing the [month] who [rely] on appearances...they have conventionally adopted): It is very likely that Ṭūsī is thinking of the Mongols when he refers to those who begin the month from the day of conjunction; Bīrjandī seems to confirm this when he ascribes this type of reckoning to the "Turks." This month and the calendar of which it is a part is ultimately Chinese in origin; for a discussion, see Kennedy [1964].

The Muslims, of course, take the first of the month to be from the first visibility of the lunar crescent; they were preceded in this by the Jews and Babylonians among others.

That Ṭūsī has anyone specifically in mind when he states that a month may be from "some other shape to its like" is unlikely; Bīrjandī calls this an "intellectual possibility" with no known examples.

III.10 [2]5–11. *wa-musta*ᶜ*milūh min ahl al-ḥisāb...wa-tusammā tilk al-ayyām kabā'is* (Those employing the [month] who [rely] on calculation...These are called intercalary days): The Islamic calendar is usually taken to be observational; in other words, the new month starts when the crescent is actually observed. There is no intercalation since on the one hand Holy Writ had forbidden its use[9] and on the other a natural calendar is obviously self-regulating. But such a calendar is very difficult, if not impossible, to set in advance; a certain month does not have the same number of days from year to year and even two contiguous regions may have different starting days for a given month. Add to this the problems of weather and religious differences concerning the number of days that Ramaḍān, the month of fasting, should have and one can see that determining a date in the future (or, for that matter, in the past) is subject to a degree of uncertainty.[10]

This state of affairs was intolerable to the astronomers, of course, and they devised a "calculated" lunar calendar.[11] In this system the odd-numbered months have 30 days, while the even-numbered months have 29. But since 12 lunar months consist of approximately $354^{11}/_{30}$ days, one will need to have 11

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[9] *Qur'ān*, ix. 35–36 (*Azhar* edition numbering).

[10] A two-day ambiguity is not uncommon; for a discussion of the problem in a modern context, see Ilyas [1984], especially pp. 58–78, 133–140.

[11] For what follows, see the excellent discussion by Ginzel, *Handbuch*, 1: 254–256; cf. Hartner, "*Zamān*," *EI*[1], p. 1211 for a synopsis.

leap years in every 30-year cycle.[12] The added day was given to *Dhū al-Ḥijja*, the last month of the Islamic calendar, usually in years 2,5,7,10,13,16,18,21,24,26,29.[13] This choice is dictated by the cumulative fractional part of a day, which exceeds ½ for the 354th day of these years. In the case of year 16, there is a difference of opinion with some opting instead for year 15, the reason being that at the end of year 15 the cumulative fractional part is exactly ½. Bīrūnī mentions another method of intercalation to obtain the 11 additional days whereby every 3rd year has 355 days, but he does not notice that in a 30-year cycle one obtains 10 rather than 11 additional days.[14]

The existence of both an observational and a calculated lunar calendar raises the issue of which is being used when one is given a *hijra* date in a medieval source. Judging from what Bīrūnī says in the *Tafhīm*, these calculated "leap years of the Arabs" (*kabā'is al-ᶜArab*) are so called not because the Arabs themselves use (or used) them but because they are the basis for dating when the writers of astronomical tables (*zīj*es) have recourse to Arabic years (p. 163). That the Islamic calendar is based on observation is also emphasized by Bīrūnī in his *Chronology*,[15] and it is telling that he does not even mention a calculated Islamic calendar there. But I do not feel this is the end of the story. One may assume, with what degree of certainty is unclear, that when one encounters a *hijra* date for an observation in an astronomical text it is the calculated date; but should this assumption carry over to the date of composition or copying? One would need to analyze a large sample of dates that give the day of the week; in these cases an exact correspondence with the Julian day can be made.

III.10 [2]11. *aw yazīdūna...wajh ākhar* (Or else they add...some other way): Exactly what Ṭūsī has in mind here is not clear. Nīsābūrī and Jurjānī think he is referring to the systems of intercalation of the Jews, "Turks," and pre-Islamic Arabs. But as Bīrjandī notes, these calendars are observational, whereas Ṭūsī is discussing intercalation in a calculated lunar calendar. Perhaps he is thinking of the variations mentioned in the previous comment, namely having the 15th rather than the 16th be a leap year in the 30-year cycle, Bīrūnī's 3rd year rule, or perhaps the 8-year cycle. Against this is the fact that he himself has not mentioned any specific rule.

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[12] This is quite a good approximation scheme since it implies a lunar synodic month of 29;31,50 days. (The standard Babylonian/Ptolemaic value is 29;31,50,8,20 days.) A somewhat less accurate 8-year cycle with 3 leap years was reportedly used by some ancient Greeks (Bickerman [1968], p. 31) as also, for a time, by the Ottomans (Ginzel, *Handbuch*, 1: 255–256.)

[13] This choice of leap years is the one used in the tables of Wüstenfeld-Mahler.

[14] Bīrjandī mentions a similar calendar, attributing it to a certain Abū al-Maḥāmid al-Ghaznawī [Bīrūnī?], and also finds the same difficulty.

[15] *Āthār*, especially pp. 29, 64, 66 (*Chron.*, pp. 34, 76, 78).

III.10 [3]. For the year, see *Alm.*, pp. 131–141 (H191–209); *HAMA*, 1: 54–55, 3: 1082–1083; and Pedersen [1974], pp. 126–134. Compare *Tafhīm*, pp. 162–165, 171.

III.10 [3]14–17. *wa-yaḥṣulu dhālik...min al-kusūr* (This occurs in 365¼ days...minus a fractional part): The following table gives the values for the tropical year as reported by Bīrjandī, who provides the fractional part to be subtracted from 365¼ days.

Table 9. Tropical Year Lengths.

Ptolemy[16]	$365^d6^h - 4\frac{4}{5}^m$ (=365;14,48^d)
Battānī (d. 929)[17]	$365^d6^h - 13\frac{3}{5}^m$ (=365;14,26^d)
Muḥyī al-Dīn al-Maghribī (d. 1283)	$365^d6^h - 12^m$ (=365;14,30^d)
Ṭūsī ("The new observations at Marāgha")	$365^d6^h - 11^m$ (=365;14,32½^d)
A "Modern"	$365^d6^h - 9\frac{3}{5}^m$ (=365;14,36^d)

III.10 [3]15–17. *wa-yatimmu fīhā...min al-kusūr* (During [a year]...minus a fractional part):

$$12 \text{ lunar months} = 354^d8^h48^m = 354;22^d$$

Subtracting this from Ptolemy's tropical year one obtains

$$365;14,48^d - 354;22^d = 10;52,48^d \ (=10^d21^h7\tfrac{1}{5}^m)$$

III.10 [3]17–10. *wa-mustaᶜmilūhā...sinīn qamariyya* (Of those employing [the year]...which they call a lunar year): The following table summarizes the various years mentioned by Ṭūsī:

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[16] *Alm.*, pp. 137, 139–140.

[17] Neugebauer (*HAMA*, 1: 307) derives a value of 365;13,7,30^d based on Battānī's *approximate* values for the lengths of the seasons; but the tropical year given here is Battānī's explicit value and is confirmed in at least two places in his *zīj* (3: 63–64 and 191 [trans., 1: 42 and 127]).

Table 10. Conventional Years.

Year	Adopted by	Reference
I. "True solar year," usually taken to begin at spring equinox		
Ia. "True solar" months, determined by arrival of sun at the same degree in each zodiacal sign	"Ancient Astrologers"	Khafrī and Bīrjandī
Ib. 12 30-day "conventional" months plus 5 epagomenal days; leap year in 4th or 5th year	"Modern Astrologers"; the Seljuq ruler Malikshāh (1072–1092) ["*Jalālī* calendar"]; and the Mongol ruler Ghāzān Khān (1295–1304) ["*Khānī* calendar"]	Khafrī and Bīrjandī; cf. Sayili, *Obs.*, p. 229
II. "Conventional solar year" (*i.e.* with conventional starting date); average length slightly longer than a tropical year (for **IIa**); 12 months, each approximately (or exactly) 30 days long		
IIa. 365-day standard year with 366-day leap year every 4th year	Greeks, Romans, "Syrians and Chaldaeans," and "modern" Copts; year of Caliph al-Muᶜtaḍid bi-Allāh (892–902) and Khwārizm-Shāh Aḥmad (calendar reform of 959)	*Tafhīm*, p. 163; *Āthār*, pp. 10, 31–33, 68, 241 (=*Chron.*, pp. 12, 36–38, 81, 229–230)
IIb. 365-day year with no leap years	"Ancient" Copts (*i.e.* Ancient Egyptians) and pre-Islamic Persians*	*Tafhīm*, p. 163; *Āthār*, 10–11 (=*Chron.*, 12–13)
III. Luni-solar calendar; month added every 3rd or 2nd year	Jews, Ṣābians, Ḥarrānians, Turks(?), and pre-Islamic Arabs (the latter not consistently)	Khafrī and Bīrjandī; *Tafhīm*, p. 164; *Āthār*, 11–12 (=*Chron.* 13–14)
IV. Pure lunar year	Muslims	III.10 [3]9–10

* The Persians, unlike the Egyptians, did try to reconcile their calendar somewhat with the seasons by adding an extra month every 120 years.

III.10 [3]22. *al-mustaraqa* (stolen): Bīrjandī insists on this vocalization, with which I agree; Steingass, however, has *mustariqa.*

III.10 [3]10–11. *wa-li-kull qawm...bi-hādhā al-ᶜilm* (Every people has an epoch...to this science): On epochs, see *Tafhīm*, pp. 171–173 and *Āthār* (*Chron.*), Chs. 3, 6, 8. Ṭūsī is again careful to differentiate *ᶜilm al-hay'a* from other disciplines; see our introduction, pp. 37–38, and cf. II.4 [12], where Ṭūsī states that the science of the fixed stars is a separate discipline. Concerning the specific question of including a lengthy discussion of chronology in a *hay'a* work, Ṭūsī may be gently chiding Khiraqī, who had devoted a large part of his *Muntahā* to chronology.

Book III, Chapter Eleven

III.11 [1]. We may define the "degree of transit" of a star as the point on the ecliptic that crosses the meridian (or "culminates") simultaneously with that star, both the point and star being assumed above the horizon.[1] In general the degree of transit will not be equal to the degree of longitude of a star, which is also a point on the ecliptic;[2] the exception will be when the star and the ecliptic poles are all on the meridian. In this latter case the star's latitude circle and the meridian coincide, and thus the degree of transit and the degree of longitude will also coincide.

To give examples of the more general case, we shall refer to Fig. C35. Both the northern celestial and ecliptic poles are visible and the ecliptic pole is east of the meridian.[3] For a star S_1 north of the ecliptic (or, more correctly, in the same direction from the ecliptic as the visible ecliptic pole), its degree of longitude is L_1, while its degree of transit is L_T; in this case L_1 will have transited before L_T. For a star S_2 south of the ecliptic (or, more correctly, in the direction opposite the visible ecliptic pole), its degree of longitude is L_2, while its degree of transit is L_T; in this case L_T will transit before L_2. If the ecliptic pole is west of the meridian, the situation of L_1 and L_2 will be reversed.

III.11 [2–3]. The degree of rising (or the degree of setting) may be defined in a way analogous to the degree of transit, *viz.* the point on the ecliptic that rises (or sets) simultaneously with a given star. For the equator the situation is straightforward. If the ecliptic poles are on the horizon circle when a star rises

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[1] Cf. Bīrūnī, *Taḥdīd*, pp. 199–200 [trans., p. 165]; Kennedy *Taḥdīd Comm.*, pp. 122–123; and Bīrūnī, *Transits*, pp. 3, 123–124. Note that Ṭūsī never really gets around to defining "degree of transit."

[2] On longitude, see II.3 [8]. When "degree" is used alone, it means "degree of longitude."

[3] For the case where the southern ecliptic pole is visible, the same result will follow if one is careful, as Ṭūsī is, to refer to stars as being in the direction (or opposite the direction) of the visible ecliptic pole rather than north or south of the ecliptic.

(or sets), the degree of rising (or setting) will be the same as the degree of longitude. For the remaining cases, we see from Fig. C36 that if a star S_1 is in the direction of the visible pole (here the northern one), then it will rise before its degree of longitude and set after it. For a star S_2 in the direction of the invisible pole, it will rise after its degree of longitude and set before it. The similarity that Ṭūsī draws between the situation of rising and setting at the equator and that of transit for the other horizons is a consequence of the fact that in general any meridian circle is also a horizon circle for the equator (cf. III.7 [3]).

It is reasonably clear that for $\phi \neq 0$, the situation for rising and setting will be the same; in other words, a star toward the visible pole will rise before its degree of longitude and set after it, while a star away from the visible pole will rise after and set before. Now for $\phi = 0$, the circuits of the ecliptic poles are bisected by the horizon circle; thus each ecliptic pole is visible half the time. As a consequence, as stated at the end of paragraph [2], the half of the ecliptic bisected by the vernal equinox will rise, while the southern half will transit during the time the northern pole is visible. But if $0 < |\phi| < \varepsilon$, the circuits of the ecliptic poles will be unequally divided by the horizon circle and as a result one will not have half of the ecliptic rising or transiting during the period of visibility of one ecliptic pole or the other. This is what Ṭūsī is trying to get at in paragraph [3]. For example, if $\phi \approx 20;16°$, Gemini 0° and Leo 0° will pass over the zenith; consequently 10 zodiacal signs will transit (or rise) when the northern ecliptic pole is visible, while only 2 zodiacal signs will transit (or rise) when the southern ecliptic pole is visible.

If I understand the last sentence of the chapter, Ṭūsī is pointing out that for $0 \leq |\phi| < \varepsilon$ one could have the situation whereby for a given star one pole is visible at rising and the other is visible at setting; thus in applying the above rule, one uses one pole for the degree of rising and the other for the degree of setting. For $|\phi| > \varepsilon$, this "variation" (*ikhtilāf*) would not arise, of course, since one ecliptic pole is permanently visible; in this case the rule may be uniformly applied with reference to that single pole.

Book III, Chapter Twelve

III.12 [1–2]. For the justification of these two methods for determining the meridian line, see Bīrūnī, *Shadows*, 1: 144–148, 2: 78–79. The first method is based upon direct observations of two equal solar altitudes on the two sides of the maximum altitude; Bīrūnī tells us they can be found by the use of the armillary sphere or the *ṣafīḥa* (astrolabe?) (ibid., 1: 150). The second, by which one finds two equal solar altitudes indirectly by observing two equal shadow lengths, is the method of the Indian circle; see ibid., 1: 150–160, 2: 80–90 and *Tafhīm*, pp. 49–50.

Taking the radius of the Indian circle to be twice the length of the gnomon (*lit.*, measuring instrument [*miqyās*]) is standard according to Bīrjandī. But such a ratio will not be applicable for all latitudes, even for all those of the inhabited quarter. Since the Indian circle method depends upon the shadow being within the circle at noon, a 2:1 ratio of radius to gnomon will not be usable for the entire year unless $\phi < 40°$. To see this one should note that at $\phi = 40°$, the minimum solar altitude at noon is about $26\frac{1}{2}°$ and $\tan(26\frac{1}{2}°) \approx \frac{1}{2}$. Bīrūnī wishes to give a rule for $\phi < 48°$ and so he is led to recommend a radius of 4 times the gnomon. For Bīrūnī's discussion and an elaboration of the method, see *Shadows*, 1: 152–153, 2: 81–82.

Although Ptolemy in the *Almagest* does not give a method for finding the meridian line,[1] Diodorus of Alexandria (1st c. B.C.) did advance one in his non-extant *Analemma*, which was known in some form to Bīrūnī. (See *Shadows*, 1: 162–166, 2: 91–93; Kennedy [1959]; and *HAMA*, 2: 841–842.) On other methods used or discussed in medieval Islam, see *Shadows*, 1: 161, 167–172, 2: 91, 94–97 (cf. *Taḥdīd*, pp. 287–288 (trans., pp. 256–258); Kennedy, *Taḥdīd Comm.*, pp. 216–219; and Kennedy [1963]).

III.12 [1]8 & [2]16. *dā'ira awwal al-sumūt; sumūt* (prime vertical circle; azimuths): For these terms, see II.3 [14] and [17].

III.12 [3–4]. For a competent overview of the determination of the *qibla*, see King, "*Ḳibla*," in which one also finds an extensive list of further references.

III.12 [3]19; [4]8&17. *samt al-qibla* (the *qibla* bearing): This phrase is usually intended to indicate the point of intersection in the direction of Mecca of the horizon with the great circle joining the local zenith with the zenith of Mecca.[2] Thus one sometimes finds this referred to as the *nuqṭa samt al-qibla* (*qibla* bearing point). The *qaws samt al-qibla* (arc of the *qibla* bearing) is defined by Bīrjandī as the arc along the horizon between the *samt al-qibla* and either the north or south point (depending on which would yield an arc less than 90°); alternatively it could be between the *samt al-qibla* and the east or west point. Bīrjandī claims that *qaws inḥirāf al-qibla* (arc of deviation of the *qibla*) is an equivalent formulation.[3] Finally the *khaṭṭ samt al-qibla* is defined as the line joining the zenith with the *qibla* bearing point. (See Fig. C37, where the above terms are indicated for a special case.)

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[1] Cf. *Alm.*, p. 62 (n. 72).

[2] See *Tafhīm*, p. 137 and Jaghmīnī, II.3 (trans., p. 271); this is also confirmed in the commentaries.

[3] King ["*Ḳibla*," p. 83 (col. 2)], however, differentiates between the *samt al-qibla*, which he takes to be equivalent to the arc along the horizon measured from the east (or west) point to the *qibla* bearing point, and the *inḥirāf al-qibla*, which for him is the arc along the horizon measured from the north (or south) point to the *qibla* bearing point. (Note that King's formulations are in terms of angles, not arcs.) It may be that these technical terms were not fixed for all times and places; as we shall see below, there is indeed ambiguity in the terminology.

These definitions *should* clear up the matter, but unfortunately Naṣīr al-Dīn is not being cooperative; although *samt al-qibla* is taken by the commentators to be a point in line 19, it almost certainly is a line or direction in line 17. Given this, I have translated *samt al-qibla* by the ambiguous "*qibla* bearing" since Ṭūsī does not seem to be using it strictly as a point.

III.12 [3]19–22. *wa-ammā samt al-qibla...wa-thulthā juz'* (As for the *qibla* bearing...21⅔°): These fairly common values for Mecca's longitude and latitude (77;10° and 21;40°, respectively) are, according to the list of Kennedy and Kennedy [1987], pp. 225–226, first found in *Al-Zīj al-jāmi*[c] of Kūshyār b. Labbān (ca. 990). It is noteworthy that the values they report for Ṭūsī's *Īlkhānī Zīj* are 77;13° and 21;40°. For the 10° longitude difference between the Eternal Islands and the "coast of the western sea," see III.1 [7].

III.12 [3]4–7. *wa-kull balda...maghrib al-i*[c]*tidāl* (Every locality...setting place of the equinox): Ṭūsī is here pointing out that if the locality has the same latitude as Mecca, Mecca's zenith will not be on the east-west line but will be north of it. To see this, we refer to Fig. C37 where a locality with zenith Z has latitude ϕ_M equal to that of Mecca. Since the maximum declination along the east-west line occurs at the local zenith, then the zenith of Mecca M will necessarily be north of the east-west line. Thus if Mecca is to the east, it will be "to the left of the rising place of the equinox" (*i.e.* the east point) and if it is to the west it will be "to the right of the setting place of the equinox" (*i.e.* the west point). This problem is akin to that old puzzler of why one (normally) flies north to go from New York to Madrid despite the fact that they are very nearly on the same latitude circle.

III.12 [4]. One also finds this technique for determining the *qibla* in Jaghmīnī, II.3 (trans., p. 272).[4] It takes advantage of the fact that Mecca is in the Tropics and thus witnesses two zenith transits of the sun every year.[5] At these two moments, the shadow of a gnomon elsewhere will be in the plane of the altitude circle that joins the local zenith with that of Mecca. By translating the longitude difference between the locality and Mecca into an exact time difference (*i.e.* hours before or after noon), one has a straightforward way of determining the *qibla*.

[4] King ["*Ḳibla*," p. 84 (col. 1)] notes that it is the basis for "several *ḳibla* methods."

[5] The ecliptic positions given by Ṭūsī, "degree 8 of Gemini" and "degree 23 of Cancer," do not at first sight seem symmetrical about the summer solstice. This is due to a peculiarity of this method of reporting coordinates. To be "in degree 8 of Gemini" actually means to be between Gemini 7° and 8°. Jaghmīnī gives the positions as Gemini 7;21° and Cancer 22;39°, which are correct for $\varepsilon = 23;35°$ and $\phi_M = 21;40°$.

We should here note several points: this is probably the simplest of the many ways of determining the *qibla*; it is based on direct observation rather than on a trigonometric or graphical (analemma) solution; and in principle it is exact. But besides not being very useful for someone in Boston, it has a number of other drawbacks. First one has only two chances during a year to use it; one must hope for sunny weather on those two days. More serious is the problem of time determination. Even if one assumes that he has a correct longitude difference,[6] one would still need a fairly precise means for determining time before or after noon since the exact formula for the *qibla* is rather sensitive to inaccuracies in longitude difference. For example, an error of only a few minutes (not unlikely considering the time-keeping devices of the medieval period) can result in a discrepancy of a degree or more depending on the latitude and longitude of the locality in question. Thus this method could have been of use to someone who wanted a reasonably accurate *qibla*, but it could hardly compete with the exact and vastly more sophisticated solutions that were developed in the Islamic Middle Ages. Again it is clear that Ṭūsī, who was fully in control of these latter methods, did not wish to burden the reader of the *Tadhkira* with such mathematical detail.

[6] This would more than likely be a false assumption insofar as the Middle Ages is concerned; see King, "*Ḳibla*," pp. 87–88.

BOOK IV

Book IV, Chapter One

No *hay'a* work would be complete without a section dealing with sizes and distances;[1] for it is here that the relative sizes and distances of the celestial bodies that have been individually derived are converted into absolute quantities so that these bodies may be fitted together to form a coherent, unified structure, or *hay'a*.

IV.1 [1]. The following is a summary of the propositions with references to works by Archimedes and Ṭūsī's recensions. Ṭūsī seems to have modified **(2)** and **(3)** so that the component variables of the areas are given as sides of rectangles. The actual Archimedean propositions, in modern symbols, are provided in brackets.

> [C = circumference of circle (or of largest circle in sphere); A = area of circle (or of largest circle in sphere); S = surface area of sphere; L = area of lune; d = diameter; r = radius; *DC* = *Dimension of the Circle*; *SC* = *On the Sphere and Cylinder*; *Ras* = Ṭūsī, *Rasā'il*.]

(1) $C \approx 3\frac{1}{7} \cdot d$	$[3\frac{10}{71} \cdot d < C < 3\frac{1}{7} \cdot d]$	*DC*, Prop. 3; *Ras*., 2(5): 129.
(2) $A = r \cdot \frac{1}{2}C$	$[A = \frac{1}{2}(r \cdot C)]$	*DC*, Prop. 1; *Ras*., 2(5): 133.
(3) $S = d \cdot C$	$[S = 4A]$	*SC* 1, Prop. 33; *Ras*., 2(5): 63.
(4) $L = d \cdot ((\alpha / 360°) \cdot C)$ [α is the angle of the lune; note that $((\alpha / 360°) \cdot C)$, the maximum inclination, is a linear measure.]		Not in the extant Archimedean corpus, but an immediate corollary of **(3)**.

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[1] A notable exception is Jaghmīnī's *Al-Mulakhkhaṣ fī al-hay'a al-basīṭa*. (For this work, see pp. 35, 56 of Volume One.) In his commentary (*sharḥ*) on the *Mulakhkhaṣ*, Qāḍīzāde al-Rūmī (ca. 1364–ca. 1436) explains that the lack of any treatment of sizes and distances was due to the difficulty (*ṣu*ᶜ*ūba*) of this subject and presumably was inappropriate for such an elementary textbook (f. 4b, lines 14–15). As it turns out, Jaghmīnī wrote a separate treatise on sizes and distance, the apparently unique manuscript being Cairo, Dār al-kutub MS Ṭalᶜat majāmīᶜ 429 (2), f. 4a–4b.

IV.1 [1]6. *muṣādarāt* (preliminary propositions): In the Arabic translations of Euclid and Aristotle, this term was used to render the αἰτήματα (postulates), but it could also be used to cover all the premises of a mathematical work.[2] Here, however, the word clearly does not have the restricted sense of unprovable premises since it refers to *propositions* (or "principles") that are "proved in another science and are taken for granted in this science" (I.Intr. [1]). Since they have not been necessary until now, Ṭūsī did not place them in I.1, which is generally restricted to geometrical definitions. Note that for his couple, Ṭūsī uses the word *muqaddama* in II.11 [2], which I have translated as lemma. Since he also uses the same word to refer to these *muṣādarāt* at the beginning of the next paragraph, he apparently does not strictly distinguish between the two terms.

IV.1 [2]. There is a serious problem in this paragraph inasmuch as the length of a meridian degree that is cited by Ṭūsī, namely 22⅖ parasangs (= 66⅔ miles), is Ptolemaic and at considerable variance with the value that one usually finds attributed to Ma'mūn's astronomers, which is something between 56 and 57 miles. Although at first sight this may appear to be a simple mistake, it turns out to be the tip of a rather unwieldy iceberg. Thus to explain this "mistake," we need to have an extended but far from complete discussion of the measurement of the Earth, an excursus for which I beg the reader's indulgence.

Although Ptolemy in the *Almagest* does not give an absolute linear value for the size of the Earth, we know from the *Planetary Hypotheses* and the *Geography*[3] that he adopted the norm of 180,000 stades for the circumference, a figure that seems to have replaced Eratosthenes's (3rd c. B.C.) 252,000 stades well before his time.[4] Now one of the equivalences that was introduced during the reign of the Ptolemies in Egypt and that spread to Syria was 1 mile = 7½ stades. From this and from Ptolemy's norm one arrives at a rather tidy circumference of 24,000 miles and thus a value of 66⅔ miles for the length of a terrestrial degree. These numbers are attested in various Syriac writers and even in the Talmud.[5] No doubt some of these accounts came to be known in Islam in addition to the direct statements from the *Planetary Hypotheses* and the *Geography*. The received conversion of stades into miles, however, was not universally accepted; we learn from Ḥabash that the Caliph Ma'mūn (reigned 813–33) was not satisfied with the answers he was given concerning the length of a Ptolemaic stade.[6] We know from a variety of sources that Ma'mūn thereupon ordered his astronomers to undertake one or more expeditions to ascertain anew the size of the Earth, this time in the standard measures of the day. But despite considerable

[2] Sabra [1969], pp. 1–2.

[3] *Plan. Hyp.*, BM 90b, line 3 (trans., p. 7); *Geog.*, VII.5 (Nobbe, 2: 179, par. 12, line 24).

[4] *HAMA*, 2: 652–654.

[5] See Nallino [1911], p. 279 (*Racc.* V, pp. 293–294); I am greatly indebted to his research for much of what follows.

[6] Langermann [1985], pp. 115, 122; this passage is cited by Bīrūnī, *Taḥdīd*, p. 213 (trans., p. 178).

research by Nallino[7] and Barani [1951], the actual details of these expeditions have been rather obscure, in large part because modern historians have until recently been forced to rely upon later sources for their information. But two contemporary sources have lately turned up, and they shed considerable, though not complete, light on Ma'mūn's attempts to determine the Earth's circumference. The first has been uncovered by Langermann [1985], and it is Ḥabash's (d. after 864) *Kitāb al-ajrām wa-'l-ab^{c}ād* (Book of Bodies and Distances). The second has hitherto gone unnoticed, at least as a source for this problem, and it is *Kitāb Ḥarakat al-aflāk* (Book of the Motion of the Orbs) by Muḥammad b. Mūsā b. Shākir (d. 873), the eldest of the Banū Mūsā.[8] Although it would take us far afield to deal adequately with these new sources and reopen the whole question of Ma'mūn's attempts to measure the Earth's size, it will be necessary to rehearse at least part of the story in order to understand Ṭūsī's apparently incorrect statement in this passage.

Ḥabash tells us[9] that Ma'mūn sent Khālid ibn ^{c}Abd al-Malik al-Marwarrūdhī,[10] ^{c}Alī ibn ^{c}Īsā al-Asṭurlābī, and Aḥmad ibn al-Buḥtarī al-Dharrā^c to the Plain of Sinjār near Mosul to make the measurements, and they arrived at a figure of 56 miles for a terrestrial degree, each mile being 4000 black cubits, which was a standard unit established by Ma'mūn. Ḥabash states that he got this information directly from Khālid, and it is clear that this text by Ḥabash served as a source for Mas^{c}ūdī (d. 957–58), Ibn Yūnus (d. 1009), and Bīrūnī (d. ca. 1050).[11]

There is little, if any, reason to doubt the authenticity of what Ḥabash tells us; the same courtesy, however, should not be extended to various other reports of Ma'mūn's scientific expeditions. Bīrūnī already cast doubt on a survey allegedly undertaken between Baghdad and Sāmarrā[12] and Nallino was skeptical of a reported survey near Kūfa.[13] Another expedition, this one between Raqqa

[7] Nallino [1893] (*Racc.* V, pp. 408–457) and [1911], pp. 276–289 (*Racc.* V, pp. 291–302).

[8] Ẓāhiriyya MS 4489 is the only known exemplar and it is incomplete with only 12 extant folios (not 137 ff. as in *GAS* VI, p. 147).

[9] Langermann [1985], pp. 115, 122.

[10] According to Nallino, this is the *nisba* of Marw al-Rūdh, a town in Khurāsān ([1911], p. 282 (n. 1) (*Racc.* V, p. 295 (n. 3)).

[11] Mas^{c}ūdī, *Murūj al-dhahab*, 1: 100–101 (French trans., 1: 76) (Barani [1951], pp. 7–8). The passage from Ibn Yūnus's *Ḥākimī Zīj* (French trans. only in [1804], pp. 95–96 (n. 2)) is in Nallino [1911], pp. 281–284 (*Racc.* V, pp. 295–298) (copied by Barani, pp. 8–10); the value in Ibn Yūnus's text is 56¼ rather than 56 miles, but I am inclined to see this as a mistake of unknown origin. For Bīrūnī, see especially *Taḥdīd*, pp. 213–214 (trans., pp. 178–180) as well as *Tafhīm*, pp. 118–119 and *Qānūn*, 1: 51–52, 2: 528–530 (Barani, pp. 17–22).

[12] *Taḥdīd*, pp. 212–213 (trans., pp. 177–178); cf. Kennedy, *Taḥdīd Comm.*, pp. 132–133.

[13] Nallino [1911], p. 286 (*Racc.* V, pp. 299–300). For the original reports, see Mas^{c}ūdī, *Tanbīh*, pp. 26–27 (French trans., p. 44) and Ibn Khallikān, *Wafayāt*, 5: 163 (trans., 3: 316); cf. Barani, p. 31.

and Palmyra in Syria, has escaped the doubts cast on these others. At first glance the evidence in its favor seems strong. Ibn Yūnus tells us that his information for this expedition comes from a text by Sanad ibn ᶜAlī (fl. 1st half 9th c.), who stated that he was part of one of two groups sent by Ma'mūn to determine a terrestrial degree by measuring it in the level area between Palmyra and Raqqa;[14] their final agreed-upon value was 57 miles/degree.[15] But despite the authenticity bestowed on this report as a result of Ibn Yūnus's claim that he is quoting directly from a work of Sanad, it does contain a number of suspicious features. First, the participants, with the exception of Sanad, are precisely the same individuals named by Ḥabash for the Sinjār expedition. Second, Ibn Yūnus, who clearly knows of the Sinjār expedition from Ḥabash's *Book of Bodies and Distances*,[16] does not list any names at all in connection with it even though Ḥabash, as we have seen, does so. Finally Ibn Yūnus is the only known source for this version of the Palmyra–Raqqa expedition, whereas several sources confirm a similar expedition to Sinjār.[17] On the other hand, Bīrūnī does mention Sanad in connection with finding the size of the Earth but the method described has to do with measuring the dip of the horizon from the summit of a high mountain. This was undertaken when Sanad was "in the company of al-Ma'mūn when he made his campaign against the Byzantines."[18]

Where does this leave us as far as the Palmyra–Raqqa expedition is concerned? Despite the suspicions raised above, there is certainly not enough evidence to discard Ibn Yūnus's account out of hand. Ma'mūn could well have had several expeditions only one of which, that to Sinjār, was widely reported in the later literature. But another possibility presents itself, namely that Ibn Yūnus has somehow conflated Ḥabash's account with Sanad's attempted measurement using the "dip of the horizon method." Yet another possibility is that he has confused the Sinjār expedition with a widely reported procedure for finding the size of the Earth that simply used well-known values for the latitudes of Raqqa and Palmyra and the distance between them. The earliest Arabic account of this known to me is due to Muḥammad ibn Mūsā in his *Ḥarakat al-aflāk* (Motion of the Orbs). The relevant passage[19] and translation is as follows:

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[14] It is not entirely clear that both groups went to the same region, but this is the most natural reading. Sanad states that he went to the area "between Raqqa [Wāma in the text] and Palmyra," whereas the other group went "in another direction" (or perhaps "to another region": *ilā nāḥiya ukhrā*).

[15] See footnote 11 for the references.

[16] Ibn Yūnus states that he got his information for the Sinjār expedition from "the book in which [Ḥabash] mentions the *mumtaḥan* observations in Damascus... ." These are described in the *Book of Bodies and Distances*; cf. Langermann [1985], pp. 120–121 and pp. 125–127. Ibn Yūnus does not quote Ḥabash directly but instead gives a rather loose paraphrase.

[17] Most later accounts, though, seem ultimately to derive from Ḥabash.

[18] *Taḥdīd*, p. 220 (trans., pp. 185–186).

[19] Ẓāhiriyya 4489, f. 12a–b; because this text is unique, I have attempted to reproduce the manuscript as closely as possible within the limits imposed by my printer.

Passage from *Ḥarakat al-aflāk* (Motion of the Orbs)

Muḥammad ibn Mūsā

f.12a Furthermore if we find two cities beneath a meridian line, which is the orb [*falak*] that cuts the ecliptic circle from south to north, their longitude being the same but they being different in latitude, one being less than the other and its northern pole being less in altitude than the other by a single degree or thereabouts, then when we measure the distance between the two cities we will find how many miles long is each degree of the circle [*istidāra*] of the sky on the Earth ∴. Now if there is between the two cities a single degree in the altitude or depression of the pole, then when you measure the distance of that you will find it to be $66\frac{2}{3}$ miles. This has been measured for two known cities, the city of Palmyra and the city of Raqqa. That is, they found the inclination of Palmyra [lacuna?] from the zenith that is the altitude of its northern pole $35\frac{1}{3}°$, and [from] Palmyra it is in the direction of the northern side. They are both beneath a meridian line. They then found what is between Raqqa and Palmyra to be 90 miles just as was found by him [or those] who had measured it before us. From
f.12b that it was learned | that each degree of the great circle [*istidāra*] of the celestial sphere [*falak*] has a length of $66\frac{2}{3}$ miles on the Earth. Furthermore: for finding the Earth's diameter we demarcate the meridian in a town whose latitude is known. Then one travels in its direction, any deviation kept to a minimum and any curving from it being guarded against. Then when we arrive at another town whose latitude is different from the latitude of the first town from which we began, we take the difference of the two latitudes and retain it. Then we take the number of parasangs and cubits that we have traversed along the Earth; then its amount to the degrees that were retained is as the entire circumference [*dawr*] of the Earth to 360 [lacuna or copyist error?] to the degrees that were retained from it; this then is the circumference of the Earth. From this we then know its diameter.

Furthermore: the construction of a figure for the circumference of the Earth. If we wish to do this we should ascend a high, towering mountain but in front of which is level land. Then we take an altitude plate [*ṣafīḥat al-irtifā*^{c}, *i.e.* of an astrolabe] and then its alidade or its line [*khaṭṭ*] is raised until our horizon is sighted, which is the place at which the sky touches the Earth at a point on [end of extant text]

f. 12a وايضا انا [!] وجدنا مدينتين تحت خط نصف النهار وهو الفلك الذي يقطع دايره البروج من الجنوب الي الشمال يكون طولهما واحد ويكونان في العرض مختلفين الواحده اقل من الاخري وقطبهما [!] الشمالي اقل ارتفاعا من الاخري بدرجة واحده او نحو ذلك فلما مسحنا بعد ما بين المدينتين علمنا كم ميلا لكل درجة من الاستداره السماوية في الارض من الطول ∴ فان كان بين المدينتين درجه واحده في ارتفاع القطب او انخفاضه فانك اذا مسحت مسافة ذلك ستجده ~~٣٦~~ ٦٦ ميلا وثلثي ميل وقد مسح ذلك في مدينتين معروفتين مدينه تدمر ومدينه الرقه وذلك انهم [؟] وجد ميل تدمر عن سمت رؤسا [!] الذي هو ارتفاع قطبها الشمال [!] ٣٤ جزوا وثلث وهي في سمت تدمر الي ناحية الشمال وهما جميعا تحت خط نصف النهار فوجدوا ما بين الرقه وتدمر f. 12b ٩٠ ميلا كما وجده من مسحه قبلنا فمن ذلك علم | ان كل درجه من استداره الفلك الاعظم يكون طولها من الارض ٦٦ ميلا وثلثى ميل وايضا في معرفة قطر الارض نستخرج حد نصف النهار في بلد معروف العرض ثم يسير على سمته وينخفض من الانحراف وينحرس من الاعواجاج [!] عنه فاذا صرنا الي بلد اخر خالف عرضه عرض البلد الاول الذي ابتدانا منه اخذنا فضل ما بين العرضين من الآخر فحفظناها ثم اخذنا عدد ما قطعنا من الفرا <سخ> والاذرع من الارض فيكون قدره من ~~الارض~~ جزا المحفوظه كقدر جميع دور الارض من ٣٦٠ الي الاجزا المحفوظه منها فتكون ذلك دور الارض ثم نعرف منه قطرها

وايضا عمل شكل في دور الارض اذا اردنا ذلك فانا نعلوا جبلا عاليا مرتفعا ولكن بين يديه استوا من الارض ثم ناخذ صفيحة الارتفاع فيرفع عضادتها او خطها حتي يري افقنا وهو الموضع الذي كان السماء يماس فيه الارض علي نقطة من

There are at least two textual problems: a lacuna in line 10 and an apparent repetition in line 21. The second is not of any significance; more serious is the first in which the latitude of Palmyra (or Raqqa) is missing. Fortunately Bīrūnī has preserved a report from a certain Muḥammad ibn ᶜAlī al-Makkī (fl. 850), who has virtually the same method as Muḥammad ibn Mūsā. The latitude of Raqqa is given there as 35;20°, while Palmyra is 34°, the figure missing from our text. We then have

$$90 \text{ miles} \div 1;20° = 67\tfrac{1}{2} \text{ miles/degree}$$

not the $66\frac{2}{3}$ miles/degree in the text;[20] nevertheless this was probably close enough to confirm the Ptolemaic standard. We can now make sense of the passage in *Firdaws al-ḥikma* in which it is clear, despite a somewhat garbled account, that the latitudes for Palmyra and Raqqa are as above and that the purpose of the "expedition" was to confirm the $66\frac{2}{3}$-mile Ptolemaic figure; as the author ᶜAlī ibn Rabban al-Ṭabarī (ca. 800–ca. 864) states: "they sought to attain that knowledge" (*istadrakū ᶜilm dhālik*) by undertaking the survey.[21]

It is important to note here that Muḥammad ibn Mūsā, Ṭabarī, and Makkī are younger contemporaries of Ma'mūn and as such we should take these accounts very seriously. By dismissing them (at least the latter two), Barani and Kennedy have interpreted these early reports of a Raqqa–Palmyra "expedition" in the light of the account by Ibn Yūnus, who lived some century and a half after the fact, and have discounted contemporaneous evidence. At one point in his analysis, Kennedy (*Taḥdīd Comm.*, p. 131) states that Makkī and Ṭabarī got only the Raqqa–Palmyra part of Ibn Yūnus's story right and that "someone then cooked up a derivation of the Ptolemaic 66;40." It is now clear that the chef was none other than Muḥammad ibn Mūsā, who, we should recall, was a protégé of Ma'mūn himself.

