Sir Isaac Newtons own copy, with notes in his hand writing on 44 pages, being either additions to, or alterations of the printed.

Case U. C.7. Barnsley

OPTICKS:

OR, A

TREATISE

OF THE

REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS

OF

LIGHT.

ALSO

Two TREATISES

OF THE

SPECIES and MAGNITUDE

OF

Curvilinear Figures.

LONDON,

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ADVERTISEMENT.

PArt of the enſuing Diſcourſe about Light was written at the deſire of ſome Gentlemen of the Royal Society, in the Year 1675. and then ſent to their Secretary, and read at their Meetings, and the reſt was added about Twelve Years after to complete the Theory; except the Third Book, and the laſt Propoſition of the Second, which were ſince put together out of ſcattered Papers. To avoid being engaged in Diſputes about theſe Matters, I have hitherto delayed the Printing, and ſhould ſtill have delayed it, had not the importunity of Friends prevailed upon me. If any other Papers writ on this Subject are got out of my Hands they are imperfect, and were perhaps written before I had tried all the Experiments here ſet down, and fully ſatisfied my ſelf about the Laws of Refractions and Compoſition of Colours. I have here Publiſhed what I think proper to come abroad, wiſhing that it may not be Tranſlated into another Language without my Conſent.

The Crowns of Colours, which ſometimes appear about the Sun and Moon, I have endeavoured to give an Account of; but for want of ſufficient Obſervations leave that Matter to be further examined. The Subject of the Third Book I have alſo left imperfect, not having tried all the

Expe-

Experiments which I intended when I was about these Matters, nor repeated some of those which I did try, until I had satisfied my self about all their Circumstances. To communicate what I have tried, and leave the rest to others for further Enquiry, is all my Design in publishing these Papers.

In a Letter written to Mr. Leibnitz *in the Year* 1676. *and published by Dr.* Wallis, *I mentioned a Method by which I had found some general Theorems about squaring Curvilinear Figures, or comparing them with the Conic Sections, or other the simplest Figures with which they may be compared. And some Years ago I lent out a Manuscript containing such Theorems, and having since met with some Things copied out of it, I have on this Occasion made it publick, prefixing to it an* Introduction *and subjoyning a* Scholium *concerning that Method. And I have joined with it another small Tract concerning the Curvilinear Figures of the Second Kind, which was also written many Years ago, and made known to some Friends, who have solicited the making it publick.*

I. N.

The FIRST BOOK OF OPTICKS.

PART I.

My Design in this Book is not to explain the Properties of Light by Hypotheses, but to propose and prove them by Reason and Experiments: In order to which, I shall premise the following Definitions and Axioms.

DEFINITIONS.

DEFIN. I.

By the Rays of Light I understand its least Parts, and those as well Successive in the same Lines as Contemporary in several Lines. For it is manifest that Light consists of parts both Successive and Contemporary; because in the same place you may stop that which comes one moment, and let pass that which comes presently after; and in the same time you may stop it in any one place, and let it pass in any other. For that part of Light which is stopt cannot be the same with that which is let pass. The least Light or part of Light, which may be stopt alone without the rest of the Light, or propagated alone, or do or suffer any thing alone, which the reſt of the Light doth not or ſuffers not, I call a Ray of Light.

DEFIN. II.

Refrangibility of the Rays of Light, is their Diſpoſition to be refracted or turned out of their Way in paſſing out of one tranſparent Body or Medium into another. And a greater or leſs Refrangibility of Rays, is their Diſpoſition to be turned more or leſs out of their Way in like Incidences on the ſame Medium. Mathematicians uſually conſider the Rays of Light to be Lines reaching from the luminous Body to the body illuminated, and the refraction of thoſe Rays to be the bending or breaking of thoſe Lines in their paſſing out of one Medium into another. And thus may Rays and Refractions be conſidered, if Light be propagated in an inſtant. But by an Argument taken from the Æquations of the times of the Eclipſes of *Jupiter*'s *Satellites* it ſeems that Light is propagated in time, ſpending in its paſſage from the Sun to us about Seven Minutes of time: And therefore I have choſen to define Rays and Refractions in ſuch general terms as may agree to Light in both caſes.

DEFIN. III.

Reflexibility of Rays, is their Diſpoſition to be reflected or *turned back into the ſame Medium from any other Medium upon whoſe Surface they fall. And Rays are more or leſs reflexible, which are returned back more or leſs eaſily.* As if Light paſs out of Glaſs into Air, and by being inclined more and more to the common Surface of the Glaſs and Air, begins at length to be totally reflected by that Surface; thoſe ſorts of Rays which at like Incidences are reflected moſt copiouſly, or by inclining the Rays begin ſooneſt to be totally reflected, are moſt reflexible.

DE-

DEFIN. IV.

The Angle of Incidence, is that Angle which the Line described by the incident Ray contains with the Perpendicular to the reflecting or refracting Surface at the Point of Incidence.

DEFIN. V.

The Angle of Reflexion or Refraction, is the Angle which the Line described by the reflected or refracted Ray containeth with the Perpendicular to the reflecting or refracting Surface at the Point of Incidence.

DEFIN. VI.

The Sines of Incidence, Reflexion, and Refraction, are the Sines of the Angles of Incidence, Reflexion, and Refraction.

DEFIN. VII.

The Light whose Rays are all alike Refrangible, I call Simple, Homogeneal and Similar; and that whose Rays are some more Refrangible than others, I call Compound, Heterogeneal and Dissimilar. The former Light I call Homogeneal, not because I would affirm it so in all respects; but because the Rays which agree in Refrangibility, agree at least in all those their other Properties, which I consider in the following Discourse.

DEFIN. VIII.

The Colours of Homogeneal Lights, I call Primary, Homogeneal and Simple; and those of Heterogeneal Lights, Heterogeneal and Compound. For these are always compounded of the colours of Homogeneal Lights; as will appear in the following Discourse.

AXIOMS.

AX. I.

THE Angles of Incidence, Reflexion, and Refraction, lye in one and the ſame Plane.

AX. II.

The Angle of Reflexion is equal to the Angle of Incidence.

AX. III.

If the refracted Ray be returned directly back to the Point of Incidence, it ſhall be refracted into the Line before deſcribed by the incident Ray.

AX. IV.

Refraction out of the rarer Medium into the denſer, is made towards the Perpendicular; that is, ſo that the Angle of Refraction be leſs than the Angle of Incidence.

AX. V.

The Sine of Incidence, is either accurately or very nearly in a given Ratio to the Sine of Refraction.

Whence if that Proportion be known in any one Inclination of the incident Ray, 'tis known in all the Inclinations, and thereby the Refraction in all caſes of Incidence on the ſame refracting Body may be determined. Thus if the Refraction be made out of Air into Water, the Sine of Incidence of the red Light is to the Sine of its Refraction as 4 to 3. If out of Air into Glaſs, the Sines are as 17 to 11. In Light of other Colours the Sines have other Proportions : but the difference is so little that it need seldom be considered.

Suppose therefore, that R S represents the Surface of stagnating Water, and C is the point of Incidence in which any Ray coming in the Air from A in the Line A C is reflected or refracted, and I would know whether this Ray shall go after Reflexion or Refraction : I erect upon the Surface of the Water from the point of Incidence the Perpendicular C P and produce it downwards to Q, and conclude by the first Axiom, that the Ray after Reflexion and Refraction, shall be found somewhere in the Plane of the Angle of Incidence A C P produced. I let fall therefore upon the Perpendicular C P the Sine of Incidence A D, and if the reflected Ray be desired, I produce A D to B so that D B be equal to A D, and draw C B. For this Line C B shall be the reflected Ray; the Angle of Reflexion B C P and its Sine B D being equal to the Angle and Sine of Incidence, as they ought to be by the second Axiom. But if the refracted Ray be desired, I produce A D to H, so that D H may be to A D as the Sine of Refraction to the Sine of Incidence, that is as 3 to 4; and about the Center C and in the Plane A C P with the Radius C A describing a Circle A B E, I draw Parallel to the Perpendicular C P Q, the Line H E cutting the circumference in E, and joyning C E, this Line C E shall be the Line of the refracted Ray. For if E F be let fall perpendicularly on the Line P Q, this Line E F shall be the Sine of Refraction of the Ray C E, the Angle of Refraction being E C Q; and this Sine E F is equal to D H, and consequently in Proportion to the Sine of Incidence A D as 3 to 4. *Fig.* 1.

In

In like manner, if there be a Prifm of Glafs (that is a Glafs bounded with two Equal and Parallel Triangular ends, and three plane and well polifhed Sides, which meet in three Parallel Lines running from the three Angles of one end to the three Angles of the other end) and if the Refraction of the Light in paffing crofs this Prifm be defi-
Fig. 2. red : Let ACB reprefent a Plane cutting this Prifm tranfverfly to its three Parallel lines or edges there where the Light paffeth through it, and let DE be the Ray incident upon the firft fide of the Prifm A C where the Light goes into the Glafs; And by putting the Proportion of the Sine of Incidence to the Sine of Refraction as 17 to 11 find E F the firft refracted Ray. Then taking this Ray for the Incident Ray upon the fecond fide of the Glafs B C where the Light goes out, find the next refracted Ray F G by putting the Proportion of the Sine of Incidence to the Sine of Refraction as 11 to 17. For if the Sine of Incidence out of Air into Glafs be to the Sine of Refraction as 17 to 11, the Sine of Incidence out of Glafs into Air muft on the contrary be to the Sine of Refraction as 11 to 17, by the third Axiom.

Fig. 3. Much after the fame manner, if A C B D reprefent a Glafs fpherically Convex on both fides (ufually called a *Lens*, fuch as is a Burning-glafs, or Spectacle-glafs, or an Object-glafs of a Telefcope) and it be required to know how Light falling upon it from any lucid point Q fhall be refracted, let QM reprefent a Ray falling upon any point M of its firft fpherical Surface ACB, and by erecting a Perpendicular to the Glafs at the point M, find the firft refracted Ray M N by the Proportion of the Sines 17 to 11. Let that Ray in going out of the Glafs be incident upon N, and then find the fecond refracted Ray N *q* by the Proportion of the Sines 11 to 17. And after the fame ſame manner may the Refraction be found when the Lens is Convex on one ſide and Plane or Concave on the other, or Concave on both Sides.

AX. VI.

perpendicularly
Homogeneal Rays which flow from ſeveral Points of any Object, and fall almoſt Perpendicularly on any reflecting or refracting Plane or Spherical Surface, ſhall afterwards diverge from ſo many other Points, ~~or be Parallel to ſo many other Lines~~, or converge to ſo many other Points, or be parallel to so many severall lines, *either accurately or without any ſenſible Error. And the ſame thing will happen, if the Rays be reflected or refracted ſucceſſively by two or three or more Plane or ſpherical Surfaces.*

The Point from which Rays diverge or to which they converge may be called their *Focus*. And the Focus of the incident Rays being given, that of the reflected or refracted ones may be found by finding the Refraction of any two Rays, as above; or more readily thus.

Caſ. 1. Let ACB be a reflecting or refracting Plane, and Q the Focus of the incident Rays, and Q*q* C a perpendicular to that Plane. And if this perpendicular be produced to *q*, ſo that *q* C be equal to QC, the point *q* ſhall be the Focus of the reflected Rays. Or if *q* C be taken on the ſame ſide of the Plane with QC and in Proportion to QC as the Sine of Incidence to the Sine of Refraction, the point *q* ſhall be the Focus of the refracted Rays. *Fig.* 4.

Caſ. 2. Let ACB be the reflecting Surface of any Sphere whoſe Center is E. Biſect any Radius thereof (ſuppoſe EC) in T, and if in that Radius on the ſame ſide the point T you take the Points Q and *q*, ſo that TQ, TE, and T*q* be continual Proportionals, and the point Q be the *Fig.* 5.

the Focus of the incident Rays, the point *q* ſhall be the Focus of the reflected ones.

Fig. 6. *Caſ.* 3. Let A C B be the refracting Surface of any Sphere whoſe Center is E. In any Radius thereof E C produced both ways take E T and C *t* ~~ſeverally~~ equal to one another in ſuch Proportion to that Radius as the leſſer of the Sines of Incidence and Refraction hath to the difference of thoſe Sines. And then if in the ſame Line you find any two Points Q and *q*, ſo that T Q be to E T as E *t* to *t q*, taking *t q* the contrary way from *t* which T Q lieth from T, and if the Point Q be the Focus of any incident Rays, the Point *q* ſhall be the Focus of the refracted ones.

And by the ſame means the Focus of the Rays after two or more Reflexions or Refractions may be found.

Fig. 7. *Caſ.* 4. Let A C B D be any refracting Lens, ſpherically Convex or Concave or Plane on either ſide, and let C D be its Axis (that is the Line which cuts both its Surfaces perpendicularly, and paſſes through the Centers of the Spheres,) and in this Axis let F and *f* be the Foci of the refracted Rays found as above, when the incident Rays on both ſides the Lens are Parallel to the ſame Axis; and upon the Diameter F *f* biſected in E, deſcribe a Circle. Suppoſe now that any Point Q be the Focus of any incident Rays. Draw Q E cutting the ſaid Circle in T and *t*, and therein take *t q* in ſuch Proportion to *t* E as *t* E or T E hath to T Q. Let *t q* lye the contrary way from *t* which T Q doth from T, and *q* ſhall be the Focus of the refracted Rays without any ſenſible Error, provided the Point Q be not ſo remote from the Axis, nor the Lens ſo broad as to make any of the Rays fall too obliquely on the refracting Surfaces.

And by the like Operations may the reflecting or refracting Surfaces be found when the two Foci are given, and

and thereby a Lens be formed, which ſhall make the Rays flow towards or from what place you pleaſe.

So then the meaning of this Axiom is, that if Rays fall upon any Plane or Spherical Surface or Lens, and before their Incidence flow from or towards any Point Q, they ſhall after Reflexion or Refraction flow from or towards the Point q found by the foregoing Rules. And if the incident Rays flow from or towards ſeveral points Q, the reflected or refracted Rays ſhall flow from or towards ſo many other Points q found by the ſame Rules. Whether the reflected and refracted Rays flow from or towards the Point q is eaſily known by the ſituation of that Point. For if that Point be on the ſame ſide of the reflecting or refracting Surface or Lens with the Point Q, and the incident Rays flow from the Point Q, the reflected flow towards the Point q and the refracted from it; and if the incident Rays flow towards Q, the reflected flow from q, and the refracted towards it. And the contrary happens when q is on the other ſide of that Surface.

A X. VII.

Wherever the Rays which come from all the Points of any Object meet again in ſo many Points after they have been made to converge by Reflexion or Refraction, there they will make a Picture of the Object upon any white Body on which they fall.

So if PR repreſent any Object without Doors, and AB be a Lens placed at a hole in the Window-ſhut of a dark Chamber, whereby the Rays that come from any Point Q of that Object are made to converge and meet again in the Point q; and if a Sheet of white Paper be held at q for the Light there to fall upon it: the Picture of that Object PR will appear upon the Paper in its proper Shape *Fig.* 3.

 and

and Colours. For as the Light which comes from the Point Q goes to the Point *q*, ſo the Light which comes from other Points P and R of the Object, will go to ſo many other correſpondent Points *p* and *r* (as is manifeſt by the ſixth Axiom;) ſo that every Point of the Object ſhall illuminate a correſpondent Point of the Picture, and thereby make a Picture like the Object in Shape and Colour, this only excepted that the Picture ſhall be inverted. And this is the reaſon of that Vulgar Experiment of caſting the Species of Objects from abroad upon a Wall or Sheet of white Paper in a dark Room.

Fig. 8. In like manner when a Man views any Object P Q R, the Light which comes from the ſeveral Points of the Object is ſo refracted by the tranſparent skins and humours of the Eye, (that is by the outward coat E F G called the *Tunica Cornea*, and by the cryſtalline humour A B which is beyond the Pupil *m k*) as to converge and meet again at ſo many Points in the bottom of the Eye, and there to paint the Picture of the Object upon that skin (called the *Tunica Retina*) with which the bottom of the Eye is covered. For Anatomiſts when they have taken off from the bottom of the Eye that outward and moſt thick Coat called the *Dura Mater*, can then ſee through the thinner Coats the Pictures of Objects lively painted thereon. And theſe Pictures propagated by Motion along the Fibres of the Optick Nerves into the Brain, are the cauſe of Viſion. For accordingly as theſe Pictures are perfect or imperfect, the Object is ſeen perfectly or imperfectly. If the Eye be tinged with any colour (as in the Diſeaſe of the *Jaundiſe*) ſo as to tinge the Pictures in the bottom of the Eye with that Colour, then all Objects appear tinged with the ſame Colour. If the humours of the Eye by old Age decay, ſo as by ſhrinking to make the *Cornea* and Coat of the *Cry-*

ſtalline

stalline humour grow flatter than before, the Light will not be refracted enough, and for want of a sufficient Refraction will not converge to the bottom of the Eye but to some place beyond it, and by consequence paint in the bottom of the Eye a confused Picture, and according to the indistinctness of this Picture the Object will appear confused. This is the reason of the decay of Sight in old Men, and shews why their Sight is mended by Spectacles. For those Convex-glasses supply the defect of plumpness in the Eye, and by encreasing the Refraction make the Rays converge sooner so as to convene distinctly at the bottom of the Eye if the Glass have a due degree of convexity. And the contrary happens in short-sighted Men whose Eyes are too plump. For the Refraction being now too great, the Rays converge and convene in the Eyes before they come at the bottom; and therefore the Picture made in the bottom and the Vision caused thereby will not be distinct, unless the Object be brought so near the Eye as that the place where the converging Rays convene may be removed to the bottom, or that the plumpness of the Eye be taken off and the Refractions diminished by a Concave-glass of a due degree of Concavity, or lastly that by Age the Eye grow flatter till it come to a due Figure: For short-sighted Men see remote Objects best in Old Age, and therefore they are accounted to have the most lasting Eyes.

A X. VIII.

An Object seen by Reflexion or Refraction, appears in that place from whence the Rays after their last Reflexion or Refraction diverge in falling on the Spectator's Eye.

If the Object A be seen by Reflexion of a Looking-glass *m n*, it shall appear, not in it's proper place A, but *Fig. 9.*

 behind

behind the Glaſs at *a*, from whence any Rays AB, AC, AD, which flow from one and the ſame Point of the Object, do after their Reflexion made in the Points B, C, D, diverge in going from the Glaſs to E, F, G, where they are incident on the Spectator's Eyes. For theſe Rays do make the ſame Picture in the bottom of the Eyes as if they had come from the Object really placed at *a* without the interpoſition of the Looking-glaſs; and all Viſion is made according to the place and ſhape of that Picture.

Fig. 2. In like manner the Object D ſeen through a Priſm appears not in its proper place D, but is thence tranſlated to ſome other place *d* ſituated in the laſt refracted Ray F G drawn backward from F to *d*.

Fig. 10. And ſo the Object Q ſeen through the Lens AB, appears at the place *q* from whence the Rays diverge in paſſing from the Lens to the Eye. Now it is to be noted, that the Image of the Object at *q* is ſo much bigger or leſſer than the Object it ſelf at Q, as the diſtance of the Image at *q* from the Lens AB is bigger or leſs than the diſtance of the Object at Q from the ſame Lens. And if the Object be ſeen through two or more ſuch Convex or Concave-glaſſes, every Glaſs ſhall make a new Image, and the Object ſhall appear in the place and of the bigneſs of the laſt Image. Which conſideration unfolds the Theory of Microſcopes and Teleſcopes. For that Theory conſiſts in almoſt nothing elſe than the deſcribing ſuch Glaſſes as ſhall make the laſt Image of any Object as diſtinct and large and luminous as it can conveniently be made.

I have now given in Axioms and their Explications the ſumm of what hath hitherto been treated of in Opticks. For what hath been generally agreed on I content my ſelf to aſſume under the notion of Principles, in order to what I have further to write. And this may ſuffice for an

Intro-

Introduction to Readers of quick Wit and good Underſtanding not yet verſed in Opticks: Although thoſe who are already acquainted with this Science, and have handled Glaſſes, will more readily apprehend what followeth.

PROPOSITIONS.

PROP. I. Theor. I.

LIGHTS which differ in Colour, differ alſo in Degrees of Refrangibility.

The Proof by Experiments.