There are several points concerning this "measurement" by Muḥammad that we should emphasize. First it is clear, as Kennedy notes, that the reported latitudes for Raqqa and Palmyra are simply lifted from Ptolemy's geography without modification. This argues both for an early date for Muḥammad's "measurement" (since beginning with Muḥammad ibn Mūsā al-Khwārizmī (fl. 813–33) the Islamic values for the latitudes of Raqqa and Palmyra are generally different from Ptolemy's[22]) and against an independent determination (for obvious reasons). The most that can be said in terms of independent observation is that Muḥammad would have us believe that the received value of 90 miles for the Raqqa–Palmyra distance has been latterly confirmed.

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[20] Cf. Kennedy, *Taḥdīd Comm.*, p. 131.
[21] Ṭabarī, *Firdaws*, p. 547; repeated by Masᶜūdī, *Murūj al-dhahab*, 1: 104 (French trans., p. 79); cf. Barani, pp. 6–7 for the text and a translation but ignore the commentary.
[22] Kennedy and Kennedy [1987], pp. 252, 281–282.

At the end of this passage Muḥammad begins to describe the method involving the measurement from a high mountain of the dip of the horizon. As we have seen, Bīrūnī associates this with Sanad and what we have here may well be related to that determination. But Bīrūnī gives no actual values arrived at by Sanad and it does not seem likely that what Muḥammad is here beginning to describe will result in numbers at odds with the Ptolemaic values. Taking all the evidence together, I am inclined to see the Palmyra–Raqqa "expedition" as little more than a youthful thought experiment (half-baked at that) by Muḥammad ibn Mūsā (who probably would have been rather too young to head scientific missions during the reign of Ma'mūn[23]). Sanad's contribution then becomes an early—and extemporaneous—attempt at the urging of Ma'mūn to measure a meridian degree, but this does not seem to have led to a result other than a confirmation of Ptolemy.[24] It may well be that at this point Ma'mūn, who as we know from Ḥabash was not averse to riding roughshod over his scientists when he wanted a job done,[25] felt that something was amiss if Ptolemy's suspicious values were being so glibly confirmed. He therefore ordered that the Sinjār expedition be undertaken as reported by Ḥabash.

The story that I have just outlined gains some support from Ibn Khallikān's biographical entry for Muḥammad ibn Mūsā.[26] Although the details are hopelessly askew, we can at least glimpse part of the process that led to the Sinjār expedition. Ma'mūn is said to have asked the Banū Mūsā about the 24,000 mile circumference, and he was informed "that the fact was certain." Curiously enough, though, Ma'mūn was not completely satisfied and insisted upon verification. Ibn Khallikān then: (1) has the Banū Mūsā leading the expedition to Sinjār; (2) confirming the 66⅔ miles for a meridian degree; and then (3) reconfirming the result in Kūfa. The first two statements are unquestionably incorrect, and the third, to say the least, is also suspect. But if we take the skeleton of the report and ignore all the uncomfortable fleshy details, we are left with an early *assertion* by the Banū Mūsā of the correctness of the Ptolemaic value of 66⅔ miles/meridian degree (confirmed in the passage above), a healthy skepticism by Ma'mūn leading to a first attempt to verify what the Banū Mūsā had stated (perhaps by Sanad using the mountain/level plain method?), and a second attempt (presumably to Sinjār, not Kūfa) that settled the matter.

Whatever the early history of the problem, it is reasonably clear that by the 10th century 56 miles (or 56⅔ miles) per meridian degree was generally

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[23] Cf. Nallino [1911], p. 286 (*Racc.* V, p. 299).

[24] But, as I have said above, one cannot completely dismiss Ibn Yūnus's account of Sanad being connected with a serious expedition to the Palmyra–Raqqa region that involved the same principals and arrived at approximately the same value as the Sinjār episode. It is curious, though, that no one besides Ibn Yūnus seems to have any inkling of it.

[25] Langermann [1985], pp. 120–121, 125–126.

[26] Ibn Khallikān, *Wafayāt*, 5: 161–163 (Engl. trans., 3: 315–317); cf. Barani pp. 23–27 and Nallino [1911], pp. 284–286 (*Racc.* V, pp. 298–300).

accepted as the Ma'mūnī value.[27] Bīrūnī seems to have played a pivotal role in making 56⅔ *the* Ma'mūn value inasmuch as his earlier wavering between 56 and 56⅔ in the *Taḥdīd*, p. 214 (trans., pp. 179–180) is superseded by his assertion of 56⅔ in his later *Tafhīm*, p. 119 and *Qānūn*, 1: 52, 2: 529.[28] By the time we reach Khiraqī (d. 1138–39),[29] 56⅔ miles/degree is accepted without question as the Ma'mūn value; he further gives the following standard equivalences:[30]

1 mile = 4000 cubits
1 cubit = 24 digits
1 digit = 6 barleycorns

Khiraqī then gives the Ptolemaic value of 66⅔ miles but adds a new twist. The equivalences are now

1 mile = 3000 cubits
1 cubit = 36 digits
1 digit = 6 barleycorns

We are at long last able to return to Ṭūsī and the passage from the *Tadhkira* that has led to our own excursion. Ṭūsī in the *Muᶜīniyya* repeats the information from the *Muntahā* with only one minor change; instead of 1 cubit = 36 digits for the Ptolemaic standard, he has 1 cubit = 32 digits.[31] (I do not know whether this is a correction or a misreading on Ṭūsī's part.[32]) But here in the *Tadhkira* Ṭūsī

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[27] Thābit ibn Qurra (d. 901), for example, accepts 56 miles/degree; we know this from Ṣaghānī's *Maqāla fī al-abᶜād wa-'l-ajrām* (Ẓāhiriyya MS 4871, f. 78b for which see Ragep and Kennedy [1981], pp. 97–98). It is confirmed by Bīrūnī (who bases himself on Ṣaghānī) in the *Taḥdīd*, p. 214 (trans., p. 179). Farghānī (fl. 833–61), however, uses 56⅔ (ibid.).

[28] This is rather puzzling in that his own measurement as reported in the *Taḥdīd*, p. 223 (trans., p. 189) and the *Qānūn*, 2: 531 is much closer to 56; cf. Kennedy, *Taḥdīd Comm.*, p. 143. The way in which Bīrūnī deals with these conflicting values deserves a much more thorough analysis than is possible here.

[29] *Muntahā*, II.17(1), ff. 35b–37a.

[30] Cf. Hinz [1955], pp. 54–55, 63.

[31] *Muᶜīniyya*, pp. 82–83; Kennedy [1984], pp. 114–115.

[32] 1 cubit = 24 digits and 1 cubit = 30 digits are attested in Babylonian sources (*HAMA*, 2: 591). The former was standard throughout the Islamic Middle Ages (Hinz [1955], p. 54). One does, though, find a *Hāshimī* cubit that is equal to 32 digits and it may date from the time of the Caliph al-Manṣūr (reigned 754–75) (ibid., p. 58). If so, it could be the cubit that was originally used to convert Ptolemy's stades into known units and is thus retained as the "Ptolemaic" cubit. At any rate it is clear that Ṭūsī, unlike Khiraqī, intends the Ptolemaic and the Ma'mūnī miles to be equivalent since each consists of 96,000 digits. Ṭūsī's version is confirmed by Shīrāzī (*Nihāya*, IV.2, f. 149b and *Tuḥfa*, IV.1(2), f. 272a), ᶜUbaydī, Khafrī, and Bīrjandī.

gives a completely different account, maintaining that the 66⅔-mile figure is what was found by the Ma'mūn expedition to Sinjār! This is obviously no slip of the pen[33] since all the other numbers are based on this value and the unit equivalences given are the "modern" ones. How are we to account for this confusion, especially in view of the contradictory information from the *Mu^{c}īniyya*? One explanation could be that Ṭūsī has simply made a silly mistake. Though I cannot discount this possibility, and I hardly wish to advocate the position that Naṣīr al-Dīn is incapable of error, I believe there is a far more interesting—and plausible—explanation that presents itself. As we have seen above, Ibn Khallikān claims that the Banū Mūsā were responsible for the Sinjār expedition and confirmed the Ptolemaic value of 66⅔ miles. As we have also seen, there is a morsel of truth here since Muḥammad ibn Mūsā did indeed "confirm" Ptolemy though he scarcely needed to leave his divan to do so. Where did Ibn Khallikān get his information? Undoubtedly some corrupt version of the passage from Muḥammad ibn Mūsā came into his possession. A likely source is Kamāl al-Dīn ibn Yūnus, the great legal scholar of Mosul whom he visited several times.[34] Ibn Yūnus was also noted for his knowledge of astronomy though his historical reconstructions are certainly suspect if Ibn Khallikān's anecdotes are any indication.[35] Ṭūsī was also a student of Kamāl al-Dīn,[36] and the latter provides a possible link between Naṣīr al-Dīn and Ibn Khallikān and thus may be the common source for both this passage in the *Tadhkira* and the information in Ibn Khallikān's entry for Muḥammad ibn Mūsā. At any rate sometime between the writing of the *Mu^{c}īniyya* in 1235 and the *Tadhkira* in 1261, Ṭūsī probably came to view the version of Ma'mūn's expedition in Khiraqī's *Muntahā* (or some similar source) as suspect and relied instead upon the bogus story we find in Ibn Khallikān (though perhaps it did not come directly from him).

The subsequent history of reconciling these contradictory stories is not without interest, but we shall content ourselves with a brief overview. Nīsābūrī and Jurjānī repeat the version in the *Tadhkira* without comment; whether they were getting tired by the end of the book or whether they believed Ṭūsī's *Tadhkira* version is unclear. What makes the latter seem unlikely is that Shīrāzī had already dealt extensively with the problem in both the *Nihāya* (IV.2) and the *Tuḥfa* [IV.1(2)] and had revived the earlier history as found in the *Mu^{c}īniyya*.[37]

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[33] That it was is the perfectly reasonable, but I believe incorrect, conjecture of Abū al-Faḍl ^{c}Allāmī in his *Ā'īn-i-Akbarī* (quoted by Barani, pp. 28–29).

[34] "Ibn Khallikān," *EI*², 3: 832.

[35] King [1981], p. 55 proposes but quickly (perhaps too quickly?) dismisses Ibn Yūnus as the source of the story found in Ibn Khallikān that Ptolemy's horse was the "inventor" of the astrolabe since he squashed a celestial sphere dropped by his master.

[36] See the biography of Ṭūsī in the introduction, pp. 7–8.

[37] The passage in the *Tuḥfa* is much more extensive and clearly derives from Ḥabash, perhaps via Bīrūnī.

But Shīrāzī adds another element to our story: he asserts in the *Nihāya* that the investigation (*baḥth*) of the ancients is more reliable (*awfā*) than that of the "moderns." (These words are echoed faithfully by ᶜUbaydī, but he uses them to make the extraordinary claim that Ṭūsī's faith in the ancients led him to assign their value to the moderns!) In the *Tuḥfa*, however, it is Ma'mūn's figure that is "correct and verified" (*ṣaḥīḥ mumtaḥan*); he seems to be influenced there by the account of the two groups in Sinjār finding approximately the same amount for a meridian degree. But despite this Shīrāzī is still not willing to abandon the ancient value, in part because it is the basis for the Ptolemaic standard measuring stick for sizes and distances. Both Khafrī and Bīrjandī quote this passage from the *Tuḥfa* but without further comment.

The length of the meridian degree is but one of several parameters (others being the obliquity of the ecliptic, the precessional rate, and the motion of the solar apogee) that could be studied as a group in order to shed light on the question of tradition and innovation in Islamic science. If nothing else, I hope the above discussion indicates both the complexity and the worth of such a venture.

IV.1 [2]6–7. *ḥaṣala miqdār quṭrihā...bi-'l-taqrīb* (one obtains for the size of the diameter...approximately): $8000 \div 3\frac{1}{7} = 2545.45$. In the Baghdad version (MSS FL), *wa-niṣf farsakh* (and half a parasang) has been added to account for the fractional part.

IV.1 [3]10–11. *wa-idhā ḍuriba al-quṭr...farsakh* (If the diameter is multiplied...[20,360,000] parasangs): The number is obtained by using Lemma 3 in IV.1 [1] as follows:

$$S = d \cdot C = 2545 \cdot 8000 = 20{,}360{,}000 \text{ sq. parasangs}$$

In the *Muᶜīniyya*, Ṭūsī gives 183,264,000 sq. miles (= $20{,}362{,}666\frac{2}{3}$ sq. parasangs).[38] The discrepancy is due to his use there of the slightly more accurate 7636 miles (= $2545\frac{1}{3}$ parasangs) for the diameter.

IV.1 [3]12, 13. *al-rubᶜ al-maskūn; al-maᶜmūr* (the populated quarter; [actually] inhabited): Ṭūsī seems to use *maskūn* to indicate the quarter of the Earth's surface where population occurs; *al-maᶜmūr* (*lit.*, the settled area) is used to designate the actual inhabited region. Compare III.1 [2–3] and [5–8].

IV.1 [3]13–20. *wa-ammā al-qadr al-maᶜmūr...wa-suds ᶜushrih* (As for the amount that is [actually] inhabited...entire surface of the Earth): Ṭūsī makes an

[38] *Muᶜīniyya*, p. 83; cf. Kennedy [1984], p. 114.

error here that was pointed out by Shīrāzī.[39] He calculates the surface area of the inhabited region by considering it a lune. Thus from IV.1 [1], Lemma 4:

$$L = d \cdot [(66;25° / 360°) \cdot C] = 2545 \cdot 1476 = 3{,}756{,}420 \text{ sq. parasangs}$$

(In the *Mu^cīniyya*, Ṭūsī has $7636 \cdot 4428 = 33{,}812{,}208$ sq. miles = 3,756,912 sq. parasangs; as in IV.1 [3]10–11, the discrepancy is due to a different value for the diameter.[40]) Shīrāzī notes that the actual area in question is not a lune, however, but half the zone[41] bounded by the equator and the day-circle with latitude 66;25°. Although he does not carry out the computation, he advises finding half the surface area of the segment whose vertex is the celestial pole and whose base is the plane of the day-circle with latitude 66;25° and then subtracting this from the area of the populated quarter.[42] Bīrjandī does the calculation and arrives at 4,665,712$\frac{1}{30}$ sq. parasangs (which is accurate to the nearest $\frac{1}{30}$ of a square parasang assuming $\pi = \frac{22}{7}$, C = 8000 parasangs and $\varepsilon = 23;35°$).

IV.1 [3]18. *wa-sitta wa-khamsūn* (fifty-six): In the earlier version (MSS DGMT), one finds 3,765,420; this has been corrected to 3,756,420 in MS L. In MS F one has 3,756,231½, which seems to be the result of using a diameter of 2545 parasangs and a rather more accurate maximum width for the lune of about 1475.926 parasangs. Shīrāzī has 3,765,420 in the *Nihāya* but may have corrected this to 3,756,420 in the *Tuḥfa*.[43] Jurjānī and Khafrī have the incorrect figure, while Nīsābūrī gives the correct value. Bīrjandī quotes the incorrect value that he found in his copy of the *Tadhkira* but corrects it.

IV.1 [4]. Abū al-Rayḥān, of course, is Bīrūnī, and his method is presented in the *Taḥdīd*, pp. 218–223 (trans., pp. 183–189) and the *Qānūn*, 2: 530–531.[44] There exist several expositions: for example, Kennedy, *Taḥdīd Comm.*, pp. 137–143; Nallino [1911], pp. 289–292 (*Racc.* V, pp. 302–305); Barani, pp. 31–44; and

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39 *Nihāya*, IV.2; *Tuḥfa*, IV.1(2). The error was also noted by Kennedy [1984], pp. 114–115, who calls Ṭūsī's rule "grossly inaccurate."

40 *Mu^cīniyya*, p. 83; Kennedy [1984], pp. 114–115.

41 He does not have a specific word for zone but uses the all-purpose *qiṭ^ca* (segment).

42 For the surface area of a segment, Shīrāzī depends on Archimedes, *Sphere and Cylinder* I, Prop. 42 (= Prop. 44 in Ṭūsī's *Taḥrīr*, p. 74). Bīrjandī uses this rule as well as the more familiar area of a zone = $2\pi rh$ where r is the radius of the sphere and h the altitude of the zone. Neither Shīrāzī nor Bīrjandī calculates the actual area directly, which is that of a zone of two bases; they seem to restrict themselves to zones of one base.

43 This is the case for Mosul, Jāmi^c al-Bāshā MS 287, f. 273a; BM MS Add. 7477, however, has the incorrect value. Bīrjandī in quoting the *Tuḥfa* gives the incorrect value as well.

44 Bīrūnī also mentioned the method in his *Risāla* (or *Kitāb*) *fī al-asṭurlāb* [Epistle (or Book) on the Astrolabe], but he had not yet done the necessary observation; see Nallino [1911], pp. 289–291 (*Racc.* V, pp. 302–304) and Barani, pp. 32–33.

Wiedemann [1909]. As we have seen (IV.1 [2]), Bīrūnī did not originate this method since he mentions in the *Taḥdīd* that Sanad ibn ᶜAlī had used this technique and furthermore we see the beginning of a description of the method in the passage by Muḥammad ibn Mūsā quoted on pp. 504–505. In Bīrūnī's *Book on the Astrolabe* there is a puzzling reference to al-Nayrīzī's inspiration in connection with the method, but there is no evidence to connect him with a determination of the size of the Earth along similar lines.

Ṭūsī again reiterates his stand against giving geometrical proofs in the *Tadhkira* (cf. I.Intr. [3]) though there have certainly been lapses (most obviously in II.11).

IV.1 [5]. The "promise" was made in II.1 [2]. We have the following proportion:

$$\tfrac{1}{2} \text{ parasang} : d = x \text{ barleycorns} : 1 \text{ cubit}$$

Since d = 2545 parasangs and 1 cubit = 144 barleycorns (according to the "modern" system), then $x \approx 1 / 35.3$ barleycorns, which, in unit fractions, is approximately equal to ($\tfrac{1}{5}$ of $\tfrac{1}{7}$) barleycorns. In passing we may note that half a parasang is something around 3 kilometers,[45] an impressive but hardly formidable size for a mountain.

Book IV, Chapter Two

IV.2 [1]. For calculating the positions of the planets, Ptolemy only needed to use relative distances; for each model he set the deferent *circle* at 60 parts.[1] In the *Almagest* he calculated absolute distances only for the sun and moon; in the *Planetary Hypotheses* he computed them for the other planets. Absolute distances were given using the Earth's radius as the measuring stick.

IV.2 [2–4]. This is a nontechnical summary of *Alm.*, V.13 (H408–416); for discussions, see *HAMA*, 1: 101–103 and Pedersen [1974], pp. 204–207.

IV.2 [2]. Note that Ptolemy gives zenith distances ($50\tfrac{11}{12}°$ and 49;48°), whereas Ṭūsī converts these into altitudes ($39\tfrac{1}{12}°$ and $40\tfrac{1}{5}°$).

IV.2 [3]1–3. *wa-qad tabayyana fī ᶜilm al-handasa...wa-zawāyāh maᶜlūma* (It may be shown in the science of geometry...and angles are known): Euclid's

[45] Hinz [1955], p. 62.

[1] For the moon only, the eccentric deferent radius is taken to be 49;41 parts and it is the "inclined" radius that is set at 60; see II.7 [4] and [5], Pedersen [1974], pp. 184–186, and *HAMA*, 1: 88.

Elements I.26 shows that two angles and a side uniquely determine a triangle, but it does not, of course, provide a method for calculating the sizes of the angles and sides. For this one needs the law of sines (or some equivalent), which was widely known and used in the Islamic Middle Ages.

IV.2 [3] Fig. T26. Compare *Alm.*, Fig. 5.10, p. 248.

IV.2 [3]10–11. *tis^ca wa-thalāthīn juz'^{an} wa-nisf wa-rub^c juz'* ((39 + ½ + ¼) parts): Bīrjandī corrects this to (39 + ½ + ⅓) parts, *i.e.* 39;50 parts; cf. Toomer, *Alm.*, p. 249 (n. 47), who has the same correction.

IV.2 [4]. For the Earth's radius $r_e = 1$ part, Ptolemy found the moon's distance for the above observation to be 39;45 parts; for the inclined radius R = 60 parts, he arrived at a distance of 40;25 parts for the same observation. One thus has a means of converting from one system to the other since one now has the ratio of r_e : R / 60, which is equal to $39;45^p / 40;25^p \approx 0.984$. We may therefore set up the following table of equivalences:

Table 11. Conversion of Lunar Parameters.

	R = 60	$r_e = 1$
Inclined Radius	60^p	59^p
Epicyclic Radius	$5\frac{1}{4}^p$	$5\frac{1}{6}^p$
Eccentricity	$10;19^p$	$10;9^p$
Farthest Distance	$65;15^p$	$64\frac{1}{6}^p$
Nearest Distance*	$34;7^p$	$33;33^p$

* MS T corrects $33;33^p$ to $33;32^p$ [$= 59^p - 2(10;9^p) - 5\frac{1}{6}^p$], but this latter is based on rounded numbers; $33;33^p$ is correct and is found in the *Almagest* (*e.g.* V.17).

Book IV, Chapter Three

For this chapter, see *Alm.*, V.14–15, pp. 251–257 (H416–425); *HAMA*, 1: 103–112; and Pedersen [1974], pp. 207–213.

IV.3 [1]16–17. *fī dhirwat al-tadwīr* (at the epicyclic apex): It would be a bit much to ask for eclipses to occur when the moon was precisely at the apex; in fact the two observations in question occurred within 30° of the apex.

IV.3 [1]23–24. *wa-huwa bi-'l-taqrīb...quṭrih* (This [radius] was approximately...⅗ times its radius): The latitude when half the moon was eclipsed was 40⅔ minutes, which Ptolemy then took to be the radius of the shadow. This radius to the radius of the moon is then

$$40\tfrac{2}{3} \text{ minutes} / 15\tfrac{2}{3} \text{ minutes} \approx 2\tfrac{3}{5}$$

IV.3 [2]. On the diameters of the sun and moon, compare commentary to II.13 [8]2–4. Ṭūsī here follows the *Almagest* in making the moon's apparent diameter at its farthest distance equal to the sun's (allegedly constant) apparent diameter; he thus ignores both his previously stated values as well as *Planetary Hypotheses*, BM 91a (trans., p. 8) in which Ptolemy gives the moon's diameter at *mean* distance (on both its epicycle and eccentric) as 1⅓ the sun's.

IV.3 [3] Fig. T27. See *Alm.*, Fig. 5.12, p. 256; cf. II.13 [7], Fig. T18. The remark at the bottom of the figure in which Ṭūsī notes that indicated diameters are not the actual diameters is a paraphrase of the last sentence of *Alm.*, V.14, p. 254; as Toomer explains in n. 64, pp. 254–255, the half-bases MN, HT, and GD of the shadow cones are not the actual radii of the Earth, moon, and sun, respectively, but the "simplifying approximation is fully justified."

IV.3 [3]. Ṭūsī is giving a summary of *Alm.*, V.15, p. 255 (H422–424), and he repeats the Ptolemaic values. However he recasts the calculation using the law of sines thus avoiding Ptolemy's dependence on chords. The proportion then becomes

$$HT / NT = \sin\angle HNT / \sin\angle NHT = \sin 0;15,40^\circ / \sin 89;44,20^\circ$$
$$\approx 0;16\tfrac{2}{5} \text{ parts} / 60 \text{ parts}$$

(Ṭūsī's statement that the actual figure is something less than 60 is correct; it is approximately equal to 59.98.)

Now then to obtain the moon's radius using e.r. = 1, one has

$$(0;16\tfrac{2}{5} \text{ parts} / 60 \text{ parts}) \cdot 64\tfrac{1}{6} \text{ e.r.} \approx 0;17,33 \text{ e.r.}$$[1]

The shadow's radius (0;45,38 e.r.) is obtained by multiplying this by 2⅗ (see IV.3 [1]).

IV.3 [4]. Ṭūsī here gives a verbal account of Ptolemy's mathematical derivation in *Alm.*, V.15, pp. 255–256 (H424–425) of the distance of the sun. In order to simplify the discussion, we shall translate the text into symbols using Fig. C38, which is extracted from Fig. T27; my clarifying additions are in brackets.

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[1] 0;17,32 would be more accurate.

[$FQ = NR = TX = r_{sh}$, $NM = r_e$, $TH = r_m$, TY = radius of shadow at moon's position, HY = excess of radius of cone at moon's position over radius of moon.]

Since TF = 2NF, [then XQ = 2RQ and thus XY = 2RM; it follows that]

$$TY - FQ = 2(NM - FQ)$$

$$FQ + TY = 2NM = \text{diameter of Earth}$$

[Since TY = TH + HY, therefore $FQ + TH + HY = 2r_e$ and]

$$HY = 2r_e - (FQ + TH) = 56'49''$$

[Now referring to Fig. T27, if we make the reasonable simplifications that $\angle NMG = 90°$, NG = ND = Earth–sun distance, GH = DT = moon–sun distance, and NH = NT = Earth–moon distance, then using triangle MNG it follows that]

$$1 / HY = ND / DT = 1 / 56'49''$$

If ND = 1, then DT = 56′49″ and NT = 3′11″.

Setting NT = $64\frac{1}{6}$ parts, we have ND = 1209.4 ≈ 1210 parts.

IV.3 [5]. Continuing, we have

$$NM / FQ = NS / SF = 1 / 45'38''$$

If NS = 1, then SF = 45′38″ and FN = 14′22″ = $(14 + \frac{1}{5} + \frac{1}{6})'$

Setting FN = $64\frac{1}{6}$ parts, we have

$$SF = 203.81 \approx (203 + \tfrac{1}{2} + \tfrac{1}{3}) \text{ parts}$$

Ṭūsī has made a mistake in this passage. In lines 18–19, instead of "the distance of the cone's apex from the center of the shadow" (SF), one has "the distance of the cone's apex from the center of the Earth" (SN) in MSS DFLT. (MS G is missing this section.) He seems to have intended to give the more interesting distance of the apex from the center of the Earth (SN = 268 parts) but mistakenly gave the distance to the shadow center (SF = 203 + ½ + ⅓). In the *Almagest*, the two numbers are juxtaposed and the magnitudes are designated by XP and XN; it is easy to see how one could make a careless error. In MS M, "Earth" (*arḍ*) has been replaced by "shadow" (*ẓill*) and in the margin one reads: "for the center of the Earth it is 268." But this manuscript is based on one in

Shīrāzī's hand, and my inclination is to see this as his correction rather than Ṭūsī's; at any rate the correction did not make it into the Baghdad version (MSS FL) but Shīrāzī gets it right as early as the *Nihāya* (IV.4). The commentators have quietly corrected the manuscripts. For confirmation that this error originated with Ṭūsī, see IV.5 [6].

Book IV, Chapter Four

For this chapter, see *Alm.*, V.16, p. 257 (H426–427).

IV.4 [1]3–5. *thabata fī ᶜilm al-manāẓir...ilā buᶜd al-abᶜad* (It has been established in the science of optics...to the distance of the farther): Bīrjandī provides the simple proof for this theorem after correctly noting that it does not actually occur in Euclid's *Optics*. Ptolemy states it and outlines a proof in *Plan. Hyp.*, BM 91a (trans., p. 8).

IV.4 [1]. We have

$$0;17,33^p / r_{sun} = 64\tfrac{1}{6}^p / 1210^p$$

$$r_{sun} \approx 5\tfrac{1}{2}^p, \text{ where } r_{Earth} = 1^p$$

$$\text{For } d_{moon} = 1^p,\ d_{Earth} \approx 3\tfrac{2}{5}^p \text{ and } d_{sun} \approx 18\tfrac{4}{5}^p$$

All these values are from the *Almagest*.

IV.4 [2]. For the proposition, see *Elements*, XII.18. Technically speaking, though, Euclid has $S_1 / S_2 = (d_1 / d_2)^3$, whereas Ṭūsī gives $S_1 / S_2 = d_1^3 / d_2^3$. Bīrjandī is not one to let this pass and upbraids the other commentators for doing so. To reach Ṭūsī's proposition, he proposes using in addition *Elements* XI.33[1] and V.11, which allow one to equate $(d_1 / d_2)^3$ and d_1^3 / d_2^3. [2]

Turning to the volumes, one has

$V_{sun} / V_{Earth} = (11)^3 / (2)^3 = 166.375 = 166 + \tfrac{1}{4} + \tfrac{1}{8}$ [*Alm.*: 170]

$V_{sun} / V_{moon} = (18.8)^3 / (1)^3 = 6644.672 \approx 6644\{\tfrac{2}{3}\}$ [*Alm.*: 6644½]

$V_{Earth} / V_{moon} = (3.4)^3 / (1)^3 = 39.304 \approx 39;18 = 39+\tfrac{1}{4}+\{\tfrac{1}{2} \text{ of } \tfrac{1}{10}\}$ [*Alm.*: 39¼]

[1] The manuscript has XI.36.

[2] He also gives VIII.12 to use as an alternative.

In the *Planetary Hypotheses*, Ptolemy gives the volume of the sun as 166⅓ and that of the moon as $\frac{1}{40}$, where the Earth's volume is 1. [3]

Some variants should be noted. For (166 + ¼ + ⅛), MS F gives (169 + ½ + $\frac{1}{7}$); a similar number (169 + (½ of $\frac{1}{7}$)) is given as a variant by MS T. I do not have a plausible explanation for either number. Only MSS DF add ⅔ to 6644 and both do so in the margin. MSS L and F (the latter in the margin) add (½ of $\frac{1}{10}$) to 39¼.

Book IV, Chapter Five

This chapter, as well as the following two, does not correspond to anything in the *Almagest*, but much of the information may be found in the *Planetary Hypotheses*. Note, though, that Ṭūsī himself does not refer to the *Planetary Hypotheses* but instead uses some circumlocution such as "they have stated" or "it seemed likely to them"; see, for example, IV.5 [5] and [7], IV.6 [2], [5] and [7], and IV.7 [2].

IV.5 [1]. As is his usual practice, Ṭūsī simply reports Ptolemy and does not attempt to modify parameters based upon the observations of the "moderns." Thus Ptolemy's solar eccentricity of 2½ parts is used here rather than the 2;05 that was quoted in II.6 [4]. Since 2½ / 60 = $\frac{1}{24}$, then where e.r. = 1,

$$\text{solar eccentricity} = \tfrac{1}{24}(1210 \text{ e.r.}) \approx 50 \text{ e.r.}$$

Thus the farthest distance is 1260 e.r. and the nearest distance is 1160 e.r. as one finds in *Planetary Hypotheses*, BM 89b (trans., p. 7).

IV.5 [2]. This is the basic "nesting" principle for the orbs and as such is a cornerstone of *hay'a*. Compare *Planetary Hypotheses*, BM 90b (trans., p. 8) where Ptolemy makes the interesting remark that even "if there is space or emptiness between the [spheres], then it is clear that the distances cannot be smaller, at any rate, than those mentioned." Ṭūsī echoes this statement here but he does not allow for the possibility of empty space. Ptolemy is compelled to do so since he finds the farthest distance of Venus to be 1079 e.r., whereas the nearest distance of the sun is 1160;[1] he does make the suggestion that the gap could be filled in by slightly increasing the distance of the moon.[2] As we shall see, Ṭūsī is able to sidestep this problem, and thus avoid the unthinkable void, by correcting an error made by Ptolemy in the ratio of Venus's nearest to farthest distances.[3]

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[3] *Planetary Hypotheses*, BM 91b (trans., p. 9).

[1] *Planetary Hypotheses*, BM 89b (trans., p. 7).

[2] For how this would work, see Goldstein's notes to *Plan. Hyp.*, pp. 10–11; cf. Van Helden [1985], pp. 22–23.

[3] See commentary to IV.5 [4–5].

IV.5 [3]. All the values are Ptolemaic.[4] For the farthest distance one has

$$60 + 43\tfrac{1}{6} + 1\tfrac{1}{4} = 104 + \tfrac{1}{4} + \tfrac{1}{6}$$

for the nearest distance

$$60 - (43\tfrac{1}{6} + 1\tfrac{1}{4}) = 15 + \tfrac{1}{3} + \tfrac{1}{4}$$

Then

$$15\tfrac{7}{12} : 104\tfrac{5}{12} \approx 0.15 = \tfrac{1}{10} + \tfrac{1}{20}$$

In the *Planetary Hypotheses* (BM 89b [trans., p. 7]), this latter value is given as 16 : 104.

IV.5 [4–5]. The values for the eccentricity and the epicycle radius are from the *Almagest*. In the case of Mercury, the distance from the Earth to the deferent center is 3 times the eccentricity or 9 parts (see Fig. T8); thus the farthest distance is $60 + 9 + 22\tfrac{1}{2} = 91\tfrac{1}{2}$ parts. Because the nearest distance for Mercury occurs at about 120° from the farthest distance, rather than at 180° as Ptolemy found for the other planets (with the exception of the moon), one cannot calculate the nearest distance of Mercury in the simple way we did for Venus. Presumably by successive approximation, Ptolemy finds 55;34 parts for the least distance of Mercury's epicycle center from the Earth;[5] subtracting the epicycle radius ($22\tfrac{1}{2}$ parts) from this, one obtains 33;4 parts as in the text. Then

$$33;4 : 91\tfrac{1}{2} = 0.36 \approx \tfrac{1}{5} + \tfrac{1}{6}$$

For the ratio of Mercury's nearest distance to Venus's farthest distance, one may designate the former by $(\tfrac{1}{5} + \tfrac{1}{6})z$ and the latter by $(\tfrac{20}{3})z$, where z is both Mercury's farthest distance and Venus's nearest distance. Then

$$(\tfrac{1}{5} + \tfrac{1}{6})z : (\tfrac{20}{3})z = 11 : 200 \approx 1 : 18$$

Now from IV.5 [1], the sun's nearest distance is 1160 e.r. The ratio of the moon's farthest distance to this is then

$$64\tfrac{1}{6} : 1160 \approx 1 : 18$$

It is important to note here that these two equal ratios were arrived at from values totally independent of one another; this "incredible numerical accident,"

[4] That is, they are found both in the *Almagest* and the *Planetary Hypotheses*; see *HAMA*, 2: 908.

[5] *Alm.*, p. 460 (H282), p. 546 (H431).

as Neugebauer calls it,[6] was one of the strongest bits of evidence for the standard medieval ordering of the planets as well as for the entire system of contiguous orbs. Ṭūsī says as much here and in II.2 [4] to which he is referring.

The numerical basis for fitting Mercury and Venus below the sun, however, has a rather checkered career. In the *Planetary Hypotheses*, Ptolemy gave 34 : 88 as the ratio of the least to greatest distance for Mercury. Now Ptolemy had there modified his parameters for Mercury so that the epicycle radius became 22;15 (instead of 22;30 in the *Almagest*) and the distance between the dirigent center and the deferent and equant centers each became 2;30 (instead of 3). (The distance from the center of the World to the equant center remained 3.)[7] Now subtracting 22;15 from 55;34 yields 33;19, and one may surmise that Ptolemy carelessly rounded up instead of down.[8] The 88, however, is a real puzzle. As Goldstein has noted, even by using these new parameters one will arrive at 90;15 (= 60 + 3 + 2;30 + 2;30 + 22;15) and not 88 for the farthest distance.[9] My only suggestion, one I shall not insist on, is that Ptolemy simply forgot to add one of the eccentricities (*i.e.* 2;30 or 3) and, arriving at 87;45 (or 87;15), again rounded up, this time to 88.[10]

What makes this mistake of more than marginal interest is that without it Ptolemy could have avoided a good portion of the grief brought on by the gap between the orbs of Venus and the sun. To see this, we note that a 34 : 88 ratio yields a farthest distance for Mercury of 166 e.r. and a farthest distance for Venus of 1079 e.r.(assuming a farthest distance for the moon of 64 e.r. and a ratio for Venus of 16 : 104). Using Ṭūsī's ratio of 11 : 30 for Mercury, and the somewhat more accurate values of $64\frac{1}{6}$ e.r. and 3 : 20 for the moon distance and Venus's ratio, respectively, one comes up with 175 e.r. for Mercury's farthest

[6] *HAMA*, 2: 917; cf. Hartner's [1964], p. 274 "really astounding accident."

[7] Heiberg [1907], pp. 86, 88 (Greek) 87, 89 (German) (= *Plan. Hyp.*, BM 84b, lines 7–12, 23–24); cf. Goldstein [1967], pp. 9–10 and Hartner [1964], pp. 266–267.

[8] Or else, as Goldstein and Swerdlow [1970], p. 140 surmise, Ptolemy used the parameters from the *Planetary Hypotheses* to find a new minimum value for the Earth–epicycle center distance that resulted in a least distance of 33;49; he then correctly rounded up. If so, the care and effort taken by Ptolemy to obtain this difficult quantity makes his mistake in deriving the much easier to calculate greatest distance even more perplexing (see following discussion).

[9] Goldstein [1967], p. 10.

[10] Although this may seem implausible, it is at least easier to swallow than Hartner's number juggling that would require Ptolemy to have derived the incorrect 34 : 88 ratio after having first used the correct ratio of 33;4 : 91;30 with an incorrect maximum lunar distance of 60 to obtain the incorrect maximum distance for Mercury of 166. Ptolemy may not have been perfect but I think we may safely exclude such a procedure from his *modus operandi*. In fairness to Hartner, he does not attribute this mathematical wizardry to Ptolemy himself (since he did not know of the relevant section of the *Hypotheses* when he wrote the article) but instead postulates a series of discrete events leading to the final result ([1964], pp. 268–269).

distance and 1167 e.r. for Venus's farthest distance, which is only slightly greater than the sun's nearest distance.[11]

The involved history of the attempts to fit Mercury and Venus between the sun and moon would take us far afield; accounts may be found in Swerdlow [1968], pp. 99–106, 121–165, 194–203; Van Helden [1985], pp. 20–33; and *HAMA*, 2: 917–921. For our purposes it is worth mentioning that Proclus (5th c. A.D.) had already plugged the gap, and more so, between Venus and the sun in his *Hypotyposes* where he finds Venus's farthest distance to be 1190 e.r. based upon exact figures from the *Almagest*.[12] It is also noteworthy that in the *Muᶜīniyya*, Ṭūsī gives Ptolemy's value for the farthest distance of Mercury (166 e.r.) but by a sleight of hand he manages to reach 1160 for Venus's farthest distance by using an approximate ratio for Venus of 1 : 7.[13]

One cannot avoid the impression from this rather facile number juggling that the subject of sizes and distances was not afforded great importance by astronomers before Ṭūsī—or indeed by Ṭūsī himself. An insight into the reason this may have been so is offered by Bīrjandī in his comment to IV.5 [2]14–16. He tells us that since the most knowledgeable realized that certainty was not possible in this matter—one could only set the lower limit as we have seen—the basic goal was to know the sizes and distances in a general way (*ᶜalā al-ijmāl*); thus, he continues, a great deal of simplification occurred both in calculation and in the conceptualization of the problem. As an example of the latter, Bīrjandī notes that the distance of the convex surface of each orb should be equal to the concave surface of the orb immediately above it; but most astronomers did not take into account such things as the radii of the planets themselves,[14] the thickness of the parecliptic of the moon,[15] and part of the thickness of the complementary bodies of Mercury.[16]

This latter is particularly interesting since it illustrates so well the inability, or unwillingness, to resolve the different approaches of the *Almagest* and the *Planetary Hypotheses* before the 13th c. When we examine a standard *hay'a* diagram for Mercury, for example Fig. T8, it should be readily apparent that if we are to take the orbs seriously, then the nearest distance for Mercury should be the radius of the concave surface of the parecliptic orb. This radius, 28½

[11] In IV.5 [6], he takes 1160 e.r. as given and then finds Mercury's nearest distance to be 174 e.r. Compare Proclus's more accurate 177;33 e.r. in the *Hypotyposes*, which is based on the exact ratio (*HAMA*, 2: 920).

[12] It is interesting that Proclus did not feel it necessary to modify the sun's nearest distance of 1160 e.r.; see *HAMA*, 2: 920.

[13] The values from the *Almagest* would give 1 : 6.7.

[14] But cf. ᶜUrḍī, who seems to have been the first to do so (see Goldstein and Swerdlow [1970], especially pp. 143–145 and 148–151).

[15] This gives rise to the motion of the nodes of the moon; see II.7 [3] and [7].

[16] Bīrjandī is here considering only the classical Ptolemaic models; for the non-Ptolemaic models of the 13th c. and later, one also has such things as the enclosing sphere (*muḥīṭa*). See II.11 [4] and [5] and especially my commentary to II.11 [5]16–17.

parts, is readily calculated [60 – (3 · 3 + 22½)];[17] but as far as I can tell no one before the 13th c. did this but instead took the least distance of the *planet*, which, as we have seen, is a difficult quantity to obtain. Of course once Ptolemy had set the pattern of using this latter approach in the *Planetary Hypotheses*, it would take on a life of its own but I still think this is an excellent example of how long it took for the *hay'a* approach (not to mention the *Planetary Hypotheses*) to be taken seriously enough to investigate all its consequences.[18]

These consequences could be unfortunate if not embarrassing to the *hay'a* enterprise. ᶜUrḍī in *Kitāb al-hay'a*,[19] who seems to be the first astronomer to have this new sensitivity to the distances of the orbs, makes the rather startling discovery that by taking the ratio for Mercury to be 28½ : 91½, Venus can no longer be fitted between Mercury and the sun since its farthest distance then becomes about 1380 e.r.,[20] which is considerably larger than the sun's minimum, or for that matter maximum, distance. But in putting Venus above the sun, one is faced with an even larger gap than the one Ptolemy had to contend with. Furthermore the aesthetic arrangement by which the sun was the medial orb is lost;[21] how the later *hay'a* tradition coped with these unpleasant "realities" is a subject for future research.[22]

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[17] This, of course, is based on the parameters of the *Almagest*; from those of the *Planetary Hypotheses* we would have $29\frac{3}{4}^{p}$.

[18] This applies to modern historians as well. Even Goldstein and Swerdlow [1970], pp. 139, 156; who have done so much in recent years to unravel the mysteries of sizes and distances, failed to understand why ᶜUrḍī used 28½ for Mercury's nearest distance and considered it an "error" that is "especially surprising since...the author presented an extensive and reasonably correct description of Mercury's model and its effect on relative distance." But of course calculating the nearest distance of the planet Mercury from the Earth is a completely different problem from finding the amount of space occupied by Mercury's orbs. We may be excused for saying that Swerdlow's error is especially surprising in view of his later analysis (both extensive and more than reasonably correct) of the importance of the nesting principle for Copernicus (see especially his [1973] and [1976]).

[19] The relevant parts of the work are translated and commented on by Goldstein and Swerdlow [1970], pp. 147–149, 156–159; the identification of the author as ᶜUrḍī is due to Saliba [1979].

[20] Because of slightly modified parameters, Goldstein and Swerdlow [1970], p. 159 have a reconstructed value of 1387 e.r.; Shīrāzī in the *Nihāya* IV.9, who presumably depends on ᶜUrḍī for his discussion of the problem, has 1388 e.r.

[21] See II.2 [4].

[22] In his final comment on this chapter, Bīrjandī criticizes Shīrāzī for oversimplifying the problem and, relying on Kāshī's *Sullam al-samā'*, shows how one can still fit Mercury and Venus between the sun and moon by using Kāshī's improved parameters for the distances of the two luminaries.

IV.5 [6]. For Venus's nearest distance, which is Mercury's farthest distance, Ṭūsī has

$$(\tfrac{1}{10} + \tfrac{1}{20}) \cdot 1160 \text{ e.r.} = 174 \text{ e.r.}$$

In calculating this from the other direction, we have

$$(91;30 / 33;4) \cdot 64;10 \text{ e.r.} \approx 177\tfrac{1}{2} \text{ e.r.}$$

or, less exactly,

$$[1 / (\tfrac{1}{5} + \tfrac{1}{6})] \cdot 64;10 \text{ e.r.} = 175 \text{ e.r.}$$

Turning to the shadow cone: one has for the mean distance of Venus[23]

$$(174 \text{ e.r.} + 1160 \text{ e.r.}) / 2 = 667 \text{ e.r.}$$

Ṭūsī's value for the height of the shadow cone (203^{+} e.r.) then falls between the near and mean distances for Venus. But Ṭūsī here repeats the error that he made in IV.3 [5], namely of confusing the Ptolemaic value for the distance from the Earth to the cone's apex (268 e.r.) with the distance from the moon (or "the center of the shadow") to the apex (203;50 e.r.). This, of course, has no practical significance for the gross limits he uses in this paragraph to locate the disappearing point of the shadow.

The thickness of Venus's orb is

$$1160 \text{ e.r.} - 174 \text{ e.r.} = 986 \text{ e.r.}$$

This distance is between the convex and concave surfaces of Venus's parecliptic (see. Fig. T10). In the case of Mercury, however, Ṭūsī gives the diameter of the convex surface of the parecliptic, namely

$$2 \cdot (174 \text{ e.r.}) = 348 \text{ e.r.}$$

The reason he compares the thickness of Venus's orb with this diameter for Mercury, the latter including not only the orbs of Mercury but also the orbs of the moon and the levels of the Earth, is unknown to me.[24]

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[23] A more exact calculation starting from the moon and using the actual space occupied by Mercury's orbs (see pp. 520–521) results in 683 e.r.

[24] Bīrjandī suggests that this has something to do with Ṭūsī's awareness of the problem of the discrepancy between Mercury's (*i.e.* the planet's) nearest distance and the nearest distance to its orbs (see previous commentary); I remain unconvinced.

At any rate, the ratio of Mercury's orb "plus what is contained inside it" to Venus's orb is

$$348 \text{ e.r.} / 986 \text{ e.r.} \approx \tfrac{1}{3}$$

and, finally, one finds Mercury's nearest distance to be

$$(\tfrac{11}{30}) \cdot 174 \text{ e.r.} \approx 64 \text{ e.r.}$$

which is close to the $64\tfrac{1}{6}$ e.r. found for the moon's farthest distance (IV.2 [4]).

IV.5 [7–8]. The apparent diameters are from the *Planetary Hypotheses* (BM 90b–91a [trans., p. 8]), but the actual diameters and volumes vary from those of Ptolemy due to Ṭūsī's different distances. Variations will be noted below.[25]

For Venus, one has

$$667 \text{ e.r.}^{26} : 1210 \text{ e.r.} = \text{Venus's diameter} : \tfrac{1}{10} \text{ sun's diameter}$$

$$1 : 1;49 \approx \text{Venus's diameter} : \tfrac{1}{10} \text{ sun's diameter}$$

$$1 : 18\tfrac{1}{6} = \text{Venus's diameter} : \text{sun's diameter}$$

Since the ratio of the diameter of the Earth to that of the sun is 2 : 11 (IV.4 [1]), then the ratio of Venus's diameter to the Earth's is[27]

$$1 : (18\tfrac{1}{6} \cdot \tfrac{2}{11}) \approx 1 : 3\tfrac{3}{10}$$

The ratio of the volume of Venus to that of the Earth is then[28]

$$1^3 : (3\tfrac{3}{10})^3 = 1 : 35;56 \approx 1 : 36$$

[25] Values from the *Plan. Hyp.* occur on BM 90b–91b (trans., p. 8–9).

[26] *Plan. Hyp.*: $622\tfrac{1}{2}$ e.r.

[27] The *Planetary Hypotheses* gives Venus's diameter as $(\tfrac{1}{4} + \tfrac{1}{20})$ of the Earth's, which results in a ratio of $1 : 3\tfrac{1}{3}$. A ratio more in conformity with Ptolemy's own parameters, however, should be closer to $1 : 3\tfrac{1}{2}$, which is equivalent to Venus's diameter being $(\tfrac{1}{4} + \tfrac{1}{30})$ of the Earth's; cf. Goldstein's commentary to *Plan. Hyp.*, p. 12.

[28] Ptolemy has 1 : 44, which would be the result of using $(\tfrac{1}{4} + \tfrac{1}{30})$ instead of $(\tfrac{1}{4} + \tfrac{1}{20})$; this was pointed out by Goldstein (see previous footnote).

Turning to Mercury, we find its mean distance to be[29]

$$(64 \text{ e.r.} + 174 \text{ e.r.}) \div 2 = 119 \text{ e.r.}$$

Thus

$$119 : 1210 = \text{Mercury's diameter} : \tfrac{1}{15} \text{ sun's diameter}$$

$$1 : 153 \approx \text{Mercury's diameter} : \text{sun's diameter}$$

Converting this to a ratio of Mercury's diameter to the Earth's, one has[30]

$$1 : (153 \cdot \tfrac{2}{11}) \approx 1 : 28$$

The ratio of the volume of Mercury to that of the Earth is then[31]

$$1^3 : 28^3 = 1 : 21{,}952 \approx 1 : 22{,}000$$

Book IV, Chapter Six

IV.6 [1]. The parameters are from the *Almagest*; Mars's boundaries of 1260 e.r. and 8820 e.r. are from the *Plan. Hyp.*, BM 90a (trans., p. 7).

IV.6 [2]. The 1 : 20 ratio comes from the *Plan. Hyp.*, BM 91a (trans., p. 8). Mars's average distance is 5040 e.r., which is about $4\frac{1}{6}$ times 1210 e.r., *i.e.* the sun's mean distance. Now at the sun's mean distance, Mars's apparent diameter would be

$$\tfrac{1}{20} \cdot 5\tfrac{1}{2}^{p} = 16\tfrac{1}{2} \text{ minutes}$$

Adjusting for Mars's mean distance, one has

$$0;16\tfrac{1}{2}^{p} \cdot 4\tfrac{1}{6} \approx 1;9^{p}$$

which is the diameter of Mars in terms of Earth diameters (not radii!).[1]

The volume of Mars is then $(1;9)^3$ or 1;31 times the volume of the Earth.[2]

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[29] *Plan. Hyp.*, BM 91a (trans., p. 8) has (64 e.r. + 166 e.r.) ÷ 2 = 115 e.r.

[30] *Plan. Hyp.*, BM 91b (trans., p. 8) has 1 : 27.

[31] *Plan. Hyp.*, BM 91b (trans., p. 9) has $1^3 : 27^3 = 1 : 19{,}683$.

[1] *Plan. Hyp.*, BM 91b (trans., p. 8) has $1\frac{1}{7}$ Earth diameters.

[2] *Plan. Hyp.*, BM 91b (trans., p. 9) has $1\frac{1}{2}$.

IV.6 [3]. The "thickness" (*thikhan*) of Mars's orb is arrived at by subtracting its nearest distance (1260 e.r.) from its farthest distance (8820 e.r.). The diameter of the sun's "sphere" (*kura*) is the orb of the sun plus everything contained inside it, *i.e.* the orbs of Venus, Mercury, and the moon as well as the levels of the four elements; its measure is simply twice the sun's farthest distance of 1260 e.r.

Ṭūsī engages in a bit of hand waving here since the fact that Mars's orb is 3 times the "sphere" of the sun is hardly an "elucidation" (*bayān*) much less an explanation for the problem originally stated in II.9 [14], namely that Mars is always closer to the sun at opposition than it is at combust. Using Ptolemaic parameters, we may supplement the text as follows: during combust, Mars is at the apex of its epicycle and thus its distance from the Earth can vary between $(60 + 6 + 39\frac{1}{2})^{p}$ and $(60 - 6 + 39\frac{1}{2})^{p}$, the radius of the deferent circle being 60. Converting these into absolute distances, one has a range between 8820 e.r. and 7817 e.r. Since the sun ranges between 1260 e.r. and 1160 e.r., the maximum and minimum distances between the sun and Mars at combust are 7660 e.r. and 6557 e.r. At opposition, Mars is at the perigee of its epicycle and thus its relative distance varies between $(60 + 6 - 39\frac{1}{2})^{p}$ and $(60 - 6 - 39\frac{1}{2})^{p}$. Converting these to absolute distances, one has a range between 2303 e.r. and 1260 e.r. The maximum and minimum distances between the sun and Mars at opposition are then 3563 e.r. and 2420 e.r., which are considerably less than the distances at combust.

IV.6 [4]. The basic parameters are from the *Almagest*. Instead of a ratio of $74\frac{1}{4} : 45\frac{3}{4}$, Ptolemy uses 74 : 46 in the *Plan. Hyp.*, which he reduces to 37 : 23.[3] Thus Ptolemy's farthest distance is 14,187[4] rather than Ṭūsī's 14,259.

IV.6 [5]. That Jupiter's diameter is $\frac{1}{12}$ that of the sun is from *Plan. Hyp.*, BM 91a (trans., p. 8). Because Ṭūsī's value for Jupiter's farthest distance differs from that of Ptolemy, the mid-distance also differs—Ṭūsī has 11,540 e.r., whereas Ptolemy has 11,504 e.r. (ibid.). At the sun's mean distance, the diameter of Jupiter's planetary body would be

$$\tfrac{1}{12} \cdot 5\tfrac{1}{2}\text{ e.d.} = 0;27\tfrac{1}{2}\text{ e.d.}$$

At Jupiter's mean distance it becomes[5]

$$0;27\tfrac{1}{2}\text{ e.d.} \cdot (9 + \tfrac{1}{3} + \tfrac{1}{5}) = 4.37\text{ e.d.} \approx (4 + \tfrac{1}{5} + \tfrac{1}{6})\text{ e.d.}$$

By cubing this last quantity, one obtains Jupiter's volume, which is approximately equal to $83\frac{1}{4}$ Earth volumes.[6]

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[3] *Plan. Hyp.*, BM 90a (trans., p. 7); cf. Goldstein's commentary, p. 11.

[4] Goldstein surmises that this was probably 14,189 in the original Greek.

[5] *Plan. Hyp.* has $(4 + \frac{1}{3} + \frac{1}{40})$.

[6] Actual volume based on Ṭūsī's diameter for Jupiter: 83.26 e.v.; *Plan. Hyp.* has $(82 + \frac{1}{2} + \frac{1}{4} + \frac{1}{20})$. Note that the β version (MSS FL) has $83\frac{1}{4}$; earlier versions (MSS DMT) have $82\frac{1}{4}$. 82 has been corrected above the line to 83 in MS D.

IV.6 [6]. The basic parameters are from the *Almagest*. That the farthest distance of Saturn is $1\frac{2}{5}$ times the nearest is in the *Plan. Hyp.*[7] Because of the difference in the farthest distance for Jupiter, Ṭūsī has 19,963 for the farthest distance of Saturn, whereas Ptolemy arrives at 19,865.[8]

IV.6 [7]. That Saturn's diameter is $\frac{1}{18}$ that of the sun is from *Plan. Hyp.*, BM 91a (trans., p. 8). For Ṭūsī's mean distance of 17,111 e.r., Ptolemy has 17,026 e.r. At the sun's mean distance, the diameter of Saturn's planetary body would be

$$\tfrac{1}{18} \cdot 5\tfrac{1}{2} \text{ e.d.} = 0;18\tfrac{1}{3} \text{ e.d.}$$

At Saturn's mean distance, it becomes[9]

$$0;18\tfrac{1}{3} \text{ e.d.} \cdot 14 = 4.28 \text{ e.d.} \approx 4\tfrac{1}{4} \text{ e.d.}$$

By cubing this last quantity, one obtains Saturn's volume, which is approximately equal to 77 Earth volumes.[10]

Book IV, Chapter Seven

IV.7 [2]. The 1 : 20 ratio for the apparent size of a first magnitude fixed star in relation to the sun comes from *Plan. Hyp.*, BM 91a (trans., p. 8); Ṭūsī takes this to be for an "average size" first magnitude star but Ptolemy implies in two places that this is a minimum value (BM 91b, lines 12–14, 19–20). Now the fixed stars are 19,963 e.r. from the Earth, which is about $16\frac{1}{2}$ times 1210 e.r., *i.e.* the sun's mean distance. At the sun's mean distance, the apparent diameter of a first magnitude star would then be

$$\tfrac{1}{20} \cdot 5\tfrac{1}{2}^{\text{p}} = 16\tfrac{1}{2} \text{ minutes}$$

Adjusting for the star's actual distance, one has

$$0;16\tfrac{1}{2}^{\text{p}} \cdot 16\tfrac{1}{2} = 4;32,15^{\text{p}} \approx (4 + \tfrac{1}{3} + \tfrac{1}{5})^{\text{p}}$$

which is the diameter of the star in terms of Earth diameters.[1] The volume of the star is therefore $(4 + \frac{1}{3} + \frac{1}{5})^3$ or about 93 times the volume of the Earth.[2]

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[7] *Plan. Hyp.*, BM 90a (trans., p. 7); Ptolemy gives it as the ratio of 7 : 5.

[8] *Plan. Hyp.*, BM 90a (trans., p. 7); this is no doubt based on a farthest distance for Jupiter of 14,189 rather than the 14,187 of the text; cf. footnote 4.

[9] Ptolemy has $(4\frac{1}{4} + \frac{1}{20})$ e.d. (*Plan. Hyp.*, BM 91b [trans., p. 8]).

[10] Actual value based on Ṭūsī's diameter for Jupiter: 76.77; *Plan. Hyp.* has $79\frac{1}{2}$.

[1] *Plan. Hyp.*, BM 91b (trans., p. 9) has "at least $4\frac{1}{2} + \frac{1}{20}$" (= 4;33).

[2] *Plan. Hyp.*, BM 91b (trans., p. 9) has "at least $94\frac{1}{6} + \frac{1}{8}$."

IV.7 [3]. The calculated results of Ṭūsī's approach are displayed in Table 13.[3] Note that he correlates an increasing magnitude with a linearly decreasing volume; this widely-held medieval interpretation of Hipparchus's magnitude is apparently due to Farghānī.[4] The division of each magnitude into large, mean, and small could well be due to Ṭūsī himself since it is otherwise unattested before him.[5]

We should emphasize here that in ancient, medieval, and early modern astronomy, stellar magnitudes were generally correlated with size, not brightness.[6] But the convention apparently adopted by Hipparchus and reiterated by Ptolemy was that magnitudes were measures of stellar diameters, not volumes. And since the only reported observations, due evidently to Hipparchus,[7] had the fixed stars ranging in apparent diameter from $\frac{1}{20}$ to $\frac{1}{30}$ of the apparent diameter of the sun, the scale based on volume makes little sense. This criticism is stated explicitly by Jurjānī and is expanded on by Bīrjandī, who notes correctly that a fixed star that is $\frac{1}{30}$ the diameter of the sun will have a volume of approximately $27\frac{2}{3}$ Earth volumes, which would be a 5th magnitude star in Ṭūsī's scheme. Bīrjandī rather mysteriously attributes the magnitude scale based on volume to Ptolemy and notes that it is quite popular among the moderns despite the problem associated with it. Less mysteriously, he traces the scale based on diameters to Hipparchus and reports the clumsy relationship whereby a 2nd magnitude star is $\frac{1}{22}$ the sun's diameter, a 3rd magnitude star is $\frac{1}{24}$, and so on. This latter relationship, of course, cannot be attributed to Hipparchus; indeed it seems to be due to ᶜUrḍī.[8]

Before leaving this topic, I should mention another scale based on diameters that Bīrūnī tells us he found in Abū Jaᶜfar al-Khāzin's *Book on Sizes and Distances*.[9] The scale is as follows:[10]

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[3] These values agree with those of Bīrjandī.

[4] *Elements*, ch. 22, pp. 84–85; cf. Swerdlow [1968], pp. 173, 175.

[5] Swerdlow [1968], pp. 189–190 only finds this scheme in Bar Hebraeus (d. 1286) and suggests that he may have been responsible for it; it is more reasonable to assume that he got it from Naṣīr al-Dīn.

[6] The defining relationship for the modern logarithmic stellar magnitude scale, whereby a brightness ratio of 100 is made to correlate *exactly* with a Hipparchan magnitude difference of 5, dates only from the last century; see Pannekoek [1961], pp. 444–447.

[7] *Plan. Hyp.*, BM 90b–91a (trans., p. 8).

[8] Swerdlow [1968], pp. 209–210; the attribution of the text to ᶜUrḍī is due to Saliba [1979].

[9] *Qānūn*, 3: 1312; on al-Khāzin's nonextant work, see *GAS*, 5: 299 and 6: 190.

[10] The same scale can be found in the *Tafhīm*, p. 115, except that Bīrūnī there uses Ptolemy's $\frac{1}{20}$ for first magnitude stars. In the *Tafhīm* he does not mention al-Khāzin in connection with the scale and rather carelessly seems to be attributing it to Ptolemy's *Manshūrāt* (*Planetary Hypotheses*); the sloppiness is corrected in the later *Qānūn*.

Table 12. Al-Khāzin's Magnitude Scale.

Star	Diameter	
	Qānūn	*Tafhīm*
Sun's Diameter	1	1
1st magnitude	1 / 17	1 / 20
2nd magnitude	1 / 20¼	4 / 81
3rd magnitude	1 / 21⅘	5 / 109
4th magnitude	1 / 24	1 / 24
5th magnitude	1 / 27½	2 / 55
6th magnitude	1 / 36	1 / 36

Bīrūnī rather caustically remarks that al-Khāzin "does not attribute this [scale] to himself nor to anyone else; neither does he indicate the way in which it was derived or discovered." Although Swerdlow believes these values to be "the result of some sort of computation rather than observation,"[11] they are so odd that I doubt that there is any computational rationale for them. (The fact that they defeated both Bīrūnī and Swerdlow is eloquent testimony to this.) I am inclined to see them as the result of some kind of observation, but I am at a loss to understand what sort of instrumental scale could have given al-Khāzin, or whomever, both 1 / 20¼ and 1 / 21⅘.

IV.7 [4]. The ordering of bodies is basically correct except that large and average 2nd magnitude stars should have been placed between Jupiter and Saturn (see Table 13). This ordering, with the exception of those fixed stars with magnitudes other than 1 for which Ptolemy gives no accounting, is the same as in the *Planetary Hypotheses*, BM 91b (trans., p. 9). This is something of a coincidence since in giving his volumes, Ptolemy, unlike Ṭūsī, has the moon larger than Venus. What seems to have happened is that Ptolemy ordered the celestial bodies on the basis of his true diameters rather than his volumes; accordingly Venus would be larger than the moon. The reason for the discrepancy is that Ptolemy's stated diameter for Venus is not in conformity with the amount he gives for its volume.[12]

IV.7 [5]. Ṭūsī uses 1273 parasangs for the Earth's radius and 33;33 e.r. for the moon's nearest distance to obtain 42,709 parasangs. Subtracting 1273 parasangs from this results in 41,436 parasangs for this distance measured from the surface of the Earth. For the distance of the fixed stars one has

$$19{,}963 \text{ e.r.} \cdot 1273 \text{ parasangs/e.r.} = 25{,}412{,}899 \text{ parasangs}$$

[11] Swerdlow [1968], p. 183.

[12] See footnotes 27 and 28 to commentary IV.5 [7–8].

Table 13. Sizes and Distances.[1]

Celestial Body	Nearest Distance (in Earth radii) Ṭūsī [Ptol]	Apparent Diameter[2] (in apparent solar diameters) Ṭūsī [Ptol]	True Diameter (in Earth diameters) Ṭūsī [Ptol]	Volume (in Earth volumes) Ṭūsī [Ptol]
Moon	33;33 [33][3]	1 [$1\frac{1}{3}$][4]	0;17,33 [$\frac{1}{4}+\frac{1}{24}$][5]	$1 / (39+\frac{1}{4}+\{\frac{1}{20}\})$ [$\frac{1}{40}$][6]
Mercury	$64\frac{1}{6}$ [64][7]	$\frac{1}{15}$ [$\frac{1}{15}$]	$\frac{1}{28}$ [$\frac{1}{27}$]	$\frac{1}{21{,}952}$ [$\frac{1}{19{,}683}$]
Venus	174 [166][8]	$\frac{1}{10}$ [$\frac{1}{10}$]	$1 / 3\frac{3}{10}$ [$\frac{1}{4}+\frac{1}{20}$]	$\frac{1}{35;56}$ [$\frac{1}{44}$]
Sun	1160 [1160][9]	1 [1]	$5\frac{1}{2}$ [$5\frac{1}{2}$]	$166+\frac{1}{4}+\frac{1}{8}$ [$166\frac{1}{3}$][10]
Mars	1260 [1260][9]	$\frac{1}{20}$ [$\frac{1}{20}$]	1;9 [$1\frac{1}{7}$]	1;31 [$1\frac{1}{2}$]
Jupiter	8820 [8820]	$\frac{1}{12}$ [$\frac{1}{12}$]	$4+\frac{1}{5}+\frac{1}{6}$ [$4+\frac{1}{3}+\frac{1}{40}$]	$83\frac{1}{4}$ [$82+\frac{1}{2}+\frac{1}{4}+\frac{1}{20}$]
Saturn	14,259 [14,187]	$\frac{1}{18}$ [$\frac{1}{18}$]	$4\frac{1}{4}$ [$4+\frac{1}{4}+\frac{1}{20}$]	77 [$79\frac{1}{2}$]
1st Mag. Stars	19,963 [19,865][11]	$\frac{1}{20}$ [$\frac{1}{20}$][12]	$4+\frac{1}{3}+\frac{1}{5}$ [$4+\frac{1}{2}+\frac{1}{20}$][13]	$98\frac{1}{6}$ 93 [$94+\frac{1}{6}+\frac{1}{8}$][13] $< 87\frac{5}{6} >$
2nd Mag. Stars	19,963 [19,865]			$< 82\frac{2}{3} >$ $< 77\frac{1}{2} >$ $< 72\frac{1}{3} >$
3rd Mag. Stars	19,963 [19,865]			$< 67\frac{1}{6} >$ $< 62 >$ $< 56\frac{5}{6} >$
4th Mag. Stars	19,963 [19,865]			$< 51\frac{2}{3} >$ $< 46\frac{1}{2} >$ $< 41\frac{1}{3} >$
5th Mag. Stars	19,963 [19,865]			$< 36\frac{1}{6} >$ $< 31 >$ $< 25\frac{5}{6} >$
6th Mag. Stars	19,963 [19,865]	[$\frac{1}{30}$][14]		$< 20\frac{2}{3} >$ $< 15\frac{1}{2} >$ $10\frac{1}{3}$

Notes to Table 13.

[1] For Ptolemy all numbers are from the *Planetary Hypotheses*; differing *Almagest* values are noted.
[2] At mean distances except for Ṭūsī's moon and for the fixed stars.
[3] *Alm.*: 33;33. The distance of 94;36 myriad stades given for the boundary of the level of fire and the lunar orb (*Plan. Hyp.*, BM 90b [trans., p. 8]) is equivalent to 33 e.r.
[4] Ṭūsī in IV.3 [2] follows the *Almagest* in giving the moon's apparent diameter at its farthest distance as 1 apparent solar diameter; he thus ignores both his previously stated values in II.13 [8]2–4 and the diameter of 1⅓ occurring at mean distance as given in the *Hypotheses*.
[5] *Alm.* has 0;17,33 for the moon at its farthest distance.
[6] *Alm.* has 1 / 39¼.
[7] Ṭūsī finds the moon's farthest distance to be 64⅙ (as in *Alm.*) and Mercury's nearest distance 64.
[8] The farthest distance for Venus is 1079 e.r. in *Plan. Hyp.*, BM 89b (trans., p. 7).
[9] In the *Almagest*, the only distance given for the sun is 1210 e.r.
[10] *Alm.* has 170.
[11] Ptolemy probably used 20,000 to calculate the true diameter and volume.
[12] Despite Ṭūsī's claim in IV.7 [2] that "they" have given 1⁄20 for "average-size stars of first magnitude," Ptolemy simply gives this value for first magnitude stars without further qualification; cf. *Plan. Hyp.*, BM 91a (trans., p. 8).
[13] Ptolemy gives these as minimum values (*Plan. Hyp.*, BM 91b [trans., p. 9]).
[14] This is attributed to Hipparchus (*Plan. Hyp.*, BM 90b [trans., p. 8]).

Part V

Critical Apparatus

531 a

Explanation of Signs and Conventions Used in Apparatus

Numbers in right column indicate page numbers of the edited text.

/2–24/	Refers to lines of the edited text; here from line 24 to line 2 of the following page (reading right to left)
[	Separates reading in edition from any variant
:	Separates variant and manuscript *sigla*
+	Added in
–	Missing from
=	Indicates another variant
(...)	Editor's comments
(٢)	Indicates variant is the second occurrence of the word or phrase on the line
ض	Istanbul, Feyzullah MS 1330, 1
ط	Istanbul, Ahmet III MS 3453, 19 (= Topkapı Saray MS 7005, 19)
غ	St. Petersburg (a.k.a. Leningrad), Oriental Institute MS A 437
ف	Vatican, ar. MS 319, 1
ل	Leiden, University Library MS or. 905 (= 1093)
م	Istanbul, Lâleli MS 2116
با	بياض (blank)
تا	تحت السطر في (under the line in)
خا	خرم (hole)
شا	مشطوب في (crossed out in)
طا	مطموس ، غير مقروء ، إلخ (smudged, unreadable, etc.)
فا	فوق السطر في (above the line in)
ها	هامش (margin)

532 e

§1. Text Apparatus

الديباجة والمقدمة

91 /1/ الرحيم] +رب تمّم بالخير : ض = +رب زدني علما : غ = +وبه نستعين : ف = +رب يسر : ل . /2/ الحمد] طاف . /5/ نورد] نحرر : م . من] +من : ض = على : ط . لبعض] طاض . /6–7/ فلنورد ما قصدناه] طاف . /8–9/ الباب الأول فيما] طاف . /9/ تقديمه] طاف . /10/ لكل علم موضوع] طاف . يُبحث] +فيه عن اعراضه الذاتيه : شاض . عنه] عن اعراضه الذاتية : ط . /10–11/ عنه ...بينة] طاف . /11/ تُبيّن في علم] طاف . /12/ العلم ... ومسائل] طاف . تُبيّن] تتبين (بإهمال التاءين) : ف . في ذلك العلم] طاف . /13/ الهيئة] طاف . /13–14/ البسيطة ...كمياتها] طاف . /14/ وحركاتها] وحركتها (؟) : ض . /15/ إلى البيان] طاف . تتبين] تُبيّن : غ (بدون الضمة) ، م . علوم ثلاثة] طاض . /16/ ومسائلها معرفة] طاف . /17/ وكيفية ...وحركاتها] طاف . ومقادير الحركات] طاف . الحركات] طاض . /18/ وعلل] وعللُ : ف . اختلاف الأوضاع] الاختلاف والاوضاع : غ .
93 /1/ والفن الذي] طاف . نريد ...تقرير] طاض ، طاف . /2/ وتُبيّن] فاض (بخط غير الناسخ) = وتتبيّن : ض (بإعجام التاء الثاني فقط) ، غ . وتُبيّن تفاصيلها] طاف . /4/ ما] -ض ، -غ ، -ف . عما ثبت] طاف . /5/ فيه] -ض ، -ط ، -غ ، هاف . تعرّف] هاض (بخط غير الناسخ) = تعريف : ض (بإعجام الفاء فقط) . /6/ سبيل التصدير] طاف . بيانها] تبيانها : ف . /7/ مواضع] هاض (بخط غير الناسخ) . بياناتها تنقسم] طاف . /8/ فلنقدم] ولنقدم : ط .

الباب الأول ، الفصل الأول

/13–14/ أي ... بالحس] -غ . /14/ وهي] هي : غ . وهو] هو : غ . طول
فقط] طاض . /15/ وهو] فام . وينتهي(٢)] طاض . /16/ بالسطح]
طاض . /18/ هو] وهو : ف . يتحاذى] طاض . /19/ التي تفرض] م =
المفروضة : هام . عليه] + بعضها لبعض : م . /19–20/ هو الذي يكون فرض
الخطوط المستقيمة عليه في جميع الجهات ممكناً] هاط (مع رمز "خ") ، ف ،
ل = هو الذي يمكن أن تخرج فيه (= منه : م) الخطوط المستقيمة في جميع
الجهات : م ، (كذا مقيّدة في شروح الجرجاني والخفري والبيرجندي) = هو
الذي تكون الخطوط المفروضة عليه في جميع الجهات مستقيمة : ض ("هو
95 الذي تكون" مطموس) ، ط ، غ . /2/ أن يتحدا] طاف . أحاط] (كذا في
كل النسخ) . /3/ أن] هاض . /7/ مستقيم(١)] -ط . مستقيم(٢)]
-ط ، فاغ . وحدثت] طاض = وحدث : ف . /9/ والتي هي] طاض . /10/
أعظم] اكبر : غ . /11/ مع] طاض . /12/ السطع] سطح : ض . /13/
سطح(١)] طاض . /14/ فيهما من أي] طاض . /15/ يتقاطعان على قوائم]
طاض . /16–17/ التي ... أخرجت] طاض . /17/ نهاية] النهاية : ض ، غ .
هي] تسمى : غ . /17–18/ المتوازية وكذلك السطوح] طاض . /18/ في جميع
الجهات] -غ . /19/ نهاية] النهاية : ض ، غ . /23/ وتلك] في تلك : ف .
97 الخارجة] + منها : غ . /24/ قطر لها] قطرها : غ = قطر : ل . /2/ يُفرَز]
تفرز : م . /2–4/ ونصف ... القوس] هاض . /2/ القوس] قوس : هاض =
قوسه : ط . /3/ من] طاهاض . /3–4/ لنصف القوس] م = للقوس : هام
(مع رمز "ظ" (= أظنه؟) ولعله بخط غير الناسخ) . /4/ القوس] + ايضا : ط .
/10/ فهي] وهي : ل . /11/ تلك] -غ . وتمر] ط = ويمر : م . /12/ فعلت]
ف ، ل ، م = فعل : ض ، ط ، غ . /12–13/ ترسم عليها بحركتها] م =
تتحرك بحركتها : فام (مع رمز "ظ" ولعله بخط غير الناسخ) . /13/ عليها]
+ دايرة : غ . كل] غ ، ف = -ض ، -ط ، -ل ، -م . تامة] -غ . /17/
متوازية وموازية] موازية : غ . للمنطقة] للنقطة : ل . والمحور] فالمحور : ض .
عموداً] (ال "ا" مشطوب في ض) = عمود : غ . /18/ متساويتي]
غ ، ف ، ل ، م : متساويي] ض ، ط . /19/ متساويان] متساويتان : غ .
عظمى أو صغرى في الكرة] ل = -غ = عظمى : ض (التصحيح في الهامش

مطموس) = عظمى : ط («وصغرى» فوق السطر مع رمز «صح») = عظمى وصغرى : ف («في الكرة» في الهامش مع رمز «صح») = عظمى : م («اذالصغرى»(؟) في هامش م مع رمز «ظ» ولعله بخط غير الناسخ) . /21/ دائرتان] تاض (بخط غير الناسخ) ، هاض (بعض الكلمة مطموس) ، -ط ، هام (مع رمز «خ» ولعله بخط غير الناسخ) . /22/ خطأ] فام . /22–23/ ماراً بالمركز] -غ ، -ف = فاض (بخط غير الناسخ) ،
هاط ، هال ، تام (بخط غير الناسخ) . /23/ كالبعد بين] هام . /1/ 99
تقاطعتا] تقاطعا : ض ، ط ، غ . /2/ الأخرى] الاخر : غ . /3/ الفلك] والفلك : ض . مركزاهما] ف ، ل ، (كذا في شروح النيسابوري والجرجاني والخفري) = مركزهما : ض ، ط ، غ ، م ، (كذا في شرح البيرجندي) . /8–9/ ويكون ... قائمة] هاض ، هاط (بدون «ويكون الخط الواصل» ونجد رمز «خ اصح») ، ف ، ل = ويكون الخط الواصل بين المركزين عمودا على سطحي الدايرتين وهو سهم الاسطوانة : ض ، ط ، م = ويكون الخط الواصل بين المركزين محيطاً على سطح الدايرتين وهو سهم الاسطوانة : غ . /9/ قائمة] + والا فمائلة : هاض (لعله بخط غير الناسخ) . /10/ مستدير] + صِنوبرى : هاض (بخط غير الناسخ) . /12–13/ يكون سهمه ... قائماً] هاض (بدون «يكون» وبخط غير الناسخ) ، هاط (مع رمز «خ اصح») ، ف ، ل ، م («سهمه فان كان» في هامش م ؛ وأما «كان المخروط قايما» فلعله بخط غير الناسخ ومن المحتمل أنه بدل «وهو سهمه» الذي ربما قد شُطب) ، (كذا في شرح النيسابوري إلا أنه يضيف «وهو سهمه») = يكون عمودا على قاعدته وهو سهمه : ض ، ط ، غ . /13/ قائماً] + والا فمائلا : هاض (بخط غير الناسخ) .