Exper. 1. I took a black oblong ſtiff Paper terminated by Parallel Sides, and with a Perpendicular right Line drawn croſs from one Side to the other, diſtinguiſhed it into two equal Parts. One of theſe Parts I painted with a red Colour and the other with a blew. The Paper was very black, and the Colours intenſe and thickly laid on, that the Phænomenon might be more conſpicuous. This Paper I viewed through a Priſm of ſolid Glaſs, whoſe two Sides through which the Light paſſed to the Eye were plane and well poliſhed, and contained an Angle of about Sixty Degrees: which Angle I call the refracting Angle of the Priſm. And whilſt I viewed it, I held it before a Window in ſuch manner that the Sides of the Paper were parallel to the Priſm, and both thoſe Sides and the Priſm parallel to the Horizon, and the croſs Line also parallel ~~perpendicular~~ to it; and that the Light which fell from the Window

upon

upon the Paper made an Angle with the Paper, equal to that Angle which was made with the ſame Paper by the Light reflected from it to the Eye. Beyond the Priſm was the Wall of the Chamber under the Window covered over with black Cloth, and the Cloth was involved in Darkneſs that no Light might be reflected from thence, which in paſſing by the edges of the Paper to the Eye, might mingle it ſelf with the Light of the Paper and obſcure the Phænomenon thereof. Theſe things being thus ordered, I found that if the refracting Angle of the Priſm be turned upwards, ſo that the Paper may ſeem to be lifted upwards by the Refraction, its blew half will be lifted higher by the Refraction than its red half. But if the refracting Angle of the Priſm be turned downward, ſo that the Paper may ſeem to be carried lower by the Refraction, its blew half will be carried ſomething lower thereby than its red half. Wherefore in both caſes the Light which comes from the blew half of the Paper through the Priſm to the Eye, does in like Circumſtances ſuffer a greater Refraction than the Light which comes from the red half, and by conſequence is more refrangible.

Fig. 11. *Illuſtration.* In the Eleventh Figure, M N repreſents the Window, and D E the Paper terminated with parallel Sides D J and H E, and by the tranſverſe Line F G diſtinguiſhed into two halfs, the one D G of an intenſely blew Colour, the other F E of an intenſely red. And B A C *c a b* repreſents the Priſm whoſe refracting Planes A B *b a* and A C *c a* meet in the edge of the refracting Angle A *a*. This edge A *a* being upward, is parallel both to the Horizon and to the parallel edges of the Paper D J and H E. And *de* repreſents the Image of the Paper ſeen by Refraction upwards in ſuch manner that the blew half D G is carried higher to *d g* than the red half F E is to *f e*, and therefore ſuffers

ſuffers a greater Refraction. If the edge of the refracting Angle be turned downward, the Image of the Paper will be refracted downward ſuppoſe to $\delta\epsilon$, and the blew half will be refracted lower to $\delta\gamma$ than the red half is to $\phi\epsilon$.

Exper. 2. About the aforeſaid Paper, whoſe two halfs were painted over with red and blew, and which was ſtiff like thin Paſtboard, I lapped ſeveral times a ſlender thred of very black Silk, in ſuch manner that the ſeveral parts of the thred might appear upon the Colours like ſo many black Lines drawn over them, or like long and ſlender dark Shadows caſt upon them. I might have drawn black Lines with a Pen, but the threds were ſmaller and better defined. This Paper thus coloured and lined I ſet againſt a Wall perpendicularly to the Horizon, ſo that one of the Colours might ſtand to the right hand and the other to the left. Cloſe before the Paper at the confine of the Colours below I placed a Candle to illuminate the Paper ſtrongly: For the Experiment was tried in the Night. The flame of the Candle reached up to the lower edge of the Paper, or a very little higher. Then at the diſtance of Six Feet and one or two Inches from the Paper upon the Floor I erected a glaſs Lens four Inches and a quarter broad, which might collect the Rays coming from the ſeveral Points of the Paper, and make them converge towards ſo many other Points at the ſame diſtance of ſix Feet and one or two Inches on the other ſide of the Lens, and ſo form the Image of the coloured Paper upon a white Paper placed there; after the ſame manner that a Lens at a hole in a Window caſts the Images of Objects abroad upon a Sheet of white Paper in a dark Room. The aforeſaid white Paper, erected perpendicular to the Horizon and to the Rays which fell upon it from the Lens, I moved ſometimes towards the Lens, ſometimes from it, to find

the

the places where the Images of the blew and red parts of the coloured Paper appeared moſt diſtinct. Thoſe places I eaſily knew by the Images of the black Lines which I had made by winding the Silk about the Paper. For the Images of thoſe fine and ſlender Lines (which by reaſon of their blackneſs were like Shadows on the Colours) were confuſed and ſcarce viſible, unleſs when the Colours on either ſide of each Line were terminated moſt diſtinctly. Noting therefore, as diligently as I could, the places where the Images of the red and blew halfs of the coloured Paper appeared moſt diſtinct, I found that where the red half of the Paper appeared diſtinct, the blew half appeared confuſed, ſo that the black Lines drawn upon it could ſcarce be ſeen; and on the contrary where the blew half appeared moſt diſtinct the red half appeared confuſed, ſo that the black Lines upon it were ſcarce viſible. And between the two places where theſe Images appeared diſtinct there was the diſtance of an Inch and a half: the diſtance of the white Paper from the Lens, when the Image of the red half of the coloured Paper appeared moſt diſtinct, being greater by an Inch and an half than the diſtance of the ſame white Paper from the Lens when the Image of the blew half appeared moſt diſtinct. In like Incidences therefore of the blew and red upon the Lens, the blew was refracted more by the Lens than the red, ſo as to converge ſooner by an Inch and an half, and therefore is more refrangible.

Fig. 12. *Illuſtration.* In the Twelfth Figure, DE ſignifies the coloured Paper, DG the blew half, FE the red half, MN the Lens, HJ the white Paper in that place where the red half with its black Lines appeared diſtinct, and *hi* the ſame Paper in that place where the blew half appeared diſtinct. The place *hi* was nearer to the Lens MN than the place HJ by an Inch and an half.

Scholium.

Scholium. The ſame things ſucceed notwithſtanding that ſome of the Circumſtances be varied : as in the firſt Experiment when the Priſm and Paper are any ways inclined to the Horizon, and in both when coloured Lines are drawn upon very black Paper. But in the Deſcription of theſe Experiments, I have ſet down ſuch Circumſtances by which either the Phænomenon might be rendred more conſpicuous, or a Novice might more eaſily try them, or by which I did try them only. The ſame thing I have often done in the following Experiments : Concerning all which this one Admonition may ſuffice. Now from theſe Experiments it follows not that all the Light of the blew is more Refrangible than all the Light of the red ; For both Lights are mixed of Rays differently Refrangible, So that in the red there are ſome Rays not leſs Refrangible than thoſe of the blew, and in the blew there are ſome Rays not more Refrangible than thoſe of the red ; But theſe Rays in Proportion to the whole Light are but few, and ſerve to diminiſh the Event of the Experiment, but are not able to deſtroy it. For if the red and blew Colours were more dilute and weak, the diſtance of the Images would be leſs than an Inch and an half ; and if they were more intenſe and full, that diſtance would be greater, as will appear hereafter. Theſe Experiments may ſuffice for the Colours of Natural Bodies. For in the Colours made by the Refraction of Priſms this Propoſition will appear by the Experiments which are now to follow in the next Propoſition.

PROP. II. Theor. II.

The Light of the Sun consists of Rays differently Refrangible.

The Proof by Experiments.

Exper. 3. IN a very dark Chamber at a round hole about one third part of an Inch broad made in the Shut of a Window, I placed a Glass Prism, whereby the beam of the Sun's Light which came in at that hole might be refracted upwards toward the opposite Wall of the Chamber, and there form a coloured Image of the Sun. The Axis of the Prism (that is the Line passing through the middle of the Prism from one end of it to the other end Parallel to the edge of the Refracting Angle) was in this and the following Experiments perpendicular to the incident Rays. About this Axis I turned the Prism slowly, and saw the refracted Light on the Wall or coloured Image of the Sun first to descend and then to ascend. Between the Descent and Ascent when the Image seemed Stationary, I stopt the Prism, and fixt it in that Posture, that it should be moved no more. For in that posture the Refractions of the Light at the two sides of the Refracting Angle, that is at the entrance of the Rays into the Prism and at their going out of it, were equal to one another. So also in other Experiments as often as I would have the Refractions on both sides the Prism to be equal to one another, I noted the place where the Image of the Sun formed by the refracted Light stood still between its two contrary Motions, in the common Period of its progress and egress; and when the Image fell upon that place, I made fast the Prism. And in this posture, as the

the most convenient, it is to be understood that all the Prisms are placed in the following Experiments, unless where some other posture is described. The Prism therefore being placed in this posture, I let the refracted Light fall perpendicularly upon a Sheet of white Paper at the opposite Wall of the Chamber, and observed the Figure and Dimensions of the Solar Image formed on the Paper by that Light. This Image was Oblong and not Oval, but terminated with two Rectilinear and Parallel Sides, and two Semicircular Ends. On its Sides it was bounded pretty distinctly, but on its Ends very confusedly and indistinctly, the Light there decaying and vanishing by degrees. The breadth of this Image answered to the Sun's Diameter, and was about two Inches and the eighth part of an Inch, including the Penumbra. For the Image was eighteen Feet and an half distant from the Prism, and at this distance that breadth if diminished by the Diameter of the hole in the Window-shut, that is by a quarter of an Inch, subtended an Angle at the Prism of about half a Degree, which is the Sun's apparent Diameter. But the length of the Image was about ten Inches and a quarter, and the length of the Rectilinear Sides about eight Inches; And the refracting Angle of the Prism whereby so great a length was made, was 64 degr. With a less Angle the length of the Image was less, the breadth remaining the same. If the Prism was turned about its Axis that way which made the Rays emerge more obliquely out of the second refracting Surface of the Prism, the Image soon became an Inch or two longer, or more; and if the Prism was turned about the contrary way, so as to make the Rays fall more obliquely on the first refracting Surface, the Image soon became an Inch or two shorter. And therefore in trying this Experiment, I was as curious as I could be in placing the Prism by the above-mentioned Rule exactly in ſuch a poſture that the Refractions of the Rays at their emergence out of the Priſm might be equal to that at their incidence on it. This Priſm had ſome Veins running along within the Glaſs from one end to the other, which ſcattered ſome of the Sun's Light irregularly, but had no ſenſible effect in encreaſing the length of the coloured Spectrum. For I tried the ſame Experiment with other Priſms with the ſame Succeſs. And particularly with a Priſm which ſeemed free from ſuch Veins, and whoſe refracting Angle was $62\frac{1}{2}$ Degrees, I found the length of the Image $9\frac{3}{4}$ or 10 Inches at the diſtance of $18\frac{1}{2}$ Feet from the Priſm, the breadth of the hole in the Window-ſhut being $\frac{1}{4}$ of an Inch as before. And becauſe it is eaſie to commit a miſtake in placing the Priſm in its due poſture, I repeated the Experiment four or five times, and always found the length of the Image that which is ſet down above. With another Priſm of clearer Glaſs and better Polliſh, which ſeemed free from Veins and whoſe refracting Angle was $63\frac{1}{2}$ Degrees, the length of this Image at the ſame diſtance of $18\frac{1}{2}$ Feet was alſo about 10 Inches, or $10\frac{1}{8}$. Beyond theſe Meaſures for about $\frac{1}{4}$ or $\frac{1}{3}$ of an Inch at either end of the Spectrum the Light of the Clouds ſeemed to be a little tinged with red and violet, but ſo very faintly that I ſuſpected that tincture might either wholly or in great meaſure ariſe from ſome Rays of the Spectrum ſcattered irregularly by ſome inequalities in the Subſtance and Poliſh of the Glaſs, and therefore I did not include it in theſe Meaſures. Now the different Magnitude of the hole in the Window-ſhut, and different thickneſs of the Priſm where the Rays paſſed through it, and different inclinations of the Priſm to the Horizon, made no ſenſible changes in the length of the Image. Neither did the different matter of

the Priſms make any : for in a Veſſel made of poliſhed Plates of Glaſs cemented together in the ſhape of a Priſm and filled with Water, there is the like Succeſs of the Experiment according to the quantity of the Refraction. It is further to be obſerved, that the Rays went on in right Lines from the Priſm to the Image, and therefore at their very going out of the Priſm had all that Inclination to one another from which the length of the Image proceeded, that is the Inclination of more than two Degrees and an half. And yet according to the Laws of Opticks vulgarly received, they could not poſſibly be ſo much inclined to one another. For let E G repreſent the Window- *Fig.* 13.
ſhut, F the hole made therein through which a beam of the Sun's Light was tranſmitted into the darkned Chamber, and ABC a Triangular Imaginary Plane whereby the Priſm is feigned to be cut tranſverſly through the middle of the Light. Or if you pleaſe, let A B C repreſent the Priſm it ſelf, looking directly towards the Spectator's Eye with its nearer end : And let X Y be the Sun, M N the Paper upon which the Solar Image or Spectrum is caſt, and P T the Image it ſelf whoſe ſides tovvards ~~V~~ and ~~W~~ are Rectili- v w
near and Parallel, and ends tovvards P and T Semicircular. Y K H P and X L J T are ~~the~~ two Rays, the firſt of which comes from the lower part of the Sun to the higher part of the Image, and is refracted in the Priſm at K and H, and the latter comes from the higher part of the Sun to the lower part of the Image, and is refracted at L and J. Since the Refractions on both ſides the Priſm are equal to one another, that is the Refraction at K equal to the Refraction at J, and the Refraction at L equal to the Refraction at H, ſo that the Refractions of the incident Rays at K and L taken together are equal to the Refractions of the emergent Rays at H and J taken toge-

ther :

ther: it follows by adding equal things to equal things, that the Refractions at K and H taken together, are equal to the Refractions at J and L taken together, and therefore the two Rays being equally refracted have the same Inclination to one another after Refraction which they had before, that is the Inclination of half a Degree answering to the Sun's Diameter. For so great was the Inclination of the Rays to one another before Refraction. So then, the length of the Image P T would by the Rules of Vulgar Opticks subtend an Angle of half a Degree at the Prism, and by consequence be equal to the breadth *v w*; and therefore the Image would be round. Thus it would be were the two Rays X L J T and Y K H P and all the rest which form the Image P *w* T *v*, alike Refrangible. And therefore seeing by Experience it is found that the Image is not round but about five times longer than broad, the Rays which going to the upper end P of the Image suffer the greatest Refraction, must be more Refrangible than those which go to the lower end T, unless the inequality of Refraction be casual.

This Image or Spectrum P T was coloured, being red at its least refracted end T, and violet at its most refracted end P, and yellow green and blew in the intermediate spaces. Which agrees with the first Proposition, that Lights which differ in Colour do also differ in Refrangibility. The length of the Image in the foregoing Experiments I measured from the faintest and outmost red at one end, to the faintest and outmost blew at the other end.

Exper. 4. In the Sun's beam which was propagated into the Room through the hole in the Window-shut, at the distance of some Feet from the hole, I held the Prism in such a posture that its Axis might be perpendicular to that beam. Then I looked through the Prism upon the hole,

hole, and turning the Prism to and fro about its Axis to make the Image of the hole ascend and descend, when between its two contrary Motions it seemed stationary, I stopt the Prism that the Refractions on both sides of the refracting Angle might be equal to each other as in the former Experiment. In this Situation of the Prism viewing through it the said hole, I observed the length of its refracted Image to be many times greater than its breadth, and that the most refracted part thereof appeared violet, the least refracted red, the middle parts blew green and yellow in order. The same thing happened when I removed the Prism out of the Sun's Light, and looked through it upon the hole shining by the Light of the Clouds beyond it. And yet if the Refraction were done regularly according to one certain Proportion of the Sines of Incidence and Refraction as is vulgarly supposed, the refracted Image ought to have appeared round.

So then, by these two Experiments it appears that in equal Incidences there is a considerable inequality of Refractions: But whence this inequality arises, whether it be that some of the incident Rays are refracted more and others less, constantly or by chance, or that one and the same Ray is by Refraction disturbed, shattered, dilated, and as it were split and spread into many diverging Rays, as *Grimaldo* supposes, does not yet appear by these Experiments, but will appear by those that follow.

Exper. 5. Considering therefore, that if in the third Experiment the Image of the Sun should be drawn out into an oblong form, either by a Dilatation of every Ray, or by any other casual inequality of the Refractions, the same oblong Image would by a second Refraction made Sideways be drawn out as much in breadth by the like Dilatation of the Rays or other casual inequality of the Refractions

fractions Sideways, I tried what would be the Effects of such a second Refraction. For this end I ordered all things as in the third Experiment, and then placed a second Prism immediately after the first in a cross Position to it, that it might again refract the beam of the Sun's Light which came to it through the first Prism. In the first Prism this beam was refracted upwards, and in the second Sideways. And I found that by the Refraction of the second Prism the breadth of the Image was not increased, but its superior part which in the first Prism suffered the greater Refraction and appeared violet and blew, did again in the second Prism suffer a greater Refraction than its inferior part, which appeared red and yellow, and this without any Dilation of the Image in breadth.

Fig. 14. *Illustration.* Let S represent the Sun, F the hole in the Window, A B C the first Prism, D H the second Prism, Y the round Image of the Sun made by a direct beam of Light when the Prisms are taken away, P T the oblong Image of the Sun made by that beam passing through the first Prism alone when the second Prism is taken away, and *pt* the Image made by the cross Refractions of both Prisms together. Now if the Rays which tend towards the several Points of the round Image Y were dilated and spread by the Refraction of the first Prism, so that they should not any longer go in single Lines to single Points, but that every Ray being split, shattered, and changed from a Linear Ray to a Superficies of Rays diverging from the Point of Refraction, and lying in the Plane of the Angles of Incidence and Refraction, they should go in those Planes to so many Lines reaching almost from one end of the Image P T to the other, and if that Image should thence become oblong; those Rays and their several parts tending towards the several Points of the

the Image P T ought to be again dilated and ſpread Side-ways by the tranſverſe Refraction of the ſecond Priſm, ſo as to compoſe a fourſquare Image, ſuch as is repreſented at πτ. For the better underſtanding of which, let the Image PT be diſtinguiſhed into five equal Parts PQK, KQRL, LRSM, MSVN, NVT. And by the ſame irregularity that the Orbicular Light Y is by the Refraction of the firſt Priſm dilated and drawn out into a long Image P T, the the Light P Q K which takes up a ſpace of the ſame length and breadth with the Light Y ought to be by the Refraction of the ſecond Priſm dilated and drawn out into the long Image *π q k p*, and the Light K Q R L into the long Image *k q r l*, and the Lights L R S M, M S V N, N V T into ſo many other long Images *l r s m*, *m s v n*, *n v t τ*; and all theſe long Images would compoſe the fourſquare Image πτ. Thus it ought to be were every Ray dilated by Refraction, and ſpread into a triangular Superficies of Rays diverging from the Point of Refraction. For the ſecond Refraction would ſpread the Rays one way as much as the firſt doth another, and ſo dilate the Image in breadth as much as the firſt doth in length. And the ſame thing ought to happen, were ſome Rays caſually refracted more than others. But the Event is otherwiſe. The Image P T was not made broader by the Refraction of the ſecond Priſm, but only became oblique, as 'tis repreſented at *p t*, its upper end P being by the Refraction tranſlated to a greater diſtance than its lower end T. So then the Light which went towards the upper end P of the Image, was (at equal Incidences) more refracted in the ſecond Priſm than the Light which tended towards the lower end T, that is the blew and violet, than the red and yellow; and therefore was more Refrangible. The ſame Light was by the Refraction of the firſt Priſm tranſlated further from the place Y to which it tended before Refraction; and therefore suffered as well in the first Prism as in the second a greater Refraction than the rest of the Light, and by consequence was more Refrangible than the rest, even before its incidence on the first Prism.

Sometimes I placed a third Prism after the second, and sometimes also a fourth after the third, by all which the Image might be often refracted sideways: but the Rays which were more refracted than the rest in the first Prism were also more refracted in all the rest, and that without any Dilatation of the Image sideways: and therefore those Rays for their constancy of a greater Refraction are deservedly reputed more Refrangible.

Fig. 15. But that the meaning of this Experiment may more clearly appear, it is to be considered that the Rays which are equally Refrangible do fall upon a circle answering to the Sun's Disque. For this was proved in the third Experiment. By a circle I understand not here a perfect Geometrical Circle, but any Orbicular Figure whose length is equal to its breadth, and which, as to sense, may seem circular. Let therefore A G represent the circle which all the most Refrangible Rays propagated from the whole Disque of the Sun, would illuminate and paint upon the opposite Wall if they were alone; E L the circle which all the least Refrangible Rays would in like manner illuminate and paint if they were alone; B H, C J, D K, the circles which so many intermediate sorts of Rays would successively paint upon the Wall, if they were singly propagated from the Sun in successive Order, the rest being always intercepted; And conceive that there are other intermediate Circles without number which innumerable other intermediate sorts of Rays would successively paint upon the Wall if the Sun should successively emit every sort apart.