الباب الأول ، الفصل الثاني

/19/ تسليمه] تسلّمه : غ . /21/ من] عن : ط ، غ ، ف . /1/ قد] هاض . 101
وقد يصير] ويصير : ط («وقد» في الهامش مع رمز «خ») . /2/ والفلكي] والاوّل : غ . هو] فاط . /4/ والحيوانات] والحيوان : غ . /5/ ولكل] لكل : ط ، غ ، ل ، م . يفارقه] يفارق : غ . /6/ يتحرك] متحرك : ل . وإن] فإن : غ . نُسب] نسبت : ض ، ل . /10/ بغيره] بغير : غ . /12/

إلى(١)] -غ . /13/ وتختص] ويختص : ل . /16/ بسيطة] بسيط : غ ، ف . تصدر] يصدر : غ . /17/ أو] و : م . تقطع] يقطع : ف . /18/ عن] من : ط . /21/ وكل] كل : ط ، غ ، ل ، م . /22/ تنخرق] تتخرق : ف = ينخرق : ل . /23/ تنمو] تنمُوا (بإهمال التاء) : ل . /24/ في حركاتها ولا تضعف] ولا تضعف في حركاتها : غ . حركاتها] حركتها : ف . /25/ حركتها] حركاتها : م . /26/ المتشابهة] المتشابه : ض .

الباب الثاني ، الفصل الأول

103 /2/ العلوية] العِلْويّة : ف ، م = العُلويّة (كذا في مخطوط ٣٢١٧ ، مكتبة أحمد الثالث) = +وفيه : غ . /3/ أربعة عشر فصلاً] هاض . /11/ بعينهما] بعينها : م . /14/ زمان] -غ . إلى أن ينتهي إلى] ض ، ط ، ف = الى : غ ، ل = ينتهي الى : م . /15/ قليلاً] +قليلا : غ ، هاف . ثم] -غ . دورة] (بإعجام التاء في ض ، ف ، م) . /20/ من جرمه] هاض . /21/ جميع] فال . الأفق] هاض (مع رمز «صح») = الارض : ض . تراكم] تَراكُمَ : ل . المرتفعة] طاف . /22/ يُري] يُرِى : ض ، ط ، ف ، م = يرى : غ

105 (بإهمال الياءين) = تُرى : ل . /3/ يدل] تدل : ط . /7/ الجنوبية] +في : غ . /8/ وغولهما] وغولها : ل . /16/ الواقف] -غ . /21/ في] على : ط . متقابلين] متساويين : غ . /22/ التي] هاف . /25/ ومما]

107 وما : ف . /2/ السطح] سطح : غ . /3/ القمر فلها قدر] طاف . ولذلك] فلذلك : ف . /4/ وسنبيّن] سيتبيّن : ل . /8/ يقع] تقع : غ . أو] (ال «ا» مشطوب في ض) . /9/ أن] +تكون] ض ، ط ، هام (مع رمز «خ») . /10/ أبطأ] ابطاءُ : ض ، م . /11/ بها] (كذا في كل النسخ) . /14/ ثبت] (كذا في كل النسخ) . والسماء] -ض ، -ط ، -ل ، -م . والماء] -غ ، هاف . /16/ جوانب الأرض] الجوانب : غ . ما(٢)] مما : ض ، ل . /17/ ما] مما : ض ، ل . مركز] -غ . /18/ أقطار لها] أقطارها : غ ، ف . /19/ ماءً] ما : ط ، ف = مَآً : م . الماء] (بدون الهمزة في ط) . /21/ مثلاً] هاف . /23/ إنّية] اِنيّة : ض ، ف ، ل = انيّه : ط ، غ = اَنَيّةٌ : م . تُفيد (موقعان)] يُفيد : ض . /24/ اللِميات] اللمّيات : ض = اللميّات : غ ، ف ، م = اللميَات : ل .

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109 /4/ بالحركة] الحركة : غ ، ل . ما يطلع منها] منها ما يطلع : غ . /5/ فيه] منه : ط . /6/ طلع] يطلع : غ ، ف . لا] -غ . /10/ الإحساس] الاجناس : م . /11/ بأعيانها] باعيانهما : غ . /12/ إن] وان : غ . إلى] على : ض . /13/ على] الي : ل . /14/ متشابهتان] -ط . /15/ علواً] عِلواً : ض ، م . /18/ اثنين] اثنان : ض ، ط ، ل . /18–19/ اثنين منها] -غ . /19/ للسيّارات] للسيارة : غ . تكن] يكن : ل ، م . /20/ حركة] -ط . اكتفوا] + في : شاط . بأحد] باحدى : ض ، غ ، ف ، ل ، م (الياء مشطوب) . فلكيهما] فلكهما : ض = فلكيها : ط . /22/ الأوليين] الاولين : ض . /22–23/ لو لا الحركة الأخرى] -ض ، -ط ، -م (ونجد التالي في هامش م : «لم يكن ممتنعا لو لم تكن الحركة الثانية لكنهم لم يذهبوا الى ذلك لوجودها هكذا في بعض النسخ والظاهر انه لا حاجة الى التقييد والصحّاح [كذا] ما في المتن») . /22/ لا(٢)] لم تكن : غ . /23/ الأخرى] الثانية : غ . لوجودها] -ض ، -ط ، -م .

111 /2/ وسموه] ويسمى : ض (بإهمال الياء) . /3/ وسموه] وسموا : ل . /7/ وجعلوا] وجعلو : ف . الفلك] -غ . /8/ لم] + تكن : ض ، ط ، هال ، م . /10/ والسفليان] والسفليتان : غ . والقمر بوجه] + آخر : م . /12/ الزهرة] + قد : ل . بعديها] بعدها : ل . الأبعد والأقرب] الاقرب والابعد : غ ، ف . /14/ المركبة] م = المولفة : هام (مع رمز "خ") . وسيأتي] فسياتى : ط . /15/ فهذه] وهذه : غ . /16/ وأما] اما : غ . /17/ الفلكيات] الافلاك : غ . /18/ طبقة(١)] -غ . للنار] النار : ط . الصرف] الصرفة : ف . /19/ الحار] الحارة : غ . تتلاشى] يتلاشى : ف . فيها] منها : ض . /20/ تتكوّن] يتكون : م . /22/ هي] -غ .

113 /1/ بغيرها] بغيره : ل ، م = لغيره : ض ، ط . فيها تتولد] تتولد فيها : ض ، م . /2/ النباتيات] النباتات : غ ، ل . والحيوانات] والحيوانيات : م .

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/5/ المشهورة] المشهور : ف . /7/ وستين] وستون : م . /8/ الربع] ربع : م . /10/ عنه] هاض . /12/ وتسمى] ويسمى : ف ، ل . /15/ ويسمى]

وتُسمى : م . /18/ ويسمى] وتسمى : ل ، م . جميعها] جميعاً : غ (بدون التنوين) ، م . /21/ ويحدث] وتحدث : ض . /22/ يسميان] تسميان : ض .

115 /1/ جازته] جاوزته : ف . /4/ فتتوهم] فيتوهم : ض . /7/ عندهما] وعندهما : م . تُربَع] تتربّع : غ = يتربع : ف . وتسميان] وتسمى : ط (بإهمال التاء) = ويسميان : ف . /11/ قطب] قطبي : ف . والمنطقة] ومنطقة : غ ، ل ، م . /12/ وتقسم] ونُقسِم : ل . /13/ الاثنا] الاثنى : م . /16/ فكل] وكل : ل . /21/ منها] -ل . /22/ وهي من الميول الجزئية] -غ .

117 الجزئية و] + القوس : هاف . /26/ توهمت] طاف . /2/ ذلك الجزء] طاف . ميلا اولَ] ض = الميل الاول : ط = ميلا اولا : غ ، ف ، ل ، م . /3/ يتحدان] تتحدان : ل . /5/ وبين فلك] وفلك : ف . /6/ قطب] + فلك : ف . /8/ إن] وأن : غ . /9/ تقطع] تقع : غ . /10/ اعتبر] (كذا في كل النسخ ولكن نجد تاء فوق السطر في م) . /11/ الاعتدال] فام . /12/ العروض] العرض : ف ، م . /13/ وتكون] ويكون : م . إحداها] احديهما : ف . المارة] المار : ض . /14/ كل قسم] + منها : م . /16/ البرج] البروج : ض ، ف . /17/ تسمى أيضاً] ايضا تسمى : غ ، ف . أيضاً] -ط . /20/ الأربعة] -ض ، -ط ، -ل . واثنتان] واثنان : م . نوعان] هاف . وهما] وهي : ل .

119 /22/ وأما] اما : غ . /7–8/ بأسرها أيضاً] ف ، ل = ايضا باسرها : ض ، ط ، م . /7/ بأسرها] -غ . /8/ تمرّان] يمران : ل . /9/ بقطبيها] بقطبيهما : م . /10/ وتسميان] ويسميان : ف . /11/ أو بين قطب الأفق] فال . /12/ أو المنطقتين] والمنطقتين : م . /20/ سماء] السماء : ف ، م . /21/ وبقطبي] هام . وهي] وهو : غ . النصفين]

121 نصفين : غ . /22/ وتسمى] ويسمى : ض ، ف . /1/ التي] -ل . /2/ النقطة] النقط : غ . /11/ من] هال .

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/16–17/ وقد يظن] طاف . /18/ أقدم] + زمانا : غ . /19/ ونصف جزء] هاف . /20/ جزءاً] -ف . /23/ يتحرك فلكُ البروج بتلك الحركة] ف (بدون الضمة) ، ل = يُحرك فلكَ البروج تلك الحركة] ض ، ط ، غ ، م (الحركات

123 موجودة في ض فقط) . /2–16/ وتلك الغاية ...في بقعة بعينها] هاض (من

«مرتين» (سطر 3) إلى «التقدير» (سطر 15)) ، هاط (من «مرتين» (سطر 3) إلى «بعينها» (سطر 16) ؛ مع رمز «خ اصح») ، ف ، ل = وتلك الغاية يمكن ان تكون بعد انطباقها على معدل النهار ومفارقتها* اياه ويمكن ان تكون حال انطباقها ويمكن ان تكون قبل انطباقها وعلى التقدير الاول يمكن تبادل نصفي فلك البروج اعني الشمالي والجنوبي بالتمام وعلى التقدير الثاني يمكن ذلك في البعض وعلى التقدير الثالث لا يمكن ذلك الا ان النهار والليل يصيران متساويين عند الانطباق في جميع الاحوال* وتبطل فصول السنة وعلى التقدير الرابع† لا يكون ذلك الا ان الارتفاعات ومقادير الايام والليالي تزيد وتنقص في بقعة بعينها : ض (مشطوب من «ومفارقتها*» إلى «الاحوال*» ؛ «الرابع†» مغيرة إلى «الثامن») ، ط ، م ، (كذا في شرح النيسابوري) ، (كذا موجودة كعبارة النسخة القديمة في شرح البيرجندي إلاّ أنّ «اعني الشمالي والجنوبي» ناقصة) = وتلك الغاية يمكن ان تكون حال انطباقها الاول ويمكن ان تكون قبل انطباقها الاول وعلى تقدير الاول والثاني اذا انطبق مرتين يمكن تبادل نصفي فلك البروج اعني الشمالي والجنوبي من سطحه ومنطقته بالتمام وعلى التقديرات الباقية يمكن ذلك في السطح بالعرض اما ان لم ينطبق عليه غير مرة واحدة من التقدير الثاني يمكن التبادل في المنطقة فقط وعلى التقدير الثالث لا يمكن ذلك الا ان النهار والليل يصيران متساويين عند الانطباق في جميع الاحوال وتبطل فصول السنة وعلى التقدير الرابع لا يكون ذلك الا ان الارتفاعات ومقادير الايام والليالى تزيد وتنقص في بقعة بعينها : غ = (نجد التالية في شرح الخفري كعبارة النسخة القديمة : «ثم المنطقة ان تحركت فاما ان تتم الدورة او لا بل تتحرك الى غاية ما ثم تعود وتلك الغاية اما بعد انطباقها على معدل النهار ومفارقتها اياه مرة او مرتين واما حال احد الانطباقين واما قبله») . /5/ تصل] يصل : هاط . /6/ الأول] طا ها ض . /7/ الأوَل] الاولى : هاط . /9/ الأوَل] الاولى : ف . /11/ الثلاثة] الثلث : هاض . الأوَل] الاولى : هاط . /12/ السبعة الأوَل] طاف . الأوَل] الاولى : هاط . /13/ منطقة] فلك : هاض . المجاور] المجاوز : ل . /14/ النهار والليل] الليل والنهار : هاض . /14–15/ في جميع البقاع وتبطل فصول السنة] طا ها ض . /18/ واحداً] هاف . /19/ ست] ستة : ل ، م .

125 وجدوها] طام . /2/ منهما] منها : ل . كل] ف ، ل = -ض ، -ط ، -غ ، -م . /4/ تبطؤ] (بدون الهمزة في ض ، ط ، غ) =

تَبطو : ف ، ل = تبطُؤُ : م (ولعل المقصود تُبطئ) . /5/ وتسرع] وَتسْرَعُ : ل = وتُسرِعُ : م . /6/ ظنوا] ظنوه : ف . /8/ للاختلافين] فاض . /9/ يحرك] تحرك : ف . /10/ نصفيه] (كذا في كل النسخ) . /13/ فهذا] وهذا : غ . وهيئاته] وهيئته : غ . /14/ تحقّق] تحقيق : غ . /15/ بملازمة] يلازمه : غ . /17/ بحركته] -ض . /18/ حركته] طاف . به] -غ ، -ل . /20/ وإذا] واذ : ط . /22/ مداراتها] مدارتها : غ . /25/ فكل] وكل : ف . /2/ عنه]

127 -ف . النصف] -غ . /3/ أيضاً معدل النهار] ف ، م = معدل النهار ايضا : ض ، ط ، غ ، ل . /5/ أعظمهما] اعظمها : غ . جهة العرض] + فان كان العرض شماليا فيكون راس السرطان في منتصف القسم الاكبر من مداره وان كان جنوبيا فيكون منتصفه راس الجدي واصغر المدارات اليومية هو الذي يمر(؟) بمنتصف القسم الاكبر مرة واحدة : ها ط (مع رمز "خ") . يساوي] مساو : ف . /11/ المدارات] المدرات : ض . لكل كوكب] ها ل . /13/ أو إلى] والى : ف . بالضد] يبعد : غ . /16/ صيرورة] + تمام : غ . /18/ أو أبدي] وابدي : غ . /19–22/ مساوياً ... معدل النهار] -ط . /20/ أو الخفي] والخفي : غ . /21/ أو الخفاء] والخفاء : غ . /24/ قَنْطُورِس] م

129 (بإهمال الفتحة) = قنْطُورس : ض = قَنطورس : ف ، ل . /2/ فعرف] (كذا في ض ، ط ، ف ، ل ، م) = تعرف : غ . /3/ وتوهموا] فتوهموا : غ . /5/ رجل] -غ . وكانت] وكان : ض . /6/ ثمانية] (كذا في كل النسخ) . إحدى] احد : ف . /7/ والتِنّين] طاغ . وقَيْقاوُس] (مشكلة كذا في ض ، م) . /8/ والفَكّة] والفيكة : ض . رُكْبَتَيه] ركبته : غ ، ف ، ل . وشِلْياق] وشِلياق : ض = والشلياق : غ = وشَلْياف : ف = وشُلْيَاق : م . /10/ والدُلْفِين] والدُلْفَيْن : م . /11/ واثنتا عشرة] ط ، غ = واثنتا عشر : ض = واثنى عشر : ف = واثنا عشر : ل = واثنا عشرة : م . /12/ وخمس عشرة] م = وخمسة عشر : ض ، ط ، غ ، ف ، ل . /13/ قِيْطُس] ض = قَيْطُس : م . /15/ وقَنْطُورِس] وقَنطورس : ض = وقنطورِس : ف = وقَنْطورِس : م . /20/ متقاربة] -غ . وصغرها] وصغارها : ف . /23/ جعلته] (كذا في كل النسخ) . الأقسام] للاقسام : ف . /24/ لتكون] ليكون : ض ، م . /26/ فنّ] فهي : ض . مفرد] منفرد : ض (بإهمال النون) . /27/ هاهنا] منها : ف .

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131 /5/ تتشابه] يتشابه : ف ، م . أيضاً] فال . /8–10/ الذي نحن ... بمركز
العالم] -غ . /10/ وإما] او : غ . /15/ فتكون] ويكون : غ (بإهمال الياء) ، م .
هي(٢)] هاض . /16/ أقرب] + منه : ف . /17/ يُرى] يَرى : ف . /18/
وبمركز] ومركز : ض . /21/ ووصل] ووُصل : م . /22/ تكون] يكون : م .
133 /6/ فُرض] + فلك : ض . /9/ المركز] -ط . وفي] في : غ . جهته] (كذا في
137 كل النسخ) . /12/ جهتها] جهته : غ . /2/ تكون] يكون : م . من]
+ القطعة : ط . فإنّ] وان : م . /4/ بمختلفين] (كذا في كل النسخ) .
/6–8/ نسبة (-هاط) الخط الواصل بين (= من : هاط) مركز
(= المركز : هاض) الموافق وحضيض الخارج إلى نصف قطر الخارج كنسبة
الخط الواصل بين مركز الموافق وحضيض التدوير] هاض (مع ملاحظة "ح
في الاصلاح الجديد") ، هاط (مع رمز "خ") ، ف = نسبة الخط الواصل بين
مركز الموافق والبعد الاقرب من الخارج الى نصف قطر الخارج كنسبة الخط
الواصل بين مركز الموافق والبعد الاقرب من التدوير : ل = نسبة نصف قطر
الخارج المركز الى ما بين المركزين كنسبة نصف قطر
الحامل : ض ، ط ، غ ، م (انظر شرحنا والمقدمة) . /8/ قطر التدوير] هال .
/9/ الموافقي] موافقي : ض . /10/ في] -ل ، -م . /11/ متشابهتان]
متشابهتين : غ . /12/ يخلو] تخلو : م . /14/ الخارج] خارج : غ . /17/
ينقص] يُنقَصُ : ل . /18/ القطعة] + في القطعة : شاض . حركة الخارج]
الحركة الخارج : ض ، م . /19/ ينقص] يُنقَصُ : ل . /21/ وفي] + القطعة : غ .
139 /24/ وقوفين] الوقوفين : ف . /1/ وليخرج] ولنَخرُجْ : ف . عن مركزي] من
مركزي : م . /5/ مساوية] مساويا : ل . /8/ كل] -غ ، -ل . /9/ والتدوير]
او التدوير : ف . ممكن] -غ = ممكنا : ف . /12/ غايته] (كذا في كل
النسخ) . /13/ الأقرب] هاض . بطء] + متدرج : غ . /13–14/ وعند وصوله
141 إلى الخط الثاني] -غ . /14/ وقوفاً] -ل . /1/ ويكون] فيكون : غ ، م ("و"
فوق السطر وفوقه "خ") . /3/ في التدوير] هاض ، -ط ، هاف ، -م . /5/
كان] كانت : ض ، ط ، م . /8/ أوردناها] اوردنا : م . ههنا] -ف = هنا : غ =
هاهنا : م . /10/ للناظر] -غ . /11–12/ فلا بدّ ... بتلك الحركات] (مكرر
ومشطوب في ط) . /11/ فلا بدّ] + له : ف . /12/ تظهر] يظهر : ف ، م .

/13/ كلاً] كل : غ . /14–16/ المركز فلكاً ... واحد خارج] -غ . /15/ الموافق]
موافق : ف . مركزاهما] مركزهما : ط . /16/ مركز] المركز : غ . /17/
نقطة(١)] هاف . هي] وهي : غ ، ل . عليه] فاض . /18/ من] + من : ف .
مماسّ] يماس : غ . لمقعّر] بمقعر : غ . واحدة] + هي : ف . /21/
ومنطقته] ومنطقتَهُ : ف . ومنطقة] ومنطقةَ : ف . /24/ وفلكَ] ف =
143 وفلكُ : ل . كرة] كرةً : ف = كرةٌ : ل . /25/ محدّبها] محدبه : ل . /1/
سطحه] سطحَه : ف . /2/ محدّب] محدبَ : ض = محدبُ : ف . /3/
ومنطقة] ومنطقةَ : ف = ومنطقةُ : ل . /4/ الموافق] هاض . انفصال]
نقصان : ف . /9/ فيهما] منهما : غ . /10/ البعد الأبعد] البعد الاقرب : غ .
/11/ وهذه صورتهما] وصورتهما ما قد مضى : غ .

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145 /3/ تؤمّل] تأمل : غ . /5/ مركز جرمها] مركزها : غ . /6/ تعرّف]
ض ، ف ، ل ، م = يعرّف : ط . /7/ & /8/ أواسط] اوساط : ف . /8/ أصغر]
+ منه : فام . /9/ في(١)] فاغ . /12/ انتقالات] انقالات : ل . /13/ مركز]
هاض . /14/ تكون (بإهمال التاء)] ط ، ل = يكون : ف = وتكون :
ض ، غ ، م . /14–15/ وهي تتحرك وتحرك] وهو يتحرك ويحرك : ف
(ونجد «هو» في شرح الخفري) . /15/ توالي البروج] التوالي : ف . /16/
وتسمى] ويسمى : ف . حركة(٢)] حركةُ : ف ، ل . /18/ في] على : ط .
النصف] نصف : ف . /20/ لتتم] ليتم : ف = ليتمم : م . وتحدث]
147 + وتحدث : ض . /21/ تكون] -ف . /25/ الخارج] خارج : ف . /3/
فالحامل] والحامل : ط . /5/ عرض] عرضاً : ف . فلكيها] ط (ال «ا»
مضاف؟) ، ل = فلكيه : ض ، غ ، ف ، م . /7/ بقدره] بقدر ما : ض ، ط . /9/
فلكيها] ط (ال «ا» مضاف؟) ، ل = فلكيه : ض ، غ ، ف ، م . إليه] -غ =
اليها : ط (ال «ا» مضاف؟) . ويصير] وتصير : ل . /10/ يمكن] يكون : غ .
وينعدم] وتنعدم : ل . /11/ ويكون] طاف . بين] -ط . /13/ ستين]
+ جُزءاً : ل . متقدم] مقدم : ط . /14/ ونصف] -ض = + جزء : ف . /15/
149 بقيد] مقيّد : ف . /3/ هو] -ط ، -غ . /8/ الشمس(١)] + ومنه الى الممثل
على التوالي : ل . /8/ & /9/ دامت] دام : ض .

الباب الثاني ، الفصل السابع

/17/ عنها] عنه : ض . /19/ عائدٌ] عايدٌ : ض = عايدة : ط ، غ ، ف ، ل ، م .
/21/ بزمان قليل] بقليل : ط . مختلفاً] مختلف : ل . /23/ ومقابلتها] او
151 مقابلتها : ض . بعد أبعد] ابعد بعد : غ . /1/ كذلك] لذلك : ض ، ط ، ل ، م .
تربيعه] تربيعيه : ط ، ف . للشمس] -غ . /2/ مختلف] مختلفُ : ل =
مختلفَ : ف . /4/ فأثبتوا] فاثبتو : ض . /5/ هو] وهو : غ . /6/ المثل
لعطارد] ممثل عطارد : غ . /7/ الفلك] فلك : ض . /9/ مائلة] مايلا : غ .
/11/ فلك خارج] فلك الخارج : ط ، م . /12/ في سطح] -ض . سطح] -غ .
/13/ تدوير] التدوير : غ ، ل = التدويرِ : م (ال «ال» مشطوب) . /14/
لمنطقته] في المنطقة : غ . /16/ تتقاطعان] متقاطعان : ض ، ل =
يتقاطعان : ف . /17/ والجَوْزَهْر] والجوزهرين : م . جاوزها]
جازها : ض ، ط ، غ . /18/ هي(١)] هو : ل ، م . هي(٢)] -ل ، -م . /23–24/
153 في القمر] -ض ، هال . /25/ يتكثر] يكثر : غ . /1–2/ الكسوفات
والخسوفات] طاف . /2/ عن] من : ل . /3/ الجوزهر لاتّحاد موضوعيهما]
طاف . /4/ الجوزهر] الجوهر : ل . /5/ البسيطة] هام (مع رمز "خ") =
البطية : م . /7/ أيضاً] -ط . إحدى عشرة] احدى عشر : غ . /8/ الحركة]
-ض ، -ط ، -غ ، -م . /10/ أربعاً وعشرين] اربعٌ وعشرون : م = اربعة
وعشرين : ل . درجة وثلاثاً وعشرين] هال . وثلاثاً] وثلث : م = طا ها ل .
وعشرين(٢)] وعشرون : م . /11/ به] (كذا في كل النسخ) . /12/ ولكون]
ويكون : غ . /13/ واثنتا] واثنتى : ض ، ط ، ل ، م = واثنى : ف = وثنتا : غ .
/14/ هذا(١)] هام = ذلك : م . /15/ من] في : ل . /16/ الأوليين]
الاولين : ل . ثلاث] ثلثة : غ . عشرة(١)] عشر : ف . /17/ حركة]
الحركة : ض ، ل . تكون] -ط . /19–20/ اثنتي عشرة] اثنى
عشرة : غ ، م = اثنى عشر : ف . /20/ عن] من : ل . /24/ تسمى]
155 يسمى : ف ، م . /25/ مضعّفاً] مضعفها : ط . /1/ هذا] هذه : ف .
الوَسطيّين] الوسطين : ف . /2/ خارج] الخارج : غ . /3/ الحركات]
+ متشابهة : ف . /10/ حركته] هاض ، -ط . /12/ والتربيعين]
والحضيض : ط ، ف . /14/ والسرعة] او السرعة : ف . بأعيانها] بعينها : غ .
/16/ خارج مركز] خارج المركز : غ . /20/ إلى بطء أكثر] الى اكثر : ل =

157 اكثر : م . /21/ وغيرهما] وغيرها : ل . /1/ تلزمه] تلزمها : ض . الحركات] الحركة : غ . /2/ الأول] + هو : ل ، م . قطر] قال . /2–3/ في الاجتماعات والاستقبالات] ف ، ل ، شام = في الاجتماع والاستقبال : غ = -ض ، -ط . /4/ القمر] + يكون في الاجتماعات والاستقبالات] ض ، ط ، م . /4–5/ وتكون غايته] تام . /6/ وربعاً] وربع : ض ، ط ، م . /14/ ولما] ولِما : ض ، ف ، م . /15/ الأول] -غ . اختـــلاف] الاختـــلاف : ض . /17/ تثليثهـــا] تثليثـــه :
159 ض ، ط ، غ ، م . /18/ حركته] حركة : ل . /1–2/ كبعد ...المحاذاة] هام . /3/ قطر] القطر : ف . ستون] + جزءاً : ل . /5/ وكذلك] فكذلك : غ . /8/ المركز] + في : شاط . أو الحضيض] والحضيض : ط ، ل . ويكون زائداً] هاض . /9/ تعديل] التعديل : غ . /14/ وأما] اما : غ . /16/ وجنوبياً] جنوبيا : غ . /19/ فسيجيء] فيجي : ض . باب] فصل : ل . /22/ تدويره]
161 التدوير : غ . /25/ الخارج] خارج : ط . /4/ الحركة] الجرم : غ . /8/ التركيب] التركب : ف ، م . /9/ تعالى] -ض . /10/ سيجيء] وسيجي : ض . وصفه]
163 وضعه : ض = + ان شا الله تعالى : ف . /5/ الدورة] الذروة : ل = دوره : غ . /7/ من المثل] -ض . /9/ تتغير] تزول : ض . /10/ هو] -ف . /12/ المائل (١)]
165 الحامل : م . /1/ ذروته] دورته : م . /2/ فيه] + ومما : ف . /3/ حركته] حركة : غ . خاصته] الخاصةُ : ض . /5/ تقاطع] + عليها : ف . المثل (١)] للممثل : ف . /6/ وهي] وهو : ض ، ط ، غ . /7/ التوالي] + والله اعلم : ف .

الباب الثاني ، الفصل الثامن

/11/ حواليها] حواليه : ض ، غ ، م . منها] منه : ض ، غ ، ل ، م . شمالها] شماله : ض ، م = الشمال : غ . جنوبها] جنوبه : ض ، م = الجنوب : غ . /12/ هو] -ف . /14/ ويختفي] ويخفى : ض . فتسبقه] يسبقه : ض . /15/ ثم يقف ويستقيم] ويقف ثم يستقيم : ض ، ط . /17/ ولا] فلا : ض . /18/ وإذا] اذا : ف . /19/ لم توجد] لا توجد : ل = لم يوجد : م . متشابهة] متشاهد : ط . /20/ قدراً] دورا : ض ، ظ . /21/ أكثر] + واكبر : ض .
167 الجزء] -ض . /1/ تثليثيه] تثليثه : ط . مقابلة ذلك(٢)] مقابلته : ل = مقابله : م . /2/ ولكن] لكن : غ . في تلك] الى تلك : ل . /4/ لمقعّر] بمقعر : غ . فلك] -غ . /5/ ممثل] + فلك : ف . /6/ يسمى] ويسمى : غ .

/8/ المركز] طاف . /10/ يقاطع] تقاطع : ض . زوايا] طاف . /11/ فتَحدث] فيحدث : م . /13/ فلكه] فلك : غ ، ل ، م . /15/ ويكون] يكون : ط . في ثخن(٢)] طاف . /17/ وتكون] ويكون : ل . أربعةُ] اربع : غ ، ف . /22/ حركة] -ض . /23/ وتظهر] ويظهر : ض ، م . /25/ مركز] هال . /27/
169 حول مركزه] -غ . وتظهر] ويظهر : ض ، م . /12–13/ كما ... التدوير] -غ . /15/ ربع آخر] الربع الآخر : غ . /17/ ويعودان] ويعود : غ . /21/ بمتساويين] متساويين : ض ، ط . من أوج] في اوج : ط . /22/ مقابلته]
171 طاف = مقابليه : ل . /24/ وحركة الأوج] هام . /1/ والحركة] + والحركة : ض . فلك] -غ . /5/ من] عن : ض . بقدر] -غ . /7/ مركزه] مركزها : ض ، غ ، ل ، م . /10–11/ حركة مركز التدوير و] ف = -ض ، هاط ، -غ ، -ل ، هام . /12/ وتسمى] ويسمى : ض ، ط ، ف ، م . وتتوهم] ويتوهم : ف ، م = وتوهم : ض = فتتوهم : غ . /13/ تسمى] يسمى : ض ، ف ، م . /15/ مركز معدل] معدل مركز : غ . مركز(٢)] -ط . /16/ الوسطيان] الوسطيّان : ط ، ل . /19/ دورة] دور : ل . معدل] المعدل : م (ال «ال» مشطوب) . /20/ عند] + ما : شاف . /21/ وعند] عند : ط . /24/ اللازمة
173 لحركاته] هاض . اللازم] اللازمة : ف . /1/ على مركز العالم تَحدث] تحدث على مركز العالم : غ . /5/ الثاني] -ض ، -ط ، -غ ، -م = هاف . /8/ أبعد] + منه : ف ، ل . /9/ فينقص] فينتقص : م . /10/ على] + على : ف . /13/ الذروتين] الذورتين : م . /17/ والثاني] والآخر : ل . معدل] المعدل : م (ال «ال» مشطوب) . /18/ المركز] + زايدا : شاط . /22/ المذكور]
175, 177 المذكورة : غ . /1/ مختلفتين] -ط . /3/ وشكل مدار مركز التدوير] هاض . /4/ إلى] -ط . هكذا] + صورته في مقابلة هذه الصفحة : م . /6/ يجيء] سيجى : ف . باب] فصل : ل . مفرد] + والله اعلم : ف .

الباب الثاني ، الفصل التاسع

179 /4/ وجدوا] وجد : غ . /7/ بقليل] هاض . /9/ بعده] قبله : ض ، ط ، غ ، ل . /10/ فتخفى] فتختفي : ف . /11/ استقاماتها] استقامتها : ل . وإذا] فاذا : ط . /12/ بأعيانها] بعينها : ل . /14/ البعدُ] -ض ، فاط . /16/ عنه(١)] عنها : ل . /18/ ووجدوا] ووجد : غ . /19/ مقابل] يقابل : ط .

181 /1/ محدبه لزُحَل] لزحل محدبه : ف . /2/ ممثل(١)] + فلك : غ . /10/ مركزهُ فيه] فيه مركزه : م . /11/ ثابتة] + الميل : غ . /12/ منطقة] -غ . الممثل(٢)] + بحركة مركز التدوير : غ ، شاف ، هام = + بسبب حركة مركز التدوير : هاط . دائرةٌ] -ض ، هاط . مركزها مركز العالم] هاف ، ل = -ض ، -ط ، -غ ، -م . /15/ باب] فصل : ط ، ل . /16/ وأما] اما : ف . فالأولى] فا الاولى : ض . /17/ وفي العقدتين] والعقدتين : ض ، ط . /21/ ولذلك] وكذلك : م . التدوير] الكوكب : ض ، غ ، م . /24/ عمّا] مما : ط .

183 /1/ لِما] بما : ض . /6/ تلك(١).] ذلك : غ . /9/ وإذا] فاذا : ف . أُضيفت] اضفت : غ = اضيف : ض . /11/ وهي] وهو : غ . /16/ نسبة(٢)] -ض . /18/ الوسطى] المرئية : ل . /19/ فضل] + حركة : ف . /20–19/ على ... الشمس] فام . /21/ مراكز] مركز : ل . الوسطى] المرئية : ل . /22/ أواسط] اوساط : ل . رجوعاتها] رجوعها : غ . /23/ ويعود] وتعود : ف ، ل ، م . /24/ تدويرها] التدوير : ف . /26/ تبعد] يبعد : م .

185 /1/ بالرصد] هال . /2/ ونصف(١)] -غ . وللمشتري] والمشتري : م . أحد] احدى : غ ، م (ال "ى" مشطوب) . /4/ ستين] + جزءا : غ ، ل . /9–8/ أعظم ... المريخ] -ض . /9/ فقالوا] وقالوا : ط . على] -غ . /11/ لكون المريخ] لكونه : ض ، ط ، غ ، م . /12/ ذروة تدويره] ذروة تدويرها : ض . /15/ في هذا العلم] هاط . العلم] الفن : غ . /16/ الاختلافات] اختلافات :

187 ض . /18/ غير مركز] هاض . /19/ فيه] -ل = بعينه : ط . /1/ من] + هذه : ض . /2/ على] + هذه : ض . /3/ والمائل] هاض . ومعدل المسير] والمعدل للمسير : غ . /5/ فهذا] وهذا : غ . هذا] فال . أفلاك] + هذا : ط . الكواكب] الكوكب : م .

الباب الثاني ، الفصل العاشر

189 /8/ فتقرب] فيقرب : ف ، ل . /9/ تفارقها] يفارقها : ل . /11–9/ غاية ... عنها] -ض . /10/ تنطبق] ينطبق : ف . /11/ النصفان] نصفان : ل . /12/ وتتم] ويتم : ض ، م = فتتم : غ . /15–14/ أو ذنبيهما] وذنبيهما : ض ، ط ، م . /15/ فإذا] + كان فاذا : شاض . /17/ فيصير] ويصير : ض ، ط ، غ ، م . النصف] نصف : ف . /18/ النصف] نصف : ف . الميل] + غايته : غ . /20/

191 الميلان] المَيَلاَن : ف . /22/ على] الى : ض . /24–2/ شمالياً ... مركزه إليه] -غ . /1/ يصير] هام . /4/ يبلغا] م (ال «ا» مضاف) . /7/ دائماً] -غ . /8/ يذكرهما] يذكرها : غ . /9/ متقدم] مقدّم : ف . /9–10/ متقدم ... زحل] -ض . /10/ وذنب زحل] وذنبه : ل . وذنب ... درجة] -ط ، هام ، (يوجد بين "بسبعين درجة" و "وذنب المشتري" في غ) . /11/ متقدم ... المشتري] -ض . وذنب المشتري] وذنبه : ل . /11–12/ وذنب ... درجات] -ط ، هام . /12/ درجات] درجة : غ . /19/ للعلوية] العلوية : ض ، ط . وللسفليين] وللسفلية : ض . /23/ تكون] يكون : ل ، م . /25/ في غاية] غايته في : ط .

193 /1/ الجنوبي] الجنوب : غ . ثمانياً] ثمان : ض ، ط = ثمانى : غ ، ل ، م = ثمانية : ف . في غاية] غايته في : ط . /2/ الجنوبي] الجنوب : غ . /3/ في غاية] غايته في : ط . /4/ الجنوبي] الجنوب : غ . /5/ في غاية] غايته في : ط . الجنوبي] الجنوب : غ . ثمانياً] ثمان : ض ، ط = ثمانى : غ ، ف ، ل ، م . /6/ الشمالي] + ثلثة اجزاء و : شاف . اثنتين] اثنين : ض ، غ ، ف ، ل . /7/ الجنوبي] الجنوب : غ . /8/ في غاية] غايته في : ط . واثنتين] واثنين : غ ، ف ، ل ، م = وثنتين : ض . /9/ الجنوبي] الجنوب : غ . السفليان] السفليات : م . فالزهرة] هاط . /11/ مركزه] مركزها : غ . /12/ مالت] مال : غ . /15/ غايتي] غاية : ض . /18/ وليس] + وليس : م . /21/ يكون] -ل . /22/ المثلة] المثل : ف . تدويريهما] تدويرهما : ض ، ط ، ف ، م . /24/ ينحرف] منحرف : غ ، م . /26/ ومقابلته] ومقابله : ف . /27/ يجاوز] تجاوز : ض . وينتقص] وينقص : ل . /7–8/ والحضيض ... الأوج] هال ، هام . /12/ تعالى] -ض ، -ط .