And

And ſeeing the Sun emits all theſe ſorts at once, they muſt all together illuminate and paint innumerable equal circles, of all which, being according to their degrees of Refrangibility placed in order in a continual ſeries, that oblong Spectrum P T is compoſed which I deſcribed in the third Experiment. Now if the Sun's circular Image Y which is made by an unrefracted beam of Light was by any dilatation of the ſingle Rays, or by any other irregularity in the Refraction of the firſt Priſm, converted into the Oblong Spectrum, P T : then ought every circle A G, B H, C J, &c. in that Spectrum, by the croſs Refraction of the ſecond Priſm again dilating or otherwiſe ſcattering the Rays as before, to be in like manner drawn out and transformed into an Oblong Figure, and thereby the breadth of the Image P T would be now as much augmented as the length of the Image Y was before by the Refraction of the firſt Priſm ; and thus by the Refractions of both Priſms together would be formed a fourſquare Figure *p π t τ* as I deſcribed above. Wherefore ſince the breadth of the Spectrum P T is not increaſed by the Refraction ſideways, it is certain that the Rays are not ſplit or dilated, or otherways irregularly ſcattered by that Refraction, but that every circle is by a regular and uniform Refraction tranſlated entire into another place, as the circle A G by the greateſt Refraction into the place *ag*, the circle B H by a leſs Refraction into the place *bh*, the circle C J by a Refraction ſtill leſs into the place *ci*, and ſo of the reſt ; by which means a new Spectrum *p t* inclined to the former P T is in like manner compoſed of circles lying in a right Line ; and theſe circles muſt be of the ſame bigneſs with the former, becauſe the breadths of all the Spectrums Y, P T and *pt* at equal diſtances from the Priſms are equal.

Fig. 14, 15

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I conſidered further that by the breadth of the hole F through which the Light enters into the Dark Chamber, there is a Penumbra made in the circuit of the Spectrum Y, and that Penumbra remains in the rectilinear Sides of the Spectrums P T and *pt*. I placed therefore at that hole a Lens or Object-glaſs of a Teleſcope which might caſt the Image of the Sun diſtinctly on Y without any Penumbra at all, and found that the Penumbra of the Rectilinear Sides of the oblong Spectrums P T and *pt* was alſo thereby taken away, ſo that thoſe Sides appeared as diſtinctly defined as did the Circumference of the firſt Image Y. Thus it happens if the Glaſs of the Priſms be free
- and
from veins, and their Sides be accurately plane and well poliſhed without thoſe numberleſs waves or curles which uſually ariſe from Sand-holes a little ſmoothed in poliſhing with Putty. If the Glaſs be only well poliſhed and free from veins and the Sides not accurately plane but a little Convex or Concave, as it frequently happens; yet may the three Spectrums Y, P T and *pt* want Penumbras, but not in equal diſtances from the Priſms. Now from this want of Penumbras, I knew more certainly that every one of the circles was refracted according to ſome moſt regular, uniform, and conſtant law. For if there were any irregularity in the Refraction, the right Lines A E and G L which all the circles in the Spectrum P T do touch, could not by that Refraction be tranſlated into the Lines *a e* and *g l* as diſtinct and ſtraight as they were before, but there would ariſe in thoſe tranſlated Lines ſome Penumbra or crookedneſs or undulation, or other ſenſible Perturbation contrary to what is found by Experience. Whatſoever Penumbra or Perturbation ſhould be made in the circles by the croſs Refraction of the ſecond Priſm, all that Penumbra or Perturbation would be conſpicuous in

the

the right Lines *a e* and *g l* which touch thoſe circles. And therefore ſince there is no ſuch Penumbra or Perturbation in thoſe right Lines there muſt be none in the circles. Since the diſtance between thoſe Tangents or breadth of the Spectrum is not increaſed by the Refractions, the Diameters of the circles are not increaſed thereby. Since thoſe Tangents continue to be right Lines, every circle which in the firſt Priſm is more or leſs refracted, is exactly in the ſame Proportion more or leſs refracted in the ſecond. And ſeeing all theſe things continue to ſucceed after the ſame manner when the Rays are again in a third Priſm, and again in a fourth refracted Sideways, it is evident that the Rays of one and the ſame circle as to their degree of Refrangibility continue always Uniform and Homogeneal to one another, and that thoſe of ſeveral circles do differ in degree of Refrangibility, and that in ſome certain and conſtant Proportion. Which is the thing I was to prove.

There is yet another Circumſtance or two of this Ex- *Fig.* 16.
periment by which it becomes ſtill more plain and convincing. Let the ſecond Priſm D H be placed not immeately after after the firſt, but at ſome diſtance from it; Suppoſe in the mid-way between it and the Wall on which the oblong Spectrum P T is caſt, ſo that the Light from the firſt Priſm may fall upon it in the form of an oblong Spectrum, πτ Parallel to this ſecond Priſm, and be refracted Sideways to form the oblong Spectrum *p t* upon the Wall. And you will find as before, that this Spectrum *p t* is inclined to that Spectrum P T, which the firſt Priſm forms alone without the ſecond; the blew ends P and *p* being further diſtant from one another than the red ones T and *t*, and by conſequence that the Rays which go to the blew end π of the Image πτ and which therefore ſuffer the greateſt Refraction in the firſt Priſm, are again in the ſecond Priſm more refracted than the reſt.

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Fig. 17. The ſame thing I try'd alſo by letting the Sun's Light into a dark Room through two little round holes F and φ made in the Window, and with two Parallel Priſms ABC and $\alpha\beta\gamma$ placed at thoſe holes (one at each) refracting thoſe two beams of Light to the oppoſite Wall of the Chamber, in ſuch manner that the two colour'd Images P T and *m n* which they there painted were joyned end to end and lay in one ſtraight Line, the red end T of the one touching the blew end *m* of the other. For if theſe two refracted beams were again by a third Priſm D H placed croſt to the two firſt, refracted Sideways, and the Spectrums thereby tranſlated to ſome other part of the Wall of the Chamber, ſuppoſe the Spectrum P T to *p t* and the Spectrum M N to *m n*, theſe tranſlated Spectrums *p t* and *m n* would not lie in one ſtraight Line with their ends contiguous as before, but be broken off from one another and become Parallel, the blew end of the Image *m n* being by a greater Refraction tranſlated farther from its former place M T, than the red end *t* of the other Image *p t* from the ſame place MT, which puts the Propoſition paſt diſpute. And this happens whether the third Priſm D H be placed immediately after the two firſt or at a great diſtance from them, ſo that the Light refracted in the two firſt Priſms be either white and circular, or coloured and oblong when it falls on the third.

Exper. 6. In the middle of two thin Boards I made round holes a third part of an Inch in Diameter, and in the Window-ſhut a much broader hole, being made to let into my darkned Chamber a large beam of the Sun's Light; I placed a Priſm behind the Shut in that beam to refract it towards the oppoſite Wall, and cloſe behind the Priſm I fixed one of the Boards, in ſuch manner that the middle of the refracted Light might paſs through the hole

made

made in it, and the reſt be intercepted by the Board. Then at the diſtance of about twelve Feet from the firſt Board I fixed the other Board, in ſuch manner that the middle of the refracted Light which came through the hole in the firſt Board and fell upon the oppoſite Wall might paſs through the hole in this other Board, and the reſt being intercepted by the Board might paint upon it the coloured Spectrum of the Sun. And cloſe behind this Board I fixed another Priſm to refract the Light which came through the hole. Then I returned ſpeedily to the firſt Priſm, and by turning it ſlowly to and fro about its Axis, I cauſed the Image which fell upon the ſecond Board to move up and down upon that Board, that all its parts might ſucceſſively paſs through the hole in that Board and fall upon the Priſm behind it. And in the mean time, I noted the places on the oppoſite Wall to which that Light after its Refraction in the ſecond Priſm did paſs; and by the difference of the places I found that the Light which being moſt refracted in the firſt Priſm did go to the blew end of the Image, was again more refracted in the ſecond Priſm than the Light which went to the red end of that Image, which proves as well the firſt Propoſition as the ſecond. And this happened whether the Axis of the two Priſms were parallel, or inclined to one another and to the Horizon in any given Angles.

Illuſtration. Let F be the wide hole in the Window-ſhut, *Fig.* 18. through which the Sun ſhines upon the firſt Priſm A B C, and let the refracted Light fall upon the middle of the Board D E, and the middle part of that Light upon the hole G made in the middle of that Board. Let this trajected part of the Light fall again upon the middle of the ſecond Board *d e* and there paint ſuch an oblong coloured Image of the Sun as was deſcribed in the third Experiment.

By

By turning the Prifm A B C flowly to and fro about its Axis this Image will be made to move up and down the Board *d e*, and by this means all its parts from one end to the other may be made to pafs fucceffively through the hole *g* which is made in the middle of that Board. In the mean while another Prifm *a b c* is to be fixed next after that hole *g* to refract the trajected Light a fecond time. And thefe things being thus ordered, I marked the places M and N of the oppofite Wall upon which the refracted Light fell,and found that whilft the two Boards and fecond Prifm remained unmoved, thofe places by turning the firft Prifm about its Axis were changed perpetually. For when the lower part of the Light which fell upon the fecond Board *d e* was caft through the hole *g* it went to a lower place M on the Wall , and when the higher part of that Light was caft through the fame hole *g*, it went to a higher place N on the Wall, and when any intermediate part of the Light was caft through that hole it went to fome place on the Wall between M and N. The unchanged Pofition of the holes in the Boards, made the Incidence of the Rays upon the fecond Prifm to be the fame in all cafes. And yet in that common Incidence fome of the Rays were more refracted and others lefs. And thofe were more refracted in this Prifm which by a greater Refraction in the firft Prifm were more turned out of the way, and therefore for their conftancy of being more refracted are defervedly called more Refrangible.

Exper. 7. At two holes made near one another in my Window-fhut I placed two Prifms , one at each, which might caft upon the oppofite Wall (after the manner of the third Experiment) two oblong coloured Images of the Sun. And at a little diftance from the Wall I placed a long flender Paper with ftraight and parallel edges, and

ordered

ordered the Prisms and Paper so, that the red Colour of one Image might fall directly upon one half of the Paper, and the violet colour of the other Image upon the other half of the same Paper; so that the Paper appeared of two Colours, red and violet, much after the manner of the painted Paper in the first and second Experiments. Then with a black Cloth I covered the Wall behind the Paper, that no Light might be reflected from it to disturb the Experiment, and viewing the Paper through a third Prism held parallel to it, I saw that half of it which was illuminated by the Violet-light to be divided from the other half by a greater Refraction, especially when I went a good way off from the Paper. For when I viewed it too near at hand, the two halfs of the Paper did not appear fully divided from one another, but seemed contiguous at one of their Angles like the painted Paper in the first Experiment. Which also happened when the Paper was too broad.

Sometimes instead of the Paper I used a white Thred, and this appeared through the Prism divided into two Parallel Threds as is represented in the 19th Figure, where *Fig.* 19.
D G denotes the Thred illuminated with violet Light from D to E and with red Light from F to G, and *d e f g* are the parts of the Thred seen by Refraction. If one half of the Thred be constantly illuminated with red, and the other half be illuminated with all the Colours successively, (which may be done by causing one of the Prisms to be turned about its Axis whilst the other remains unmoved) this other half in viewing the Thred through the Prism, will appear in a continued right Line with the first half when illuminated with red, and begin to be a little divided from it when illuminated with Orange, and remove further from it when illuminated with Yellow, and still

further when with Green, and further when with Blew, and go yet further off when illuminated with Indigo, and furthest when with deep Violet. Which plainly shews, that the Lights of several Colours are more and more Refrangible one than another, in this order of their Colours, Red, Orange, Yellow, Green, Blew, Indigo, deep Violet; and so proves as well the first Proposition as the second.

Fig. 17. I caused also the coloured Spectrums P T and M N made in a dark Chamber by the Refractions of two Prisms to lye in a right Line end to end, as was described above in the fifth Experiment, and viewing them through a third Prism held Parallel to their length, they appeared no longer in a right Line, but became broken from one another, as they are represented at *p t* and *m n*, the violet end *m* of the Spectrum *m n* being by a greater Refraction translated further from its former place M T than the red end *t* of the other Spectrum *p t*.

Fig. 20. I further caused those two Spectrums P T and M N to become co-incident in an inverted order of their Colours, the red end of each falling on the violet end of the other, as they are represented in the oblong Figure P T M N; and then viewing them through a Prism D H held Parallel to their length, they appeared not co-incident as when viewed with the naked Eye, but in the form of two distinct Spectrums *p t* and *m n* crossing one another in the middle after the manner of the letter X. Which shews that the red of the one Spectrum and violet of the other, which were co-incident at P N and M T, being parted from one another by a greater Refraction of the violet to *p* and *m* than of the red to *n* and *t*, do differ in degrees of Refrangibility.

I illuminated also a little circular piece of white Paper all over with the Lights of both Prisms intermixed, and

when

when it was illuminated with the red of one Spectrum and deep violet of the other, so as by the mixture of those Colours to appear all over purple, I viewed the Paper, first at a less distance, and then at a greater, through a third Prism; and as I went from the Paper, the refracted Image thereof became more and more divided by the unequal Refraction of the two mixed Colours, and at length parted into two distinct Images, a red one and a violet one, whereof the violet was furthest from the Paper, and therefore suffered the greatest Refraction. And when that Prism at the Window which cast the violet on the Paper was taken away, the violet Image disappeared; but when the other Prism was taken away the red vanished: which shews that these two Images were nothing else than the Lights of the two Prisms which had been intermixed on the purple Paper, but were parted again by their unequal Refractions made in the third Prism through which the Paper was viewed. This also was observable that if one of the Prisms at the Window, suppose that which cast the violet on the Paper, was turned about its Axis to make all the Colours in this order, Violet, Indigo, Blew, Green, Yellow, Orange, Red, fall successively on the Paper from that Prism, the violet Image turning successively to Indigo, Blew, Green, Yellow & Red changed Colour accordingly, and in changing Colour came nearer & more to the red ~~one~~ Image made by the other Prism, until when it was also red ~~they~~ Images both became fully co-incident.

I placed also two paper circles very near one another, the one in the red Light of one Prism, and the other in the violet Light of the other. The circles were each of them an Inch in Diameter, and behind them the Wall was dark that the Experiment might not be disturbed by any Light coming from thence. These circles thus illuminated, I viewed through a Prism so held that the Refraction might be made towards the red circle, and as I went from them they came nearer and nearer together, and at length became co-incident; and afterwards when I went ſtill further off, they parted again in a contrary order, the violet by a greater Refraction being carried beyond the red.

Exper. 8. In Summer when the Sun's Light uſes to be ſtrongeſt, I placed a Priſm at the hole of the Window-ſhut, as in the third Experiment, yet ſo that its Axis might be Parallel to the Axis of the World, and at the oppoſite Wall in the Sun's refracted Light, I placed an open Book. Then going Six Feet and tvvo Inches from the Book, I placed there the abovementioned Lens, by vvhich the Light reflected from the Book might be made to converge and meet again at the diſtance of ſix Feet and tvvo Inches behind the Lens, and there paint the Species of the Book upon a ſheet of vvhite Paper much after the manner of the ſecond Experiment. The Book and Lens being made faſt, I noted the place vvhere the Paper vvas, vvhen the Letters of the Book, illuminated by the fulleſt red Light of the Solar Image falling upon it, did caſt their Species on that Paper moſt diſtinctly; And then I ſtay'd till by the Motion of the Sun and conſequent Motion of his Image on the Book, all the Colours from that red to the middle of the blew paſs'd over thoſe Letters; and when thoſe Letters were illuminated by that blew, I noted again the place of the Paper when they caſt their Species moſt diſtinctly upon it: And I found that this laſt place of the Paper was nearer to the Lens than its former place by about two Inches and an half, or two and three quarters. So much ſooner therefore did the Light in the violet end of the Image by a greater Refraction converge and meet, than the Light in the red end. But in trying this the Chamber was as dark as I could make it. For if theſe Colours be diluted and weakned by the mixture of any adventitious Light, the diſtance

between

between the places of the Paper will not be ſo great. This diſtance in the ſecond Experiment where the Colours of natural Bodies were made uſe of, was but an Inch and a half, by reaſon of the imperfection of thoſe Colours. Here in the Colours of the Priſm, which are manifeſtly more full, intenſe, and lively than thoſe of natural Bodies, the diſtance is two Inches and three quarters. And were the Colours ſtill more full, I queſtion not but that the diſtance would be conſiderably greater. For the coloured Light of the Priſm, by the interfering of the Circles deſcribed in the ~~11~~2th Figure of the fifth Experiment, and alſo 2) by the Light of the very bright Clouds next the Sun's Body intermixing with theſe Colours, and by the Light ſcattered by the inequalities in the poliſh of the Priſm, was ſo very much compounded, that the Species which thoſe faint and dark Colours, the Indigo and Violet, caſt upon the Paper were not diſtinct enough to be well obſerved.

Exper. 9. A Priſm, whoſe two Angles at its Baſe were equal to one another and half right ones, and the third a right one, I placed in a beam of the Sun's Light let into a dark Chamber through a hole in the Window-ſhut as in the third Experiment. And turning the Priſm ſlowly about its Axis until all the Light which went through one of its Angles and was refracted by it began to be reflected by its Baſe, at which till then it went out of the Glaſs, I obſerved that thoſe Rays which had ſuffered the greateſt Refraction were ſooner reflected than the reſt. I conceived therefore that thoſe Rays of the reflected Light, which were moſt Refrangible, did firſt of all by a total Reflexion become more copious in that Light than the reſt, and that afterwards the reſt alſo, by a total Reflexion, became as copious as theſe. To try this, I made the reflected Light paſs through another Priſm, and being refra-

cted

cted by it to fall afterwards upon a ſheet of white Paper placed at ſome diſtance behind it, and there by that Refraction to paint the uſual Colours of the Priſm. And then cauſing the firſt Priſm to be turned about its Axis as above, I obſerved that when thoſe Rays which in this Priſm had ſuffered the greateſt Refraction and appeared of a blew and violet Colour began to be totally reflected, the blew and violet Light on the Paper which was moſt refracted in the ſecond Priſm received a ſenſible increaſe above that of the red and yellow, which was leaſt refracted; and afterwards when the reſt of the Light which was green, yellow and red began to be totally reflected in the firſt Priſm, the light of thoſe Colours on the Paper received as great an increaſe as the violet and blew had done before. Whence 'tis manifeſt, that the beam of Light reflected by the Baſe of the Priſm, being augmented firſt by the more Refrangible Rays and afterwards by the leſs Refrangible ones, is compounded of Rays differently Refrangible. And that all ſuch reflected Light is of the ſame Nature with the Sun's Light, before its Incidence on the Baſe of the Priſm, no Man ever doubted: it being generally allowed, that Light by ſuch Reflexions ſuffers no Alteration in its Modifications and Properties. I do not here take notice of any Refractions made in the Sides of the firſt Priſm, becauſe the Light enters it perpendicularly at the firſt Side, and goes out perpendicularly at the ſecond Side, and therefore ſuffers none. So then, the Sun's incident Light being of the ſame temper and conſtitution with his emergent Light, and the laſt being compounded of Rays differently Refrangible, the firſt muſt be in like manner compounded.

Fig. 21. *Illuſtration.* In the 21th Figure, A B C is the firſt Priſm, B C its Baſe, B and C its equal Angles at the Baſe, each of of 45 degrees, A its Rectangular Vertex, F M a beam of the Sun's Light let into a dark Room through a hole F one third part of an Inch broad, M its Incidence on the Base of the Prism, M G a less refracted Ray, M H a more refracted Ray, M N the beam of Light reflected from the Base, V X Y the second Prism by which this beam in passing through it is refracted, N *t* the less refracted Light of this beam, and N *p* the more refracted part thereof. When the first Prism A B C is turned about its Axis according to the order of the Letters A B C, the Rays M H emerge more and more obliquely out of that Prism, and at length after their most oblique Emergence are reflected towards N, and going on to *p* do increase the number of the Rays N *p*. Aftervvards by continuing the motion of the first Prism, the Rays M G are also reflected to N and increase the number of the Rays N *t*. And therefore the Light M N admits into its Composition, first the more Refrangible Rays, and then the less Refrangible Rays, and yet after this Composition is of the same Nature vvith the Sun's immediate Light F M, the Reflexion of the specular Base B C causing no Alteration therein.

Exper. 10. Two Prisms, which were alike in shape, I tied so together, that their Axes and opposite Sides being Parallel, they composed a Parallelopiped. And, the Sun shining into my dark Chamber through a little hole in the Window-shut, I placed that Parallelopiped in his beam at some distance from the hole, in such a posture that the Axes of the Prisms might be perpendicular to the incident Rays, and that those Rays being incident upon the first Side of one Prism, might go on through the two contiguous Sides of both Prisms, and emerge out of the last Side of the second Prism. This Side being Parallel to the first Side of the first Prism, caused the emerging Light to be Parallel

to

to the Incident. Then, beyond these two Prisms I placed a third, which might refract that emergent Light, and by that Refraction cast the usual Colours of the Prism upon the opposite Wall, or upon a sheet of white Paper held at a convenient distance behind the Prism for that refracted Light to fall upon it. After this I turned the Parallelopiped about its Axis, and found that when the contiguous Sides of the two Prisms became so oblique to the incident Rays that those Rays began all of them to be reflected, those Rays which in the third Prism had suffered the greatest Refraction and painted the Paper with violet and blew, were first of all by a total Reflexion taken out of the transmitted Light, the rest remaining and on the Paper painting their Colours of Green, Yellow, Orange, and Red as before; and afterwards by continuing the motion of the two Prisms, the rest of the Rays also by a total Reflexion vanished in order, according to their degrees of Refrangibility. The Light therefore which emerged out of the two Prisms is compounded of Rays differently Refrangible, seeing the more Refrangible Rays may be taken out of it while the less Refrangible remain. But this Light being trajected only through the Parallel Superficies of the two Prisms, if it suffered any change by the Refraction of one Superficies it lost that impression by the contrary Refraction of the other Superficies, and so being restored to its pristine constitution became of the same nature and condition as at first before its Incidence on those Prisms; and therefore, before its Incidence, was as much compounded of Rays differently Refrangible as afterwards.

Fig. 22. *Illustration.* In the 22th Figure A B C and B C D are the the two Prisms tied together in the form of a Parallelopiped, their Sides B C and C B being contiguous, and their Sides A B and C D Parallel. And H J K is the third Prism,

Prism, by which the Sun's Light propagated through the hole F into the dark Chamber, and there passing through those sides of the Prisms AB, BC, CB and CD, is refracted at O to the white Paper PT, falling there partly upon P by a greater Refraction, partly upon T by a less Refraction, and partly upon R and other intermediate places by intermediate Refractions. By turning the Parallelopiped ACBD about its Axis, according to the order of the Letters A, C, D, B, at length when the contiguous Planes BC and CB become sufficiently oblique to the Rays FM, which are incident upon them at M, there will vanish totally out of the refracted Light OPT, first of all the most refracted Rays OP, (the rest OR and OT remaining as before) then the Rays OR and other intermediate ones, and lastly, the least refracted Rays OT. For when the Plane BC becomes sufficiently oblique to the Rays incident upon it, those Rays will begin to be totally reflected by it towards N; and first the most Refrangible Rays will be totally reflected (as was explained in the preceding experiment) and by consequence must first disappear at P, and afterwards the rest as they are in order totally reflected to N, they must disappear in the same order at R and T. So then the Rays which at O suffer the greatest Refraction, may be taken out of the Light MO whilst the rest of the Rays remain in it, and therefore that Light MO is Compounded of Rays differently Refrangible. And because the Planes AB and CD are parallel, and therefore by equal and contrary Refractions destroy one anothers Effects, the incident Light FM must be of the same kind and nature with the emergent Light MO, and therefore doth also consist of Rays differently Refrangible. These two Lights FM and MO, before the most refrangible Rays are separated out of the emergent Light MO agree in Colour, and in all other properties so far as my observation reaches, and therefore are deservedly reputed of the same Nature and Constitution, and by consequence the one is compounded as well as the other. But after the most Refrangible Rays begin to be totally reflected, and thereby separated out of the emergent Light MO, that Light changes its Colour from white to a dilute and faint yellow, a pretty good orange, or a very full red successively and then totally vanishes. For after the most Refrangible Rays which paint the Paper at P with a Purple Colour, are by a total reflexion taken out of the Beam of light M O, the rest of the Colours which appear on the Paper at R and T being mixed in the light M O compound there a faint yellow, and after the blue and part of the green which appear on the Paper between P and R are taken away, the rest which appear between R and T (that is the Yellow, Orange, Red and a little Green) being mixed in the Beam M O compound there an Orange; and when all the Rays are by reflexion taken out of the Beam MO, except the least Refrangible, which at T appear of a full Red, their Colour is the same in that Beam M O as afterwards at T, the Refraction of the Prism H J K serving only to separate the differently Refrangible Rays, without making any alteration in their Colours, as shall be more fully proved hereafter. All which confirms as well the first Proposition as the second.

Scholium. If this Experiment and the former be conjoyned
Fig. 22. and made one, by applying a fourth Prism V X Y to refract the reflected Beam M N towards *tp*, the conclusion will be clearer. For then the light N*p* which in the 4th Prism is more refracted, will become fuller and stronger when the Light O P, which in the third Prism H J K is more refracted, vanishes at P; and afterwards when the less refracted

refracted Light O T vaniſhes at T, the leſs refracted Light N*t* will become encreaſed whilſt the more refracted Light at *p* receives no further encreaſe. And as the trajected Beam M O in vaniſhing is always of ſuch a Colour as ought to reſult from the mixture of the Colours which fall upon the Paper P T, ſo is the reflected Beam MN always of ſuch a Colour as ought to reſult from the mixture of the Colours which fall upon the Paper *pt*. For when the moſt refrangible Rays are by a total Reflexion taken out of the Beam MO, and leave that Beam of an Orange Colour, the exceſs of thoſe Rays in the reflected Light, does not only make the Violet, Indigo and Blue at *p* more full, but alſo makes the Beam M N change from the yellowiſh Colour of the Sun's Light, to a pale white inclining to blue, and afterward recover its yellowiſh Colour again, ſo ſoon as all the reſt of the tranſmitted light MOT is reflected.

Now ſeeing that in all this variety of Experiments, whether the trial be made in Light reflected, and that either from natural Bodies, as in the firſt and ſecond Experiment, or Specular, as in the Ninth; or in Light refracted, and that either before the unequally refracted Rays are by diverging ſeparated from one another, and loſing their whiteneſs which they have altogether, appear ſeverally of ſeveral Colours, as in the fifth Experiment; or after they are ſeparated from one another, and appear Coloured as in the ſixth, ſeventh, and eighth Experiments; or in Light trajected through Parallel ſuperficies, deſtroying each others Effects as in the 10th Experiment; there are always found Rays, which at equal Incidences on the ſame Medium ſuffer unequal Refractions, and that without any ſplitting or dilating of ſingle Rays, or contingence in the inequality of the Refractions, as is proved in the fifth and ſixth Ex-

 periments;

periments ; And ſeeing the Rays which differ in Refrangibility may be parted and ſorted from one another, and that either by Refraction as in the third Experiment, or by Reflexion as in the tenth, and then the ſeveral ſorts apart at equal Incidences ſuffer unequal Refractions, and thoſe ſorts are more refracted than others after ſeparation, which were more refracted before it, as in the ſixth and following Experiments, and if the Sun's Light be trajected through three or more croſs Priſms ſucceſſively, thoſe Rays which in the firſt Priſm are refracted more than others are in all the following Priſms, refracted more then others in the ſame rate and proportion, as appears by the fifth Experiment ; it's manifeſt that the Sun's Light is an Heterogeneous mixture of Rays, ſome of which are conſtantly more Refrangible then others, as was ~~to be~~ propoſed.

PROP. III. Theor. III.

The Sun's Light conſiſts of Rays differing in Reflexibility, and thoſe Rays are more Reflexible than others which are more Refrangible.

THIS is manifeſt by the ninth and tenth Experiments : For in the ninth Experiment, by turning the Priſm about its Axis, until the Rays within it which in going out into the Air were refracted by its Baſe, became ſo oblique to that Baſe, as to begin to be totally reflected thereby ; thoſe Rays became firſt of all totally reflected, which before at equal Incidences with the reſt had ſuffered the greateſt Refraction. And the ſame thing happens in the Reflexion made by the common Baſe of the two Priſms in the tenth Experiment.

PROP. IV. Prob. I.

To ſeparate from one another the Heterogeneous Rays of Compound Light.

THE Heterogeneous Rays are in ſome meaſure ſeparated from one another by the Refraction of the Priſm in the third Experiment, and in the fifth Experiment by taking away the Penumbra from the Rectilinear ſides of the Coloured Image, that ſeparation in thoſe very Rectilinear ſides or ſtraight edges of the Image becomes perfect. But in all places between thoſe rectilinear edges, thoſe innumerable Circles there deſcribed, which are ſeverally illuminated by Homogeneal Rays, by interfering with one another, and being every where commixt, do render the Light ſufficiently Compound. But if theſe Circles, whilſt their Centers keep their diſtances and poſitions, could be made leſs in Diameter, their interfering one with another and by conſequence the mixture of the Heterogeneous Rays would be proportionally diminiſhed. In the 23th Figure let A G, B H, C J, D K, E L, F M be the Circles which ſo many ſorts of Rays flowing from the ſame Diſque of the Sun, do in the third Experiment illuminate; of all which and innumerable other intermediate ones lying in a continual Series between the two Rectilinear and Parallel edges of the Sun's oblong Image P T, that Image is compoſed as was explained in the fifth Experiment. And let *ag*, *bh*, *ci*, *dk*, *el*, *fm* be ſo many leſs Circles lying in a like continual Series between two Parallel right Lines *af* and *gm* with the ſame diſtances between their Centers, and illuminated by the ſame ſorts of Rays, that is the Circle *ag* with the ſame ſort by which the correſponding Circle

Fig. 23.

Circle A G was illuminated, and the Circle *bh* with the ſame ſort by which the correſponding Circle B H was illuminated, and the reſt of the Circles *c i*, *d k*, *e l*, *f m* reſpectively, with the ſame ſorts of Rays by which the ſeveral correſponding Circles C J, D K, E L, F M were illuminated. In the Figure P T compoſed of the greater Circles, three of thoſe Circles A G, B H, C J, are ſo expanded into one another, that the three ſorts of Rays by which thoſe Circles are illuminated, together with other innumerable ſorts of intermediate Rays, are mixed at Q R in the middle of the Circle B H. And the like mixture happens throughout almoſt the whole length of the Figure P T. But in the Figure *p t* compoſed of the leſs Circles, the three leſs Circles *a g*, *b h*, *c i*, which anſwer to thoſe three greater, do not extend into one another; nor are there any where mingled ſo much as any two of the three ſorts of Rays by which thoſe Circles are illuminated, and which in the Figure P T are all of them intermingled at B H.

Now he that ſhall thus conſider it, will eaſily underſtand that the mixture is diminiſhed in the ſame Proportion with the Diameters of the Circles. If the Diameters of the Circles whilſt their Centers remain the ſame, be made three times leſs than before, the mixture will be alſo three times leſs; if ten times leſs, the mixture will be ten times leſs, and ſo of other Proportions. That is, the mixture of the Rays in the greater Figure P T will be to their mixture in the leſs *p t*, as the Latitude of the greater Figure is to the Latitude of the leſs. For the Latitudes of theſe Figures are equal to the Diameters of their Circles. And hence it eaſily follows, that the mixture of the Rays in the refracted Spectrum *p t* is to the mixture of the Rays in the direct and immediate Light of the Sun, as the breadth of that Spectrum is to the difference between the length and breadth of the ſame Spectrum.

So

So then, if we would diminiſh the mixture of the Rays, we are to diminiſh the Diameters of the Circles. Now theſe would be diminiſhed if the Sun's Diameter to which they anſwer could be made leſs than it is, or (which comes to the ſame purpoſe) if without Doors, at a great diſtance from the Priſm towards the Sun, ſome opake body were placed, with a round hole in the middle of it, to intercept all the Sun's Light, excepting ſo much as coming from the middle of his Body could paſs through that hole to the Priſm. For ſo the Circles A G, B H and the reſt, would not any longer anſwer to the whole Diſque of the Sun, but only to that part of it which could be ſeen from the Priſm through that hole, that is to the apparent magnitude of that hole viewed from the Priſm. But that theſe Circles may anſwer more diſtinctly to that hole a Lens is to be placed by the Priſm to caſt the Image of the hole, (that is, every one of the Circles A G, B H, *&c.*) diſtinctly upon the Paper at P T, after ſuch a manner as by a Lens placed at a Window the Species of Objects abroad are caſt diſtinctly upon a Paper within the Room, and the Rectilinear Sides of the oblong ſolar Image in the fifth Experiment became diſtinct without any Penumbra. If this be done it will not be neceſſary to place that hole very far off, no not beyond the Window. And therefore inſtead of that hole, I uſed the hole in the Window-ſhut as follows.

Exper. 11. In the Sun's Light let into my darkned Chamber through a ſmall round hole in my Window-ſhut, at about 10 or 12 Feet from the Window, I placed a Lens, by which the Image of the hole might be diſtinctly caſt upon a ſheet of white Paper, placed at the diſtance of ſix, eight, ten or twelve Feet from the Lens. For according to the difference of the Lenſes I uſed various

diſtances,

diſtances, which I think not worth the while to deſcribe. Then immediately after the Lens I placed a Priſm, by which the trajected Light might be refracted either upwards or ſideways, and thereby the round Image which the Lens alone did caſt upon the Paper might be drawn out into a long one with Parallel Sides, as in the third Experiment. This oblong Image I let fall upon another Paper at about the ſame diſtance from the Priſm as before, moving the Paper either towards the Priſm or from it, until I found the juſt diſtance where the Rectilinear Sides of the Image became moſt diſtinct. For in this caſe the circular Images of the hole which compoſe that Image after the ſame manner that the Circles *ag*, *bh*, *ci*, &c. do
Fig. 23. the Figure *pt*, were terminated moſt diſtinctly without any Penumbra, and therefore extended into one another the leaſt that they could, and by conſequence the mixture of the Heterogeneous Rays was now the leaſt of all. By this
Fig. 23, *and* 24. means I uſed to form an oblong Image (ſuch as is *pt*) of circular Images of the hole (ſuch as are *ag*, *bh*, *ci*, &c.) and by uſing a greater or leſs hole in the Window-ſhut, I made the circular Images *ag*, *bh*, *ci*, &c. of which it was formed, to become greater or leſs at pleaſure, and thereby the mixture of the Rays in the Image *pt* to be as much or as little as I deſired.

Fig. 24. *Illuſtration.* In the 24th Figure, F repreſents the circular hole in the Window-ſhut, M N the Lens whereby the Image or Species of that hole is caſt diſtinctly upon a Paper at J, A B C the Priſm whereby the Rays are at their emerging out of the Lens refracted from J towards another Paper at *pt*, and the round Image at J is turned into an oblong Image *pt* falling on that other Paper. This Image *pt* conſiſts of Circles placed one after another in a Rectilinear order, as was ſufficiently explained in the fifth

Experiment;

Experiment; and these Circles are equal to the Circle I, and consequently answer in Magnitude to the hole F; and therefore by diminishing that hole they may be at pleasure diminished, whil'st their Centers remain in their places. By this means I made the breadth of the Image *p t* to be forty times, and sometimes sixty or seventy times less than its length. As for instance, if the breadth of the hole F be $\frac{1}{10}$ of an Inch, and MF the distance of the Lens from the hole be 12 Feet; and if *p* B or *p* M the distance of the Image *pt* from the Prism or Lens be 10 Feet, and the refracting Angle of the Prism be 62 degrees, the breadth of the Image *p t* will be $\frac{1}{12}$ of an Inch and the length about six Inches, and therefore the length to the breadth as 72 to 1, and by consequence the Light of this Image 71 times less compound than the Sun's direct Light. And Light thus far Simple and Homogeneal, is sufficient for trying all the Experiments in this Book about simple Light. For the composition of Heterogeneal Rays is in this Light so little that it is scarce to be discovered and perceived by sense, except perhaps in the Indigo and Violet; for these being dark Colours, do easily suffer a sensible allay by that little scattering Light which uses to be refracted irregularly by the inequaliteis of the Prism.

Yet instead of the circular hole F, 'tis better to substitute an oblong hole shaped like a long Parallelogram with its length Parallel to the Prism A B C. For if this hole be an Inch or two long, and but a tenth or twentieth part of an Inch broad or narrower: the Light of the Image *p t* will be as Simple as before or simpler, and the Image will become much broader, and therefore more fit to have Experiments tried in its Light than before.

Instead of this Parallelogram-hole may be substituted a Triangular one of equal Sides, whose Base for instance is

about the tenth part of an Inch, and its height an Inch or more. For by this means, if the Axis of the Priſm be Parallel to the Perpendicular of the Triangle, the Image
Fig. 25. *p t* will now be formed of Equicrural Triangles *a g*, *b h*, *ci*, *dk*, *e l*, *f m*, &c. and innumerable other intermediate ones anſwering to the Triangular hole in ſhape and bigneſs, and lying one after another in a continual Series between two Parallel Lines *a f* and *g m*. Theſe Triangles are a little intermingled at their Baſes but not at their Vertices, and therefore the Light on the brighter ſide *a f* of the Image where the Baſes of the Triangles are is a little compounded, but on the darker ſide *g m* is altogether uncompounded, and in all places between the ſides the Compoſition is Proportional to the diſtances of the places from that obſcurer ſide *g m*. And having a Spectrum *p t* of ſuch a Compoſition, we may try Experiments either in its ſtronger and leſs ſimple Light near the ſide *a f*, or in its weaker
gm and ſimpler Light near the other ſide *hm*, as it ſhall ſeem moſt convenient.

But in making Experiments of this kind the Chamber ought to be made as dark as can be, leaſt any forreign Light mingle it ſelf with the Light of the Spectrum *p t*, and render it compound; eſpecially if we would try Experiments in the more ſimple Light next the ſide *g m* of the Spectrum; which being fainter, will have a leſs Proportion to the forreign Light, and ſo by the mixture of that Light be more troubled and made more compound. The Lens alſo ought to be good, ſuch as may ſerve for Optical Uſes, and the Priſm ought to have a large Angle, ſuppoſe of 70 degrees, and to be well wrought, being made of Glaſs free from Bubbles and Veins, with its ſides not a little Convex or Concave as uſually happens but truly Plane, and its polliſh elaborate, as in working Optick-

glaſſes,

glasses, and not such as is usually wrought with Putty, whereby the edges of the Sand-holes being worn away, there are left all over the Glass a numberless company of very little Convex polite risings like Waves. The edges also of the Prism and Lens so far as they may make any irregular Refraction, must be covered with a black Paper glewed on. And all the Light of the Sun's beam let into the Chamber which is useless and unprofitable to the Experiment, ought to be intercepted with black Paper or other black Obstacles. For otherwise the useless Light being reflected every way in the Chamber, will mix with the oblong Spectrum and help to disturb it. In trying these things so much Diligence is not altogether necessary, but it will promote the success of the Experiments, and by a very scrupulous Examiner of things deserves to be applied. It's difficult to get glass Prisms fit for this purpose, and and therefore I used sometimes Prismatick Vessels made with pieces of broken Looking-glasses, and filled with rain Water. And to increase the Refraction, I sometimes impregnated the Water strongly with *Saccharum Saturni.*

PROP. V. Theor. IV.

Homogeneal Light is refracted regularly without any Dilatation splitting or shattering of the Rays, and the confused Vision of Objects seen through Refracting Bodies by Heterogeneal Light arises from the different Refrangibility of several sorts of Rays.

THE first Part of this Proposition has been already sufficiently proved in the fifth Experiment, and will further appear by the Experiments which follow.

Exper. 12. In the middle of a black Paper I made a round hole about a fifth or ſixth part of an Inch in Diameter. Upon this Paper I cauſed the Spectrum of Homogeneal Light deſcribed in the former Propoſition, ſo to fall, that ſome part of the Light might paſs through the hole of the Paper. This tranſmitted part of the Light I refracted with a Priſm placed behind the Paper, and letting this refracted Light fall perpendicularly upon a white Paper two or three Feet diſtant from the Priſm, I found that the Spectrum formed on the Paper by this Light was not oblong, as when 'tis made (in the third Experiment) by Refracting the Sun's compound Light, but was (ſo far as I could judge by my Eye) perfectly circular, the length being no greater than the breadth. Which ſhews that this Light is refracted regularly without any Dilatation of the Rays.

Exper. 13. In the Homogeneal Light I placed a paper Circle of $\frac{1}{4}$ of an Inch in Diameter, and in the Sun's unrefracted Heterogeneal white Light I placed another Paper Circle of the ſame bigneſs. And going from the Papers to the diſtance of ſome Feet, I viewed both Circles through a Priſm. The Circle illuminated by the Sun's Heterogeneal Light appeared very oblong as in the fourth Experiment, the length being many times greater than the breadth: but the other Circle illuminated with Homogeneal Light appeared Circular and diſtinctly defined as when 'tis viewed with the naked Eye. Which proves the whole Propoſition.