الباب الثاني ، الفصل الحادي عشر

/16/ الإشكالات] المشكلات : غ . /18/ هيئة] فام . /20/ هي] وهي : ط . /21/ الأخرى] الآخر : ض ، ط (بدون المَدة) . وفُرضتا] وفرضنا : ض ، غ . /22–23/ الدائرة الصغيرة] دايرة صغيرة : غ . /23/ ولتكن] وليكن : ض ، م . /23–1/ ولتكن ... الصغيرة] -غ . /1/ تكون] يكون : ف . /2/ فتتم]

197 فيتم : ض ، ف ، م . /3/ المارّ] المارة : غ . /4/ أربعة] ض ، غ ، ف ، ل ، م = اربعا : ط . /5/ وهي هذه] -ض ، -ط ، -غ ، -م . هذه] هذا : ل . /2/

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فلتكن] فليكن : ض ، ف ، م . /3/ جـ ه د] جـ د ه : ف . /4/ وقطرها] طاف .
/5/ نقطة(٢)] -ف . ولتكن] وليكن : م . معهما] طاف . /6/ لتتحرك]
ليتحرك : ض . جـ ه د] جـ د ه : ف . ولتنتقل] ولينتقل :ض ، ف ، م .
/6–7/ أن تقطع قوسَ] طاف . /7/ ولتتحرك] وليتحرك : ض ، ف . /8/ تلك
الحركة ولينتقل] طاف . /10/ ضعف] اضعف : م . لأجل الحركتين]
طاف . /11/ المتساويتين] المتساويين : ض . /12/ جـ د ه] زده : ل . /14/
201 دائماً] -ط . /2/ الدايرة] ف ، ل = منطقة : ض ، ط ، غ ، م . مدارَ]
مدارا : ط . /3/ التدوير] + فيها : ض ، ا : ض ، ط ، غ ، شاف ، م . /3–4/
ومن...الصغيرة] هاط . /3/ الدائرة] ف ، ل = المنطقة : ض =منطقة : ط ، غ ، م .
/4/ الدائرة] ف ، ل = منطقة : ض ، ط ، غ ، م . /7/ حركة] الحركة : غ .
لترد القطر] طاف . /9/ فيها] فيه : م . قطر(١)] -ض . الدائرة : ف ، ل =
المنطقة : غ = منطقة : ض ، ط ، م . قطر(٢)] -ض . /10/ الدائرة] ف ، ل =
منطقة : ض ، ط ، غ ، م . /11/ منطبق] + قطرها : ل . مترددة] مترددا : ل .
/12/ طرفيه] طرفيها : م = طرفها : ض ، ط ، غ . زائلة] زايلٍ : ل . /14–15/
مركزه ... القمر] ف ، ل = -ض ، -ط ، -غ ، -م . /16/ وينبغي]
فينبغي : غ . لئلاّ] ليلا : غ ، ف ، م . تشغل] يشغل : ف . /17/ كثيراً (كذا
في ف ، م) . /18–20/ والأخرى ... المركزين] -غ . /19–20/ مركزها ... فيكون]
-ض ، -ط ، -م . /21/ يحيط] محيط : ل . /22/ المحيط] المحيطة : غ .
عند] هاط (مع رمز «صح») ، ف ، ل = بقرب من : ض ، ط ، غ ، م . /23/
203 قطر] قطرَ : ل . /2/ مع] + تمام : ض ، ط ، غ ، هام . للحامل]
الحامل : غ . /6/ نزل] لم يزل : ط . الكرة] -ف = الدايرة : ل . /8/ وتلي]
ويلي : ض ، ط ، ف ، ل . /10/ شبيه] شبيهة : غ . بمحيط] لمحيط : غ . /13/
وتماسّ] ويماس : ف ، ل . المحيطةُ] المحيط : ض . /14/ بقرب] + من :
شاض . فكان التدوير] -غ . /16/ تتحرك] يتحرك : ف . القطر] م =
الوجه : هام (مع رمز «خ») . /17/ عن] من : ض . /19/ نقطة منه] منه
نقطة : ط . /20/ وتقابلها] ويقابلها : ل . /21/ وتكون] ويكون : ف ، ل ، م .
205, 207 /4/ الفلك] فلك : غ . /5/ المركز] فام . /3/ تربيع الأوج] التربيع للاوج : غ .
209 /5–6/ وكان...والأقرب] هاض . /6/ البعد] طاف . /1/ فيه] + وبين
منتصف البعدين الآخرين : م . مما] من نصف ما : م . /3/ مطابقاً]
للأصل] -ط . /3–5/ مطابقة...عليه] هاغ . /4/ الحساب] الحسّاب : ف .

/4–5/ في هذا ... الحساب] هام . /6/ وغايته تكون] وتكون غايته : ف .
تكون] يكون : ل ، م . /9/ الدايرة] هاط (مع رمز "خ اصح") ، ف ، ل =
منطقة : ط = منطقة الكرة : ض ، غ ، م . /10/ ومعدل المسير] والمعدل
للمسير : غ . الدايرة] فاط (مع رمز "خ اصح") ، ف ، ل = منطقة
الكرة : ض ، ط ، غ ، م . /10–12/ ضعف ... الكبيرة] -غ . /11/ فلكاً]
فلك : ض . مركزها] ض ، ف ، ل ، م = مركزه : هاط (مع رمز "صح") .
/12/ الكرة] هاف = الكرةَ : ل . فيه] (كذا في كل النسخ) . /16/ ثلاث]
ثلثة : غ . معدل] المعدل : ض . /18/ في] -غ . /19/ تتركب] يتركب : ف .
/21/ بذلك] بهذا : ض ، ط ، غ ، م . الموضع ... تعالى] تام . تعالى(٢)]
-ض . /23/ العلم] الفن : غ . النقطة (توجد كلمة مشطوبة بعدها في ض) .
211 /1/ الأوسطين] الوسطين : ض . /2/ يُبيّن] يتبين : ض = + لي : ط . /4/
وأنا أقول] طاض . تداوير] التداوير : غ (ال "ال" مشطوب) . الكواكب]
كواكب : ف . /5/ عرضية] عرضي : غ . تَخرج] يَخرُج : ل . بها] هاض .
/7/ فليتوهم] فاليُتوهم : ف . /8/ المنطقة به] به المنطقة : ط ، غ ، م .
سطحها] سطحه : ض ، ط ، غ ، م . هي] هو : ض ، ط ، غ ، م . /9/
مواضعها] موضعها : ض . تلتوي على نفسها] طاف . /12/ والثانية]
والاخرى : ل . /14/ الذروة] هاض . /19/ وطرف] فطرف : م . /23/
بالمراكز] -ف ، -م . فكان] وكانّ : ط = وكان : غ ، (يوجد "و" فوق السطر
213 مع رمز "خ" في م) . متحركاً] متحرك : ض ، ط ، م . /5/ أقطار] -ل . /6/
تداوير] التداوير : غ ، ف . /8/ ميول] الميول : ض . مساوية]
متساوية : غ . /10/ تتشابه] يتشابه : ف . نقط] نقطة : ض ، ط ، غ ، ف .
/11/ تتشابه(١)] يتشابه : ف ، م . الدوائر] التداوير : غ . تتشابه(٢)]
يتشابه : ف . /12/ نقط] نقطة : ط ، غ ، ف . الدوائر] التداوير : غ . /13/
النقط] النقطة : ض ، ط ، غ ، ف . /14/ التداوير] التدوير : ط . الحوامل]
الحامل : ف . /15/ لتكون] ليكون : ف . /17–18/ عن ... الدوائر] هاض .
/18/ بقدر] قدر : هاض . الدوائر] التداوير : غ . /21/ للسفليين]
+ اقول : ط . /23/ هيئة] هذه : غ . /24/ يُضعّف] + هذا : غ . جميع]
215 فيه : غ . /1/ تتغير] يتغير : ف . /2/ النقط] النقطة : ط ، ف = نقطة : ض .
/5/ كرة] كرةً : ف ، ل ، م . /6/ ويكون] فيكون : غ . القطر] قطر : ض .
/9/ حركة] حركةً : ف ، ل . الذي] التي : ض ، ط ، م . المذكورة] -غ .

/12/ للدائرة] في الدايرة : ض ، غ ، ل . تلزم] يلزم : ض . /13/ القطر] للقطر : غ . /15/ التدوير] التداوير : ط . كرة] كرةً : ف ، ل . /16/ قطباها] وقطباها : غ . /17/ حركة] حركةً : ف ، ل . مساوية] مساويةً : ل ، م . /19/ كادت] كانت : غ . /20/ الكرة] الحركة : غ . /21/ القطر] قطر : غ . /22/ أخريان] -ف . لأجل] م = لاصل : هام (مع رمز "خ") .

217 /24/ باقي] ما في : غ . تصير] يصير : ط ، ف ، م . /1–25/ العلوية ... من] -غ . /1/ السفليين] + على : ف . /2/ إثبات] -ل . /3/ لتم] ليتم : م . /4/ أصول] + اهل : ض . /5/ إن] اذا : م . /6/ مساوٍ] مساويا : ط ، غ . /7/ لتمّ] ليتم : م . /8/ كرة] كرةً : ف . /9/ ذكرنا من] ذكرناه : م . نقطة]

219 -ض . /12/ الميل] ميل : ط . /2/ ونفرض دائرة] هاض . /3/ وبنقطتي] ونقطتي : غ . ولتكن] وليكن : م . ونفصل] ونُفصِّل : ض . /4/ منهما] منها : م . مساويتين] متساويتين ومساويين : ل = متساويين : ط ، ف . /5/ تكون] يكون : م . نقطتا] نقطتي : غ . للتدوير] التدوير : غ . /7/ النقطتين] القطبين : غ . /9/ للتدوير] -غ . نسميها] ونسميها : غ .

221 تتحرك] يتحرك : ف . /1/ مدارا] مدار : غ . /2/ يماسّانهما] يماسانه : ض = يماسانه منهما : غ . /5/ لضعفها] لنصفها : ض . /6/ قوسي] -ض . /8/ طرف(١)] -غ . /9/ بميلهما] مثلهما : غ . أضيف] (كذا في كل النسخ) . /11/ ولا بالعكس] وبالعكس : غ . /14/ نزيد ثلاث] يزيد ثلثُ : ل . /17/ تنطبق] ينطبق : ف . /18/ الجانب] جانب : ف . فتنطبق] فينطبق : م . /22/ تدوير] فاغ . /23/ الطولي] -غ . /24/ الوسطيين] الوسطين : ض . /25/ الفلك] فلك : غ . أيضاً] فال . /26/ تحيط] محيطة : ف = يحيط : م .

223 /1/ بخلاف ذلك] بخلافه : ف . /2/ المركز] -ط . /3/ هو] تاف . /5–4/ بالصغر والكبر] بالكبر والصغر : ض ، غ ، ل = بالصغير والكبير : م . /6/ حركة] حركتي : غ . /7/ وحركة] حركة : غ . /10/ لحلّ] يحل : ف . /11/ الباقي] الثاني : غ .

الباب الثاني ، الفصل الثاني عشر

/15/ مواضعُها(١)] مواضعه : غ . /16/ أقطار : ل = -ض ، -ط ، -غ ، -م ، هاف . /17/ فإنّ] بان : غ . /18/ ينتهي] وينتهي : غ . /19/ ينتهي]

وينتهي : غ . /21/ تمرّ] يمر : ل ، م . /22/ أقرب] فاط . /23/ زاوية الاختلاف] هاط . /1/ وهذه صورته] وصورته في الصفحة الاخرى : غ . /4/ 225
وأكثره] واكثر : غ . /5/ بين] من : غ . /8/ بالقياس] فاط . أقطار] ل = -ض ، -ط ، -غ ، هاف ، هام . /2/ في الحقيقة] هاط . /4/ ما بينهما] 227
منهما : غ . /5/ اختلف] اختلفت : ف . /6/ العرض] -ض . /7/ والقوسين] والقوسان : غ = + هما : فام . /9/ سماء] السماء : م (ال «ال» مشطوب) . /11/ اختلافه] اختلاف : غ . وفي] في : غ . /12/ الموضع(١)] غ ، ف ، ل = الوضع : ض ، ط ، م . /18/ الموضع] غ ، ف ، ل = الوضع : ض ، ط ، م . /19/ يكون له] -غ . /21–23/ اللّهم...الحقيقي] هاض . /22/ عن] من : غ . /23/ هناك يكون] يكون هناك : غ . يكون أيضاً] ايضاً يكون : ط ، م (؟) .
229 أيضاً] طاض ، هام . كان] فام . /24/ الحقيقي] -غ . /3/ وبرَصَد] (كذا مشكّلة في ف ، ل ، م) . يُتوصَّل] (كذا مشكّلة في ض) . /4/ يجيء] سيجيء : غ ، ط ، م . /5/ الحساب] الحسّاب : ض ، غ ، ف . /6/ يُخرج] + له : ض ، ط ، غ ، م .

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/10/ وفي] في : غ . /11/ وضعه] هاط . /14/ نصفه] + ابدا : غ . /17/ تتطابقان] يتطابقان : ف ، م . /18/ ويكون] يكون : غ . منه] منها : غ . /19/ وهو البَدْر] هاط . وتتقاطعان] ويتقاطعان : ف ، م . /22/ الذي(٢)]
231, 233 هاض . /2/ له] طاط = لها : ف . /4/ ليلاً] هاض . /1/ وكلما كان] طاف . /3/ ويدلّ ذلك] وذلك يدلّ : ف . على] + على : غ . /4/ لأنّ] + نور : ض . /5/ فكان...الأرض] -غ . زاد (١)] ازداد : ض . /7/ أيضاً] -غ . /10/ لستر ظلّها] يسترُ ظلَّها : ض . /12/ عليها] عليه : غ . /14/ تُسمى] فيسمى : ض = يسمى : ط ، م . /16/ الظلّ] + للقمر : ف . للقمر] -ف . /19–20/ نصف...انخسف] هال . /24/ لأنّ عرضه] لانه : ل . زاد]
235 + عرضه : ل . /26/ أيضاً] -ف . /1/ ويُجزّأ] ويُجزّى : ض (بدون الفتحة) ، ف = ويجزّء : غ = ويجزءُ : ل . /2/ تسمى] يسمى : غ ، ف ، م . وتقيّد] ويقيّد : ط . /3/ والجرمية] والحرويّة (؟) : غ (بإهمال الياء) . /4/ اثنتي عشرة] اثنى عشر : ط ، غ ، ف ، ل . /6/ ووقع] وقع : ف . /9/

لموضع] موضع : م . /12/ بحركته] بحركتها : ط . /18/ المرئي] للمراى : ف .
237 /25/ أكثر] اكبر : غ . /3/ أربعة] (كذا في كل النسخ) . وثلاثين]
239 + دقيقة : ف . قطر] طريق : ل . /4/ ست] ستة : م . /1/ انكسفت]
انكسف : ض ، غ . /2/ كلها] كله : ض . أكبر] اكثر : ل . /3/ بقي] (كذا
في كل النسخ) . وتسمى] ويسمى : م . /9/ وكان] كان : ض . يزاد]
تزاد : ط . /10/ تكون] يكون : ف . /11/ يكون] + امكان : ض ، ط ، غ ،
شاف ، م . /12/ ثماني عشرة] ثمانية عشر : ض ، غ ، ل ، م = ثماني
عشر : ط ، ف . /15/ طرفي] فاط ، ف = -ض ، -غ ، -ل ، -م . /16/ طرفي]
طرف : ف . /17/ استقبال واجتماع] اجتماع واستقبال : ط ، م (نجد ميمين
(= منعكس ؟) فوق الكلمتين) . /19–20/ الداخل في الخسوف والكاسف]
غ ، ف = الكاسف والداخل في الخسوف : ض ، ط ، ل ، م . /20/ أبداً
شرقيّهُ] شرقيّه ابداً : غ . /21/ غربي] غربية : غ .

الباب الثاني ، الفصل الرابع عشر

241 /2–3/ والاقترانات وأحوال الظهور والاختفاء] ف = والاقترانات وأحوال الظهور
والاختفاء والقرانات : م = وأحوال الظهور والاختفاء والقرانات : غ = وأحوال
الظهور والاختفاء والاقترانات : ض ، ط ، ل . /5/ البعيدة والقريبة] القريبة
والبعيدة : ف . /7/ إمّا] امّا : ض (غير مشكّلة في النسخ الأخرى) . /8/
مركزي] مركز : ف . /9/ وإمّا] وامِا : ض (غير مشكّلة في النسخ الأخرى) .
/13/ صاعد] صاعداً : م . مستعل] مستعلٍ : غ (بدون التنوين) ، ف . /16/
الكواكب] الكوكب : ض . /17/ مقادير] -ط . /20/ وعشياً] ض (بدون
التنوين) ، ف ، ل = وعشية : ط ، غ ، م . /21/ راجعة] ف ، ل = مستقيمة :
243 ض ، ط ، غ ، م . /1/ فارقتها] فارقها : ض . /3/ تقابلها] يقابلها : ف .
تغرب] فاط . /4/ تُرى] -ف . /6/ ويختفيا] ويختفى : ض (بإهمال الياء
الأول) . /8/ الاختلافات] الاختلاف : ط ، ف . /9/ الشمس] هاف . /10/
حدود] -ل . /13/ أحد] احدى : ف . ونصفاً] ونصف : ف . /16/ من
أحد القطبين] -ض . /18/ & /17/ يمرّ] تمر : ف . /18/ الناظر] الناظرين :
ض ، ط ، غ ، (ال "ين" فوق السطر في م) . إليهما] اليها : ط = + والله
اعلم : ف .

الباب الثالث ، الفصل الأول

245 /4/ اثنا عشر فصلاً] وفيه فصول : هاض = (يوجد قبل عنوان الباب في م) .
/6/ جمل] جملة : م . هيئة] هيات : ض . /7/ أول] اوايل : غ ، ل =
اولى : ف . /8/ المحيط وهو] -غ . /9/ هو (٢)] -م . /10/ الفلك] فلك : ض .
/12/ عن] من : ف . /13/ فسار] وسار : ف . /14/ ورجع] فرجع : ف . فيَ]
الى : ط ، غ ، ف ، م . المغرب] الغرب : ط ، غ ، ل ، م . /15/ الشرق]
المشرق : غ ، ف . في] الى : ط ، غ ، ف ، م . المشرق] الشرق : ض ، ط ، غ ،
ل ، م . المشرق] + إليه : ط ، غ ، ف ، م . الغرب] المغرب : ف . /18/ مما]
247 -غ . /19/ التي] هاض، -ط ، -غ ، -ل . /20/ تسمى] ويسمى : ف . /1/
انقسمت] اقتسمت : ط . بهما] بها : ف . /2/ غير مسكونة] -غ . /4/ إلى]
+ الى : ف . دوائر] دايرة : ض . /5/ وتتوهّم] ويتوهّم : ف = فتتوهّم : غ
(بإهمال التاءين) . /6/ لتمكّن] ليمكن : م . /9/ تقدّم] + في : فاض ، غ ، م .
/10/ زائداً] زايد : غ . اثنتي عشرة] اثنى عشر : غ ، ف = اثنتى
عشر : ض . /11/ من] + من : ف . /12/ توجد] يوجد : م . نهار]
النهار : ف . /13/ جنوبية] جنوبياً : ط (بدون تنوين) ، ل ، م . /14/
وغيرها] وغيرهما : ف . حُكي] يحكى : ف . /15/ نيّف] نَيْف : ض ، ف
(غير مشكّلة في النسخ الباقية ؛ ويقول ابن منظور : «ونيْف وهو لحن عند
الفصحاء») . درجات] درجة : غ . يُسكن] + فيها : ف . /16/ جاوز]
جاوزه : غ . /18/ المغرب] الغرب : ض . والشمال] والشمالي : ف . لا سيّما
الشرقي منه] هال . الشرقي] الشرقية : غ . /21/ عشرة] عشر : غ . /22/
249 في] من : غ . بحر] البحر : ف . /23/ أيضاً] -ف . /2/ ورنك] وَرْنَك : ف =
وَرِنك : ل . /4/ موانع] مواضع : غ . /5/ والرمال] -ط . كثير] كثيرة : غ .
/6/ يعرّفها] فعرفها : ل . /7/ قد] فاط . هذا] هاض ، -ل . /8/ الناحية]
الجانب : ف . /11/ ببيّن] يبيّن : ض = يتبيّن : ف (بإهمال الياءين) . /14/
ذلك] هاض . /17/ تجذب] تحدث : غ . الرطوبة] الرطوبات : غ . /19/
وتنتقل] وينتقل : م . وهذا] هذا : ف . /21/ أيضاً] -ط . /22/ هبوطي]
هبوط : ف . وتُسمّى] ويسمى : ض ، ف ، م . /24/ خرافات] (كذا في كل
النسخ إلاّ أنّ البيرجندي وضع «جزافات» وأورد «خرافات» أيضاً ويقول إنها
«في بعض النسخ») . /26/ بها] ض = به : هاض . أوضاعهما]
251 أوضاعها : غ ، ف . /27/ السماويّات] السماوات : م . /1/ في] + في : ض .

طرف] اطراف : م . يقع] -ف . /2/ فقسمها] فقسَمُوها : ل . /3/ طولاً]
شاض = عرضا لا طولا : هاض . فتتشابه] فيتشابه : ف ، ل = متشابه : غ =
فتشابه : ط (؟) ، م (؟) . /5/ ويكون] -ف . /6/ مقادير] مقدار : غ .
الأطوال] الاطول : ل . /7/ المغرب] الغرب : غ ، م . الطول] الاطوال : ل .
/8/ العروض] العرض : ف . متعيّن] فاغ . وقد] فقد : غ ، ل ، م (نجد "و"
فوق السطر مع رمز "خ") . /11/ جعلوا] -ط = + مبدا الطول : غ .
ساحل] الساحل : غ (ال "ال" مشطوب) . مبدأه] -غ . /13/ علمائهم]
+ من : غ . كَنْكدِز] (كذا في ف) = كنكْدزْ : م . /17/ وأوساطها]
واواسطها : ط ، م = واوسطها : غ ، ف . العروض] العرض : ط ، ل ، م ،
(نجد "و" فوق السطر في ض) . /18/ فهي هذه] فهذه : غ . /19/ اثنتا
عشرة(١)] اثنا عشر : غ . /19–20/ وثُلثا درجة] هال . /20/ ثلاث عشرة]
ثلثة عشر : ط ، غ ، ف = ثلاثة عشر : ل . ساعة] هاض ، -ل . /20–21/
ستّ عشرة] ستة عشر : ض ، ط ، ل = ستة عشرة : غ . /21–22/ ثلاث
عشرة] ثلاثة عشرة : غ . /22–23/ وربع والعرض ... ثلاث عشرة] -غ . /23/
أربع] اربعة : غ . /24/ سُدس] ض = وسدس : ط ، غ ، ف ، ل ، م . ثلاث
عشرة] ثلثة عشر : ط ، غ . عشرة] + ساعة : ف . /25/ ورُبع ... ونصف]
هال . سبع] سبعة : ض ، ط ، غ ، ف ، هال . /25–26/ أربع عشرة] اربعة
عشر : غ . /27/ عشرة] عشر : غ ، ف = + نصف : م . /1/ عشرة] عشر : غ . 253
ستّ] ستة : ط . /2/ أربع عشرة] اربعة عشر : غ . عشرة] هاض .
ونصف] -م . /4/ خمس عشرة] خمسة عشر : غ . /5/ وأربعون]
+ وثلث : غ . /6/ خمس عشرة] خمسة عشر : غ . /6–9/ والعرض ... ستّ
عشرة] هاط . /7/ وعُشر] وثمن ووسطه حيث النهار خمس واربعون ونصف
وعشر : غ . /8/ عشرة] عشر : غ . /9/ عشرة] + وربع : ض = عشر : غ .
ثمان] ثماني : ض ، ط ، غ ، ل ، م . /10/ عشرة] عشر : غ . /14/ يبلغ]
-غ . سبع] سبعة : غ ، ف . /15/ ثماني] ثمان : ف . عشرة] عشر : غ .
ثمان] (ثمانى في كل النسخ) . /16/ تسع] تسعة : غ . /18/ اثنتين]
اثنى : ف . /19/ ستّ] ستة : غ ، ف ، ل . /19–20/ ستّ ... العرض] هام .
/21/ العرض] -ف . سبع] سبعة : ف . /21–22/ سبع ... العرض(١)] -غ .
/22/ ثلاث] ثلاثة : ف . /23/ وأربعة ... ونصف] -غ . ثمان] ثمانى : ض ،
ط ، ف ، ل ، م . /26/ ولنشرع] ونشرع : ف .

الباب الثالث ، الفصل الثاني

255 /3/ دوائر ... الاستواء] -غ . تكون] هال . /5/ الليل والنهار] ف ، ل =
النهار والليل : ض ، ط ، غ ، م . /6/ ظهور] -غ . /6–8/ فإن ... محسوساً]
هاض . /7/ بسبب] طا ها ض = بحسب : ف . /8/ في السنة مرّتين] مرتين
في السنة : ف . /9/ نقطة] ف ، ل = نقطتي : ض ، ط ، غ ، م . /11/ عن]
من : ف . /14/ منطقة] هام (مع رمز "خ") = فلك : ط ، ف ، م =
المنطقة : غ . للأفق] والافق : غ . /15/ النصف] نصف : غ . /16/ النصف]
نصف : غ ./18/ تكون] يكون : ف . /20/ الاعتداليين] الاعتدال : ض ، ط ،
257 غ ، م . /21/ مبادئ] مبداء : ف . /22/ أوساط] اواسط : غ . /1/ تقطع]
يقطع : ف . وتسمى] ويسمى : ض ، ف ، ل ، م . /3/ مشرق] هاف .
تكون(٢)] فام ./4/ ومطلع] وبين مطلع : غ . /7/ تلبث] تثبت : غ (بإهمال
التاءين) = («تمكث» في شروح الجرجاني والخفري والبيرجندي ؛ «تلبث» في
شرح النيسابوري) . /9/ تكون(٢)] يكون : ف ، م . تكون(٣)]
يكون : ف ، م . /13/ تنكسر] ينكسر : ف ، م . /14/ بالآخر] بالاخرى : ط
(لعل ال "ى" مضاف) . /20/ تبعد] يبعد : ض . فهي] وهي : ل . /21/
259 بها] بها ما : ض ، ط ، ل ، م . /23/ فيُعلم] فليعلم : ض ، م . /3/ يدلّ]
ويدل : ط . عليه] -غ . /4/ جُعودة] جعود : غ . /6/ الإقليم] اقليم : غ .
تدل] يدل : ف ، ل ، م . /7/ سائر] غيرها من : غ . /10/ الفَجاجة] ل =
الفِجاجة : ف . اللازمين] اللازمتين : ل .

الباب الثالث ، الفصل الثالث

/12/ المواضع] -ف . عرض] عروض : ف . /13/ وتُسمّى] ويسمى : ض ،
ف ، م . /14/ المدارات] مدارات : غ . /15/ الأولى] تاض ، ف = -ط ، -غ ،
-ل ، -م . /17/ وكان] واذا كان العرض غير زايد على الميل الكلى
كان : ض . /20/ مختلفين] المختلفين : ف . أعظمهما] اعظمها : ل . /22/
261 متساويي] متساوي : ض ، ف ، م . على] عن : غ ، ل . /1/ للّيل]
باللّيل : غ ./3/ دائرتا] دايرة : ض . تمرّان] تمر : ض . /4/ من الكواكب]
263 -ف . /5/ تينك] -ف . تينك الدائرتين] تلك الدايرة : ض ، غ ، م . /1/

يكون] -ف . بُعده] فاط (مع رمز «صح») ، ف ، ل = -ض ، -غ ، -م .
من] عن : ل . النهار] + يكون : ف . /2/ يصل] + اليه : شاط . /5/ بُعده]
+ عن ذلك : شال . /7/ يقطع] + دايرة : م . /9/ الكوكب] الكواكب : ل .

الباب الثالث ، الفصل الرابع

/15/ وفي] في : ط ، ف ، م . /19/ انتصاف] انصاف : ف . /20–21/ يقع ...
265 الظاهر] هال . /21/ هو] وهو : غ . /23/ أعني] يعني : غ . /1/ الخفي ...
جهة القطب] هام . ويكون] فيكون : غ . /3/ تكون] يكون : م . /4/ تكن]
يكن : ض ، م . /6/ وفي] في : غ . /8/ ويصير] فيصير : م (نجد «و» فوق
الكلمة) . /9/ دورتهما] دوريهما : ل . نقطة] فاغ . /10/ الذي] التي : ل .
/11/ تقطع] يقطع : ف . فقط] -ض . وتصير] ويصير : ط ، م . /13/
ترجع] يرجع : ف ، ل . وتتناقص] ويتناقص : ف ، ل . تعود] يعود : ف ،
م . /14/ وتصير] ويصير : م . /16/ من] عن : ض ، ف . تنتهي] ينتهي :
ف ، م . /17/ لها] له : ض ، ط ، ل ، م . يكون(٢)] -غ . /18/ وتمام عرض
البلد] -غ . /20/ السيّارة] السيارات : غ . /22/ ساوى] يساوي : ف . /23/
267 والمغرب] هام . /23–24/ بازدياد العرض] -ض . /1/ عرضها] ط (ال «ا»
مضاف؟) ، ف = عرضه : ض ، غ ، ل ، م . /5/ فإذا] واذا : غ . ماسّةُ]
ماست : ض . /6/ قطب] فاط .. وماسّه] وماست : ط ، م . /8/ وانطبقت]
وانطبق : ف . /9/ عنه] -ف . /10/ دفعة] + واحدة : م . التالي] الثاني : ط .
/11/ & /12/ يريد] (علامة الإهمال فوق الراء في ف) = يُريد : م (بإهمال
الياءين) . /11/ الغروب] الغروبَ : ل . التالي] الثاني : ط (بإهمال الثاء
والنون) . /12/ الآخر] الاخرى : غ . /14/ أو الخريفي] والخريفي : ف .
/15–17/ في جميع ... بعد جزء] -غ . /17/ كذلك] وكذلك : ط ، غ ، م . في]
-ف . اليوم] يوم : غ . أن] -ط . حاله] الحالة : ط . /18/ هناك] -ط .
/19/ أن] -م . /20/ يحدث] حدث : غ . ليل] الليل : ف . /22/ يأخذ]
هاف .

الباب الثالث ، الفصل الخامس

269 /5/ يتساوى] تتساوى : ض . /6/ لها] -ط . متقابلتين] مقابلتين : ض ، ط . /7/ وتنقسم] وينقسم : ف . قسي] قسيٍ : م . /10/ وطرفا] فطرفا : ف . /11/ الباقيتان] الباقيان : غ . /13/ وتطلعَ] ويطلع : ض . /14/ والتي] والذي : غ . /15/ أعلى] فا ط . /17/ في] وفي : غ . وأسفل] فال . /19/ أيضاً] -ط . /20/ بقدر] -ف . /21/ بقدر] -م . /23/ المتبادلين] المتبادلتين : غ ، ف . /25/ ولكي] ولكن : ض . نتصور] لتصوّر : ض ، غ = (بإهمال النون في ط ، ف ، ل ، م) . هذه العروض] هذا العرض : ط = هذا العروض : ف . نمثّل] (بإهمال النون في ض ، ل) . /26/ العرض]
271 العروض : ف . الظهور] -غ . /1/ والسرطان] -غ . والقوس الأبدية] والابدية : ف . /3/ معكوسة] معكوسا : غ . /4/ على] + دايرة (بإهمال الياء) : هاف . /8/ وربع وسدس] وسدس وربع : م . /8/ & /9/ يريد (موقعان)] ويُريد : ض (بإهمال الياء الثاني) = (علامة الإهمال فوق الراء في ف) = يُريد : م (بإهمال الياءين) . /9/ الغروب] الغرُوبَ : ل . /10/ على
273 هذه الصورة] وهذه صورته : ط ، م = هكذا : ل . /2/ مستويين] -ض = مستويتين : ف . الربع] الربعَ : ض . /2/ & /3/ سعة] سعةُ : ف . /3/ مستويين] مستويتين : ف . الربع] الربعَ : ض ، ف . /4/ الانحطاط] انحطاط : ض ، ط ، م . /5/ الارتفاع] ارتفاع : ض ، ط ، غ ، م . /6/ نقطة]
275 -ض . /8/ الجانب] جانب : ف . /1/ يطلع] -غ . /6/ الغربي] -ل . سعة] لسعة : م . /8/ وقطب] هام . إلى] في : ط ، م . /9/ ست] ستة : غ ، ف .
277 /10/ الظاهر] -ض . /11/ للمعهود] المعهود : غ = للمعهودة : م . /1–8/ ثم ... الصورة] هام . /2/ مشرقهما] مشرقها : ف . /3/ إلى(١)] فاغ . ثم آخر] واخر : غ . /6/ مماساً] مماس : غ . الظاهر] هاط = الظاهرة : غ . /8/
279 المغرب] الغرب : ل . /1/ وارتفع] وليرتفع : غ . الأفق] + الشمال : ف . /2/ فتطلع] فيطلع : ض ، ف ، ل ، م . أجزاء] آخر : م . /6/ ثم ينتهي] وينتهي : ض . /7/ ويصير] فيصير : غ . /11/ ويتّضح] فيتضح : غ . وصفناه] وضعناه : ف . /14/ لقرب] بقرب : ض ، ط (بإهمال الباء) ، غ (بإهمال الباء) . /15/ من الأفق] هاط . /17/ من] في : غ . /18/ المغرب] الغرب : ل . وهذا] وهذه : غ . الأسئلة] («الاسولة» في كل النسخ) .

الباب الثالث ، الفصل السادس

/20/ المواضع] الموضع : ض . التي] الذي : ض . /21/ عرضها] عرضه : ض .
281 /2/ في] من : غ . /3/ يكون] -غ . /6/ فتكون] فيكون : ض . /9/ أكبر]
أكثر : غ ، ف . /11/ وتكون] فتكون (بإهمال التاء) : ط ، ل ، م . طلوع]
غروب : غ . /12/ تتبين] (بإهمال النون الأول في ض ، ط ، غ ، ل) =
نُبيّن : ف . /13/ انحطاطها] انحطاطهما : ض . غاية(٢)] الغاية : ض . /15/
عرضها] عروضها : غ . كله] الكلي : ل . /16/ بُعد] -ط . /17/ مساوٍ]
مساوى : ض . /18/ تماس] يماس : ف . دور واحد] دورة واحده : غ .
/19/ للتي] التي : ط ، ل ، م = للذي : غ . الكلي] كلى : ض . /20/ تكون]
+ لها : شاض . وليتذكر] (بإعجام الياء في ض ، ف ، م) = وتتذكر : غ .
/21/ وليحكم] ف ، م = ولنحكم : ل = (بإهمال الياء في النسخ الباقية) .
هاهنا] -ض = ههنا : ط ، غ ، ف .

الباب الثالث ، الفصل السابع

283 /2/ مَطالِع] (كذا مشكّلة في ل) . /3/ تطلع] يطلع : ض = تقع : غ (بإهمال
التاء) . /4/ مَطالِع] (كذا مشكّلة في م) . /5/ الدرَج] (ال «ال» مضاف في
م) = درج : ط ، ل . السَوَاء] بالسواء : ض = السما : ط = السوا : غ ، ف ،
ل = السواءِ : م . /6/ تختلف] (بإعجام التاء الأول في م) . خط] -غ . /7/
فكل] وكل : ل . /8/ معاً] -ض . /10/ فتكون] فيكون : م . الحدٰان]
-غ . /11/ الأرباع] الاوضاع : ل . /12/ نقط] نقطة : غ . سدس نصف]
نصف سدس : ض ، ط ، غ . نصف] ها ط . /14/ البرج] البروج : غ . أحد
حدّيهما] احديهما : غ (بإهمال الياء) . /16/ معه] -غ . /17/ بها]
بهما : ض . /18/ والباقيتان] والباقيان : غ . فلكون] فتكون : غ (بإهمال
التاء) . البرج] البروج : غ . /19/ حادةٍ] قايمه : ض . البرج] البروج : غ .
القول] القوس : غ . /20/ ومطالعهما] ومطالعها : غ . /21/ لأنّ] انّ : غ
285 (بدون الشدة) ، ل . /23/ تكون] يكون : م . /4/ تكون] يكون : ف ، م .
أواسط] اوساط : ض ، ل . ويكون] فيكون : ض (بإهمال الياء) . /7/
ومنطقة] ومَنطقة : ل . /10/ والمغارب] والغارب : ض . كالمطالع] كالطالع :

ض . /11/ وأما] اما : غ . /13/ متحدّدين] مُتحدين : ض . الاعتدالين]
الاعتدال : ض . تلي] يلي : ف . /19/ الاعتدال] الاعتدالين : ف. /20/
قطعتين] قطبين : غ (بإهمال الباء والياء) . الذي] التي : ف . /22/
تكون(٢)] يكون : ف . /23/ كمطالع] لمطالع : ض . /25/ نظير] نظر : ض .
/26/ فيها] -ض . مدارا] مدار : ض ، ف ، ل . /27/ أعظم] اعظمى : غ .
287 فقد] قد : م . /3/ تكون] هال . /4/ تمثّلنا] تمتثل (؟) : غ (بإهمال التاء
الأول) . /6/ فإذا] واذا : غ . /8/ يبتدئ] تبتدئ : ف . /11/ فتعود]
فيعود : ف ، م (؟) . /12/ على] الى : ض .

الباب الثالث ، الفصل الثامن

/15/ بين] من : م . /18/ تقطعها] يقطعها : م . /20/ تقطعها]
يقطعها : ف ، م . الشمس] -ض . /21/ الأرض] -ض . /23/ فإنّه] وانه : ل .
289 تكون] لكون : غ = يكون : ل ، م . /3/ الوسطى] الوسطىَ : ف . /6/ معرفة]
فاط . /7/ الذي] التي : ف . بسبب] بحسب : غ . /8/ الأوسط] الأبعد : غ
(هناك إشارة فوقها ولكن لم نجد تصحيح) . /13/ ويكون] فيكون : م (يوجد
رمز "خ و" فوق السطر) . /15/ التفاوت] الاختلاف : ل . جعل] جُعل : ف .
/16/ بحسب] بسبب : غ . /18/ انتهاءها] ض = انتهاؤها : ط = انتهاها : غ
= وانتهاؤُها : ف = انتهاوها : ل = انتهاوُها : م . /20/ انتهاءها]
انتهاها : ض ، غ = انتهاؤها : ط = انتهاوها : ف ، ل = انتهاُها : م . /21/
جعل] جُعل : م . انتهاءها] انتهاوها : ض ، ط ، ل ، م = انتهاها : غ =
وانتهاوها : ف . /24/ ينقسم] تنقسم : ض . /25/ تزيدان] يزيدان : م .
291 مطالعيهما] مطالعهما : ل . /27/ مطالعها] مطالعيهما : ل = مطالعهما : غ . /1/
وهما] هال . /2/ من] عن : ف . /4/ كل] (مكرّر في ض) . مطالعها]
مَطالعهما : ل = مطالعيهما : غ . /8/ الأيّام(١)] ايام : ف . الحقيقية]
الحقيقه : م . /10/ للأيّام] لايام : م . /14/ كانت] (مكرّر في ض) . ناقصة]
ناقصاً : ل . /15/ زائدة] زايدا : غ . /16/ واتفق] ف ، ل = فاتفق : ض ،
293 ط ، غ ، م . /1/ أواخر] غ ، ف = آخر : ض ، ط ، ل ، م . /2/ بسبب]
بحسب : غ .

الباب الثالث ، الفصل التاسع

295 /4/ المرئي] المرأى : ض ، ط ، ف (بدون الهمزة في ض ، ط) . /8/ وضلعاه]
وضلعه : ف . الضلع] الظلع : غ (العصا مشطوب) . /10/ فإذن] فاذاً : ف .
/11/ الارض] ف ، ل ، م = الافق : ض ، ط ، غ . /12/ الأفق] ط =
الارض : فاط (مع رمز "خ") . /15/ يصدُق] يَصدُق : ل = يصْدق : م (غير
مشكّلة في النسخ الباقية) . /16/ منه] -ض = عنه : ف . والشمس] -غ .
297 /1/ قربت] قرب : ف . فصار] وصار : ض . /2/ بعكس] كعكس : ل ، م(؟) .
/3/ الأفق] ض ، غ ، ف ، هام (مع رمز "خ") = الارض : ط ، ل ، م . /4/
299 يكون] + يكون : ض . /5/ درجة] غ ، ف = -ض ، -ط ، -ل ، -م . /1/
بالصبح] + الكاذب : ف . /3/ القدر] بالقدر : ف . ويتبيّن] وتبيّنَ : م .