Exper. 14. In the Homogeneal Light I placed Flies and ſuch like Minute Objects, and viewing them through a Priſm, I ſaw their Parts as diſtinctly defined as if I had viewed them with the naked Eye. The ſame Objects placed in the Sun's unrefracted Heterogeneal Light which was white I viewed alſo through a Priſm, and ſaw them moſt

confuſedly

confusedly defined, so that I could not distinguish their smaller Parts from one another. I placed also the Letters of a small Print one while in the Homogeneal Light and then in the Heterogeneal, and viewing them through a Prism, they appeared in the latter case so confused and indistinct that I could not read them; but in the former they appeared so distinct that I could read readily, and thought I saw them as distinct as when I viewed them with my naked Eye. In both cases I viewed the same Objects through the same Prism at the same distance from me and in the same Situation. There was no difference but in the Light by which the Objects were illuminated, and which in one case was Simple and in the other Compound, and therefore the distinct Vision in the former case and confused in the latter could arise from nothing else than from that difference of the Lights. Which proves the whole Proposition.

And in these three Experiments it is further very remarkable, that the Colour of Homogeneal Light was never changed by the Refraction.

PROP. VI. Theor. V.

The Sine of Incidence of every Ray considered apart, is to its Sine of Refraction in a given Ratio.

THAT every Ray considered apart is constant to it self in some certain degree of Refrangibility, is sufficiently manifest out of what has been said. Those Rays which in the first Refraction are at equal Incidences most refracted, are also in the following Refractions at equal Incidences most refracted; and so of the least Refrangible, and the rest which have any mean degree of Refran-

Refrangibility, as is manifest by the 5th, 6th, 7th, 8th, and 9th Experiments. And those which the first time at like Incidences are equally refracted, are again at like Incidences equally and uniformly refracted, and that whether they be refracted before they be separated from one another as in the 5th Experiment, or whether they be refracted apart, as in the 12th, 13th and 14th Experiments. The Refraction therefore of every Ray apart is regular, and what Rule that Refraction observes we are now to shew.

The late Writers in Opticks teach, that the Sines of Incidence are in a given Proportion to the Sines of Refraction, as was explained in the 5th Axiom; and some by Instruments fitted for measuring Refractions, or otherwise experimentally examining this Proportion, do acquaint us that they have found it accurate. But whilst they, not understanding the different Refrangibility of several Rays, conceived them all to be refracted according to one and the same Proportion, 'tis to be presumed that they adapted their Measures only to the middle of the refracted Light; so that from their Measures we may conclude only that the Rays which have a mean degree of Refrangibility, that is those which when separated from the rest appear green, are refracted according to a given Proportion of their Sines. And therefore we are now to shew that the like given Proportions obtain in all the rest. That it should be so is very reasonable, Nature being ever conformable to her self: but an experimental Proof is desired. And such a Proof will be had if we can shew that the Sines of Refraction of Rays differently Refrangible are one to another in a given Proportion when their Sines of Incidence are equal. For if the Sines of Refraction of all the Rays are in given Proportions to the Sine of Refraction

of

of a Ray which has a mean degree of Refrangibility, and this Sine is in a given Proportion to the equal Sines of Incidence, those other Sines of Refraction will also be in given Proportions to the equal Sines of Incidence. Now when the Sines of Incidence are equal, it will appear by the following Experiment that the Sines of Refraction are in a given Proportion to one another.

Exper. 15. The Sun shining into a dark Chamber through a little round hole in the Window-shut, let S represent his round white Image painted on the opposite Wall by his direct Light, P T his oblong coloured Image made by refracting that Light with a Prism placed at the Window; and *pt*, or 2*p* 2*t*, or 3*p* 3*t*, his oblong coloured Image made by refracting again the same Light sideways with a second Prism placed immediately after the first in a cross Position to it, as was explained in the fifth Experiment: that is to say, *pt* when the Refraction of the second Prism is small, 2*p* 2*t* when its Refraction is greater, and 3*p* 3*t* when it is greatest. For such will be the diversity of the Refractions if the refracting Angle of the second Prism be of various Magnitudes; suppose of fifteen or twenty degrees to make the Image *p t*, of thirty or forty to make the Image 2*p* 2*t*, and of sixty to make the Image 3*p* 3*t*. But for want of solid Glass Prisms with Angles of convenient bignesses, there may be Vessels made of polished Plates of Glass cemented together in the form of Prisms and filled with Water. These things being thus ordered, I observed that all the solar Images or coloured Spectrums P T, *pt*, 2*p* 2*t*, 3*p* 3*t* did very nearly converge to the place S on which the direct Light of the Sun fell and painted his white round Image when the Prisms were taken away. The Axis of the Spectrum P T, that is the Line drawn through the middle of it Parallel to its

Fig. 26.

its Rectilinear Sides, did when produced paſs exactly through the middle of that white round Image S. And when the Refraction of the ſecond Priſm was equal to the Refraction of the firſt, the refracting Angles of them both being about 60 degrees, the Axis of the Spectrum $3p\ 3t$ made by that Refraction, did when produced paſs alſo through the middle of the ſame white round Image S. But when the Refraction of the ſecond Priſm was leſs than that of the firſt, the produced Axes of the Spectrums tp or $2t\ 2p$ made by that Refraction did cut the produced Axis of the Spectrum TP in the Points m and n, a little beyond the Center of that white round Image S. Whence the Proportion of the Line $3t$T to the Line $3p$P was a little greater than the Proportion of $2t$T to $2p$P, and this Proportion a little greater than that of tT to pP. Now when the Light of the Spectrum P T falls perpendicularly upon the Wall, thoſe Lines $3t$T, $3p$P, and $2t$T, $2p$P and tT, pP, are the Tangents of the Refractions; and therefore by this Experiment the Proportions of the Tangents of the Refractions are obtained, from whence the Proportions of the Sines being derived, they come out equal, ſo far as by viewing the Spectrums and uſing ſome Mathematical reaſoning I could Eſtimate. For I did not make an Accurate Computation. So then the Propoſition holds true in every Ray apart, ſo far as appears by Experiment. And that it is accurately true may be demonſtrated upon this Suppoſition, *That Bodies refract Light by acting upon its Rays in Lines Perpendicular to their Surfaces.* But in order to this Demonſtration, I muſt diſtinguiſh the Motion of every Ray into two Motions, the one Perpendicular to the refracting Surface, the other Parallel to it, and concerning the Perpendicular Motion lay down the following Propoſition.

If any Motion or moving thing whatſoever be incident with any velocity on any broad and thin Space terminated on both ſides by two Parallel Planes, and in its paſſage through that ſpace be urged perpendicularly towards the further Plane by any force which at given diſtances from the Plane is of given quantities; the perpendicular Velocity of that Motion or Thing, at its emerging out of that ſpace, ſhall be always equal to the Square Root of the Summ of the Square of the perpendicular Velocity of that Motion or Thing at its Incidence on that ſpace; and of the Square of the perpendicular Velocity which that Motion or Thing would have at its Emergence, if at its Incidence its perpendicular Velocity was infinitely little.

And the ſame Propoſition holds true of any Motion or Thing perpendicularly retarded in its paſſage through that ſpace, if inſtead of the Summ of the two Squares you take their difference. The Demonſtration Mathematicians will eaſily find out, and therefore I ſhall not trouble the Reader with it.

Suppoſe now that a Ray coming moſt obliquely in the Line MC be refracted at C by the Plane RS into the Line CN, and if it be required to find the Line CE into which any other Ray AC ſhall be refracted; let MC, AD, be the Sines of incidence of the two Rays, and NG, EF, their Sines of Refraction, and let the equal Motions of the Incident Rays be repreſented by the equal Lines M C and AC, and the Motion MC being conſidered as parallel to the refracting Plane, let the other Motion AC be diſtinguiſhed into two Motions AD and DC, one of which AD is parallel, and the other DC perpendicular to the refracting Surface. In like manner, let the Motions of the emerging Rays be diſtinguiſh'd into two, whereof the per- *Fig.* 1.

 pendicular

perpendicular ones are $\frac{MC}{NG}$ CG and $\frac{AD}{EF}$ CF. And if the force of the refracting Plane begins to act upon the Rays either in that Plane or at a certain diſtance from it on the one ſide, and ends at a certain diſtance from it on the other ſide, and in all places between thoſe two Limits acts upon the Rays in Lines perpendicular to that rafracting Plane, and the Actions upon the Rays at equal diſtances from the refracting Plane be equal, and at unequal ones either equal or unequal according to any rate whatever; that motion of the Ray which is Parallel to the refracting Plane will ſuffer no alteration by that force; and that motion which is perpendicular to it will be altered according to the rule of the foregoing Propoſition. If therefore for the perpendicular Velocity of the emerging Ray CN you write $\frac{MC}{NG}$ CG as above, then the perpendicular Velocity of any other emerging Ray CE which was $\frac{AD}{EF}$ CF, will be equal to the ſquare Root of $CDq + \frac{MCq}{NGq} CGq$. And by ſquaring theſe equals, and adding to them the Equals ADq and $MCq - CDq$, and dividing the Summs by the Equals $CFq + EFq$ and $CGq + NGq$, you will have $\frac{ADq}{EFq}$ equal to $\frac{MCq}{NGq}$. Whence AD, the Sine of Incidence, is to EF the Sine of Refraction, as MC to NG, that is, in a given *ratio*. And this Demonſtration being general, without determining what Light is, or by what kind of force it is refracted, or aſſuming any thing further than that the refracting Body acts upon the Rays in Lines perpendicular to its Surface; I take it to be a very convincing Argument of the full Truth of this Propoſition.

So

So then, if the *ratio* of the Sines of Incidence and Refraction of any ſort of Rays be found in any one Caſe, 'tis given in all Caſes; and this may be readily found by the Method in the following Propoſition.

PROP. VII. Theor. VI.

The Perfection of Teleſcopes is impeded by the different Refrangibility of the Rays of Light.

THE imperfection of Teleſcopes is vulgarly attributed to the ſpherical Figures of the Glaſſes, and therefore Mathematicians have propounded to Figure them by the Conical Sections. To ſhew that they are miſtaken, I have inſerted this Propoſition; the truth of which will appear by the meaſures of the Refractions of the ſeveral ſorts of Rays; and theſe meaſures I thus determine.

In the third experiment of the firſt Book, where the refracting Angle of the Priſm was $62\frac{1}{2}$ degrees, the half of that Angle 31 deg. 15 min. is the Angle of Incidence of the Rays at their going out of the Glaſs into the Air; and the Sine of this Angle is 5188, the Radius being 10000. When the Axis of this Priſm was parallel to the Horizon, and the Refraction of the Rays at their Incidence on this Priſm equal to that at their Emergence out of it, I obſerved with a Quadrant the Angle which the mean refrangible Rays (that is, thoſe which went to the middle of the Sun's coloured Image) made with the Horizon and by this Angle and the Sun's altitude obſerved at the ſame time, I found the Angle which the emergent Rays contained with the incident to be 44 deg. and 40 min. and the half of this Angle added to the Angle of Incidence 31 deg. 15 min. makes the

Angle of Refraction, which is therefore 53 deg. 35 min. and its Sine 8047. These are the Sines of Incidence and Refraction of the mean refrangible Rays, and their proportion in round numbers is 20 to 31. This Glass was of a colour inclining to green. The last of the Prisms mentioned in the third Experiment was of clear white Glass. Its refracting Angle $63\frac{1}{2}$ degrees. The Angle which the emergent Rays contained, with the incident 45 deg. 50 min. The Sine of half the first Angle 5262. The Sine of half the Summ of the Angles 8157. And their proportion in round numbers 20 to 31 as before.

From the Length of the Image, which was about $9\frac{3}{4}$ or 10 Inches, subduct its Breadth, which was $2\frac{1}{8}$ Inches, and the Remainder $7\frac{3}{4}$ Inches would be the length of the Image were the Sun but a point, and therefore subtends the Angle which the most and least refrangible Rays, when incident on the Prism in the same Lines, do contain with one another after their Emergence. Whence this Angle is 2 deg. 0.′ 7.″ For the distance between the Image and the Prism where this Angle is made, was $18\frac{1}{2}$ Feet, and at that distance the Chord $7\frac{3}{4}$ Inches subtends an Angle of 2 deg. 0.′ 7.″ Now half this Angle is the Angle which these emergent Rays contain with the emergent mean refrangible Rays, and a quarter thereof, that is 30.′ 2.″ may be accounted the Angle which they would contain with the same emergent mean refrangible Rays, were they co-incident to them within the Glass and suffered no other Refraction then that at their Emergence. For if two equal Refractions, the one at the incidence of the Rays on the Prism, the other at their Emergence, make half the Angle 2 deg. 0.′ 7.″ then one of those Refractions will make about a quarter of that Angle, and this quarter added to

and

and ſubducted from the Angle of Refraction of the mean refrangible Rays, which was 53 deg. 35′, gives the Angles of Refraction of the moſt and leaſt refrangible Rays 54 deg. 5′ 2″, and 53 deg. 4′ 58″, whoſe Sines are 8099 and 7995, the common Angle of Incidence being 31 deg. 15′ and its Sine 5188; and theſe Sines in the leaſt round numbers are in proportion to one another as 78 and 77 to 50.

Now if you ſubduct the common Sine of Incidence 50 from the Sines of Refraction 77 and 78, the remainders 27 and 28 ſhew that in ſmall Refractions the Refraction of the leaſt refrangible Rays is to the Refraction of the moſt refrangible ones as 27 to 28 very nearly, and that the difference of the Refractions of the leaſt refrangible and moſt refrangible Rays is about the $27\frac{1}{2}$th part of the whole Refraction of the mean refrangible Rays.

Whence they that are skilled in Opticks will eaſily underſtand, that the breadth of the leaſt circular ſpace into which Object-Glaſſes of Teleſcopes can collect all ſorts of Parallel Rays, is about the $27\frac{1}{2}$th part of half the aperture of the Glaſs, or 55th part of the whole aperture; and that the Focus of the moſt refrangible Rays is nearer to the Object-Glaſs than the Focus of the leaſt refrangible ones, by about the $27\frac{1}{2}$th part of the diſtance between the Object-Glaſs and the Focus of the mean refrangible ones.

And if Rays of all ſorts, flowing from any one lucid point in the Axis of any convex Lens, be made by the Refraction of the Lens to converge to points not too remote from the Lens, the Focus of the moſt refrangible Rays ſhall be nearer to the Lens than the Focus of the leaſt refrangible ones, by a diſtance which is to the $27\frac{1}{2}$th part of the diſtance of the Focus of the mean refrangible Rays from the Lens as the diſtance between that Focus and the lucid

point

point from whence the Rays flow is to the diſtance between that lucid point and the Lens very nearly.

Now to examine whether the difference between the Refractions which the moſt refrangible and the leaſt refrangible Rays flowing from the ſame point ſuffer in the Object-Glaſſes of Teleſcopes and ſuch like Glaſſes, be ſo great as is here deſcribed, I contrived the following Experiment.

Exper. 16. The Lens which I uſed in the ſecond and eighth Experiments, being placed ſix Feet and an Inch diſtant from any Object, collected the Species of that Object by the mean refrangible Rays at the diſtance of ſix Feet and an Inch from the Lens on the other ſide. And therefore by the foregoing Rule it ought to collect the Species of that Object by the leaſt refrangible Rays at the diſtance of ſix Feet and $3\frac{2}{3}$ Inches from the Lens, and by the moſt refrangible ones at the diſtance of five Feet and $10\frac{2}{3}$ Inches from it: So that between the two Places where theſe leaſt and moſt refrangible Rays collect the Species, there may be the diſtance of about $5\frac{1}{3}$ Inches. For by that Rule, as ſix Feet and an Inch (the diſtance of the Lens from the lucid Object) is to twelve Feet and two Inches (the diſtance of the lucid Object from the Focus of the mean refrangible Rays) that is, as one is to two, ſo is the $27\frac{1}{2}$th part of ſix Feet and an Inch (the diſtance between the Lens and the ſame Focus) to the diſtance between the Focus of the moſt refrangible Rays and the Focus of the leaſt refrangible ones, which is therefore $5\frac{17}{55}$ Inches, that is very nearly $5\frac{1}{3}$ Inches. Now to know whether this meaſure was true, I repeated the ſecond and eighth Experiment of this Book with coloured Light, which was leſs compounded than that I there made uſe of: For I now ſeparated the

hetero-

heterogeneous Rays from one another by the Method I described in the 11th Experiment, so as to make a coloured Spectrum about twelve or fifteen times longer than broad. This Spectrum I cast on a printed book, and placing the above-mentioned Lens at the distance of six Feet and an Inch from this Spectrum to collect the Species of the illuminated Letters at the same distance on the other side, I found that the Species of the Letters illuminated with Blue were nearer to the Lens than those illuminated with deep Red by about three Inches or three and a quarter: but the Species of the Letters illuminated with Indigo and Violet appeared so confused and indistinct, that I could not read them: Whereupon viewing the Prism, I found it was full of Veins running from one end of the Glass to the other; so that the Refraction could not be regular. I took another Prism therefore which was free from Veins, and instead of the Letters I used two or three Parallel black Lines a little broader than the stroakes of the Letters, and casting the Colours upon these Lines in such manner that the Lines ran along the Colours from one end of the Spectum to the other, I found that the Focus where the Indigo, or confine of this colour and Violet cast the Species of the black Lines most distinctly, to be about 4 Inches or $4\frac{1}{4}$ nearer to the Lens than the Focus where the deepest Red cast the Species of the same black Lines most distinctly. The violet was so faint and dark, that I could not discern the Species of the Lines distinctly by that Colour; and therefore considering that the Prism was made of a dark coloured Glass inclining to Green, I took another Pism of clear white Glass; but the Spectrum of Colours which this Prism made had long white Streams of faint Light shooting out from both ends of the Colours, which made me conclude that something was amiss; and viewing

ing the Prism, I found two or three little Bubbles in the Glass which refracted the Light irregularly. Wherefore I covered that part of the Glass with black Paper, and letting the Light pass through another part of it which was free from such Bubles, the Spectrum of Colours became free from those irregular Streams of Light, and was now such as I desired. But still I found the Violet so dark and faint, that I could scarce see the Species of the Lines by the Violet, and not at all by the deepest part of it, which was next the end of the Spectrum. I suspected therefore that this faint and dark Colour might be allayed by that scattering Light which was refracted, and reflected irregularly partly by some very small Bubbles in the Glasses and partly by the inequalities of their Polish: which Light, tho' it was but little, yet it being of a White Colour, might suffice to affect the Sense so strongly as to disturb the Phænomena of that weak and dark Colour the Violet, and therefore I tried, as in the 12th, 13th, & 14th Experiments, whether the Light of this Colour did not consist of a sensible mixture of heterogeneous Rays, but found it did not. Nor did the Refractions cause any other sensible Colour than Violet to emerge out of this Light, as they would have done out of White Light, and by consequence out of this Violet Light had it been sensibly compounded with White Light. And therefore I concluded, that the reason why I could not see the Species of the Lines distinctly by this Colour, was only the darkness of this Colour and Thinness of its Light, and its distance from the Axis of the Lens; I divided therefore those Parallel Black Lines into equal Parts, by which I might readily know the distances of the Colours in the Spectrum from one another, and noted the distances of the Lens from the Foci of such Colours as cast the Species of the

Lines

Lines diſtinctly, and then conſidered whether the difference of thoſe diſtances bear ſuch proportion to 5$\frac{1}{2}$ Inches, the greateſt difference of the diſtances which the Foci of the deepeſt Red and Violet ought to have from the Lens, as the diſtance of the obſerved Colours from one another in the Spectrum bear to the ~~like~~ greateſt diſtance of the deepeſt Red and Violet meaſured in the rectilinear ſides of the Spectrum, that is, to the length of thoſe ſides or exceſs of the length of the Spectrum above its breadth. And my Obſervations were as follows.

When I obſerved and compared the deepeſt ſenſible Red, and the Colour in the confine of Green and Blue, which at th~~at~~e rectilinear ſides of the Spectrum was diſtant from it half the length of thoſe ſides, the Focus where the confine of Green and Blue caſt the Species of the Lines diſtinctly on the Paper, was nearer to the Lens then the Focus where the Red caſt thoſe Lines diſtinctly on it by about 2$\frac{1}{2}$ or 2$\frac{3}{4}$ Inches. For ſometimes the Meaſures were a little greater, ſometimes a little leſs, but ſeldom varied from one another above $\frac{1}{3}$ of an Inch. For it was very difficult to define the Places of the Foci, without ſome little Errors. Now if the Colours diſtant half the length of the Image, (meaſured at its rectilinear ſides) give 2$\frac{1}{2}$ or 2$\frac{3}{4}$ difference of the diſtances of their Foci from the Lens, then the Colours diſtant the whole length ought to give 5 or 5$\frac{1}{2}$ Inches difference of thoſe diſtances.

But here it's to be noted, that I could not ſee the Red to the full End of the Spectrum, but only to the Center of the Semicircle which bounded that End, or a little farther ; and therefore I compared this Red not with that Colour which was exactly in the middle of the Spectrum, or confine of Green and Blue, but with that which verged a little more to the Blue than to the Green : And as I reckoned the whole length of the Colours not to be the whole length of the Spectrum, but the length of its rectilinear sides, so completing the Semicirlar Ends into Circles, when either of the observed Colours fell within those Circles, I measured the distance of that Colour from the semicircular End of the Spectrum, and subducting half the distance from the measured distance of the two Colours, I took the remainder for their corrected distance; and in these Observations set down this corrected distance for the difference of their distances of their foci from the Lens. For as the length of the rectilinear sides of the Spectrum would be the whole length of all the Colours, were the Circles of which (as we shewed) that Spectrum consists contracted and reduced to Physical Points, so in that Case this corrected distance would be the real distance of the two observed Colours.

When therefore I further observed the deepest sensible Red, and that Blue whose corrected distance from it was $\frac{7}{12}$ parts of the length of the rectilinear sides of the Spectrum, the difference of the distances of their Foci from the Lens was about $3\frac{1}{4}$ Inches, and as 7 to 12 so is $3\frac{1}{4}$ to $5\frac{4}{7}$.

When I observed the deepest sensible Red, and that Indigo whose corrected distance was $\frac{8}{12}$ or $\frac{2}{3}$ of the length of the rectilinear sides of the Spectrum, the difference of the distances of their Foci from the Lens, was about $3\frac{2}{3}$ Inches, and as 2 to 3 so is $3\frac{2}{3}$ to $5\frac{1}{2}$.

When I observed the deepest sensible Red, and that deep Indigo whose corrected distance from one another was $\frac{9}{12}$ or $\frac{3}{4}$ of the length of the rectilinear sides of the Spectum, the difference of the distances of their Foci from the Lens was about 4 Inches; and as 3 to 4 so is 4 to $5\frac{1}{3}$.

When I observed the deepest sensible Red, and that part of the Violet next the Indigo whose corrected distance from the Red was $\frac{10}{12}$ or $\frac{5}{6}$ of the length of the rectilinear sides of

the

the Spectrum, the difference of the diſtances of their Foci from the Lens was about $4\frac{1}{2}$ Inches; and as 5 to 6, ſo is $4\frac{1}{2}$ to $5\frac{2}{5}$. For ſometimes when the Lens was advantagiouſly placed, ſo that its Axis reſpected the Blue, and all things elſe were well ordered, and the Sun ſhone clear, and I held my Eye very near to the Paper on which the Lens caſt the Species of the Lines, I could ſee pretty diſtinctly the Species of thoſe Lines by that part of the Violet which was next the Indigo; and ſometimes I could ſee them by above half the Violet. For in making theſe Experiments I had obſerved, that the Species of thoſe Colours only appeared diſtinct which were in or near the Axis of the Lens: So that if the Blue or Indigo were in the Axis, I could ſee their Species diſtinctly; and then the Red appeared much leſs diſtinct than before. Wherefore I contrived to make the Spectrum of Colours ſhorter than before, ſo that both its Ends might be nearer to the Axis of the Lens. And now its length was about $2\frac{1}{2}$ Inches and breadth about $\frac{1}{5}$ or $\frac{1}{6}$ of an Inch. Alſo inſtead of the black Lines on which the Spectrum was caſt, I made one black Line broader than thoſe, that I might ſee its Species more eaſily; and this Line I divided by ſhort croſs Lines into equal Parts, for meaſuring the diſtances of the obſerved Colours. And now I could ſometimes ſee the Species of this Line with its diviſions almoſt as far as the Center of the Semicircular Violet End of the Spectrum, and made theſe further Obſervations.

When I obſerved the deepeſt ſenſible Red, and that part of the Violet whoſe corrected diſtance from it was about $\frac{8}{9}$ Parts of the rectilinear ſides of the Spectrum the difference of the diſtances of the Foci of thoſe Colours from the Lens, was one time $4\frac{2}{3}$, another time $4\frac{3}{4}$, another time $4\frac{7}{8}$, Inches, and as 8 to 9, ſo are $4\frac{2}{3}$, $4\frac{3}{4}$, $4\frac{7}{8}$, to $5\frac{1}{4}$, $5\frac{11}{32}$ $5\frac{31}{64}$ reſpectively.

 When

When I observed the deepest sensible Red, and deepest sensible Violet, (the corrected distance of which Colours when all things were ordered to the best advantage, and the Sun shone very clear, was about $\frac{11}{12}$ or $\frac{15}{16}$ parts of the length of the rectilinear sides of the coloured Spectrum,) I found the difference of the distances of their Foci from the Lens sometimes $4\frac{3}{4}$ sometimes $5\frac{1}{4}$, and for the most part 5 Inches or thereabouts: and as 11 to 12 or 15 to 16, so is five Inches to $5\frac{1}{2}$ or $5\frac{1}{3}$ Inches.

And by this progression of Experiments I satisfied my self, that had the light at the very Ends of the Spectrum been strong enough to make the Species of the black Lines appear plainly on the Paper, the Focus of the deepest Violet would have been found nearer to the Lens, than the Focus of the deepest Red, by about $5\frac{1}{3}$ Inches at least. And this is a further Evidence, that the Sines of Incidence and Refraction of the several sorts of Rays, hold the same proportion to one another in the smallest Refractions which they do in the greatest.

My progress in making this nice and troublesome Experiment I have set down more at large, that they that shall try it after me may be aware of the Circumspection requisite to make it succeed well. And if they cannot make it succeed so well as I did, they may notwithstanding collect by the Proportion of the distance of the Colours in the Spectrum, to the difference of the distances of their Foci from the Lens, what would be the success in the more distant Colours by a better Trial. And yet if they use a broader Lens than I did, and fix it to a long streight Staff by means of which it may be readily and truly directed to the Colour whose Focus is desired, I question not but the Experiment will succeed better with them than it did with me. For I directed the Axis as nearly as I could to the

middle

middle of the Colours, and then the faint Ends of the Spectrum being remote from the Axis, cast their Species less distinctly on the Paper than they would have done had the Axis been successively directed to them.

Now by what has been said its certain, that the Rays which differ in refrangibility do not converge to the same Focus, but if they flow from a lucid point, as far from the Lens on one side as their Foci are one the other, the Focus of the most refrangible Rays shall be nearer to the Lens than that of the least refrangible, by above the fourteenth part of the whole distance: and if they flow from a lucid point, so very remote from the Lens that before their Incidence they may be accounted Parallel, the Focus of the most refrangible Rays shall be nearer to the Lens than the Focus of the least refrangible, by about the 27th or 28th part of their whole distance from it. And the Diameter of the Circle in the middle space between those two Foci which they illuminate when they fall there on any Plane, perpendicular to the Axis (which Circle is the least into which they can all be gathered) is about the 55th part of the Diameter of the aperture of the Glass. So that 'tis a wonder that Telescopes represent Objects so distinct as they do. But were all the Rays of Light equally refrangible, the Error arising only from the sphericalness of the Figures of Glasses would be many hundred times less. For if the Object-Glass of a Telescope be Plano-convex, and the Plane side be turned towards the Object, and the Diameter of the Sphere whereof this Glass is a segment, be called D, and the Semidiameter of the aperture of the Glass be called S, and the Sine of Incidence out of Glass into Air, be to the Sine of Refraction as I to R: the Rays which come Parallel to the Axis of the Glass, shall in the Place where the Image of the Object is most distinctly made, be scattered all over a little

Circle

Circle whose Diameter is $\frac{R}{I} \times \frac{S\ cub.}{D\ quad.}$ very nearly, as I gather by computing the Errors of the Rays by the method of infinite Series, and rejecting the Terms whose quantities are inconsiderable. As for instance, if the Sine of Incidence I, be to the Sine of Refraction R, as 20 to 31, and if D the Diameter of the Sphere to which the Convex side of the Glass is ground, be 100 Feet or 1200 Inches, and S the Semidiameter of the aperture be two Inches, the Diameter of the little Circle (that is $\frac{R \times S\ cub.}{I \times D\ quad.}$) will be $\frac{31 \times 8}{20 \times 1200 \times 1200}$ (or $\frac{31}{3600000}$) parts of an Inch. But the Diameter of the little Circle through which these Rays are scattered by unequal refrangibility, will be about the 55th part of the aperture of the Object-Glass which here is four Inches. And therefore the Error arising from the spherical Figure of the Glass, is to the Error arising from the different Refrangibility of the Rays, as $\frac{31}{3600000}$ to $\frac{4}{55}$ that is as 1 to 8151 : and therefore being in Comparison so very little, deserves not to be considered.

But you will say, if the Errors caused by the different refrangibility be so very great, how comes it to pass that Objects appear through Telescopes so distinct as they do ? I answer, 'tis because the erring Rays are not scattered uniformly over all that circular space, but collected infinitely more densely in the Center than in any other part of the Circle, and in the way from the Center to the Circumference grow continually rarer and rarer, so as at the Circumference to become infinitely rare; and by reason of their rarity are not strong enough to be visible, unless in the Center and very near it. *Fig. 27.* Let ADE represent one of those Circles described with the Center C and Semidiameter AC, and let BFG be a smaller Circle concentric to the former, cutting

with

with its Circumference the Diameter AC in B, and besect AC in N, and by my reckoning the density of the Light in any place B will be to its density in N, as AB to BC; and the whole Light within the lesser Circle BFG, will be to the whole Light within the greater AED, as the Excess of the Square of AC above the Square of AB, is to the Square of AC. As if BC be the fifth part of AC, the Light will be four times denser in B than in N, and the whole Light within the less Circle, will be to the whole Light within the greater, as nine to twenty five. Whence it's evident that the Light within the less Circle, must strike the sense much more strongly, than that faint and dilated light round about between it and the Circumference of the greater.

But its further to be noted, that the most luminous of the prismatick Colours are the Yellow and Orange. These affect the Senses more strongly than all the rest together, and next to these in strength are the Red and Green. The Blue compared with these is a faint and dark Colour, and the Indigo and Violet are much darker and fainter, so that these compared with the stronger Colours are little to be regarded. The Images of Objects are therefore to be placed, not in the Focus of the mean refrangible Rays which are in the confine of Green and Blue, but in the Focus of those Rays which are in the middle of the Orange and Yellow; there where the Colour is most luminous and fulgent, that is in the brightest Yellow, that Yellow which inclines more to Orange than to Green. And by the Refraction of these Rays (whose Sines of Incidence and Refraction in Glass are as 17 and 11) the Refraction of Glass and Crystal for optical uses is to be measured. Let us therefore place the Image of the Object in the Focus of these Rays, and all the Yellow and Orange will fall within a Circle, whose Diameter is about the 250th part of the Diameter of the aperture.

ture of the Glaſs. And if you add the brighter half of the Red, (that half which is next the Orange, and the brighter half of the Green, (that half which is next the Yellow,) about three fifth parts of the Light of theſe two Colours will fall within the ſame Circle,and two fifth parts will fall without it round about ; and that which falls without will be ſpread through almoſt as much more ſpace as that which falls within, and ſo in the groſs be almoſt three times rarer. Of the other half of the Red and Green, (that is of the deep dark Red and Willow Green) about one quarter will fall within this Circle, and three quarters without, and that which falls without will be ſpread through about four or five times more ſpace than that which falls within ; and ſo in the groſs be rarer, and if compared with the whole Light within it, will be about 25 times rarer than all that taken in the groſs ; or rather more than 30 or 40 times rarer, becauſe the deep red in the end of the Spectrum of Colours made by a Priſm is very thin and rare, and the Willow Green is ſomething rarer than the Orange and Yellow. The Light of theſe Colours therefore being ſo very much rarer than that within the Circle, will ſcarce affect the Senſe eſpecially ſince the deep Red and Willow Green of this Light, are much darker Colours then the reſt. And for the ſame reaſon the Blue and Violet being much darker Colours than theſe, and much more rarified, may be neglected. For the denſe and bright Light of the Circle, will obſcure the rare and weak Light of theſe dark Colours round about it, and render them almoſt inſenſible. The ſenſible Image of a lucid point is therefore ſcarce broader than a Circle whoſe Diameter is the 250th part of the diameter of the aperture of the Object Glaſs of a good Teleſcope, or not much broader, if you except a faint and dark miſty light round about it, which a Spectator will ſcarce regard. And therefore in a Teleſcope whoſe

whose aperture is four Inches, and length an hundred Feet, it exceeds not $2'' 45'''$, or $3''$. And in a Telescope whose aperture is two Inches, and length 20 or 30 Feet, it may be $5''$ or $6''$ and scarce above. And this Answers well to Experience: For some Astronomers have found the Diameters of the fixt Stars, in Telescopes of between twenty and sixty Feet in length, to be about $4''$ or $5''$ or at most $6''$, in Diameter. But if the Eye-Glass be tincted faintly $8''$ or $10''$
with the smoke of a Lamp or Torch, to obscure the Light of the Star, the fainter Light in the circumference of the Star ceases to be visible, and the Star (if the Glass be sufficiently soiled with smoke) appears something more like a Mathematical Point. And for the same reason, the enormous part of the Light in the Circumference of every lucid Point ought to be less discernable in shorter Telescopes than in longer, because the shorter transmit less Light to the Eye.

Now if we suppose the sensible Image of a lucid point, to be ~~even~~ even 250 times narrower than the aperture of the
yet this image would be still much greater than if it arose only from the spherical figure of the glass. For
Glass: ~~yet~~ were it not for the different refrangibility of the Rays, its breadth in an 100 Foot Telescope whose aperture is 4 Inches would be but $\frac{31}{3600000}$ parts of an Inch, as is manifest by the foregoing Computation. And therefore in this Case the greatest Errors arising from the spherical Figure of the Glass, would be to the greatest sensible Errors arising from the different refrangibility of the Rays as $\frac{31}{3600000}$ to $\frac{4}{250}$ at most, that is only as 1 to 1826. And this sufficiently shews that it is not the spherical Figures of Glasses but the different refrangibility of the Rays which hinders the perfection of Telescopes.

There is another Argument by which it may appear that the different refrangibility of Rays, is the true Cause of the imperfection of Telescopes. For the Errors of the Rays arising from the spherical Figures of Object-Glasses, are as the Cubes of the apertures of the Object-Glaſſes; and thence to make Teleſcopes of various lengths, magnify with equal diſtinctneſs, the apertures of the Object-Glaſſes, and the Charges or magnifying Powers, ought to be as the Cubes of the ſquare Roots of their lengths; which doth not anſwer to Experience. But the errors of the Rays ariſing from the different refrangibility, are as the apertures of the Object-Glaſſes, and thence to make Teleſcopes of various lengths, magnify with equal diſtinctneſs, their apertures and charges ought to be as the ſquare Roots of their lengths; and this anſwers to experience as is well known. For inſtance, a Teleſcope of 64 Feet in length, with an aperture of $2\frac{2}{3}$ Inches, magnifies about 120 times, with as much diſtinctneſs as one of a Foot in length, with $\frac{1}{3}$ of an Inch aperture, magnifies 15 times.

Now were it not for this different refrangibility of Rays, Teleſcopes might be brought to a greater Perfection than we have yet deſcribed, by compoſing the Object-Glaſs of two Glaſſes with Water between them. Let ADFC repre-
Fig. 28. ſent the Object-Glaſs compoſed of two Glaſſes ABED and and BEFC, alike convex on the outſides AGD and CHF, and alike concave on the inſides BME, BNE, with Water in the concavity BMEN. Let the Sine of Incidence out of Glaſs into Air be as I to R and out of Water into Air as K to R, and by conſequence out of Glaſs into Water, as I to K: and let the Diameter of the Sphere to which the convex ſides AGD and CHF are ground be D, and the Diameter of the Sphere to which the concave ſides BME and BNE are ground be to D, as the Cube Root of KK—KI to the Cube Root of RK—RI: and the Refractions on the concave ſides of the Glaſſes, will very much correct the Errors of the Refractions on the convex ſides, ſo far as they ariſe from the ſphericalneſs of the Figure. And by this means

might

might Teleſcopes be brought to ſufficient perfection, were it not for the different refrangibility of ſeveral ſors of Rays. But by reaſon of this different refrangibility, I do not yet ſee any other means of improving Teleſcopes by Refractions alone than that of increaſing their lengths, for which end the late contrivance of *Hugenius* ſeems well accommodated. For very long Tubes are cumberſome, and ſcarce to be readily managed, and by reaſon of their length are very apt to bend, and ſhake by bending ſo as to cauſe a continual trembling in the Objects, whereby it becomes difficult to ſee them diſtinctly: whereas by his contrivance the Glaſſes are readily manageable, and the Object-Glaſs being fixt upon a ſtrong upright Pole becomes more ſteddy.

Seeing therefore the improvement of Teleſcopes of given lengths by Refractions is deſperate; I contrived heretofore a Perſpective by reflexion, uſing inſtead of an Object Glaſs a concave Metal. The diameter of the Sphere to which the Metal was ground concave was about 25 Engliſh Inches, and by conſequence the length of the Inſtrument about ſix Inches and a quarter. The Eye-Glaſs was plano-convex, and the Diameter of the Sphere to which the convex ſide was ground was about $\frac{1}{5}$ of an Inch, or a little leſs, and by conſequence it magnified between 30 and 40 times. By another way of meaſuring I found that it magnified about 35 times. The Concave Metal bore an aperture of an Inch and a third part; but the aperture was limited not by an opake Circle, covering the Limb of the Metal round about, but by an opake circle placed between the Eye-Glaſs and the Eye, and perforated in the middle with a little round hole for the Rays to paſs through to the Eye. For this Circle by being placed here, ſtopt much of the erroneous Light, which otherwiſe would have diſturbed the Viſion. By comparing it with a pretty good Perſpective of four Feet in length, made with a concave Eye-Glaſs, I could read at a greater diſtance with my own Inſtrument than with the Glaſs. Yet Objects appeared much darker in it than in the Glaſs, and that partly becauſe more Light was loſt by reflexion in the Metal, then by refraction in the Glaſs, and partly becauſe my Inſtrument was overcharged. Had it magnified but 30 or 25 times it would have made the Object appear more brisk and pleaſant. Two of theſe I made about 16 Years ago, and have one of them ſtill by me by which I can prove the truth of what I write. Yet it is not ſo good as at the firſt. For the concave has been divers times tarniſhed and cleared again, by rubbing it with very ſoft Leather. When I made theſe, an Artiſt in *London* undertook to imitate it; but uſing another way of poliſhing them than I did, he fell much ſhort of what I had attained to, as I afterwards underſtood by diſcourſing the under-Workman he had imployed. The Poliſh I uſed was on this manner. I had two round Copper Plates each ſix Inches in Diameter, the one convex the other concave, ground very true to one another. On the convex I ground the Object-Metal or concave which was to be poliſh'd, till it had taken the Figure of the convex and was ready for a Poliſh. Then I pitched over the convex very thinly, by dropping melted pitch upon it and warming it to keep the pitch ſoft, whilſt I ground it with the concave Copper wetted to make it ſpread evenly all over the convex. Thus by working it well I made it as thin as a Groat, and after the convex was cold I ground it again to give it as true a Figure as I could. Then I took Putty which I had made very fine by waſhing it from all its groſſer Particles, and laying a little of this upon the pitch, I ground it upon the Pitch with the concave Copper till it had done making a noiſe; and then upon the Pitch I ground the Object-Metal with a brisk

Motion

Motion, for about two or three Minutes of time, leaning hard upon it. Then I put fresh Putty upon the Pitch and ground it again till it had done making a noise, and afterwards ground the Object Metal upon it as before. And this Work I repeated till the Metal was polished, grinding it the last time with all my strength for a good while together, and frequently breathing upon the Pitch to keep it moist without laying on any more fresh Putty. The Object-Metal was two Inches broad and about one third part of an Inch thick, to keep it from bending. I had two of these Metals, and when I had polished them both I tried which was best, and ground the other again to see if I could make it better than that which I kept. And thus by many Trials I learnt the way of polishing, till I made those two reflecting Pespectives I spake of above. For this Art of polishing will be better learnt by repeated Practice than by my description. Before I ground the Object Metal on the Pitch, I always ground the Putty on it with the concave Copper till it had done making a noise, because if the Particles of the Putty were not by this means made to stick fast in the Pitch, they would by rolling up and down grate and fret the Object Metal and fill it full of little holes.