الباب الثالث ، الفصل العاشر

/7/ يتركب] تركب : ف . /8/ وهي] وهو : ف . والسنون] والسنين : ض .
/9/ هي] هو : ض = وهي : ف . /10–11/ إن كان تعديل نهار] -ض . /11/
هو] (كذا في كل النسخ) . /13/ نصفه] نصفها : ف . /13–14/ أو أنقص]
هاط (مع رمز «صح») ، ف = -ض ، -غ ، -ل ، -م . /14/ تسيره] يسيره :
ف ، ل . /15/ القوسين] القوس : ض . /17/ حصلت] حصل : ل .
والمعوّجة] أو المعوجة : ض . /18/ طول] اطول : ف (ال «ا» مشطوب) .
الساعات] ساعات : ف . /19/ وعدد] والعدد : غ . /22/ أوضاعه] الاوضاع :
301 غ . /1/ دوراً] -غ . /2/ ووجوده] ووجدوه : غ . متعذر] متقدر : غ
(بإهمال القاف والدال) . تعذُره] تقدّره : غ . لاختلاف] باختلاف : غ . /4/
أو من] ومن : غ ، ف . تشكّل] شكل : ف(؟) . /5/ الفضل] + ما : شام .
/7/ فيأخذون] فيأخذونه : ض . /8/ الكسور] ط ، غ ، ف ، ل = للكسور]
(كذا في ض ، م ، وفي شرحين النيسابوري والخفري ولعل في شرح
البيرجندي) . /9/ فيصير] يصير : ف . /10/ ثلاثين سنة] ثلثين سنة : فاغ .
وتسمى] ويسمى : ف . /13/ من] عن : ف . عود] عودة : ف . /16/ اثنا]
اثنى : غ ، ف . /17/ ومستعملوها] (ال «ها» غير واضح في م) . /21/ وتسمى]
ويسمى : ف ، م . /22/ المسترَقة] (مشكّلة كذا في شرح البيرجندي) =

مسترقة : ض . واللواحق] ولواحق : ض ، غ ، م . سنوهم]
303 سنيّهم (؟) : غ (بإهمال الياء) . /1/ يوم] هام . موضع] فال . /2/ على
شهور] على مثل شهور الفرس : ل . تدور] يدور : ف . /5–6/
اصطلاحية ... شمسية] هاف . /5/ وإن] فان : هاف . /6/ الشهور القمرية]
شهور للقمرية : هاف . /7/ ثلاث] ثلاثة : ل . أو في كل سنتين] -غ .
في(٢)] -ف . /8/ الأحد] احد : غ ، ف . كسر] الكسر : ف ، ل . /10/
ويسمونها سنين] -ف .

الباب الثالث ، الفصل الحادي عشر

/15/ كان] كانا : ف = كاكان : م . /19/ يكون] فاط . /21/ الظاهر] -غ .
/24/ الخارج] (كذا في كل النسخ = الخارجة في شروح النيسابوري
والجرجاني والخفري والبيرجندي) . تلاقي] (بإعجام التاء في ض) =
305 يلاقي : م . /3/ المذكورة] المذكور : ف . /3/ & /4/ تلاقي] يلاقى : م . /7–8/
وطلوع النصف الآخر] هاغ . /8/ فالكوكب] والكوكب : ط ، غ ، م . /10/
يمرّ] تمر : غ ، ف . لِما] كما : ض . /12/ فالكوكب] والكوكب : غ . يوافي]
يُوافقُ : ل (بإهمال الياء) = يوافق : غ (بإهمال الياء والقاف) . /14–15/
الظاهر ... القطب] -غ . /17/ النهار] النها : ض . /18/ فوق] فوقُ : ف .
ظاهراً] ظاهر : غ . /20/ الكواكب] الكوكب : غ . وصفناه] (غير واضح في
ض) . /23/ وفي] في : غ . /24/ يزيد] تزيد : ف . /25/ ويطّرد]
(بالتشديد في غ ، ف ، م) .

الباب الثالث ، الفصل الثاني عشر

307 /4/ يُرصد] (بالضمة في م) . /5/ ويُخط] (بالضمة في ف) . مستوية]
مستوٍ : غ . /6/ تُنصّف] تنصّف (بإهمال التاء) : ف = ينصّف : م .
الزاويةُ] (بالضمة في ف) . بينهما] م = منهما : فام (مع رمز "خ") . /7/
الخط] -غ . /9/ مستوية] مستو : ض ، ط ، غ ، ل (بالتنوين) . /10/
وتُرسم] وترسَم : ط (بدون الفتحة) ، م = ويرسم : ف = وَتَرْسم : ل (بإهمال
التاء) . ويُرصد] (بالضمة في م) = وَيَرصُد : ل (بإهمال الياء) . /11/

الظل] + الداخل : شاف . في] -ض ، -ط ، -غ ، -م . الدائرة] لدايرة : ض . /12/ ويُعلّم] (بالتشديد في م) . وتُنصّف] (بدون تشكيل في كل النسخ) . ويوصَل] ونُوصِل : ل = وَيُوصَل : م . /14/ عموداً المارّ] عمودالمار : غ . المشرق والمغرب] المغرب والمشرق : ل . /15/ الدائرةَ] دايرة : ض . يُقسم] (بالضمة في ف) = يقسِمُ : ل . كلُ] كُلَ : ل . ربع] قسم : ف . بتسعين] تسعين : ض ، ط ، ل ، م . /16/ ليعرف] يعرف : ل (بإهمال الياء) = (بإعجام الياء في ف ، م) . /17/ نقطتي] نقطة : ط ، ل . /20/ جزءاً] -ض . /22/ وثُلثا جزء] وثلثاً : ف (بإهمال الثاء الأول) . وكل] ط ، ف ، ل =

309 فكل : ض ، غ ، م . /1/ أقل] اكثر : غ . /2/ تساوى] تساوا : غ . /4/ وشمالية] شمالية : غ . يساوي] تساوى : ض ، م . /4–5/ عرضها عرض مكّة] ف (بدون الشدة) ، ل = عرضها وعرض مكه : ض ، ط ، م = عرضها عرضها : غ . /6–7/ أقل...طولها] -ف . /8/ هاهنا] ههنا : ط ، ل = هنا : ف . /9/ الشمس] هال . بسمت] + رأس : ض . /10/ الثامنة] الثانية : م . /11/ والفضل] والفرق : غ . /11–12/ بين نصف نهارها و] هاف . /12/ وبين نصف] ونصف : ط ، غ ، ل ، م . /13/ فليؤخذ] فلياخذ : ط . وتؤخذ] وَيُوخَذ : ل . خمسة] خمس : غ . /15/ وليُرصد] ولنرصد : ط = فليرصد : غ (بإهمال الفاء والياء) . /16/ شرقية] شرقيتها : غ . كانت] كان : غ . /17/ القبلة] + والله اعلم : ف .

الباب الرابع ، الفصل الأول

311 /2/ معرفة] -ط . الأبعاد والأجرام] الاجرام والأبعاد : م . والأجرام] + وفيه : فاض . /6–7/ من ذلك ما] مما : ف . /10/ لتكسير] + تلك : م . /16/ ساره] سارة : ض . /18/ جزءاً] هاض ، -غ . /19/ جزء] -ض . وهي] هي : ض ، غ ، ل . /21/ رضي الله عنه] -ل ، فام . /23/ فوجدوه]

313 وجدوه : ض . اثنين] اثنى : غ (بإهمال الثاء) . /1/ أربعة(١)] اربع : ف . /2/ وعشرون] وعشرين : ض . أصبعاً] فرسخا : غ . /4/ حصل] حط : ض . /5/ وهو] -ل . ثلاثة] ثلث : غ . /6/ ألفين] الفان : ط (ال «ا» مضاف) ، ل . وأربعين] واربعون : ط ، ل . /7/ ونصف فرسخ] ف ، ل = -ض ، -ط ، -غ ، -م . /8/ & /9/ تقدّر] يقدّر : ف (بدون الشدة) ، م . /8/ الأبعادُ] (بالضمة

في ل) . /10/ الدائرة] دايرة : م . /12/ الربع(١)] ربع : ط ، ف . المسكون] غ ، ف = -ض ، -ط ، -ل ، -م . /13/ القدر] هاف . /15/ أيضاً] هاف . أربعة] اربع : ف . الجزء] فال . /16/ جزء] +ويكون : شام . ألف] (بضمتين في ف ، م) . /17/ وسبعون] وسبعين : ض . تقريباً] هاف (مع رمز «صح») ، ل = -ض ، -ط ، -غ ، -م . وتكسيره] وتكسير : غ . /18/ ألف] الاف : غ . وسبعمائة] وسبعه مايه : غ . /18–19/ وستة وخمسون ألفاً وأربعمائة وعشرون فرسخاً] ل = وخمسة وستين الفا واربعماية وعشرين فرسخا : ض ، ط ، غ ، م = وستة وخمسون الفا ومأيتان واحد وثلثون فرسخاً («ونصف فرسخ» في الهامش مع رمز «صح») : ف . /21/ الفراسخ] فراسخ :
315 غ . /1/ تعرف] يعرف : ض . /3/ هاهنا] غ = ههنا : ض ، ط ، ف ، م = هُنَا : ل . /4/ وأما] +معرفة : شام . وعدنا] وعدناه : ف . هذا] -ط ، -ل ، -م . /6/ نضعف] (بإعجام النون في ف ، م) . /7/ وتكون] ويكون : غ . /8/ نأخذ] (بإعجام النون في ط ، ف ، ل ، م) . وأربع] واربعة : ف . ونقسم] (بإعجام النون في ف ، م) . /9/ خمسة] خمس : غ . وهو] وهي : ط ، ل . /10/ سُبع] هاض . ذراغ] الذراع : غ .

الباب الرابع ، الفصل الثاني

/13/ من] عن : غ ، ل . /14/ مركز] مراكز : ط . /16/ حساب] حسات : ض . ولم] فلم : غ (بإهمال الفاء) . /20/ ارتفاعه] ف ، ل = ارتفاعاته : ض ، ط ، غ ، م . /21/ سدس] وسدس : ط . /22/ الوقت] -ل .
317 /2–3/ كانت ... معلومة] هاض . من أضلاعه وزواياه معلومة] معلومه من اضلاعه وزواياه : غ . /4/ في] فال . إحدى] احد : غ . زواياه] زوياها (!) : ض . وهي] هي : غ . /9/ خرج] خَرَّج : ف . بالحساب] غ (بإهمال الباء الأول) ، ف = من الحساب : ض ، ط ، ل ، م . /10/ تسعة] تسعاً : ل . /11/
319 وربع جزء] طاف . /1/ التقاويم] +اعني : ض . /3/ عشرة(٢)] عشر : ض . دقيقة] +وَ : هاف (مع رمز «صح») . /3–4/ عن مركز العالم في ذلك الوقت] في ذلك الوقت عن مركز العالم : ل . /4/ وربع] وربعا : ض . وسدس] سدس : غ . وإذا] اذا : غ . /6/ إلى التقدير] هام . /7/ واحد] واحدا : ف . /10/ وذلك] هام . /12/ وثلاثاً] +واثنين : فاط (مع رمز «صح») .

الباب الرابع ، الفصل الثالث

/14/ أقطار] قُطْرى : ل . والشمس] -ل . /15/ وأبعاد الشمس والظل] هام . /18/ ثمانية] (كذا في كل النسخ) . /19/ ونصف] ونصفا : غ . وأخذ] ف = وَاخِذَ : ل = فاخذ : ض ، ط ، غ ، م . /20/ دقيقة] -ض . /21/ فعرف] فَعُرِفَ : ل . أنّ] فال . أحد] (كذا في كل النسخ) . /24/ نصف(٢)] غ ، ل = هاض (مع رمز "صح") ، فاط (مع رمز "صح") ، هاف (مع رمز
321 "صح") ، هام (مع رمز "صح") . /1/ وَجَد] وَجَدَ : ل = (بعدم الحركات في النسخ الباقية) . الشمس] القمر : غ . /2/ بعده] البعد : ط ، ل ، م . /4/ في
323 السطح] بالسطح : ط . /1/ وفَرَض] وفُرِض(؟) : ض = وَفَرضَ : ل . عن الجانب الآخر] -ف = في الجانب الاخر : هاض (مع رمز "صح") . /2/ الأبعد] + راس مخروط الظلّ : شاض . /3/ منهما] منها : ض . /4/ وسدس] + بالفرض : فاف . /6/ وهي] وهو : ف . /8/ تصير] بقى : ف . /8–9/ قطر القمر] القطر : ط . /9/ ضلع] طاغ . تكون] -ض . /12/ عن] ض ، ف = من : ط ، غ ، ل ، م . ست عشرة] م (لعل التاء المربوطة مضافة) = ستة عشر : ض ، ط ، ل = ست عشر : غ ، ف . /12–13/ وخُمسي دقيقة] -غ . /13/ قليل] -ف . /14/ أنّ] فام . /15/ فنصف] ونصف : ف . /16–17/ يكون معلوماً ... بذلك المقدار] هاط . /16/ عشرة] عشر : غ . /17/ وثمانياً] وثماني : ض ، ط ، غ ، م = وثمانية : ف (بدون الألِف) ، ل . /19/ ولأنّ] لانّ : ل . /20/ مخروط الظل] مخروط : ض = المخروط : ل . /21/ قطر(١)] + مخروط : غ . /22/ يكون] هاط (مع رمز "صح") . مجموع] جميع : غ . /22–23/ قطر الظل وقطر مخروط الظل] قطرى الظلّ ومخروطه الذي : ض = قطر الظل وقطر المخروط : ل . /22/ قطر الظل(٢)] + وقطر الظل : شاط . /25/ عشرة] عشر : ض ، ف . ونقص] وَنُقِصَ : ل . /26/
325 من] + نصف : غ = عن : ف . ست] ستة : ف . /2/ البعد بين (موقعان)] البعدين(؟) من : ط . /3/ كنسبة] نسبة : ض . ست] ستة : ف . /6/ ويبقى] ف ، ل = وكان : ض ، ط ، غ ، م . ثلاث] ثلاثة : ل . /7/ عشرة] عشر : ض ، ف ، ل . /9/ بعده] ض ، ط ، غ ، ف ، ل = بعدها : م = ("بعدها" في شروح النيسابوري والجرجاني والخفري والبيرجندي) . /11/ نصف(١)] -ل . /12/ خمس] خمسة : غ . وثمان] ط ، ل = وثماني :

ض ، غ ، ف ، م . وثلاثون] واربعون : ط ، غ . /14/ مركز] فال . /15/ خمساً وأربعين] خمس واربعون : ف . وثمانياً وثلاثين] ط ، ل = وثمان وثلثين : ض = وثمنية وثلثون : ف = وثمان وثلثون : م . /16/ عن مركز الأرض] هال . /17/ دقيقة] فاض . /18/ وسدساً] وسدس : ط = وَسُدسَ جُزءٍ : ل . يكون] -ف . /19/ الأرض] ض ، ط ، ف ، ل = الظل : م (في الهامش : «واما من مركز الارض فهو ٢٦٨») = («الظل» في شروح النيسابوري والجرجاني والخفري والبيرجندي) .

الباب الرابع ، الفصل الرابع

327 /2/ في مقدار جرم النيّرين] ف ، م ، (كذا في شرح النيسابوري) = في مقدارى جرم النيرين : ض = في مقدارى جرمى النيرين : ط = في مقدار قُطر الشَمْس وَجرْمى النَيّرين : ل ، (كذا في شرحي الجرجاني والخفري) = في مقدار جرمى النيرين : (كذا في شرح البيرجندي) . /4/ في البعد] بالابعاد : ض . قطر] هاض (مع رمز «صح») ، هاف (مع رمز «صح») . /5/ بعد (موقعان)] البعد : ط . /8/ أربعة] اربع : ض ، ط ، ل . وسدس] + جُزء : ل . /10/ معلوماً] معلوم : ف . /11/ واحداً] واحد : ط ، ف . وخُمسَين] ووخُمسَين (كذا) : ض = وخمسى واحد : ط («واحد» فوق السطر) = وخُمسين : ف ، م = وَخَمْسِين : ل . /14/ ضربت] ضرب : ض ، ط ، ل . /15/ علم] عُلم : ل . /15–16/ مائة وستة وستون مثلاً وربع وثُمن] ض ، ط ، ل ، م = مايه وتسعه وستون مثلا ونصف سبع : هاط (مع رمز «خ») =ماية وتسعه وستون مثلاً ونصف وسبع : ف . /16/ الأرض] ف ، ل = للارض : ض ، ط ، م . آلاف وستمائة] طاط . /17/ وثلثي مثل] هاض (مع رمز «صح») ، -ط ، هاف (مع رمز «صح») ، -ل ، -م . القمر] ض ، ف = للقمر : ط ، ل ، م . وأنّ] فانّ : ل . /18/ ونصف عشر] هاف (مع رمز «صح») ، ل = -ض ، -ط ، -م . القمر] ف ، م = للقمر : ض ، ط ، ل .

الباب الرابع ، الفصل الخامس

329 /5/ بعدها الأوسط] البعد الأوسط : م . عنه] -ض . الآخرين]
+ عنه : ض ، شاف . /7/ ونصفاً] ف = ونصف : ض ، ط ، ل ، م . قطر]
فاض . /9/ وهو] هو : ض . /14/ جُعل] (مشكّلة كذا في ل) . /15/
لتكون] ليكون : ف . /17/ عُلم] (مشكّلة كذا في ل) . /18–19/ من
331 الأجزاء] بالاجزا : م . /19/ حاملها] الحامل : ض ، ط ، ل . /2/ مركز]
مركزين : ط . /4/ حامله] الحامل : م . /5/ عُرف] (مشكّلة كذا في ل) . /7/
أو أحد] وَاحَد : ل . عشر] عشرة : ف . /8/ هي(١)] من : ف . /9/ منه]
-م . /10/ ووُجد] (مشكّلة كذا في ط) . الشمس الأقرب] اقرب
الشمس : ض ، ط («الشمس» في الهامش) . /15/ ونعود] ويعود : ط(؟) .
/18/ نصف] + نصف : ط . /19/ فعُلم] (مشكّلة كذا في ل) = + منه : ض .
بعديه] بعديها : ف . /20/ منه] فام . مثلاً] مثل : ل . /21/ فلك] فاض =
333 كرة : ض . /22/ ثُلثه] ثلاثه : ل . /1/ فذكروا] فذكرَ : ف . /2/ وأنّ]
فان : ل . /4/ هو] هاف . /5/ وتكون] ويكون : ف . /8/ وتسع]
وتسعة : ف . /9/ وسدساً] وسدس : ط ، ل . /11/ حصل] حصلت : ل .
/13/ وإن] هاف = واذا : ض ، م . يصير] صار : م . خمسة] خمس : ط ،
ل ، م . /14/ بالتقريب] هال . فإذن جرم] فجرم : ض ، ف = فاذاً
جرم : ط . ستة] ست : ف . مثلاً] + بالتقريب : ض ، ط ، ل . /15/
بالتقريب] -ض ، -ط ، -ل . /19/ ضرب] ضربَ : ل . /20/ واحد]
كواحد : ض . /22/ قطر(٢)] -ض ، هاف (مع رمز «صح») . /24/ ألفاً]
الف : ف . /25/ اثنان] اثنين : ض ، ف ، م . وعشرون] وعشرين : ض ،
ف ، م .

الباب الرابع ، الفصل السادس

335 /3/ ستة] ست : ف . /4/ ونصفاً] ونصف : ط ، ل ، م = (ال «ا» مضاف في
ض ، ف) . /6/ وبعده(١)] وبعد : ف . /8/ وستون في] طاض . /9/ فهو]
وهو : ط ، ل . /10/ وذكروا] وقد ذكروا : ض . /11/ فأخذوا] فاخذ : ض .
/14/ أخذ] أُخذَ : ل . ست عشرة] ستة عشر : ض ، ل = ست عشر : ف .

/16/ أخذ مكعّبه] اخذ مكعبَهُ : ف = اخِذَ مُكعّبهُ : ل . فكان] وكان : ف .
/17/ وأحداً] وإحدى : ط = واحدا : ض ، ف = وَاحداً : ل = واحَداً : م .
فعلم] فَعُلِمَ : ل . /18/ بالتقريب] تقريباً : م . /19/ المرّيخ] + يكون : ف .
/20/ وستون] وستين : م . /21/ وخمسمائة] وخمسه : ل . /23/ باب] فاط .
337 هيئات] هيئه : ض ، ل = هياه : ط ، م = هيآت : ف . /1/ بالحساب]
-ط ، -ل . /3/ حامله] الحامل : ض . /4/ جزءاً] + ونصف : هاف (مع رمز
«صح») . /5/ ونصف] ونصفاً : ف (ال «ا» مضاف) . /6/ أخذ] هاض (مع
رمز «صح») = أُخذَ : ل . /11/ لنصف قطر الأرض] للارض : ض ، ط ، ف ،
(«لنصف قطر» في هامش م) . /13/ ونصفاً] ف = ونصف : ض ، ط ، ل ، م .
/14/ وخُمس(٢)] وخمسا : ف . /15/ فقطر] قطر : ض . /16/ واحدٍ]
-ض ، هاف (مع رمز «صح») . كُعّبا] كعبنا : ض ، ف (بتشديد العين) .
/17/ ثلاثة] ثلثه : فاض = ثلثة : ف = ثلاثه : ل = اثنين : شاض ، ط ، م .
مرة(٢)] -ض . /18/ مركزيه] + بالحساب : شال . /19/ ثلاثة] ثلث : م .
/21/ وبعده(٢)] وبعد : م . /22/ سدس] وسدس : غ ، م ، (ال «و» مشطوب
339 في ف؟) . والأبعد] فالابعد : غ ، م . /24/ وثلاثة] وثلث : م . /3–4/
مثلاً ...عشرة] -غ . /4/ عشرة] عشر : ض . /6/ ثماني عشرة] ثمنية
عشر : ض ، غ ، ف = ثماني عشر : ط ، ل . وثُلثاً] وثلث : ض ، ط ، غ ، ل ،
م . /8/ أربعة أجزاء] جزء : ض . تقريباً] بالتقريب : ض ، ط ، ل . كُعّبا]
كعبنا : ض ، ف .

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/13/ جُعل] (بالضمة في ل ، م) . من الأرض] هاض . إذ] اذا : ط . تكن]
يكن : ل . /14/ أكثر] اكبر : ل . /16/ بالقياس] -ض . بعدها] بعده : غ .
/17/ ستة] ست : غ . لبعد] بعد : غ = كبعد : ف . /18/ ست عشرة] ستة
341 عشر : غ . وإذا] فاذا : غ ، م(؟) . /21/ كُعّبا] كعبنا : غ ، ف . /1/ يقسم]
نقسم : غ = يُقسَم (بإهمال الياء) : ل . ويجعل] وتجعل : ط = ونجعل : غ =
ويُجعل (بإهمال الياء) : ف . السدس] السدسُ : ف ، ل . /2/ التفاضل]
الفاضل : ض . ويقسم] وَيُقسَم (بإهمال الياء) : ل . /3/ ويجعل]
وتجعل : ط ، ف = ونجعل : غ = ويجعَل (بإهمال الياء) : ل . أكبر كل]

كل اكبر : م . /4/ وبين أوسطه] واوسطه : ض ، غ ، ف . /5/ وسدس] وثلث (بإهمال الثاء الأول) : غ . للأرض] الارض : غ . /8/ كواكب] الكواكب : م . /9/ باقي] الباقى من : غ . /10/ الكواكب] الكل : غ . /11/ يحوّل] يجعل : ض . /12/ الأول] والاول : غ . /14/ فكان] وكان : غ . وتسع] (كذا في كل النسخ) . /16/ وأربعون] واربعين : ف . وثلاثون] وثلثين : ف . /18/ وأربعمائة] وابعماية (كذا) : ف . ألفاً] الف : غ . وثمانمائة] وثماني مايه : ط . /20/ هاهنا] ههنا : ض ، ط ، ف . تعالى] -غ . ومصلّين] ونصلى (بإهمال النون) : ض . /21/ المصطفى...الوكيل] المصطفى واله حسبنا اللّه ونعم الوكيل نعم المولى ونعم النصير : غ = محمد وآله الطّاهرين : ف = المصطفى والحمد لواهب العقل ومبدع الكل وعليه التكلان : م .

§2. Figure Apparatus

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Fig. T1 ض البعد الأوسط (موقعان في صورة أصل الخارج)] بعد الاوسط . البعد
p. 133 الأبعد (في صورة أصل الخارج)] بعد الابعد . البعد الأقرب (في صورة أصل الخارج)] بعد الاقرب . غ مركز الخارج المركز] مركز خارج المركز . الفلك الخارج المركز] (ناقص) . ف (يوجد خط يصل بين «البعد الأوسط» و «البعد الأوسط» في صورة أصل التدوير) . ل مركز الخارج المركز] مركز الخارج . مركز العالم (في أصل التدوير)] (غير مقروء) . (الصورتان على تبادل الوضع وصورة أصل التدوير مدوّرة ربع دور باتجاه معاكس لحركة عقارب الساعة) . م (صورة أصل التدوير مدوّرة ربع دور باتجاه حركة عقارب الساعة) .

Fig. T2 ض مدار الكوكب] مدار كوكب . فلك الحامل] فلك حامل . مركز مدار
p. 135 الكوكب] مركز مدار كوكب . مركز الحامل] مركز حامل . فلك التدوير] (ناقص) . مركز التدوير] (ناقص) . ط مدار الكوكب] مدار كوكب . فلك الحامل] فلك حامل . مركز مدار الكوكب] مركز مدار كوكب . مركز الحامل] مركز حامل . فلك التدوير] (نافص) . مركز التدوير] (ناقص) . غ مدار الكوكب] (توجد على دائرة فلك الحامل) . فلك الحامل] (ناقص) . مركز مدار الكوكب] مركز المدار . كوكب (في الوضع الموافق للساعة الثانية عشرة)] (ناقص) . فلك التدوير] (ناقص) . مركز التدوير] (ناقص) . (الصورة مدوّرة ربع دور باتجاه معاكس لحركة عقارب الساعة) . ف مركز مدار الكوكب] (ناقص) . كوكب (في كل موقع)] الكوكب . كوكب (في

الوضع الموافق للساعة الثانية)] (ناقص) . فلك التدوير] (ناقص) . مركز التدوير] (ناقص) . **ل** كوكب (في الوضع الموافق للساعة الثانية)] (وضع الكلمة غير دقيق) . فلك التدوير] (توجد في مخطوط ل فقط) . مركز التدوير] (توجد في مخطوط ل فقط) . (الخط الواصل بين الكوكب ومركز التدوير غير موجود في الأوضاع الموافقة للساعات الثانية والرابعة والثامنة والعاشرة) . **م** مدار الكوكب] مدار كوكب . فلك الحامل] فلك حامل . فلك التدوير] (ناقص) . مركز التدوير] (ناقص) . («مركز الحامل» و «مركز مدار الكوكب» على تبادل الوضع ولذلك يكون تقاطع الخطوط الخارجة من مركز التدوير إلى مركز الحامل عند الموضع الذي هو في الحقيقة مركز مدار الكوكب) . (الصورة مدوّرة ربع دور باتجاه حركة عقارب الساعة) .

Fig. T3 p. 139 **ض** البعد الأبعد (في صورة أصل التدوير)] (ناقص) . منتصف الوتر (في صورة أصل الخارج)] (وضع النقطتين غير دقيق ولعل الناسخ أراد أن ينصّف قطعة الخط بين مركز الموافق المركز ومحيط الموافق المركز من جانب البعد الأبعد) . **غ** البعد الأبعد (في صورة أصل التدوير)] (ناقص) . البعد الأقرب (في صورة أصل التدوير)] (ناقص) . مركز الموافق المركز (موقعان)] مركز موافق المركز . الخارج المركز] خارج المركز . مركز الخارج المركز] مركز خارج المركز . منتصف الوتر (الواقع إلى اليسار من صورة أصل الخارج)] (ناقص) . الموافق المركز (من صورة أصل الخارج)] (ناقص) . **ف** مركز الخارج المركز] (ناقص) . منتصف الوتر (موقعان في صورة أصل التدوير)] (وضع النقطتين غير دقيق بل نحو مركز موافق المركز) . (الخط من البعد الأبعد إلى مركز الموافق المركز في صورة أصل التدوير ممدود إلى محيط الحامل الموافق المركز) . **ل** مركز الخارج المركز] (ناقص) . مركز الموافق المركز (موقعان)] مركز الموافق . الحامل الموافق المركز] الحامل وهو الموافق المركز . منتصف الوتر (موقعان في صورة أصل الخارج)] (وضع النقطتين غير دقيق بل نحو جانب البعد الأبعد) . **م** البعد الأقرب (في صورة أصل التدوير)] بعد الاقرب . (توجد «نقطة وقوف» عند نقطتي تقاطع الخطين الخارجين من مركز الموافق المركز إلى التدوير الواقعتي عن الجانبين من البعد الأقرب) . الحامل الموافق المركز]

(ناقص ؟) . الخارج المركز] (مكرر) . منتصف الوتر (الواقع في يمين صورة أصل الخارج)] (ناقص) . (لا يتقاطع الوتران في أصل الخارج المركز عند مركز الموافق المركز بل يتقاطعان بين مركز الموافق المركز ومركز الخارج المركز) . (يظهر أنّ منتصف الوتر في أصل الخارج المركز يوجد في المنتصف بين مركز الموافق المركز ومحيط الموافق المركز من جانب البعد الأبعد) . (الصورتان على تبادل الوضع وصورة أصل التدوير مدوّرة باتجاه حركة عقارب الساعة إلى الوضع الموافق للساعة الثانية) .

Fig. T4 **ض** المتمّم (الموجودة بين محدّب الخارج ومحدّب الموافق)] (ناقص) .
p. 143 التدوير (في صورة أصل التدوير)] (ناقص) . الذروة] (ناقص) . (لا يوجد الخطان من مركز الحامل المماسّان للتدوير والخط الواصل بين نقطتي التماسّ في التدوير) . **ط** المتمّم (الموجودة بين مقعّر الخارج ومقعّر الموافق)] (ناقص) . مقعّر الموافق] (توجد عند مقعّر الخارج ولا تكون عبارة تعريف على مقعّر الموافق) . التدوير (في صورة أصل التدوير)] (ناقص) . (لا يوجد الخطان من مركز الحامل المماسّان للتدوير والخط الواصل بين نقطتي التماسّ في التدوير) . **غ** المتمّم (الموجودة بين محدّب الخارج ومحدّب الموافق)] (ناقص) . كوكب (في صورة أصل الخارج المركز)] (ناقص وتوجد كلمة «التدوير» بدون رسمه) . الحضيض (في صورة أصل الخارج المركز)] مركز الموافق . الموافق المركز] (ناقص) . مقعّر الموافق] (ناقص) . محدّب الخارج] (ناقص) . مقعّر الخارج] خارج المركز . التدوير (في صورة أصل الحامل)] (ناقص) . الحامل] (ناقص) . مقعّر الحامل] (ناقص) . (لا يوجد الخطان من مركز الحامل المماسّان للتدوير والخط الواصل بين نقطتي التماسّ في التدوير) . **ف** التدوير (في صورة أصل الخارج المركز)] (ناقص) . الموافق المركز] (ناقص) . الخارج المركز] (ناقص) . الحضيض (في صورة أصل الخارج المركز)] (توجد على محدّب الخارج) . مقعّر الموافق] (ناقص) . الحضيض (في صورة أصل الحامل)] (ناقص) . الذروة] (ناقص) . الحامل] (ناقص) . (الخط من ذروة التدوير إلى مركز الحامل ممدود إلى محدّب الحامل) . **ل** الخارج المركز] الخارج . الموافق المركز] (ناقص) . كوكب (موقعان)] الكوكب . الذروة] ذروة التدوير . التدوير (في صورة أصل الحامل)] (ناقص) . الحضيض (في

صورة أصل الحامل)] (ناقص) . **م** التدوير (في صورة أصل الخارج)] تدوير . الحضيض (في صورة أصل الخارج)] (ناقص) . التدوير (في صورة أصل الحامل)] (ناقص) . (الصورتان على تبادل الوضع) . (لا يوجد الخطان من مركز الحامل المماسّان للتدوير والخط الواصل بين نقطتي التماسّ في التدوير) .

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Fig. T5 **ض** الأوج] (ناقص) . زاوية الإختلاف (موقعان)] (ناقص) . (الخط بين
p. 147 البعدين الأوسطين المارّ بمركز الخارج مستقيم) . المتمم (موقعان)] (ناقص) . **ط** المتمم (موقعان)] (ناقص) . **غ** الشمس (موقعان)] شمس. مركز الخارج] مركز خارج المركز . المتمم (موقعان)] (ناقص) . (تنتهي الخطوط إلى مركز الشمس عند محدّب الخارج) . (الخط بين البعدين الأوسطين المارّ بمركز الخارج مستقيم) . **ف** مركز الخارج] مركز الخارج المركز . (يوجد «الذروة المرئية» عند طرفي الخطين الخارجين من مركز العالم إلى مركز الشمس و«الذروة الوسطى» عند طرفي الخطين الخارجين من مركز الخارج إلى مركز الشمس) . (الخط بين البعدين الأوسطين المارّ بمركز الخارج مستقيم ولا يوجد الخطان الخارجان من مركز العالم إلى البعدين الأوسطين اللذين نحو الأوج) . **ل** مقعر الخارج] (زائد) . مقعر المثل] (زائد) . محدب الخارج المركز] (زائد) . محدب المثل] (زائد) . (يكون البعدين الأوسطين اللذين نحو الحضيض على محدب المثل) . (تنتهي الخطوط إلى مركز الشمس عند محدّب الخارج) . **م** (يوجد في يسار الصورة «طرف خط مركز العالم» عند محدّب الخارج في الموضع المناسب) . (يوجد في يسار الصورة «طرف خط مركز الخارج» عند محدّب الخارج في الموضع المناسب) . البعد الأوسط (الموقعان نحو الأوج)] + اول . البعد الأوسط (الموقعان نحو الحضيض)] + ثاني . المتمم (موقعان)] (ناقص) . (تنتهي الخطوط إلى مركز الشمس عند محدّب الخارج) . (يوجد رسم زائد للشمس عند الأوج) .

Fig. T6 p. 161 ض زاوية اختلاف] (توجد في موضع الأوج فقط) . مركز الخارج] مركز خارج . نقطة المحاذاة] نقطه محاذاه . الخارج المركز] فلك خارج . الفلك المائل] فلك مايل . الفلك المثل] فلك ممثل . (موضع كل مركز نحو الأوج بقدر ما بين المركزين) . المتمم] (ناقص) . ط الخارج المركز] فلك خارج . الفلك المائل] فلك مائل . الفلك المثل] فلك ممثل . المتمم] (ناقص) . غ قمر] القمر . ذروة] الذروة . مركز الخارج] مركز الحامل . («ذروة وسطى» (موقعان) «وذروة مرئية» (موقعان) توجد على محدّب فلك المائل) . زاوية اختلاف (ثلاثة مواقع)] (ناقص) . تعديل خاصة (موقعان)] (ناقص) . ذنب] (ناقص) . رأس] (ناقص) . حضيض] الحضيض (ويوجد على محدّب فلك الخارج) . الخارج المركز] فلك خارج مركز . الفلك المائل] (ناقص) . الفلك المثل] فلك المثل . المتمم] (ناقص) . ف المتمّم (بين مقعر المائل ومقعر الخارج)] (زائد) . (يوجد خطّان من نقطة المحاذاة إلى القمر وتسمى الزاوية التي بين أحدهما وبين الخط من نقطة المحاذاة إلى مركز التدوير «زاوية الاختلاف») . ذروة وسطى (موقعان)] الذروة الوسطى (وتوجدان على محدّب المائل) . ذروة مرئية (موقعان)] الذروة المرئية (وتوجدان على محدّب المائل) . حضيض] الحضيض (ويوجد على محدّب الخارج) . الفلك المائل] فلك مائل . الفلك المثل] فلك المثل . ل ذروة] ذروة التدوير في الاوج . قمر] القمر . («الذروة الوسطى» (موقعان) و«الذروة المرئية» (موقعان) توجد على محدّب فلك المائل) . (يوجد خطّ من نقطة المحاذاة إلى القمر وتسمى الزاوية التي بينه وبين الخط من نقطة المحاذاة إلى مركز التدوير «زاوية الاختلاف») . (لا يوجد «مركز الخارج» ويكون موضع مركز العالم ونقطة المحاذاة نحو الأوج بقدر ما بين المركزين) . زاوية الاختلاف (نحو الاوج)] (ناقص) . حضيض] الحضيض . المتمم] (ناقص) . (يتقاطع محدّب فلك المثل ومحدّب فلك المائل عند الرأس والذنب) . («الفلك المثل» على محدّب الفلك المائل) . م زاوية اختلاف (كل موقع إلاّ الذي نحو الأوج)] + ثاني . تعديل خاصة] زاوية تعديل خاصة (+ «اول») . مركز الخارج] مركز خارج . ذروة وسطى (موقعان)] ذروه الوسطى (وتوجد على محدّب فلك المائل) . ذروة مرئية (موقعان)] ذروه المرئية (وتوجد على محدّب فلك المائل) . الخارج المركز] الفلك الخارج .

Fig. T7 **ض** ، **ط** ، **ف** ، **م** منطقة المثل] (ناقص) . **غ** ، **ل** (يوجد خطّ
p. 163 واصل بين «موضع مركز التدوير في الاجتماع» وبين «موضع مركز التدوير في الاستقبال») . **ف** التربيع (موقعان)] تربيع . **ل** مركز العالم] (ناقص) . منطقة المثل] منطة(!) المثل .

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Fig. T8 **ض** مركز التدوير (يميناً)] زائد . متمم المدير من المثل] متمم المدير .
p. 175 الذروة المرئية (موقعان)] ذروه مراييه (الياءان بدون نقط) . الذروة الوسطى (موقعان)] ذروه . مركز معدل المسير] مركز المعدل للمسير . («مركز المعدل للمسير» عند موضع مركز المدير الحقيقي و«مركز المدير» عند موضع مركز الحامل الحقيقي و«مركز الحامل» فوق موضعه الحقيقي بقدر ما بين المركزين) . **ط** الذروة الوسطى] ذروه وسطى . الذروة المرئية] ذروه مرييه (الياءان بدون نقط) . متمم المدير من المثل] متمم المدير . مركز التدوير (يميناً)] زائد . (كل المراكز تحت مواضعها الحقيقية بقدر ما بين المركزين) . **غ** الذروة الوسطى] ذروه وسطى . الذروة المرئية] ذروه مراي . متمم الحامل من المدير (بين مقعر الحامل ومقعر المدير)] (ناقص) . متمم المدير من المثل (بين مقعر المدير ومقعر المثل)] (ناقص) . (ينتهي الخطين الواصلين بين الذروة الوسطى وبين مركز التدوير عند مركز المدير) . («مركز معدل المسير» عند موضع مركز المدير الحقيقي) . مركز التدوير] (يوجد في التدوير الفوقاني فقط) . التدوير] (ناقص) . **ف** متمم المدير من المثل (بين مقعر المدير ومقعر المثل)] متمم المدير من الحامل . (ينتهي الخطين الواصلين بين الذروة الوسطى وبين مركز التدوير عند مركز الحامل) . (ينتهي الخطان من محيط التدوير عند مركز الحامل) . مركز التدوير] (يوجد في التدوير الفوقاني فقط) . التدوير] (ناقص) . **ل** زاوية تعديل المركز والخاصة] زاوية التعديل . متمم الحامل من المدير (بين مقعر الحامل ومقعر المدير)] (ناقص) . متمم المدير من المثل (بين مقعر المدير ومقعر المثل)] (ناقص) . مركز التدوير] (ناقص) . الذروة المرئية (يميناً)] الذروة الوسطى . (ينتهي الخطان من محيط التدوير عند مركز معدل المسير) . **م** الذروة المرئية (موقعان)] ذروة المرئية . الذروة

الوسطى (موقعان)] ذروة الوسطى . (ينتهي الخطان من محيط التدوير عند مركز معدل المسير) . (ينتهي الخطان الواصلان بين الذروة المرئية وبين مركز التدوير عند مركز معدل المسير) . (ينتهي الخطان الواصلان بين الذروة الوسطى وبين مركز التدوير عند مركز المدير) . الحضيض (عند مقعر المدير ومقعر المثل)] (زائد) . التدوير (يساراً ويميناً)] (زائد) . مركز التدوير (فوقاً ويميناً)] (زائد) .