But because Metal is more difficult to polish than Glass and is afterwards very apt to be spoiled by tarnishing, and reflects not so much Light as Glass quick-silvered over does: I would propound to use instead of the Metal, a Glass ground concave on the foreside, and as much convex on the backside, and quick-silvered over on the convex side. The Glass must be every where of the same thickness exactly. Otherwise it will make Objects look coloured and indistinct. By such a Glass I tried about five or six Years ago to make a reflecting Telescope of four Feet in length to magnify about 150 times, and I satisfied my self that there wants nothing

thing but a good Artist to bring the design to Perfection. For the Glass being wrought by one of our *London* Artists after such a manner as they grind Glasses for Telescopes, tho it seemed as well wrought as the Object Glasses use to be, yet when it was quick-silvered, the reflexion discovered innumerable Inequalities all over the Glass. And by reason of these Inequalities, Objects appeared indistinct in this Instrument. For the Errors of reflected Rays caused by any Inequality of the Glass, are about six times greater than the Errors of refracted Rays caused by the like Inequalities. Yet by this Experiment I satisfied my self that the reflexion on the concave side of the Glass, which I feared would disturb the vision, did no sensible prejudice to it, and by consequence that nothing is wanting to perfect these Telescopes, but good Workmen who can grind and polish Glasses truly spherical. An Object-Glass of a fourteen Foot Telescope, made by one of our *London* Artificers, I once mended considerably, by grinding it on Pitch with Putty, and leaning very easily on it in the grinding, lest the Putty should scratch it. Whether this way may not do well enough for polishing these reflecting Glasses, I have not yet tried. But he that shall try either this or any other way of polishing which he may think better, may do well to make his Glasses ready for polishing by grinding them without that violence, wherewith our *London* Workmen press their Glasses in grinding. For by such violent pressure, Glasses are apt to bend a little in the grinding, and such bending will certainly spoil their Figure. To recommend therefore the consideration of these reflecting Glasses, to such Artists as are curious in figuring Glasses, I shall describe this Optical Instrument in the following Proposition.

PROP.

PROP. VII. Prob. II.

To shorten Telescopes.

LET ABDC represent a Glass spherically concave on the foreside AB, and as much convex on the backside CD, so that it be every where of an equal thickness. Let it not be thicker on one side than on the other, lest it make Objects appear coloured and indistinct, and let it be very truly wrought and quick-silvered over on the backside ; and set in the Tube VXYZ which must be very black within. Let EFG represent a Prism of Glass or Crystal placed near the other end of the Tube, in the middle of it, by means of a handle of Brass or Iron FGK, to the end of which made flat it is cemented. Let this Prism be rectangular at E, and let the other two Angles at F and G be accurately equal to each other, and by consequence equal to half right ones, and let the plane sides FE and GE be square, and by consequence the third side FG a rectangular parallelogram, whose length is to its breath in a subduplicate proportion of two to one. Let it be so placed in the Tube, that the Axis of the Speculum may pass through the middle of the square side EF perpendicularly, and by consequence through the middle of the side F G at an Angle of 45 degrees, and let the side EF be turned towards the Speculum, and the distance of this Prism from the Speculum be such that the Rays of the light PQ, RS, &c. which are incident upon the Speculum in Lines Parallel to the Axis thereof, may enter the Prism at the side EF, and be reflected by the side F G, and thence go out of it through the side GE, to the point T which must be the common Focus of the Speculnm ABDC, and of a Plano-convex Eye-Glass H, through which those Rays must pass to the Eye. And let the Rays at their coming *Fig.* 29.

out

out of the Glaſs paſs through a ſmall round hole, or aperture made in a little Plate of Lead, Braſs, or Silver, wherewith the Glaſs is to be covered, which hole muſt be no bigger than is neceſſary for light enough to paſs through. For ſo it will render the Object diſtinct, the Plate in which 'tis made intercepting all the erroneous part of the Light which comes from the Verges of the Speculum AB. Such an Inſtrument well made if it be 6 Foot long, (reckoning the length from the Speculum to the Priſm, and thence to the Focus T) will bear an aperture of 6 Inches at the Speculum, and magnify between two and three hundred times. But the hole H here limits the aperture with more advantage, then if the aperture was placed at the Speculum. If the Inſtrument be made longer or ſhorter, the aperture muſt be in proportion as the Cube of the square ſquare Root of the length, and the magnifying as the aperture. But its convenient that the Speculum be an Inch or two broader than the aperture at the leaſt, and that the Glaſs of the Speculum be thick, that it bend not in the working. The Priſm EFG muſt be no bigger than is neceſſary, and its back ſide FG muſt not be quick-ſilvered over. For without quick-ſilver it will reflect all the Light incident on it from the Speculum.

In this Inſtrument the Object will be inverted, but may be erected by making the ſquare ſides EF and EG of the Priſm EFG not plane but ſpherically convex, that the Rays may croſs as well before they come at it as afterwards between it and the Eye-Glaſs. If it be deſired that the Inſtrument bear a larger aperture, that may be alſo done by compoſing the Speculum of two Glaſſes with Water between them.

THE

Fig. 1.

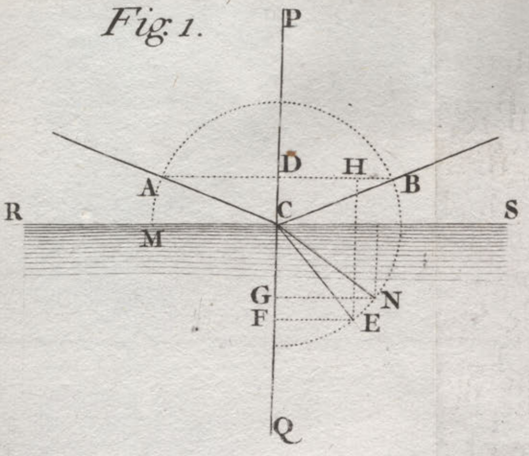
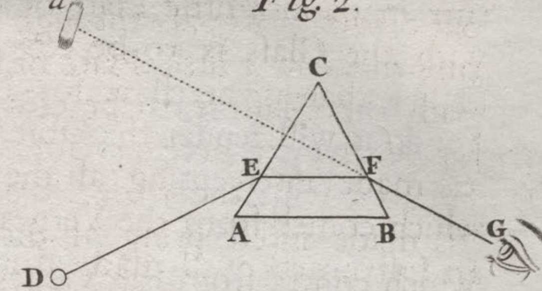


Fig. 2.



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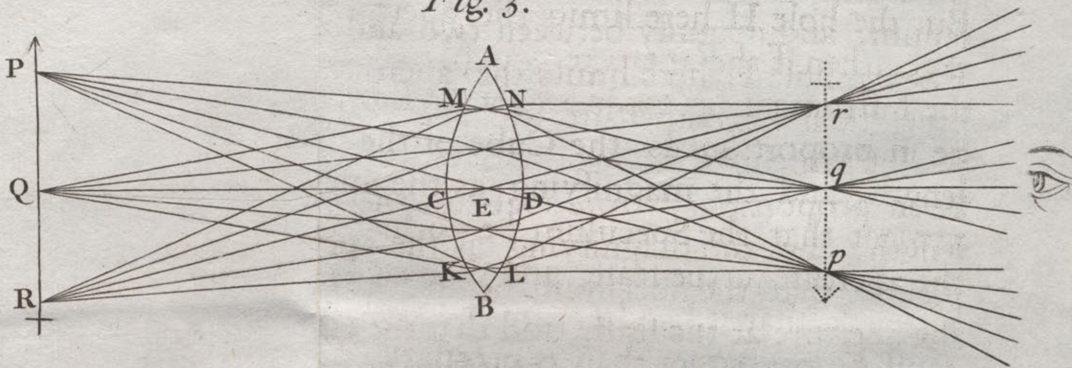

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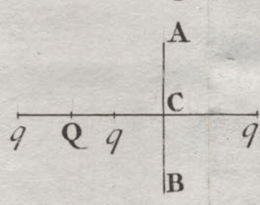

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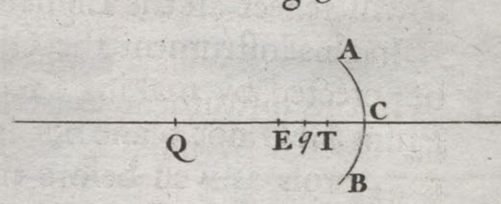

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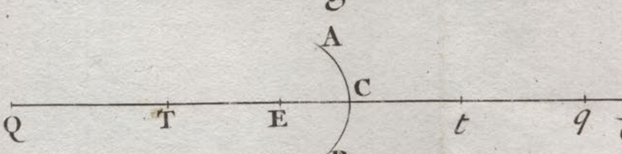

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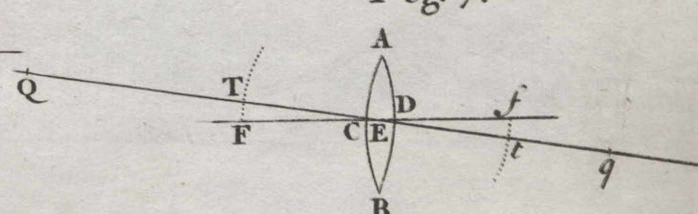

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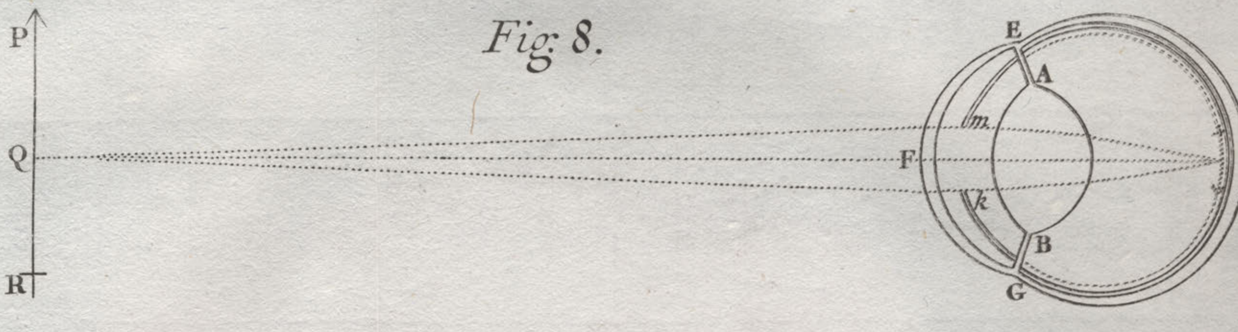

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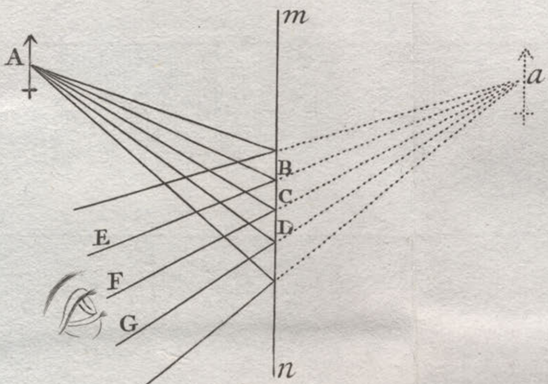

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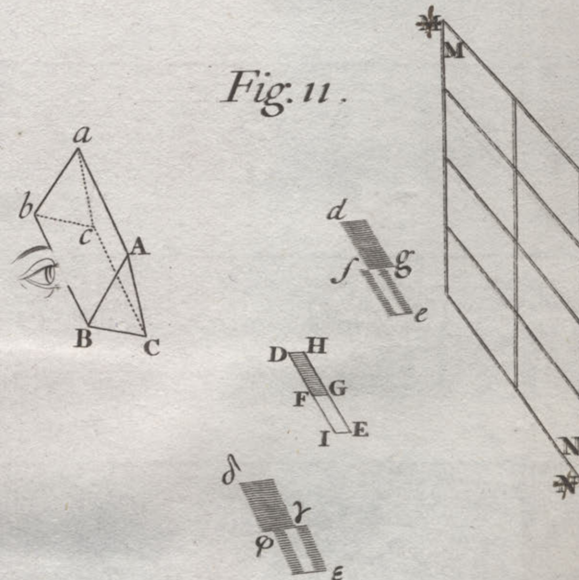
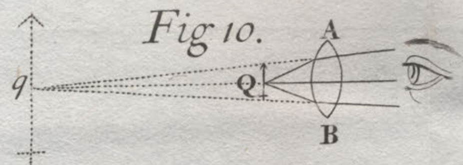


Fig 10.



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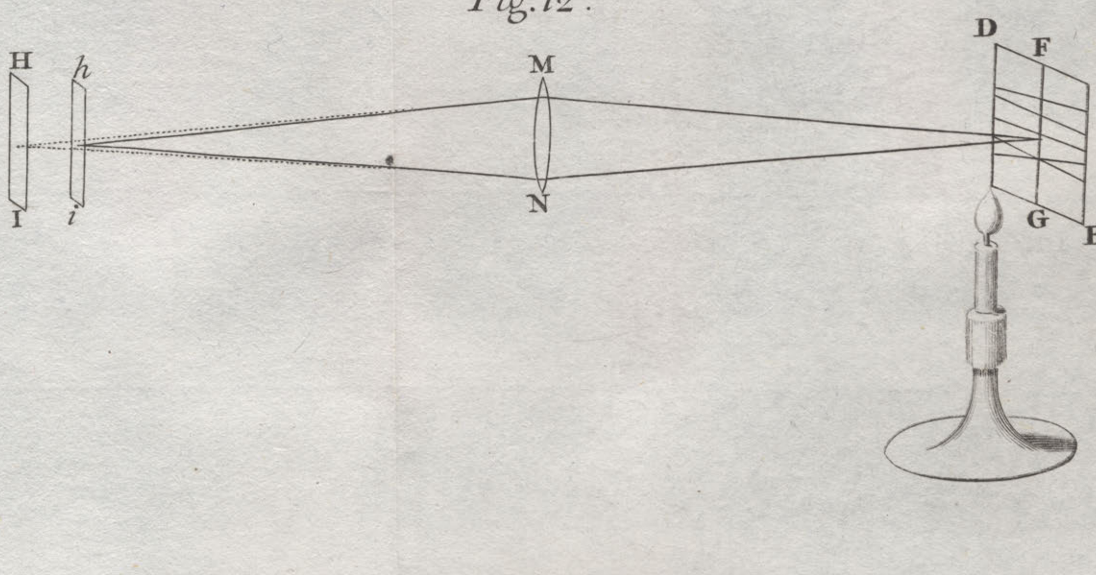

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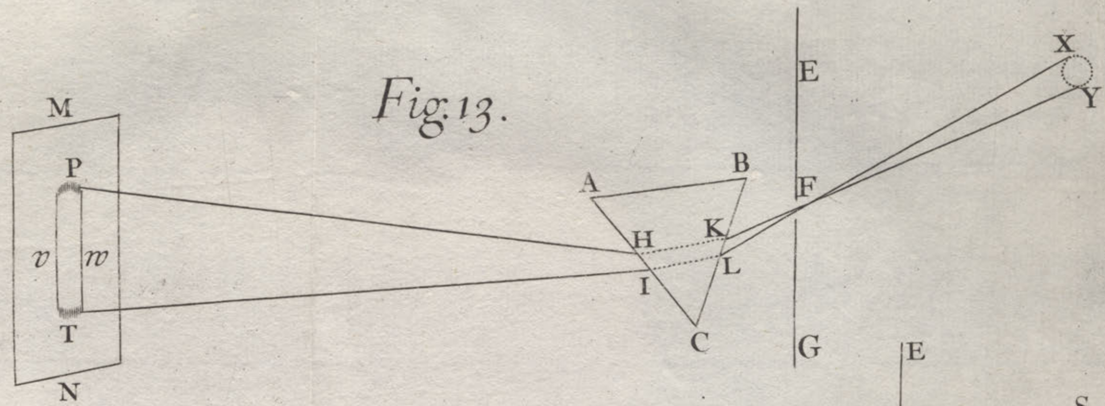

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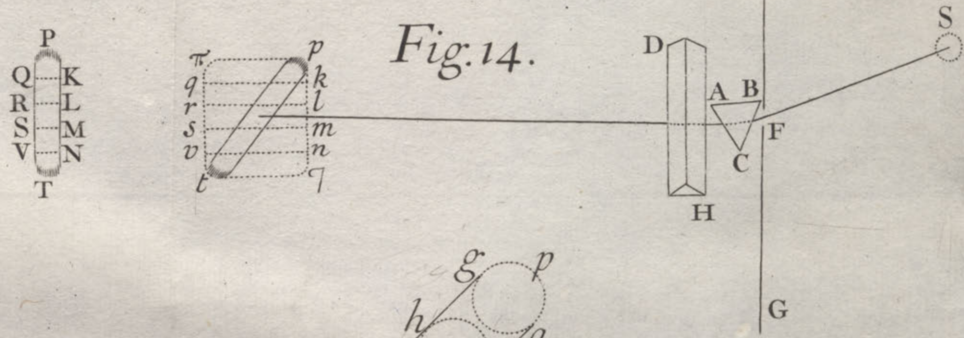


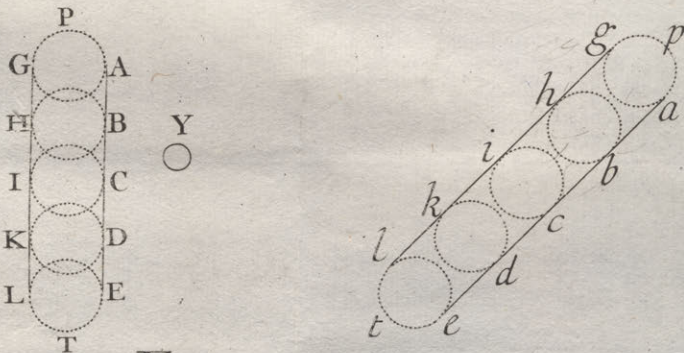


Fig. 15.

Fig. 16.

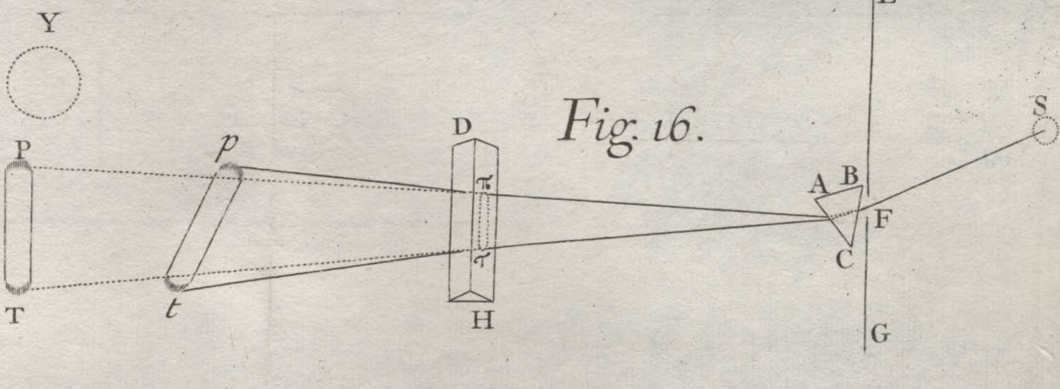


Fig. 17.

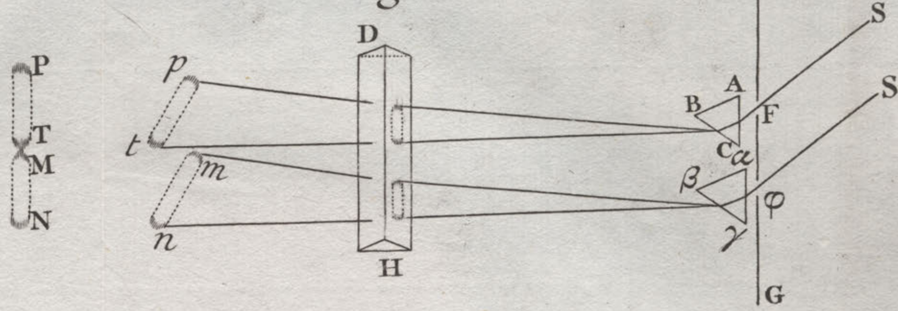
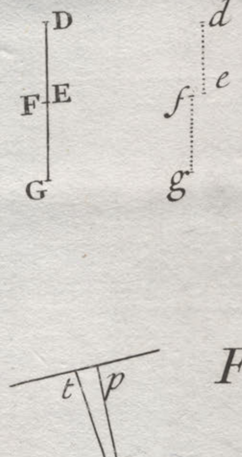
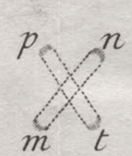


Fig. 18.