Fig. T9 p. 177 **ض** تربيع الأوج وهو حضيض الحامل (يميناً)] تربيع اوج المدير وهو حضيض الحامل . **ط** تربيع الأوج وهو حضيض الحامل (يميناً)] (يوجد على الفلك المدير) . **غ** تربيع الأوج وهو حضيض الحامل (يميناً)] تربيع الاوج وهو حضيض المدير والحامل . مدار التدوير] (ناقص) . الفلك المائل] (ناقص) . الفلك المدير] (ناقص) . **ف** مدار التدوير] (ناقص) . الفلك المائل] (ناقص) . الفلك المدير] (ناقص) . **ل** مدار التدوير] (ناقص) . **م** تربيع الأوج وهو حضيض الحامل (يميناً)] تربيع اوج المدير وهو حضيض الحامل .

Fig. T10 p. 187 **ض** زاوية اختلاف زائد] زاوية اختلاف . زاوية اختلاف ناقص] زاوية اختلاف . متمم (بين مقعر الحامل ومقعر المثل)] (ناقص) . مركز معدل المسير] مركز معدل مسير . مركز الحامل] مركز حامل . مركز العالم] (ناقص) . **غ** زاوية اختلاف زائد] (ناقص) . زاوية اختلاف ناقص] (ناقص) . متمم (بين مقعر الحامل ومقعر المثل)] (ناقص) . فلك ممثل] (ناقص) . فلك حامل] (ناقص) . **ف** زاوية اختلاف زائد] الاختلاف الزايد . زاوية اختلاف ناقص] الاختلاف الناقص . متمم (بين مقعر الحامل ومقعر المثل)] (ناقص) . فلك ممثل] (ناقص) . فلك حامل] (ناقص) . **ل** فلك ممثل] (ناقص) . فلك حامل] (ناقص) . كوكب] الكوكب . (الظاهر أن الناسخ وضع «مركز معدل المسير» عند النقطة التي هي المركز الحقيقي لدائرتي الحامل) . (ينتهي الخطان من محيط التدوير عند «مركز معدل المسير») . **م** (يكون «الذروة الوسطى» و «الذروة المرئية» على محدب المثل) . (الظاهر أن الناسخ وضع «مركز معدل المسير» عند النقطة التي هي المركز الحقيقي لدائرتي الحامل) .

Fig. T11 p. 197 غ إلى جهة يساره] الى يساره . والكبيرة نصفها] والكبيرة نصفا . ف تمّمت] تمت . ل الصغيرة نصفاً والكبيرة ربعاً] الصغيرة نصف دورة والكبيرة ربع دَورة . والكبيرة نصفها] والكبيرة نصف دَوْرَةٍ . م قطعت الصغيرة دورة] قطعت الصغيرة دوراً .

Fig. T12 p. 199 غ (الصورة مقلوبة) . (الخط بين "ز" و "ه" ناقص) .

Fig. T13 p. 205 ض منطقة الكبيرة] منطقة الكرة الكبيرة . منطقة الصغيرة] منطقة الكرة الصغيرة . الفلك المائل] (ناقص) . مركز الحامل ...] مر الحامل ... . (تتماسّ منطقتا الكبيرة والصغيرة عند ذروة التدوير بدلاً من مركز التدوير في الموضع الأول والموضع الثالث) . (لا يكون التدوير والمحيطة في جوف الكرة الصغيرة في الموضع الثاني والموضع الرابع) . ط منطقة الكبيرة] منطقة الكرة الكبيرة . منطقة الصغيرة] منطقة الكرة الصغيرة . ما هي بالسواد] ما هو بالسواد . (تتماسّ منطقتا الكبيرة والصغيرة عند ذروة التدوير بدلاً من مركز التدوير في الموضع الأول والموضع الثالث) . غ الفلك المائل] (ناقص) . منطقة الكبيرة] منطقة الكرة الكبيرة . منطقة الصغيرة] منطقة الكرة الصغيرة . مدار مركز التدوير] (ناقص) . ما هو بمنزلة المركز لمدار التدوير] ما هو مستركه (؟) المركز لمدار المدير . (تمر منطقة الكرة الصغيرة في ذروة التدوير بدلاً من مركز التدوير في الموضع الثاني والموضع الرابع) . ف منطقة الكبيرة] منطقة الكرة الكبيرة . منطقة الصغيرة] منطقة الكرة الصغيرة . المحيطة] حافظة . ما هو بمنزلة المركز لمدار التدوير] ما هو بمنزلة المركز لمدار المدير . «البعد» (عند نقطة تماسّ للكرة الكبيرة والكرة الصغيرة في الموضع الأول) و «القرب» (عند نقطة تماسّ للكرة الكبيرة والكرة الصغيرة في الموضع الثالث)] (زائد) . مركز الحامل والمائل وهو مركز العالم] مركز المايل والحامل وهو مركز العالم . م «الاوج» (عند نقطة تماسّ للكرة الكبيرة والكرة الصغيرة في الموضع الأول) و «الحضيض» (عند نقطة تماسّ للكرة الكبيرة والكرة الصغيرة في الموضع الثالث)] (زائد) . (تكون منطقة الكرة الصغيرة مماساً للكرة الصغيرة بدلاً من أن تكون موازية لها في الموضع الأول والموضع الثالث) . ما هو بمنزلة المركز لمدار التدوير] ما هو بمنزلة

المركز لمدار المدير . «الدايرة الموازية للمايل» (وهي تماسّ منطقة الكرة الكبيرة في كل المواضع)] (زائد) . الكرات المجسمة ما هي بالحمرة والدواير ما هي بالسواد (مع رمز "خ")] (في الهامش ؛ وعلى الصورة نفسها : «التي بالحمرة هي المجسّمة والتي بالسّواد هي الدايرة») .

Fig. T14 p. 207 **ض** الحامل] (ناقص) . مدار مركز التدوير] (ناقص) . (الحروف ناقصة) . **ط** الحامل] (ناقص) . مدار مركز التدوير] (ناقص) . **غ** الحامل] (ناقص) . مدار مركز التدوير] (ناقص) . منتصف ما بين البعدين] (ناقص) . (الحروف ناقصة) . **ف** الحامل] (ناقص) . مدار مركز التدوير] (ناقص) . البعد الأبعد] الابعد . البعد الأقرب] الاقرب . (الحروف ناقصة) . **ل** (توجد الحروف فقط بدون العبارات) . **م** مركز التدوير في البعد الأوسط (موقعان)] مركز التدوير وهو البعد الاوسط . (الحروف ناقصة) .

Fig. T15 p. 219 **ض** ، **ط** ، **غ** ، **ل** (الدائرتان السفليتان إلى جهة اليمين) . **غ** ح] ج . ج] ح . **ف** (كل الدوائر إلى جهة اليسار) . **م** (توجد صورتان : في الأولى الدائرتان السفليتان إلى جهة اليسار وفي الثانية هما إلى جهة اليمين) .

Fig. T16 p. 225 **ض** موضعه المرئي] موضع المراي . **ط** الأرض] (ناقص) . **غ** كوكب] الكوكب . موضع الناظر على سطح الأرض] (ناقص) . **ف** موضع الكوكب الحقيقي] الموضع الحقيقي . موضعه المرئي] الموضع المراي . مركز الأرض] مركز العالم . زاوية الاختلاف] زاوية اختلاف . الأفق المرئي والأفق الحقيقي] (مكرّران في الجانب اليميني) . **ل** موضعه المرئي] موضع الكوكب المرئي . كوكب] الكوكب . **م** نصف القطر (بين مركز الأرض وسمت الرأس)] (زائد) . موضعه المرئي] موضع المرئي .

Fig. T17 p. 231 **ض** القمر في بدء الانجلاء] القمر في تدوير(؟) الانجلاء . القمر في تمام الانجلاء] القمر في تدوير(؟) الانجلاء . نصفاً] نصفه . القمر في بدء المكث] القمر في اول(؟) الكسف . القمر في بدء الخسوف] القمر في تدوير(؟) الخسف . (الخط الواصل بين مركز الشمس ورأس المخروط ناقص) . **ط** رأس المخروط] راس مخروط الظل . مخروط النور] مخروط الظل . منطقة

مائل القمر] منطقة المايل . **غ** منطقة مائل القمر] (ناقص) . منطقة ممثل القمر] (ناقص) . عقدة] (ناقص) . مخروط النور] (ناقص) . الأرض] (ناقص) . له] (ناقص) . القمر مماس الظل ولا ينخسف] القمر مماس الارض . القمر في تمام الانجلاء] (ناقص) . دائرة الظل] (ناقص) . (الخط الواصل بين مركز الشمس ورأس المخروط ناقص) . **ف** منطقة مائل القمر] منطقة المايل . عقدة] العقدة . فلك الشمس] (يوجد على منطقة المائل) . القمر مماس الظل ولا ينخسف] القمر مماس دايرة الظل . القمر المنخسف نصفاً] (ناقص) . مخروط الظل] (ناقص) . **ل** مركز الشمس] مركزها . رأس المخروط] (ناقص) . القمر مماس الظل ولا ينخسف] القمر مماس للقمر بلا خسوف . القمر في وسط الخسوف مع العقدة] القمر في وسط الخسوف . القمر المنخسف نصفاً] القمر المنخسف نصفه . **م** فلك الشمس] (ناقص ؟) . القمر مماس الظل ولا ينخسف] القمر مماس دايرة الظل ولا ينخسف .

(«بدء» مكتوبة «بدو» في كل النسخ.)

Fig. T18 p. 237 **ض** مخروط القمر] (ناقص) . **غ** العقدة] (ناقص) . **ف** عند العقدة] في العقدة . العقدة] ذنب . مخروط القمر] (ناقص) . مخروط النور] (ناقص) . **ل** مركز الشمس] مركزها . **م** عند العقدة] مع العقدة . مخروط القمر] مخروط ظل القمر . (يوجد «منطقة المايل للقمر» و«منطقة الممثل للقمر» على نفس الدائرة) .

Fig. T19 p. 261 (لا يوجد الشكل إلاّ في مخطوط ل) .

Fig. T20 p. 271 **ض** دائرة معدل النهار] معدل النهار . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج . **ط** دائرة معدل النهار] معدل النهار . رأس الميزان] (ناقص) . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج . **غ** الجنوب] (ناقص) . الشمال] (ناقص) . المغرب] مغرب . منطقة البروج] فلك البروج . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج .

ف قطب البروج] (ناقص) . م مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج .

Fig. T21 p. 273 ض المشرق] (ناقص) . المغرب] (ناقص) . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج . ط الشمال] (ناقص) . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج . الاعتدال (موقعان)] الاعتدالين . غ الجنوب] جنوب . الشمال] شمال . أول القوس] (ناقص) . أول السرطان] (ناقص) . مغيب الاعتدال] مغيب الاعتدالين . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج . ف دائرة فلك البروج] (ناقص) . أول السرطان] (ناقص) . الاعتدال (موقعان)] الاعتدالين . ل مغيب] مغرب . م (يوجد «قطب فلك البروج» على تقاطع دائرة نصف النهار ومدار قطب فلك البروج من الجانب الجنوبي) . مدار قطب البروج] مدار قطب فلك البروج . قطب البروج] قطب فلك البروج . الاعتدال (موقعان)] الاعتدالين .

Fig. T22 p. 275 ض قطب معدل النهار] قطب الكل . مدار قطب البروج] مدار قطب فلك البروج . (مدار قطب فلك البروج مرسوم في اليمين من الصورة) . ط دائرة نصف النهار] (ناقص) . قطب معدل النهار] قطب الكل . الشمال] (ناقص) . (مدار قطب فلك البروج مرسوم في اليمين من الصورة) . مدار قطب البروج] مدار قطب فلك البروج . غ نقطة الجنوب] (ناقص) . المغرب] (ناقص) . دائرة فلك البروج] (ناقص) . قطب البروج] قطب فلك البروج . أول الحمل] (ناقص) مدار قطب البروج] مدار قطب فلك البروج . ف الجنوب] (ناقص) . الشمال] (ناقص) . (الصورة مدوّرة بحيث المغرب في أسفل الصفحة) . م قطب معدل النهار] قطب فلك معدل النهار .

Fig. T23 p. 277 ض نقطة الجنوب] (ناقص) . قطب معدل النهار] قطب الكل . أول السرطان] (ناقص) . نقطة الشمال] (ناقص) . الاعتدال (موقعان)] الاعتدالين . قطب البروج] قطب فلك البروج . مدار قطب البروج] مدار قطب فلك البروج . ط قطب معدل النهار] قطب الكل . الشمال] (ناقص) . الاعتدال (موقعان)] لاعتدالين . قطب البروج] قطب فلك البروج .

مدار قطب البروج] مدار قطب فلك البروج . **غ** نقطة الجنوب] (ناقص) . دائرة معدل النهار] (ناقص) . منطقة البروج] (ناقص) . دائرة نصف النهار] (ناقص) . قطب معدل النهار] قطب فلك البروج . قطب البروج] (ناقص) . مدار قطب البروج] (ناقص) . الشمال] (ناقص) . **ف** أول الدلو] (مطموس) . نقطة الجنوب] (ناقص) . دائرة الأفق] (ناقص) . («اول الحمل» مكرّر عند نقطة المشرق) . نقطة الشمال] (ناقص) . (الصورة مدوّرة بحيث المغرب في أسفل الصفحة) . قطب البروج] قطب فلك البروج . **ل** مغيب] مغرب . قطب معدل النهار] قطب المعدل . **م** المشرق] (غير مقروء) . الاعتدال (موقعان)] الاعتدالين . قطب البروج] قطب فلك البروج . مدار قطب البروج] مدار قطب فلك البروج .

Fig. T24 **ض** الحقيقية تزيد على الوسطى بسبب المطالع (في أسفل الصورة)]
p. 293 الحقيقية تنقص من الوسطى بسبب المطالع . (لا توجد الدائرة الصغيرة ولا الخطان الخارجان من مركزها) . **ط** (لا توجد الدائرة الصغيرة ولا الخطان الخارجان من مركزها) . **غ** وسط الأسد] (ناقص) . حضيض الشمس] الدور(؟) . تربيع الأوج (في يمين الصورة)] (ناقص) . (لا توجد الدائرة الصغيرة ولا الخطان الخارجان من مركزها) . **ف** أوج الشمس] (ناقص) . أول الميزان] (ليس بالتوسط بين أول السرطان وأول الجدي بل يقرب من أول الجدي) . أول الحمل] (ليس بالتوسط بين أول الجدي وأول السرطان بل يقرب من أول السرطان) . تربيع الأوج (موقعان)] (هما بالتوسط بين أول السرطان وأول الجدي) . الوسطى (في كل موقع)] الوسط . الحقيقية تنقص من الوسطى بسبب المطالع (في يسار الصورة)] الحقيقية تزيد على الوسط بسبب المطالع . الحقيقية تزيد على الوسطى بسبب الاختلاف] الحقيقية تنقص من الوسط بسبب الاختلاف . (الدائرة الصغيرة تماسّ فلك البروج عند أول السرطان) . **ل** وسط الأسد] اوسط الاسد . الحقيقية تنقص من الوسطى بسبب الاختلاف] الحقيقية تنقص عن الوسطى بسبب الاختلاف . الحقيقية تزيد على الوسطى بسبب الاختلاف] الحقيقية تزيد على الوسطى بسبب المطالع . (الدائرة الصغيرة تماسّ فلك البروج عند أول السرطان) . (الخطّان الخارجان من مركز الدائرة الصغيرة يمرّان في تقاطع ضلعي المربّع والخط الفاصل بين القطعة البعيدة والقطعة القريبة) .

أول الميزان] (ليس بالتوسط بين أول السرطان وأول الجدي بل يقرب من أول الجدي) . أول الحمل] (ليس بالتوسط بين أول الجدي وأول السرطان بل يقرب من أول السرطان) . تربيع الأوج (موقعان)] (هما بالتوسط بين أول السرطان وأول الجدي) . **م** أوج الشمس] أول الشمس. (لا توجد الدائرة الصغيرة ولا الخطان الخارجان من مركزها) .

الباب الثالث ، الفصل التاسع

Fig. T25
p. 297
ل مركز الشمس] شمس . الأرض] ارض . قاعدة المخروط والمثلث] (ناقص) . مثلث المخروط] (ناقص) . موضع الناظر] (ناقص) . موقع العمود ...أولاً] موضع العمود (وخط العمود نفسه ناقص) . موضع الاتصال] (ناقص) . المشرق] شرق . المغرب] غرب .

Fig. C34b
p. 369
ض طرَف الأفق المرئي] طرف الافق (وهو عند تقاطع الأرض والمثلث ولا يوجد خط من موضع الناظر إلى المثلث) . **ط** طرف الأفق المرئي] طرف الافق (وهو عند تقاطع الأرض والمثلث ولا يوجد خط من موضع الناظر إلى المثلث) . فلك الشمس] (ناقص) . **غ** موقع العمود ...أولاً] موضع العمود . فلك الشمس] (ناقص) . سطح الأفق المرئي] (ناقص ، وأيضاً لا يوجد الخط منها إلى «موضع الناظر») . **م** طرف الأفق المرئي] طرف الافق . (يمتد الخط من «طرف الأفق المرئي» إلى الضلع الآخر من المثلث) .

الباب الرابع ، الفصل الثاني

Fig. T26
p. 317
ض (يمتد الخط من «موضع الناظر» إلى «فلك البروج») . **ل** (يمتد الخط من «موضع الناظر» و«مركز الأرض» إلى «فلك البروج») . (يوجد خط عمودي قائم على الخط من «سمت الرأس» إلى «مركز الأرض» وهو يمتد من «مركز الأرض» إلى «فلك البروج» في الجهتين) .

الباب الرابع ، الفصل الثالث

Fig. T27 **ض** « ج »(؟) و«د» و«ي» و«م» و«ن»] (ناقص) . قطر الظل في بعد
p. 321 أبعد القمر من الأرض عن جانب رأس المخروط] مركز الظل في البعد الأبعد من جانب راس المخروط . نصف قطر الأرض] (ناقص) . طرف نصف قطر الشمس] (غير مقروء) . الأقطار ليست حقيقية لكن في الحس لا فرق بينها وبين الحقيقية] (ناقص) . **ط** قطر الظل في بعد أبعد القمر من الأرض عن جانب رأس المخروط] قطر الظل في بعد ابعد القمر عن جانب راس المخروط . **غ** (لا توجد الحروف الأبجدية) . رأس مخروط الظل] (ناقص) . مركز الظل عن جانب رأس المخروط] (ناقص) . قطر الظل في بعد أبعد القمر من الأرض عن جانب رأس المخروط] قطر الظل في بعد ابعد القمر . مركز القمر في البعد الأبعد وهو أيضاً مركز مخروط الظل عن جانب الشمس] مركز القمر في البعد الابعد وهو ايضا مركز محيط مخروط الظل عند القمر . نصف قطر مخروط الظل عند القمر] نصف مخروط الظل عند القمر . **ف** (لا توجد الحروف الأبجدية) . طرف نصف قطر القمر] (يوجد على «ي») . مركز القمر في البعد الأبعد وهو أيضاً مركز مخروط الظل عن جانب الشمس] مركز القمر في بعد ~~ا~~ابعد وهو ايضا مركز مخروط الظل عن جانب الشمس . نصف قطر مخروط الظل عند القمر] (ناقص) . **ل** (لا توجد الحروف الأبجدية) . نصف قطر مخروط الظل عند القمر] (ناقص) . **م** (لا توجد الحروف الأبجدية) . طرف نصف قطر القمر] طرف قطر القمر . مركز القمر في البعد الأبعد وهو أيضاً مركز مخروط الظل عن جانب الشمس] مركز القمر في البعد الابعد وهو ايضا مركز مخروط الظل عند القمر عن جانب الشمس . قطر الظل في بعد أبعد القمر من الأرض عن جانب رأس المخروط] قطر الظل في بعد ابعد القمر عن جانب راس المخروط .

Part VI

Appendices and Indices

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§1. Maps of Places Cited

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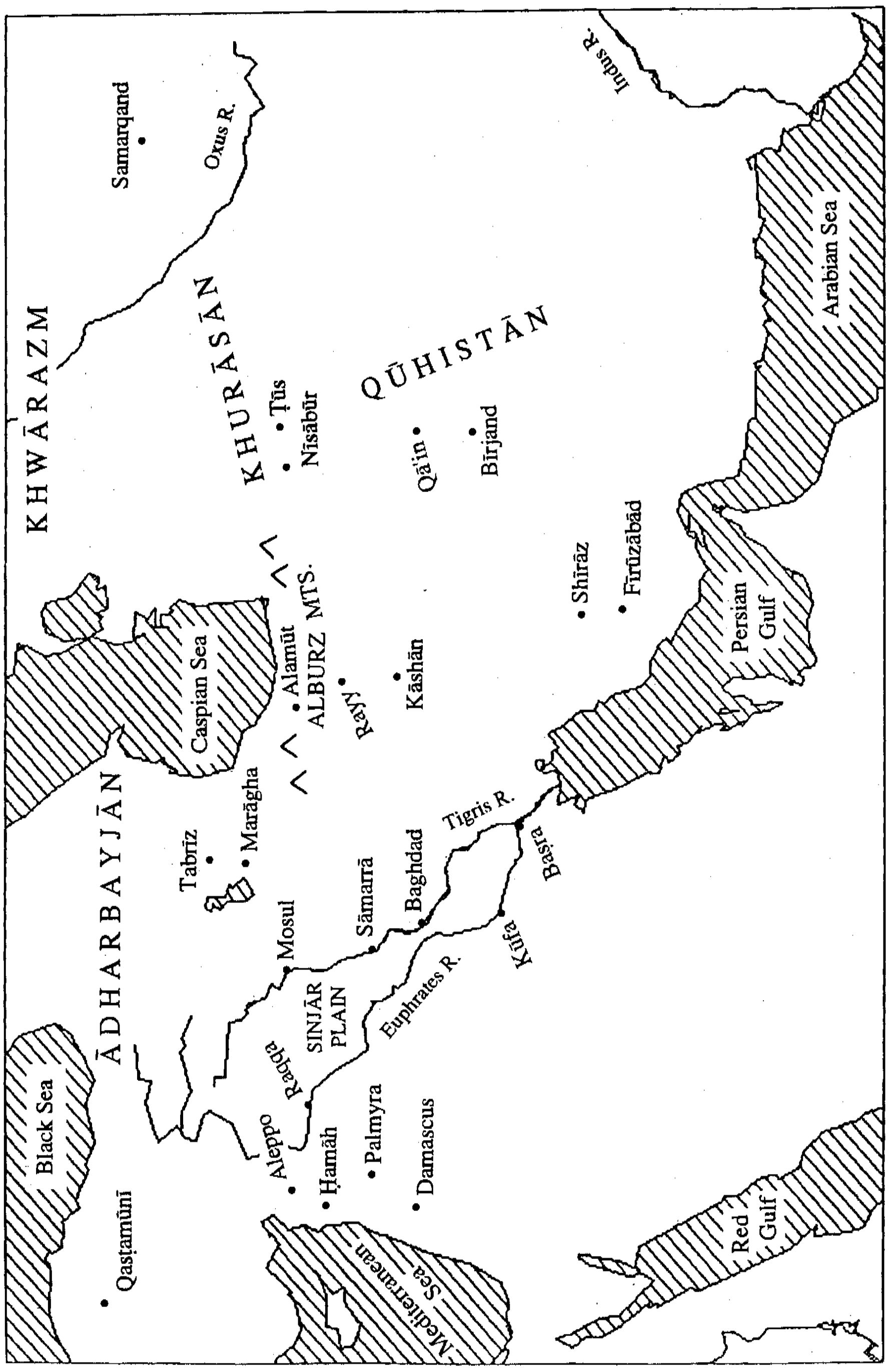
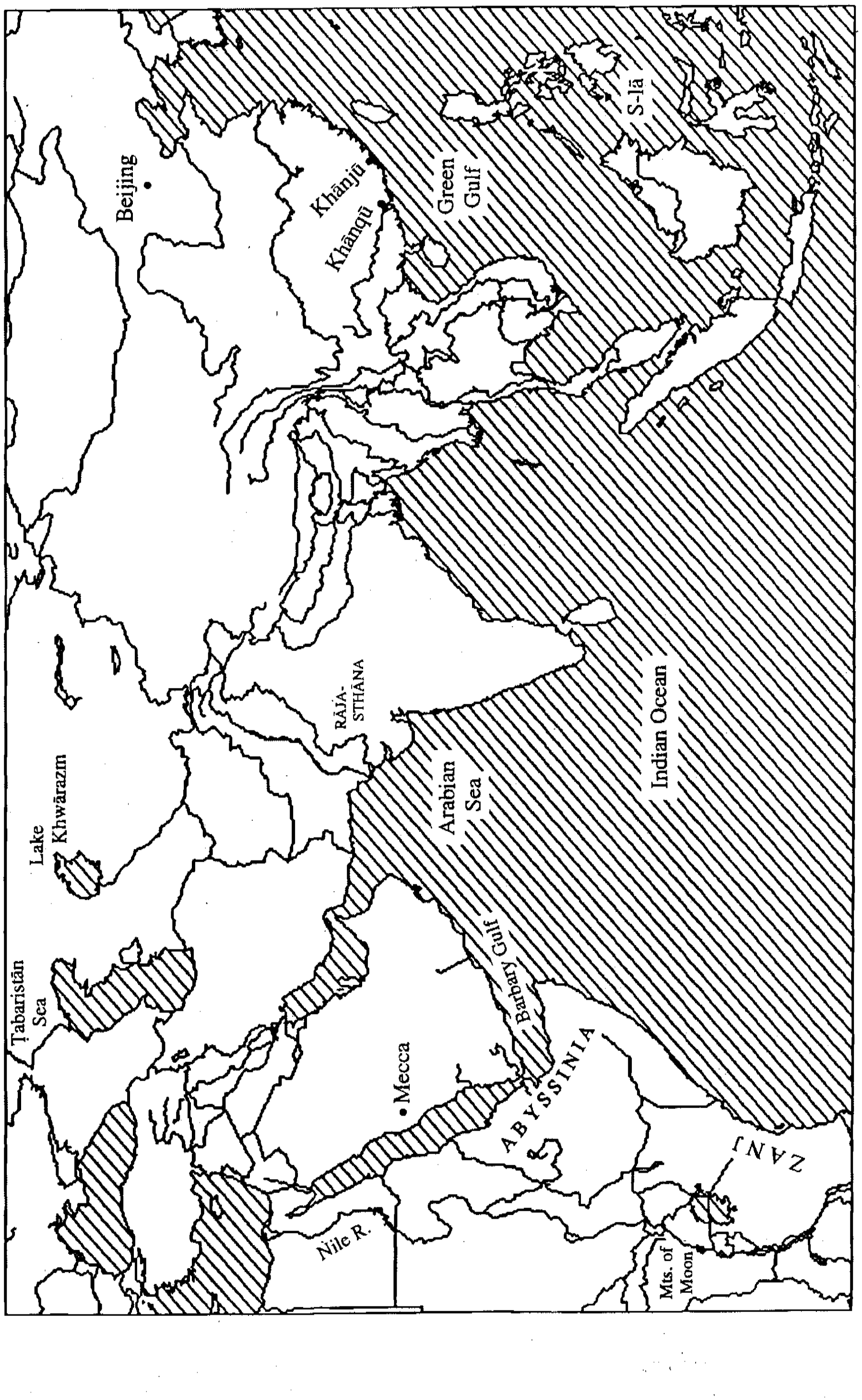


Map 1. Southwest Asia



Map 2. The Eastern Oikoumenē

§2. Conventions

For an explanation of the signs and conventions used in the apparatus, see page 532; sigla for the manuscripts are on pages 76–81.

A. Transliteration System for Arabic and Persian Words

The transliteration system is generally orthographic with no attempt being made to indicate actual pronunciation. Particles are joined by a dash with the following word; *al* becomes *’l* if preceded by a joined particle. Persian *eẓāfe* is indicated by *-i*.

ا	ā	ظ	ẓ	َ	a
ب	b	ع	c	ُ	u
پ	p	غ	gh	ِ	i
ت	t	ف	f		
ث	th	ق	q	اً	—an
ج	j	ك	k	ٍ	—in
چ	ch	گ	g		
ح	ḥ	ل	l		
خ	kh	م	m		
د	d	ن	n		
ذ	dh	ه	h		
ر	r	و	w, ū		
ز	z	ي	y, ī		
ژ	zh				
س	s	يّ	iyy		
ش	sh	ِي	ī		
ص	ṣ	ى	ā		
ض	ḍ	ء	’ (except when initial)		
ط	ṭ	ة	a; at (construct state)		

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B. Transcription System for Arabic Letters in Figures

ا	A	و	W	ك	K	ع	O
ب	B	ز	Z	ل	L	ف	F
ج	G	ح	H	م	M	ص	C
د	D	ط	T	ن	N	ق	Q
ه	E	ي	Y	س	S	ر	R

C. Abbreviations and Symbols

A	area
ar.	Arabic
b.	ibn (son of)
c.	century
C	circumference
comm.	commentary
ctr.	center
^{d}	days
d	standard arc of daylight; diameter
D	amount of daylight
e	eccentricity
earth	the element
Earth	the body
Engl.	English
e.d.	Earth diameter
e.r.	Earth radius
e.v.	Earth volume
f., ff.	folio(s)
Fr.	French
^{h}	hours
H.	*hijra* year; Heiberg (*Almagest* pagination)
H. Sh.	solar *hijra* year
Intr.	Introduction of edited text (pp. 91, 93)
L	area of lune
lit.	literally
^{m}	minutes
MS, MSS	manuscript(s)
MS 1330, 1	first treatise in MS 1330
MS 1330 (1)	first treatise in MS 1330
n.	note
n. d.	no date
O	observer, center of World

or.	Oriental
^{p}	parts
Pref.	Preface of edited text (p. 91)
pt.	point
q	equation of daylight
Q	equant
R	radius of eccentric; radius of smaller equator
r	radius of epicycle; radius
S	surface area of sphere
trans.	translation, translated by
V	volume

α	Marāgha version of *Tadhkira* (see pp. 71–75 and 85–88)
β	Baghdad version of *Tadhkira* (see pp. 70–71 and 85–88)
M	Istanbul, Lâleli MS 2116 (see pp. 85–88)

δ	declination
ΔE	equation of time
ε	obliquity of ecliptic
η	amplitude
λ	longitude
ϕ	local latitude
ϕ_e	local ecliptic latitude

A bar over a letter indicates the complement.

D. Miscellaneous

[...]	editor's/translator's additions and comments
(...)	for figures, indicates that this is missing in MSS FL
/.../	text or translation with variant reading given at foot of page
<...>	computations or insertions due to editor/translator
II.11 [9]1–2_2	standard reference notation for edited text; in this case, Book II, Chapter Eleven, paragraph 9, from line 1 of the page to line 2 of the following page
23;33,30	standard sexagesimal notation; in this case, the number represents $23x60^0 + 33x60^{-1} + 30x60^{-2}$

The first of two dates separated by a slash is the *hijra* date; the second is the corresponding Christian date. When only one date is given, it is Christian unless otherwise indicated.

§3. Glossary

II.3 [12] = first appearance or definition of term occurs in Book II, Chapter 3, paragraph 12 of edition; trans. = transitive; intrans. = intransitive; adj. = adjective; const. = constellation; ج = جمع (plural); مص = مصدر (verbal noun)

ا

permanent (II.1[1])	أبدِيّ
aether; fire (II.1[6])	الأثِير
jungle (III.1[5])	أجَمَة (ج) آجام
endpoint (III.1[8])	آخِر
last (of Aries)	آخِر (الحَمَل)
end part (III.6[2])	أواخِر
Archimedes (IV.1[1])	أرشميدس
earth (element)	أرْض
the Earth	الأرْض
Leo	الأسَد
cylinder (I.1[16])	أُسْطُوانَة
right cylinder	أُسْطُوانَة قائِمَة
circular cylinder	أُسْطُوانَة مُسْتَدِيرة
model (II.5[1])	أصْل (ج) أُصُول
horizon	أُفُق (ج) آفاق
horizon circle (II.3[12])	دائِرَة الأُفُق
Euclid (IV.4[2])	أقليدس
clime (III.1[7])	إقْلِيم (ج) أقالِيم
Andalusia	الأنْدَلُس
practitioners of the science	أهْل العِلْم
oval-shaped (II.13[1])	إهْلِيلَجِيّ الشَكْل
apogee (II.5[11])	أوْج
solar apogee (II.6[5])	أوْج الشَمْس
lunar apogee (II.7[29])	أوْج القَمَر
apogee orb (II.5[11])	فَلَك الأوْج
first (of Aries)	أوّل (الحَمَل)
beginning part (III.8[7])	أوائِل

displacing (motion) (حَرَكَة) أيْنِيّة
(I.2[3])

ب

sea بَحْر (ج) بِحار
Caspian Sea بَحْر طَبَرِسْتان
Warank (Baltic) Sea بَحْر وَرَنْك
Aral Sea بُحَيْرَة خوارِزَم
vapor (II.1[1]) بُخار (ج) أبْخِرَة
principle (I.Intr.[1]); مَبْدأ (ج) مَبادِئ
principle of motion (I.2[2]);
starting (initial) point (II.4[4]);
epoch (III.10[3])
full moon (II.13[1]) البَدْر
to interchange (II.4[3]) يَتَبادَلُ
converse(ly) (II.1[1]) عَلَى التَبادُل
desert (III.1[5]) برِّيّة (ج) بَرارِي
zodiacal sign (II.3[5]); بُرْج (ج) بُرُوج
zod. constellation (II.4[9])
the ecliptic مِنْطَقَة (فَلَك) البُرُوج
equator (orb) (II.3[3])
lightning (II.2[5]) بَرْق
proof بُرْهان (ج) بَراهِين
(demonstration) (I. Intr.[3])

simple (bodies) بَسِيط (ج) بَسائِط
(I.2[1])
the light spreads يَنْبَسِطُ النُور
(III.9[2])
the eye (III.9[1]) البَصَر
slower motion (II.5[3]); بُطْء
slowest speed (II.7[14]
channel (III.1[5]) بَطِيحَة (ج) بَطائِح
Ptolemy بطلميوس
Crater (II.4[9]) الباطِيَة
distance; interval بُعْد (ج) أبْعاد
elongation (II.9[1]) بُعْد في الطُول
farthest distance البُعْد الأبْعَد
(II.5[3–4])
nearest distance البُعْد الأقْرَب
(II.5[3–4])
mean distance (II.5[3]) البُعْد الأوْسَط
double elongation البُعْد المُضَعَّف
(II.7[11])
elongation (IV.5[1]) تَباعُد
location (II.3[2]); بُقْعَة (ج) بِقاع
locality (II.4[3])
to remain (after subtraction) يَبْقى
to result (from an arithmetical operation) يَبْلُغُ

second (II.3[1])	ثانِيَة (ج) ثَوانٍ
Taurus	الثَوْر

ج

Orion (II.4[9])	الجَبّار
mountain	جَبَل (ج) جِبال
Hercules (const.) (II.4[9])	الجاثِي عَلى رُكْبَتَيه
Capricornus	الجَدْي
Polaris (II.4[8])	الجُدَيّ
the galaxy (II.4[10])	المَجَرّة
by trial and error (III.9[2])	بالتَجْرِبَة
body; volume	جِرْم (ج) أجْرام
the simple bodies (I.Intr.[2])	الأجْرام البَسِيطَة
the celestial bodies (II.[title])	الأجْرام العَلَوِيّة / العُلْوِيّة
part (I.1[1]); unit (II.3[5])	جُزْء (ج) أجْزاء
to divide into parts	يُجَزِّئ
solid (I.1[1]); body (I.2[1])	جِسْم
spherical solid (I.1[15])	جِسْم كُرِيّ
to reappear (after being eclipsed) (II.13[9])	يَنْجَلِي

book (main division of *Tadhkira*)	باب
to prove (I.Intr.[1])	يُبَيِّنُ
proof (I.Intr.[2])	بَيان

ت

hill (III.1[5])	تَلّ (ج) تِلال
complement (II.3[1])	تَمام
complement of obliquity	تَمام المَيْل كُلّه
colatitude (II.3[7])	تَمام العَرْض
complementary body (II.5[10]); complement	المُتَمِّم
Draco (II.4[9])	التِنِّين

ث

the fixed stars (II.2[4])	الثَوابِت
thickness	ثِخَن
heaviness (I.2[3])	ثِقَل
triangle	مُثَلَّث
triangle of cone (Fig. T25)	مُثَلَّث المَخْرُوط
Triangulum (II.4[9])	المُثَلَّث
trine	تَثْلِيث

Ara (II.4[9]) المِجْمَرَة

to add يَجْمَعُ

sum مَجْمُوع

conjunction اِجْتِماع

summary (Pref.[2]) جُمَل

side (I.1[5]) جَنْبَة

side (II.1[7]) جانِب (ج) جَوانِب

south point (II.3[14]) نُقْطَة الجَنُوب

crossing point (II.7[7]) مَجاز

to exceed يُجاوِزُ

Gemini الجَوْزاء

lunar nodes (II.7[7]) الجَوْزَهَر / الجَوْزَهْر

sine (I.1[9]) جَيْب

ح

Abyssinia (III.1[3]) الحَبَشَة

definition (I.Intr.[4]); boundary (I.1[1]) حَدّ (ج) حُدُود

lunar eclipse limit (II.13[5]) حدّ الخُسوف

solar eclipse limit (II.13[9]) حدّ الكُسوف

acute (I.1[5]) حادّ

convex (I.1[15]) مُحَدَّب

to face; be aligned with يُحاذِي

alignment point (II.7[18]) نُقْطَة المُحاذاة

to face one another; to be aligned يَتَحاذَى

slant (II.10[5]) الانْحِراف

to combust (II.9[12]) يَحْتَرِقُ (مص) احْتِراقٌ

combust way (III.1[6]) الطَرِيقَة المُحْتَرِقَة

motion حَرَكَة (ج) حَرَكات

motion of the apogee (II.7[9]) حَرَكَة الأَوْج

the primary motion الحَرَكَة الأُولَى

motion of displacement (I.2[3]) حَرَكَة أَيْنِيّة

motion of the center حَرَكَة المَرْكَز

voluntary celestial motion (I.2[2]) حَرَكَة إرادِيّة فَلَكِيّة

motion by nature الحَرَكَة بالطَبْع

natural elemental motion (I.2[2]) حَرَكَة طَبِيعِيّة عُنْصُرِيّة

mean motion حَرَكَة الوَسَط / الحَرَكَة الوُسْطَى

motion in place (I.2[3]) حَرَكَة وَضْعِيّة
the daily motion الحَرَكَة اليَوْمِيّة
to move (something) يُحَرِّكُ
mover المُحَرِّك
to move يَتَحَرَّكُ
to be self-moved يَتَحَرَّكُ بِنَفْسِه
movement تَحَرُّك
mobile المُتَحَرِّك
self-moved mobile المُتَحَرِّك بِنَفْسِه
stationary غَيْر مُتَحَرِّك
the senses الحِسّ
appreciable; perceptible مَحْسُوس
perception الإحْساس
calculation حِساب
mathematician; calculator حاسِب (ج) حُسّاب
to deem elegant يَسْتَحْسِنُ
unit (IV.1[2]) حِصّة (ج) حِصَص
argument of latitude (II.7[29]) حِصّة العَرْض
to result (from an arithmetical operation) يَحْصُلُ
perigee; epicyclic perigee (II.5[11]) حَضِيض

descent (II.1[1]); decreasing altitude (II.1[2]); depression (II.3[17]) اِنْحِطاط
true حَقِيقِيّ
principle; rule; judgment حُكْم (ج) أحْكام
astrologer (III.1[6]) أحْكامِيّ
scientist (IV.1[2]) حَكِيم (ج) حُكَماء
narrative; report (I.Intr.[3]) حِكايَة
ring of light (II.13[8]) حَلْقَة النُور
Aries الحَمَل
deferent (orb) (II.5[5]) (فَلَك) حامِل
deferent orb for the deferent orb's center (II.8[9]) الفَلَك الحامِل لِمَرْكَز الفَلَك الحامِل
Perseus (II.4[9]) حامِل رأس الغُول
slanted (III.3[1]) حَمائِلِيّ
possibility (II.4[3]) اِحْتِمال
Pisces (II.14[2]) الحُوت
Piscis Austrinus الحُوت الجَنُوبِيّ
axis (I.1[12]) مِحْوَر
confines (I.2[4]) حَيِّز
to bound (I.1[3]) يُحِيطُ
circumference of a circle (I.1[8]) or sphere (I.1[10]) مُحِيط

lunar eclipse خُسُوف / اِنْخِساف (II.13[2])

to be eclipsed (moon) اِنْخَسَفَ

proper (motion) (الحَرَكَة) الخاصّة (II.7[13])

apparent proper anomaly (II.7[29]) الخاصّة المَرْئِيّة

mean proper anomaly (II.7[29]) الخاصّة الوُسْطى

compendium (II.11[3]) مُخْتَصَر

to mark (III.12[1]) يَخُطُّ

line (I.1[1]) خَطّ (ج) خُطُوط

curved line (I.1[8]) خَطّ مُسْتَدِير

(Earth's) equator (III.1[2]) خَطّ الاِسْتِواء

shadow line (III.12[2]) خَطّ الظِلّ

straight line (I.1[2]) خَطّ مُسْتَقِيم

meridian line (III.12[1]) خَطّ نِصْف النَهار

geometrically (II.5[10]) بالخُطوط

lightness (I.2[3]) خِفّة

to drop below the horizon (III.5[6]) يَنْخَفِضُ عَن الأُفُق

depressed (II.14[1]) مُنْخَفِض

enclosing sphere (II.11[4]) (كُرَة) مُحِيطَة

to convert (IV.2[4]) يُحَوِّلُ

Ophiuchus (II.4[9]) الحَوّاء

Serpens (II.4[9]) الحَيّة

animal حَيَوان (ج) حَيَوانات

خ

to extend; to result (intrans.) يَخْرُجُ

to produce or extend (a line, etc.) يُخْرِجُ

eccentric (orb) (II.5[2]) (فَلَك) خارِج مَرْكَز

exterior (angle) (II.11[3]) (زاوِيَة) خارِجَة

cone (I.1[17]) مَخْرُوط

circular cone مَخْرُوط مُسْتَدِير

shadow cone مَخْرُوط الظِلّ

lunar cone (Fig. T18) مَخْرُوط القَمَر

right cone (I.1[17]) مَخْرُوط قائِم

light cone (Figs. T17, T18) مَخْرُوط النُور

to tear (I.2[4]) يَنْخَرِقُ

to occult (II.2[4]) يَخْسِفُ

anomaly of the اِخْتِلاف البُعْد الأقْرَب
nearest distance (II.7[17])
anomaly اِخْتِلاف البُعْد الأبْعَد والأقْرَب
of the farthest and nearest
distances (II.8[16])
change of state اِخْتِلاف حال
angle of divergence زاوِيَة الاِخْتِلاف
(II.12[1]);
angle of anomaly (Fig. T6)
difference in longitude اِخْتِلاف الطُول
(II.12[4])
difference in latitude اِخْتِلاف العَرْض
(II.11[4])
parallax اِخْتِلاف المَنْظَر (ج) المَناظِر
(II.12[1]); divergence of sight
void (I.2[1]) خَلاء
mole (II.2[4]) خالَة

د

Ursa Minor (II.4[9]) الدُبّ الأصْغَر
Ursa Major (II.4[9]) الدُبّ الأكْبَر
recession (II.4[4]) إدْبار
Cygnus (II.4[9]) الدَجاجَة
interior angle (II.11[3]) زاوِيَة داخِلَة

east and west (III.1[7]) الخافِقان
to disappear; be invisible يَخْفَى
invisibility (II.14[2]) خَفاء / اِخْتِفاء
of permanent أبَدِيّ الخَفاء
invisibility (II.1[1])
period of invisibility زَمان الخَفاء
invisible خَفِيّ
gulf (III.1[5]) خَلِيج
Gulf of Barbary الخَلِيج البَرْبَري
(Aden)
Red Gulf (Sea) الخَلِيج الأحْمَر
Green Gulf الخَلِيج الأخْضَر
Persian Gulf خَلِيج فارِس
to expand (I.2[4]) يَتَخَلْخَلُ
Eternal (Islands) (جَزائِر) الخالِدات
(III.1[7])
to vary (I.2[3]); يَخْتَلِفُ
to be irregular (II.2[3])
to be invariable لا يَخْتَلِفُ
variation اِخْتِلاف (ج) اِخْتِلافات
(II.1[2]); variability (II.11[22]);
change (I.Intr.[2]);
divergence (II.4[4]); anomaly;
anomalous speed (II.7[14])

smoke — دُخان (ج) أدْخِنَة

equal degrees (III.7[1]) — دَرَج سَوَاء

degree (II.3[5]) — دَرَجَة (ج) دَرَجات

degree of transit (rising) (setting) (III.11) — دَرَجَة مَمَرّ (طُلُوع) (غُرُوب)

gradual — مُتَدَرِّج

in one stroke (III.4[5]) — دَفْعَةً

minute (II.3[1]) — دَقِيقَة (ج) دَقائق

exactly (IV.2[2]) — بالتَدْقِيق

to narrow — يَسْتَدِقُّ

evidence (II.1[6]) — دَلِيل (ج) دَلائِل

proof (II.1[8]) — دَلِيل (ج) أدِلَّة

proof of the fact (II.1[8]) — دَلِيل إنّيّ

proof of the reasoned fact (II.1[8]) — دَلِيل لِمّيّ

Delphinus (II.4[9]) — الدُلْفِين

Aquarius — الدَلْو

to rotate about itself (I.1[12]) — يَدُورُ عَلَى نَفْسِه

revolution; cycle (III.10[2]) — دَوْر

rotation (I.1[12]); revolution (II.1[1]) — دَوْرَة

circle (I.1[8]) — دائِرَة (ج) دَوائِر

small circle (I.1[13]) — دائِرَة صَغِيرَة (صُغْرَى)

great circle (I.1[13]) — دائِرَة عَظِيمَة (عُظْمَى) (ج) عِظام

Indian circle (III.12[2]) — الدائِرَة الهِنْدِيَّة

circuit (I.1[12]) — مَدار (ج) مَدارات

solar circuit (II.6[1]) — مَدار الشَمْس

parallel of latitude (II.3[5]) — مَدار عَرْضِيّ

day-circle (II.3[2]) — مَدار يَوْمِيّ

epicycle (II.5[2]) — تَدْوِير (ج) تَداوِير / تَدْوِيرات

dirigent (II.8[4]) — مُدِير

circularity; sphericity — اِسْتِدارَة

with circular (motion) (II.1[6]) — عَلَى الاِسْتِدارَة

curved (I.1[8] & [10]); circular (I.1[16] & [17]) — مُسْتَدِير

wheel-like (III.2[1]) — دُولابِيّ

ذ

to diminish (I.2[4]) — ذَبَلَ / يَذْبُلُ

cubit (IV.1[2]) — ذِراع (ج) أذْرُع / ذُرْعان

apex (of epicycle) (II.5[11]) — ذِرْوَة / ذُرْوَة

memoir ([title]); memento (Pref.[2])	تَذْكِرَة
tail (a node) (II.7[7])	ذَنَب
comet (II.2[5])	(كَوْكَب) ذُو الذَنَب (ج) ذَوات الأذْناب
quadrilateral (I.1[18])	ذُو أرْبَعَة أضْلاع
Cassiopeia (II.4[9])	ذات الكُرسِي

ر

apex (I.1[17]); head (a node) (II.7[7])	رأس (ج) رُؤُوس
head (of Aries, etc.) (Fig. T20)	رأس (الحَمَل)
apparent	مَرْئِيّ
quarter mark (II.11[9]); fourth (III.1[2])	رُبْع (ج) أرْباع
spring (*i.e.* the season)	رَبِيع
to divide into fourths (III.12[2])	يُرَبِّعُ
quadrature	تَرْبِيع
grade (II.4[9])	مَرْتَبَة (ج) مَراتِب
arrangement (II.2[title])	تَرْتِيب
to reverse direction; to retrograde	يَرْجِعُ
retrogradation (II.5[8])	رُجُوع
Rigil Centaurus (II.4[8])	رِجْل قَنْطُورِس
spinning (III.6[1])	رَحَوِيّ
to oscillate (II.11[2])	يَتَرَدَّدُ
to draw (III.12[2])	يَرْسُمُ
observation	رَصْد (ج) أرْصاد
by observation	بالرَصَد
thunder (II.2[5])	رَعْد
ascent (II.1[1]); altitude (II.3[17]); height (IV.5[6])	ارْتِفاع
altitude circle (II.3[17])	دائِرَة الارْتِفاع
compound; composed	مُرَكَّب
to be composed	يَتَرَكَّبُ
to be combined additively (III.8[6])	يَتَرَكَّبُ بالجَمْع
to have the difference taken (III.8[6])	يَتَرَكَّبُ بالتَفْرِيق
center (I.1[8])	مَرْكَز (ج) مَراكِز
solar center (II.6[5])	مَرْكَز الشَمْس
center of the World	مَرْكَز العالَم
lunar center (II.7[29])	مَرْكَز القَمَر
eccentricity (II.5[5])	ما بَيْن المَرْكَزَيْن
embedded (II.5[10])	مَرْكُوز

accumulation (II.1[1]) تَراكُم

sands (III.1[5]) رِمال

thrown object (II.1[6]) مَرْمِيّ

Lepus (II.4[9]) الأَرْنَب

voluntary (I.2[2]) إرادِيّ

ز

Saturn (II.2[4]) زُحَل

unit of time (II.3[2]) زَمَن (ج) أَزْمان

period (of time) زَمان (ج) أَزْمِنَة

intense cold (II.2[5]) الزَمْهَرِير

Zanzibar; *Zanj* (III.1[3]) الزَنْج

Venus (II.2[4]) الزُهَرَة

angle (I.1[3]) زاوِيَة (ج) زَوايا

acute angle (I.1[5]) زاوِيَة حادّة

exterior angle (II.11[3]) زاوِيَة خارِجَة

angle of divergence (II.12[1]); angle of anomaly (Fig. T6) زاوِيَة الاخْتِلاف

interior angle (II.11[3]) زاوِيَة داخِلَة

obtuse angle (I.1[5]) زاوِيَة مُنْفَرِجَة

right angle (I.1[5]) زاوِيَة قائِمَة

astronomical handbook; *zīj* زِيْج

excess زِيادَة

additive زائِد

س

problem (I.Intr.[1]) مَسْأَلَة (ج) مَسائِل

Lupus (II.4[9]) السَبُع

cloud (II.2[5]) سَحابَة (ج) سُحُب

sextile تَسْدِيس

Cancer السَرَطان

faster motion (II.5[3]); fastest speed (II.7[14]) سُرْعَة

faster سَرِيع

the stolen (epagomenal) days (III.10[3]) المُسْتَرَقَة

surface (I.1[1]) سَطْح (ج) سُطُوح

curved surface (I.1[10]) سَطْح مُسْتَدِير

plane surface (I.1[2]) سَطْح مُسْتَوٍ

the lower region (II.2[5]); (II.3[10]) السُفْل (ج) السُفْلِيّات

inferior (I.Intr.[2]) سُفْلِيّ

the two lower planets (II.2[4]) السُفْلِيّان

Argo Navis (II.4[9]) السَفِينَة

populated (III.1[2]) مَسْكُون

the populated quarter الرُبْع المَسْكُون (III.1[2])

geographical placebooks (III.1[5]) المَسالِك (والمَمالِك)

course, line; direction; azimuth (II.3[17]) سَمْت (ج) سُمُوت

circle of the initial azimuth (prime vertical) (II.3[14]) دائِرَة أوّل السُمُوت

zenith (II.3[12]) سَمْت الرأس

qibla bearing (III.12[3]) سَمْت القِبْلَة

alignment; being directly overhead (III.2[2]) مُسامَتَة

sky (II.1[title]) سَماء

the heavens (III.1[6]) السَماوِيّات

De caelo (كِتاب) السَماء والعالَم

Virgo (II.14[2]) السُنْبُلَة

year (III.10[3]) سَنَة (ج) سِنون

true (conventional) solar year (III.10[3]) سَنَة شَمْسِيّة حَقِيقِيّة (اِصْطِلاحِيّة)

lunar year (III.10[3]) سَنَة قَمَرِيّة

Plain of Sinjār بَرِّيّة سِنْجار

Canopus (II.4[8]) سُهَيل

axis سَهْم

versed sine (I.1[9]) سَهْم (لِنِصْف القَوْس)

Sagitta (II.4[9]) السَهْم

hour (III.10[1]) ساعَة (ج) ساعات

seasonal (temporal) hours (III.10[1]) ساعات زَمانِيّة

equal hours (III.10[1]) ساعات مُسْتَوِيَة

unequal (distorted) hours (III.10[1]) ساعات مُعَوَّجَة

distance مَسافَة

leg (*e.g.* of a triangle)(II.11[3]) ساق

exactly (III.6[title]) سَواءً

to equal; be equal to يُساوِي

to be equal to one another يَتَساوَى

converse equality (II.1[1]) التَساوِي عَلَى التَبادُل

equidistant مُتَساوِي البُعْد

regular order (III.5[1]) مُسْتَوٍ

motion (II.1[4]); speed (II.5[9]) سَيْر

mean speed (II.5[9]) سَيْر مُتَوَسِّط

planet (II.2[3]) سَيّارَة (ج) سَيّارات

ش

Syria	الشام
the same as	شَبِيه بِ
to be uniform (I.2[4]); to be equal (II.5[8])	يَتَشابَهُ
winter	شتاء
Hydra (II.4[9])	الشُجاع
individual; member of a class	شَخْص (ج) أشْخاص
to intensify (*e.g.* motion) (I.2[4])	يَشْتَدُّ
east-west circle (II.3[14])	دائرَة المَشْرِق والمَغْرِب
east point (II.3[13])	نُقْطَة المَشْرِق
Jupiter (II.2[4])	المُشْتَرِي
rays	شُعاع
barleycorn (IV.1[2])	شَعِيرَة
dusk (III.9[2])	شَفَق
figure	شَكْل (ج) أشْكال
difficulty	إشْكال
shape (II.13[1])	تَشَكُّل
illuminated shape	تَشَكُّل نُورِيّ
Lyra (II.4[9])	شِلْياق
sun (II.1[4])	الشَمْس
north point (II.3[14])	نُقْطَة الشَمال
shooting star (II.2[5])	شِهاب (ج) شُهُب
month (III.10[2])	شَهْر (ج) شُهور
true (conventional) solar month (III.10[3])	شَهْر شَمْسِيّ حَقِيقِيّ (اِصْطِلاحِيّ)
true (mean) lunar month (III.10[2])	شَهْر قَمَرِيّ حَقِيقِيّ (وَسَطِيّ)
to conform (II.1[6])	يُشايِعُ

ص

dawn (III.9)	صُبْح
first dawn (III.9[1])	الصُبْح الأوّل
true dawn (III.9[2])	الصُبْح الصادِق
false dawn (III.9[1])	الصُبْح الكاذِب
morning (endpoint) (II.10[5])	(الطَرَف) الصَباحِيّ
digit (II.13[5]); (IV.1[2])	إصْبَع (ج) أصابِع
area (body) digit	إصْبَع جِرْمِيّ
linear (diameter) digit	إصْبَع قُطْرِيّ
preliminary propositions (IV.1[1])	مُصادَرات

طَبِيعيّ	natural (I.2[2])
الطَبِيعِيّات	natural philosophy (I.Intr.[2])
ما بَعْد الطَبِيعَة	metaphysics (I.Intr.[2])
طَبَقَة (ج) طَبَقات	level (II.2[5])
يُطابِقُ	to coincide or be consistent with
يَتَطابَقُ	to coincide with one another
اِنْطِباق	coincidence
طَرَف (ج) أطْراف	endpoint
الطَرَف المُتَأخِّر	posterior endpoint (II.10[5])
الطَرَف المُتَقَدِّم	anterior endpoint (II.10[5])
الطَرَف الصَباحِيّ	morning endpoint (II.11[14])
الطَرَف المَسائِيّ	evening endpoint (II.11[14])
طَلِسْمات	talismans (II.4[4])
يَطْلُعُ (مص) طُلُوع	to rise
وَقْت الطُلُوع	rising time
مَطْلَع	rising place
مَطالِع	co-ascension (III.7[1])
صِرْف	pure (II.2[5])
صاعِد	ascending (II.5[11])
صاعِقَة (ج) صَواعِق	lightning bolt (II.2[5])
صَفْحَة	surface (of a planet); disk
صِقالَة	smoothness (II.13[1])
صَقِيل	smooth
صُورَة (ج) صُوَر	form; illustration; constellation (II.4[9])
صَيْف	summer

ض

يَضْرِبُ في	to multiply by
تَضارِيس	undulations (II.1[2])
يَضْعُفُ	to weaken (*e.g.* motion)
ضِلْع (ج) أضْلاع	side
ذو أرْبَعَة أضْلاع	quadrilateral
ضَوء	light (II.13[1])
مُضِيء	illuminated (II.13[1])

ط

طَبْع	a nature (I.2[2])
بِالْطَبْع	by nature; natural (I.2[2])
طَبِيعَة	a nature (I.2[1])

ascendent (II.3[17])	الطالِع
length (I.1[1]); longitude (II.3[8])	طُول
linear (IV.1[3])	طُولِيّ

ظ

shadow	ظِلّ (ج) أظْلال
shadow cone (II.13[4])	مَخْرُوط الظِلّ
shadow line (III.12[2])	خَطّ الظِلّ
shadow circle (II.13[4])	دائِرَة الظِلّ
dark	مُظْلِم
to be visible; to appear	يَظْهَرُ
visibility (II.14[2])	ظُهُور
of permanent visibility (II.1[1])	أبَدِيّ الظُهُور
period of visibility	زَمان الظُهُور
visible	ظاهِر

ع

most temperate (III.2[2])	أعْدَل
to adjust (a position)	يُعَدِّلُ
equation	تَعْدِيل
second equation (II.8[15])	التَعْدِيل الثانِي
equation of the proper (motion) (II.7[18])	تَعْدِيل الخاصّة
equation of the center and the proper (motion) (II.8[17])	تَعْدِيل المَرْكَز والخاصّة
independent equation (II.7[16])	التَعْدِيل المُفْرَد
equation of daylight (III.3[2])	تَعْدِيل نَهار
equation of the nychthemeron (III.8[8])	تَعْدِيل الأيّام بِلَيالِيها
equant (center) (II.8[14])	(مَرْكَز) مُعَدِّل المَسِير
equant (orb) (II.8[14])	(فَلَك) مُعَدِّل المَسِير
equinoctial (II.3[2])	مُعَدِّل النَهار
equality	تَعادُل
temperateness (III.2[4])	اِعْتِدال
equinox (point) (II.3[3])	(نُقْطَة) الاِعْتِدال
autumnal equinox (point)	(نُقْطَة) الاِعْتِدال الخَرِيفِيّ

vernal equinox (point) (نُقْطَة) الاِعْتِدال الرَبِيعِيّ

average (IV.1[2]) مُعْتَدِل

to disappear (*e.g.* a value) يَنْعَدِمُ

mineral مَعْدِن (ج) مَعادِن

width (I.1[1]); latitude (II.3[7]) عَرْض

local ecliptic latitude (II.3[16]) عَرْض إقْلِيم الرُؤْيَة

circle of local ecliptic latitude (II.3[16]) دائِرَة عَرْض إقْلِيم الرُؤْيَة

local latitude (II.3[13]) عَرْض البَلَد

latitude circle (II.3[7]) دائِرَة العَرْض

parallel of latitude (II.3[5]) مَدار عَرْضِيّ

characteristics عَرَض (ج) أعْراض

accidental (I.2[2]) عَرَضِيّ

Mercury (II.2[4]) عُطارِد

to turn (I.2[4]) يَنْعَطِفُ

Aquila (II.4[9]) العُقاب

node (II.7[7]) عُقْدَة

Scorpius العَقْرَب

conversely; opposite; reverse; vice versa بِالْعَكْس

reverse order (III.5[1]) مَعْكُوس

to be reflected (II.13[1]) يَنْعَكِسُ

the converse does not hold (I.2[3]) لا يَنْعَكِسُ

a reason عِلَّة (ج) عِلَل

science; discipline عِلْم (ج) عُلُوم

the World العالَم

world of generation and corruption (IV.7[5]) عالَم الكَوْن والفَساد

to mark (III.12[2]) يُعَلِّمُ

superior (I.Intr.[2]); celestial (II.1[title]) عَلَوِيّ / عُلْوِيّ / عِلْوِيّ

the upper planets (II.2[4]) (الكَواكِب) العَلَوِيّة

celestial bodies (III.[title]) العَلَوِيّات

elevated (II.14[1]) مُسْتَعْلٍ

perpendicular (I.1[5]) عَمُود

inhabited world (III.1[3]) المَعْمُور

depth (I.1[1]) عُمْق

element عُنْصُر (ج) عَناصِر / عُنْصُرِيّات

heavy element (I.2[3]) العُنْصُر الثَقِيل

light element (I.2[3]) العُنْصُر الخَفِيف

divine providence (III.1[6]) العِنايَة الإلٰهِيّة

to return يَعُودُ

to be given or assumed يُفْرَضُ

to become positionally separate from (I.2[2]) يُفارِقُ بِالْوَضْع

to cut or divide (I.1[18]) يَفْصِلُ

chapter; season فَصْل (ج) فُصُول

common part (I.1[4]) فَصْل مُشْتَرَك

excess فَضْل

difference تَفاضُل

Corona Borealis (II.4[9]) الفَكّة

orb (I.1[15]) فَلَك (ج) أفْلاك / فَلَكِيّات

the celestial sphere (II.1[4]); (II.3[17]) الفَلَك

the celestial bodies (I.2[3]) الفَلَكِيّات

celestial (I.2[1]) فَلَكِيّ

atlas orb (II.2[4]) فَلَك الأطْلَس

the ecliptic orb; the ecliptic equator (II.1[5]) فَلَك البُرُوج

orb of the fixed stars (II.2[4]) فَلَك الثَوابِت

solid orb (II.9[18]) فَلَك مُجَسَّم

orb of orbs (II.2[4]) فَلَك الأفْلاك

(horizon of) the right orb (أُفُق) الفَلَك المُسْتَقِيم

scientific discipline or exposition (I.Intr.[3]) فَنّ (ج) فُنُون

Boötes (II.4[9]) العَوّاء

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to set يَغْرُبُ (مص) غُرُوب

west point (II.3[13]) نُقْطَة المَغْرِب

the Maghrib المَغْرِب

co-descension (III.7[3]) مَغارِب

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to span (III.5[3]) يَسْتَغْرِقُ

breadth (IV.6[3]) غِلَظ

dense (II.1[1]) غَلِيظ

to widen يَسْتَغْلِظُ

depression (in the Earth) (II.1[2]) غَوْر (ج) أغْوار

maximum; limit (II.4[2]) غايَة

northern (southern) limit (II.10[4]) غايَة البُعْد الشَمالِيّ (الجَنُوبِيّ)

setting place مَغِيب

basin (III.1[5]) مَغِيض (ج) مَغائِض

ف

obtuse (I.1[5]) مُنْفَرِجَة

Pegasus (II.4[9]) الفَرَس الأعْظَم

parasang (IV.1[2]) فَرْسَخ (ج) فَراسِخ

difference تَفاوُت

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rule (II.5[10]) قانُون (ج) قَوانِين

cupola of the Earth (III.1[7]) قُبَّة الأَرْض

curvature (II.1[3]); cupola (II.1[7]) تَقْبِيب

to undergo (motion) (I.2[3]) يَقْبَلُ

qibla bearing (III.12[3]) سَمْت القِبْلَة

line of the *qibla* bearing خَطّ سَمْت القِبْلَة

arc of the *qibla* bearing قَوْس سَمْت القِبْلَة

arc of deviation of the *qibla* قَوْس اِنْحِراف القِبْلَة

qibla bearing point نُقْطَة سَمْت القِبْلَة

to face يُقابِلُ

to face one another يَتَقابَلُ

opposition مُقابَلَة / اِسْتِقْبال

accession (II.4[4]) إقْبال

trepidation الإقْبال والإدْبار

size; magnitude (of a star); scale (IV.2[4]) قَدْر (ج) أَقْدار

size; quantity; amount; extent; rate; measure; standard (IV.1[2]) مِقْدار (ج) مَقادِير

to assign a value; to measure يُقَدِّرُ

assumption (II.4[3]); unit of measure (IV.2[4]) تَقْدِير

lemma (II.11[2]) مُقَدِّمَة

by successive approximation (IV.5[4]) بِالاِسْتِقْراء

approximately بِالتَقْرِيب / تَقْرِيباً

to be in conjunction with يُقارِنُ

conjunction مُقارَنَة / اِقْتِران

true (apparent) latitudinal conjunction (II.14[3]) اِقْتِران عَرْضِيّ حَقِيقِيّ (مَرْئِيّ)

by compulsion (I.2[2]) بِالقَسْر

to divide by يَقْسِمُ عَلَى

to lead to (II.14[2]); to result in (II.13[8]); to require يَقْتَضِي

pole (I.1[12]) قُطْب (ج) أَقْطاب

diameter (I.1[8]) قُطْر (ج) أَقْطار

radius نِصْف قُطْر (ج) أَنْصاف أَقْطار

opposition (II.1[4]) مُقاطَرَة

aligned with (II.13[2]) مُقاطِر لِ

to cut off or describe (*e.g.* an arc); to traverse	يَقْطَعُ
part (I.1[9]); segment (I.1[11]); portion (IV.1[1])	قِطْعَة (ج) قِطَع
Equuleus (II.4[9])	قِطْعَة الفَرَس
to intersect	يُقاطِعُ
to intersect one another	يَتَقاطَعُ
intersection point	نُقْطَة تَقاطُع
base (I.1[16])	قاعِدَة (ج) قَواعِد
concave (I.1[15])	مُقَعَّر
solstice (point)(II.3[4])	(نُقْطَة) الانْقِلاب
winter solstice (point) (II.3[4])	(نُقْطَة) الانْقِلاب الشَتْوِيَة
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arc (I.1[9])	قَوْس (ج) قُسِيّ / قِسِيّ
Sagittarius	القَوْس
arc of night (III.10[1])	قَوْس اللَيْل
arc of daylight (III.10[1])	قَوْس النَهار
to stand erect	يَقُومُ
at right angles	عَلَى قَوائِم
to be perpendicular	يَقُومُ عَلَى زَوايا قائِمَة
true position (II.6[5])	تَقْوِيم
to undergo direct motion (II.5[9])	يَسْتَقِيمُ
gnomon (III.12[1])	مِقْياس
Cetus (II.4[9])	قِيْطُس
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ك

differing size	كبر وصغر
to intercalate (III.10[3])	يَكْبِسُ
intercalary (day) (III.10[2])	(يَوم) كَبِيس (ج) كَبائِس
practical handbook	كِتاب العَمَل
thickness (II.13[1])	كَثافَة
thick	كَثِيف
to contract (I.2[4])	يَتَكاثَفُ
sphere (I.1[10])	كُرَة (ج) كُرات
enclosing sphere (II.11[4])	الكُرَة المُحِيطَة

starless (II.2[4]) غَيْر مُكَوْكَب

proper place (I.2[1]) مَكان (ج) أَمْكِنَة

natural place (I.2[2]) مَكان بِالطَبْع

quality; كَيْفِيَّة (ج) كَيْفِيات

weather condition (III.2[2])

ل

to mend (I.2[4]) يَلْتَئِمُ

the Milky Way الدائِرَة اللَبَنِيَّة (II.4[10])

the supplementary اللَواحِق (epagomenal) days (III.10[3])

to dissolve (II.2[5]) يَتَلاشى

winding (II.10[5]) الاِلْتِفاف

to meet (trans.) يُلاقِي

to meet one another يَتَلاقَى

to meet (intrans.) يَلْتَقِي

twist (II.10[5]) الاِلْتِواء

night (time) لَيْل (ج) لَيالٍ

arc of night (III.10[1]) قَوْس اللَيْل

م

the equal of; times مِثْل (ج) أَمْثال

par- (الفَلَك) المُمَثِّل بِفَلَك البُرُوج ecliptic (orb) (II.6[3])

small sphere (Fig. T13) الكُرَة الصَغِيرَة

large sphere (Fig. T13) الكُرَة الكَبِيرَة

fraction كَسْر (ج) كُسُور

area (IV.1[1]) تَكْسِير

square (*e.g.* miles) (IV.1[3]) تَكْسِيرِيّ

to eclipse or occult يَكْسِفُ (used for the sun) (II.2[4])

solar eclipse (II.13[7]) كُسُوف

occulting (body) (II.13[9]) كاسِف

to be eclipsed or occulted يَنْكَسِفُ (used for the sun) (II.13[7])

uncovering of the Earth اِنْكِشاف الأَرْض

to cube (IV.5[7]) يُكَعِّبُ

cube (IV.4[2]) مُكَعَّب

balancing (III.2[4]) تَكافُؤ

the Universe (II.1[4]) الكُلّ

Corona Australis الإِكْلِيل الجَنُوبِيّ (II.4[9])

Canis Minor (II.4[9]) الكَلْب الأَصْغَر

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quantity كَمِّيَّة (ج) كَمِّيّات

Kangdezh (III.1[7]) كَنْكْدِز

star; planet كَوْكَب (ج) كَواكِب

المَجِسْطِي	*Almagest*
المُحاق	new moon (II.13[1])
يَمْتَحِنُ	to test (II.14[2])
مَحْو	lunar marking (II.7[23])
يَمُرُّ	to transit
مُرُور	transit; passage
الدائِرَة المارّة بالأَقْطاب الأَرْبَعَة	solstitial colure (II.3[4])
المَرْأة المُسَلْسَلَة	Andromeda (II.4[9])
المِرّيخ	Mars (II.2[4])
يُماسُّ	to touch; be tangent to; be contiguous with
يَتَماسُّ	to be tangent to one another
مِساحَة	area; measure (IV.1)
مُمْسِك العِنان	Auriga (II.4[9])
(الطَرَف) المَسائِيّ	evening (endpoint) (II.10[5])
مِصْر	Egypt (III.1[4])
ماضٍ	elapsed (III.8[7])
يَمْكُثُ	to have duration (said of eclipses) (II.13[4])
مُكْث	duration (of eclipse) (II.13[3])
ماء (ج) مِياه	water
مَيْل (ج) مُيُول	inclination; declination (II.3[6]); deviation (II.10[4])
مَيْل أوّل	first declination (II.3[7])
مَيْل ثانٍ	second declination (II.3[7])
مَيْل جُزْئِيّ	particular declination (II.3[6])
مَيْل مُسْتَقِيم	rectilinear inclination (II.1[6])
المَيْل الكُلِّيّ	total obliquity (II.3[3])
دائِرَة المَيْل (ج) دَوائِر المُيُول	declination circle (II.3[6])
(الفَلَك) المائِل	inclined (orb) (II.7[4])
(الآفاق) المائِلَة	oblique horizons (III.3[title])
مِيل (ج) أمْيال	mile (IV.1[2])

ن

نَبات (ج) نَباتات / نَباتِيّات	plants
نَباتِيّ	vegetative (I.2[2])
مَنْزِل (ج) مَنازِل القَمَر	mansions of the moon (II.4[11])
نِسْبَة (ج) نِسَب	ratio
عَلَى نِسْبَة	proportionally (II.1[1])
مَنْشَأ	place of origin (II.2[5])
مَنْشُور (ج) مَناشِير	truncated orb (II.11[16])

hemisphere (III.1[6]) نِصْف

radius نِصْف قُطْر (ج) أنْصاف أقْطار

noon نِصْف النَهار

meridian circle دائِرَة نِصْف النَهار
(II.3[13])

meridian line خَطّ نِصْف النَهار
(III.12[1])

to bisect (I.1[8]) يُنَصِّفُ

to bisect one another يَتَناصَفُ

noon (III.4[2]) انْتِصاف النَهار

midpoint (I.1[9]); مُنْتَصَف
middle part (II.3[17]);
mid-distance (IV.6[5])

arrangement; order نَضْد

(planetary) sector (II.14[1]) نِطاق

equator (I.1[13]); مِنْطَقَة (ج) مَناطِق
inner equator (II.5[10])

equator of مِنْطَقَة الكُرَة الصَغِيرَة
small sphere (II.11[4])

equator of مِنْطَقَة الكُرَة الكَبِيرَة
large sphere (II.11[4])

in appearance في النَظَر

observer (II.2[1]) ناظِر

parallax (II.12[1]) اخْتِلاف المَنْظَر

science of optics (IV.4[1]) عِلْم المَناظِر

soul (I.2[2]) نَفْس

to subtract from يَنْقُصُ مِن

subtractive ناقِص

point (I.1[1]) نُقْطَة (ج) نُقَط

to shift (II.4[4]) يَنْتَقِلُ

to grow (I.2[4]) نَما / يَنْمُو

monoformly (I.2[1]) عَلَى نَهْج واحِد

Eridanus (II.4[9]) النَهْر

day (time); نَهار (ج) أنْهُر
daylight (III.1[8])

period of daylight ساعات النَهار
(III.1[8])

equation of daylight تَعْدِيل نَهار
(III.3[2])

arc of daylight (III.10[1]) قَوْس النَهار

without limit إلَى غَيْر نِهايَة / بِلا نِهايَة

to end (I.1[1]) يَنْتَهِي (مص) انْتِهاء

to reach (II.1[1]) يَنْتَهِي إلَى

termination (III.1[8]) مُنْتَهَى

fire نار

illumination (II.13[title]) نُور

luminous نَيِّر

luminous bodies أجْرام نَيِّرَة

the two luminaries النَيِّران

illumination إنارَة

illuminated مُنِير

species; class; type نَوْع (ج) أنْواع

meteor (II.2[5]) نَيْزَك (ج) نَيازِك

the Nile (III.1[4]) النِيْل

هـ

falls of the two luminaries (III.1[6]) هُبُوطا النَيِّرَيْن

descending (II.5[11]) هابِط

crescent (III.10[2]) هِلال

crescent-shaped (II.13[1]) هِلالِيّ الشَكْل

geometry الهَنْدَسَة (ج) الهَنْدَسِيات

air هَواء

configuration; shape (II.13[3]) هَيْئَة

astronomy عِلْم الهَيْئَة

و

chord (I.1[9]); subtense (III.7[2]) وَتَر

to subtend (IV.3[3]) يُوتِرُ

face (of a celestial body) وَجْه

direction جِهَة (ج) جِهات

unit (IV.6[5]) واحِد

slope (II.10[5]) الوِراب

Libra المِيزان

to be parallel to يُوازِي

to be parallel to one another يَتَوازَى

the middle; the mean; midpoint وَسَط (ج) أوْساط

middle (of Aries, etc.) (Fig. T24) وَسَط (الحَمَل)

nodal mean (II.7[29]) وَسَط الجَوْزَهَر

solar mean (II.6[5]) وَسَط الشَمْس

lunar mean (II.7[29]) وَسَط القَمَر

ecliptic meridian circle (midheaven circle of appearances) (II.3[16]) (دائِرَة) وَسَط سَماء الرُؤيَة

orb of the middle of the zodiacal signs (II.3[9]) (فَلَك) أوْساط البُرُوج

middle; mean (adj.); average أوْسَط (ج) أواسِط

middle part (III.8[5]) أواسِط

to be at a mean; to bisect يَتَوَسَّطُ

ortive amplitude (III.2[1]) سَعَة مَشْرِق

occasive amplitude (III.2[1]) سَعَة مَغْرِب

وُقُوف	station (of a planet) (II.5[8])
عَلى (خِلاف) تَوالِي البُرُوج	in the (counter-) sequence of the zodiacal signs
مُتَوالٍ	consecutive

ي

اليَوْم بِلَيْلَتِه	nychthemeron (III.8[1])
يَوْم حَقِيقِيّ	true day (III.8[6])
يَوْم أوْسَط (ج) أيّام وُسْطَى	mean day (III.8[2])
وَصَلَ / يَصِلُ	to join
يَتَّصِلُ بِ	to be joined to; to be adjacent or contiguous
وَضْع (ج) أوْضاع	position
مَوْضِع (ج) مَواضِع	location
مَوْضِع حَقِيقِيّ	true position
مَوْضِع مَرْئِيّ	apparent position
مَوْضُوع	subject (I.Intr.[1])
(فَلَك) مُوافِق مَرْكَز	concentric (orb) (II.5[5])
وَقْت (ج) أوْقات	time

§4. Works Cited

Note that the Arabic particle *al-* has been ignored for alphabetizing the entries.

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§5. Indices

A. Subject Index

Note that the Arabic particle *al-* as well as *kitāb* (book of...) have been ignored for alphabetizing entries. References of the form II.5[2] refer to the text, translation, and, if there is any, commentary; in this case, Book II, Chapter 5, paragraph 2.

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B. Parameter Index

Ordering is by increasing numerical value. Neugebauer's sexagesimal notation has been adopted whereby 3;06° is 3 degrees, 6 minutes. Parameters in the tables and charts on pages 425, 448, 457, 460, 470, 472, 493, 508, 513, 528, and 529 have not been duplicated below.