Fig. 19.





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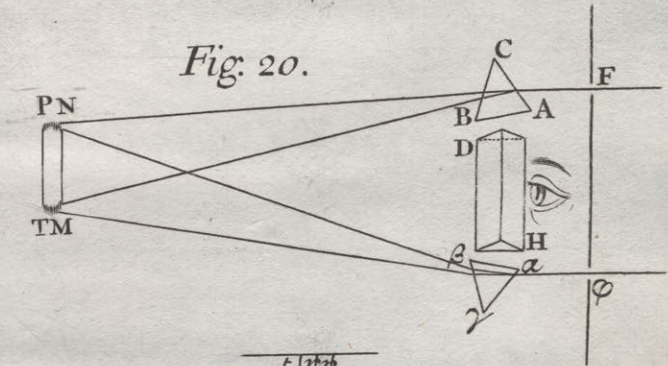

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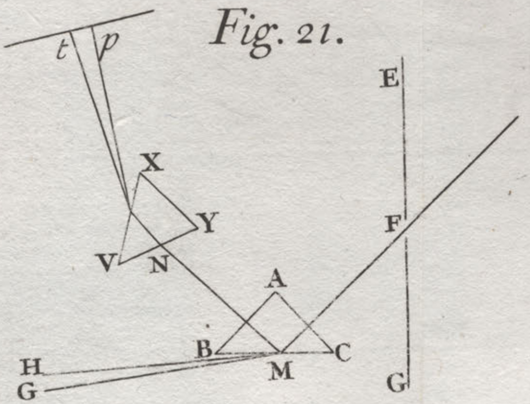

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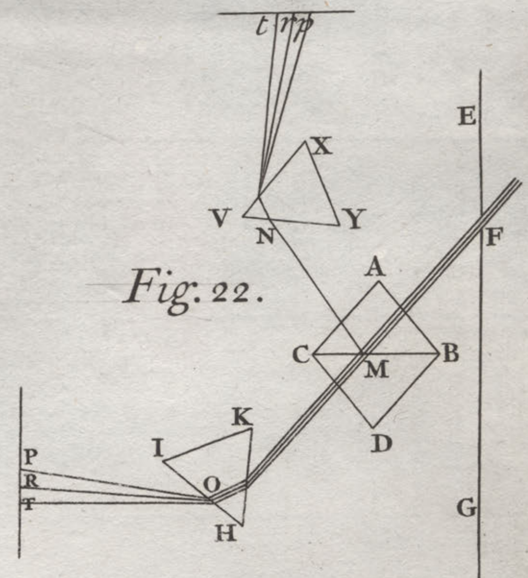

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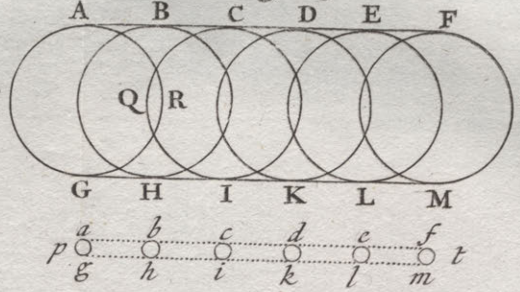

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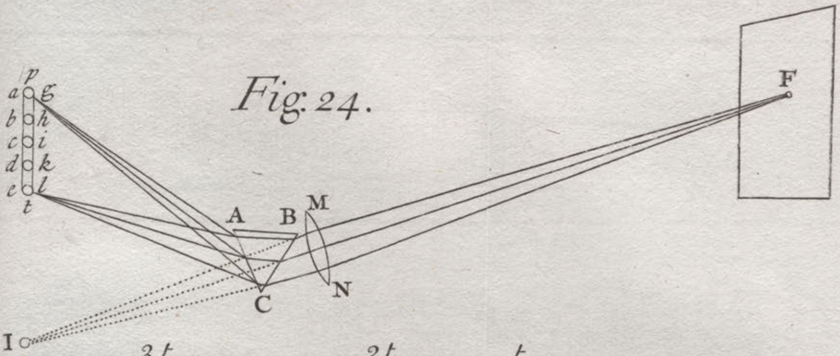

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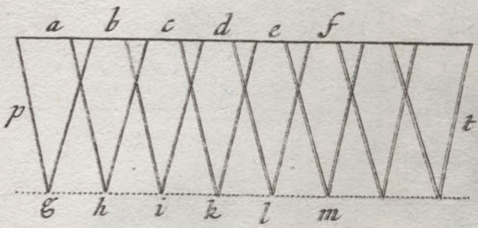


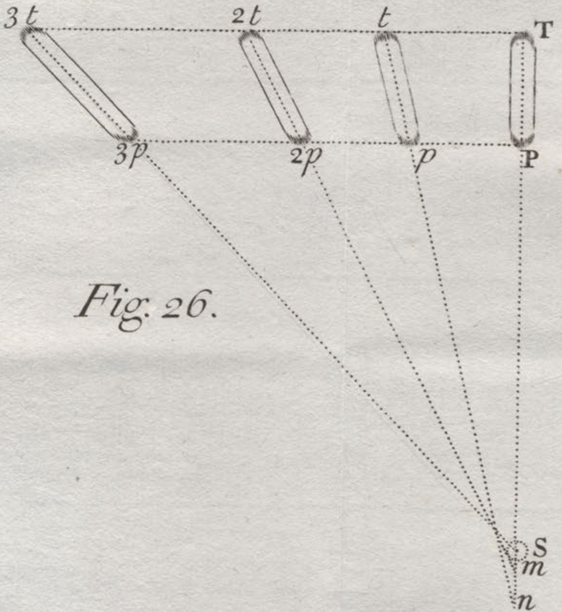
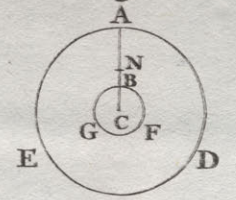


Fig: 26.

Fig: 27.



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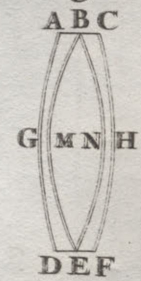

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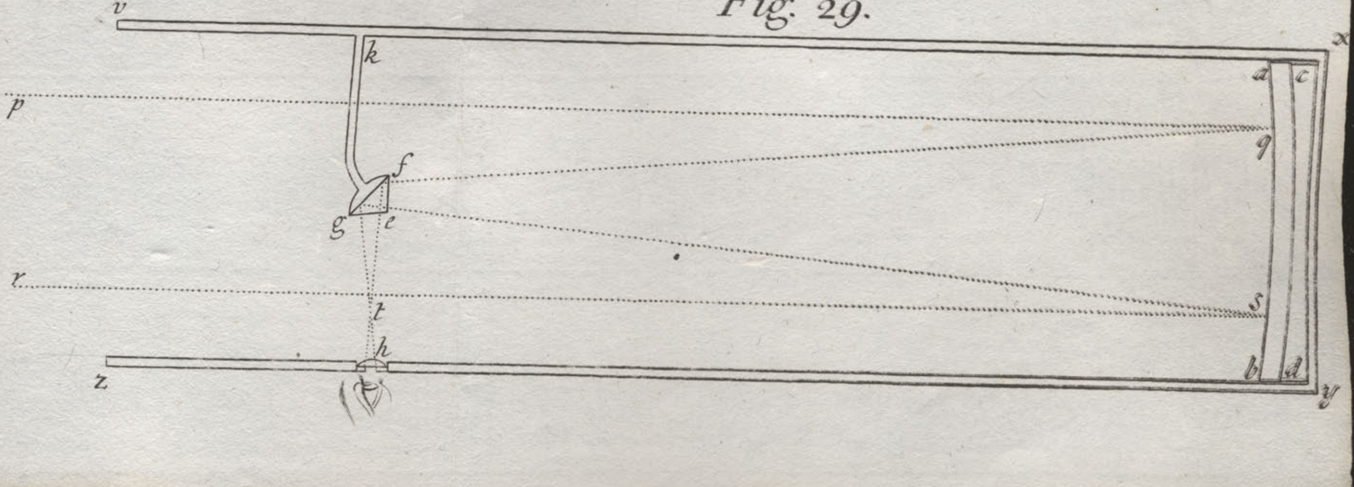

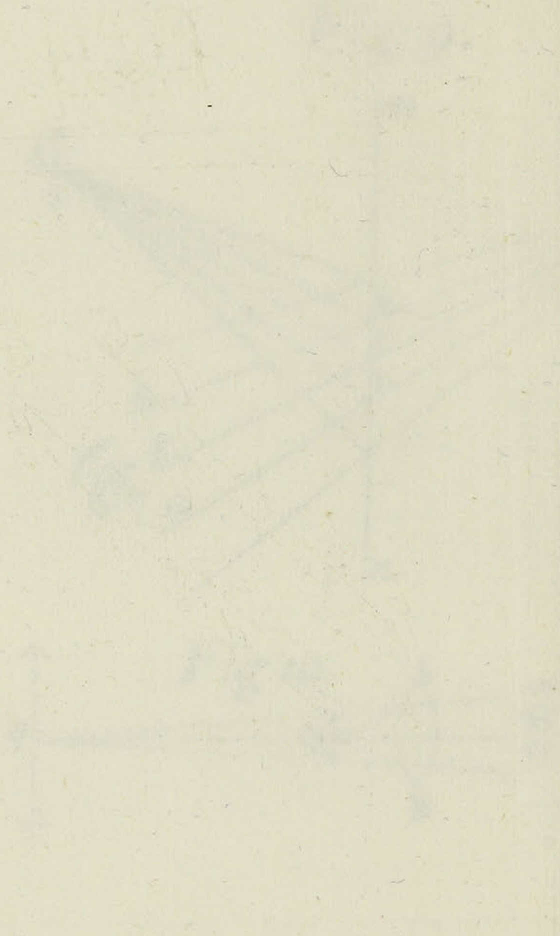

THE FIRST BOOK OF OPTICKS.

PART II.

PROP. I. THEOR. I.

The Phænomena of Colours in refracted or reflected Light are not caused by new modifications of the Light variously imprest, according to the various terminations of the Light and Shadow.

The Proof by Experiments.

EXPER. I.

FOR if the Sun shine into a very dark Chamber through an oblong Hole F, whose breadth is the sixth or eighth part of an Inch, or something less; and his Beam FH do afterwards pass first through a very large Prism ABC, distant about 20 Feet from the Hole, *Fig. 1.*

Hole, and parallel to it, and then (with its white part) through an oblong Hole H, whoſe breadth is about the fortieth or ſixtieth part of an Inch, and which is made in a black opake Body G I, and placed at the diſtance of two or three Feet from the Priſm, in a parallel ſituation both to the Priſm and to the former Hole, and if this white Light thus tranſmitted through the Hole H, fall afterwards upon a white Paper p t, placed after that Hole H, at the diſtance of three or four Feet from it, and there paint the uſual Colours of the Priſm, ſuppoſe red at t, yellow at s, green at r, blue at q, and violet at p; you may with an iron Wire, or any ſuch like ſlender opake Body, whoſe breadth is about the tenth part of an Inch, by intercepting the rays at k, l, m, n or o, take away any one of the Colours at t, s, r, q or p, whilſt the other Colours remain upon the Paper as before; or with an obſtacle ſomething bigger you may take away any two, or three, or four Colours together, the reſt remaining: So that any ~~one of~~ the Colours as well as violet may become outmoſt in the confine of the ſhadow towards p, and any one of them as well as red may become outmoſt in the confine of the ſhadow towards t, and any one of them may alſo border upon the ſhadow made within the Colours by the obſtacle R intercepting ſome intermediate part of the Light; and, laſtly, any one of them by being left alone may border upon the ſhadow on either hand. All the Colours have themſelves indifferently to any confines of ſhadow, and therefore the differences of theſe Colours from one another, do not ariſe from the different confines of ſhadow, whereby Light is variouſly modified as has hitherto been the Opinion of Philoſophers.

phers. In trying theſe things 'tis to be obſerved, that by how much the Holes F and H are narrower, and the intervals between them, and the Priſm greater, and the Chamber darker, by ſo much the better doth the Experiment ſucceed; provided the Light be not ſo far diminiſhed, but that the Colours at p t be ſufficiently viſible. To procure a Priſm of ſolid Glaſs large enough for this Experiment will be difficult, and therefore a priſmatick Veſſel muſt be made of poliſhed Glaſs-plates cemented together, and filled with Water.

EXPER. II.

The Sun's Light let into a dark Chamber through *Fig.* 2.
the round Hole F, half an Inch wide, paſſed firſt through the Priſm A B C placed at the Hole, and then through a Lens P T ſomething more than four Inches broad, and about eight Feet diſtant from the Priſm, and thence converged to O the Focus of the Lens diſtant from it about three Feet, and there fell upon a white Paper D E. If that Paper was perpendicular to that Light incident upon it, as 'tis repreſented in the poſture D E, all the Colours upon it at O appeared white. But if the Paper being turned about an Axis parallel to the Priſm, became very much inclined to the Light as 'tis repreſented in the poſitions *de* and *δε*; the ſame Light in the one caſe appeared yellow and red, in the other blue. Here one and the ſame part of the Light in one and the ſame place, according to the various inclinations of the Paper, appeared in one caſe white, in another yellow or red, in a third blue, whilſt the confine of Light and

Shadow, and the refractions of the Prism in all these cases remained the same.

EXPER. III.

Fig. 3. Such another Experiment may be more easily tried as follows. Let a broad beam of the Sun's Light coming into a dark Chamber through a Hole in the Window shut be refracted by a large Prism ABC, whose refracting Angle C is more than 60 degrees, and so soon as it comes out of the Prism let it fall upon the white Paper DE glewed upon a stiff plane, and this Light, when the Paper is perpendicular to it, as 'tis represented in DE, will appear perfectly white upon the Paper, but when the Paper is very much inclined to it in such a manner as to keep always parallel to the Axis of the Prism, the whiteness of the whole Light upon the Paper will according to the inclination of the Paper this way, or that way, change either into yellow and red, as in the posture *de*, or into blue and violet, as in the posture *δε*. And if the Light before it fall upon the Paper be twice refracted the same way by two parallel Prisms, these Colours will become the more conspicuous. Here all the middle parts of the broad beam of white Light which fell upon the Paper, did without any confine of shadow to modify it, become coloured all over with one uniform Colour, the Colour being always the same in the middle of the Paper as at the edges, and this Colour changed according the various obliquity of the reflecting Paper, without any change in the refractions or shadow, or in the Light which fell upon the Paper. And therefore these Colours are to to be derived from ſome other cauſe than the new modifications of Light by refractions and ſhadows.

If it be asked, What then is their cauſe? I anſwer, That the Paper in the poſture *de*, being more oblique to the more refrangible rays than to the leſs refrangible ones, is more ſtrongly illuminated by the latter than by the former, and therefore the leſs refrangible rays are predominant in the reflected Light. And wherever they are predominant in any Light they tinge it with red or yellow, as may in ſome meaſure appear by the firſt Propoſition of the firſt Book, and will more fully appear hereafter. And the contrary happens in the poſture of the Paper $\delta\varepsilon$, the more refrangible rays being then predominant which always tinge Light with blues and violets.

EXPER. IV.

The Colours of Bubbles with which Children play are various, and change their ſituation variouſly, without any reſpect to any confine of ſhadow. If ſuch a Bubble be covered with a concave Glaſs, to keep it from being agitated by any wind or motion of the Air, the Colours will ſlowly and regularly change their ſituation, even whilſt the Eye, and the Bubble, and all Bodies which emit any Light, or caſt any ſhadow, remain unmoved. And therefore their Colours ariſe from ſome regular cauſe which depends not on any confine of ſhadow. What this cauſe is will be ſhewed in the next Book.

To theſe Experiments may be added the tenth Experiment of the firſt Book, where the Sun's Light in a dark Room being trajected through the parallel ſuperficies of two Priſms tied together in the form of a Parallelopipede, became totally of one uniform yellow or red Colour, at its emerging out of the Priſms. Here, in the production of theſe Colours, the confine of ſhadow can have nothing to do. For the Light changes from white to yellow, orange and red ſucceſſively, without any alteration of the confine of ſhadow: And at both edges of the emerging Light where the contrary confines of ſhadow ought to produce different effects, the Colour is one and the ſame, whether it be white, yellow, orange or red; And in the middle of the emerging Light, where there is no confine of ſhadow at all, the Colour is the very ſame as at the edges, the whole Light at its very firſt emergence being of one uniform Colour, whether white, yellow, orange or red, and going on thence perpetually without any change of Colour, ſuch as the confine of ſhadow is vulgarly ſuppoſed to work in refracted Light after its emergence. Neither can theſe Colours ariſe from any new modifications of the Light by refractions, becauſe they change ſucceſſively from white to yellow, orange and red, while the refractions remain the ſame, and alſo becauſe the refractions are made contrary ways by parallel ſuperficies which deſtroy one anothers effects. They ariſe not therefore from any modifications of Light made by refractions and ſhadows, but have ſome other cauſe. What that cauſe is we ſhewed above in this tenth Experiment, and need not here repeat it.

There

There is yet another material circumſtance of this Experiment. For this emerging Light being by a third Priſm HIK refracted towards the Paper PT, and there painting the uſual Colours of the Priſm, red, yellow, green, blue, violet: If theſe Colours aroſe from the refractions of that Priſm modifying the Light, they would not be in the Light before its incidence on that Priſm. And yet in that Experiment we found that when by turning the two firſt Priſms about their common Axis all the Colours were made to vaniſh but the red; the Light which makes that red being left alone, appeared of the very ſame red Colour before its incidence on the third Priſm. And in general we find by other Experiments that when the rays which differ in refrangibility are ſeparated from one another, and any one ſort of them is conſidered apart, the Colour of the Light which they compoſe cannot be changed by any refraction or reflexion whatever, as it ought to be were Colours nothing elſe than modifications of Light cauſed by refractions, and reflexions, and ſhadows. This unchangeableneſs of Colour I am now to deſcribe in the following Propoſition. *Fig.* 22. *Part* 1.

PROP. II. THEOR. II.

All homogeneal Light has its proper Colour anſwering to its degree of refrangibility, and that Colour cannot be changed by reflexions and refractions.

In the Experiments of the 4th Propoſition of the firſt Book, when I had ſeparated the heterogeneous rays from one another, the Spectrum pt formed by the ſeparated

rated rays, did in the progreſs from its end p, on which the moſt refrangible rays fell, unto its other end t, on which the leaſt refrangible rays fell, appear tinged with this Series of Colours, violet, indico, blue, green, yellow, orange, red, together with all their intermediate degrees in a continual ſucceſſion perpetually varying: So that there appeared as many degrees of Colours, as there were ſorts of rays differing in refrangibility.

EXPER. V.

Now that theſe Colours could not be changed by refraction, I knew by refracting with a Priſm ſometimes one very little part of this Light, ſometimes another very little part, as is deſcribed in the 12th Experiment of the firſt Book. For by this refraction the Colour of the Light was never changed in the leaſt. If any part of the red Light was refracted, it remained totally of the ſame red Colour as before. No orange, no yellow, no green, or blue, no other new Colour was produced by that refraction. Neither did the Colour any ways change by repeated refractions, but continued always the ſame red entirely as at firſt. The like conſtancy and immutability I found alſo in the blue, green, and other Colours. So alſo if I looked through a Priſm upon any body illuminated with any part of this homogeneal Light, as in the 14th Experiment of the firſt Book is deſcribed; I could not perceive any new Colour generated this way. All Bodies illuminated with compound Light appear through Priſms confuſed (as was ſaid above) and tinged with various new Colours, but thoſe illuminated with homogeneal Light appeared through Prisms neither less distinct, nor otherwise coloured, than when viewed with the naked Eyes. Their Colours were not in the least changed by the refraction of the interposed Prism. I speak here of a sensible change of Colour: For the Light which I here call homogeneal, being not absolutely homogeneal, there ought to arise some little change of Colour from its heterogeneity. But if that heterogeneity was so little as it might be made, by the said Experiments of the fourth Proposition, that change was not sensible, and therefore, in Experiments where sense is judge, ought to be accounted none at all.

EXPER. VI.

And as these Colours were not changeable by refractions, so neither were they by reflexions. For all white, grey, red, yellow, green, blue, violet Bodies, as Paper, Ashes, red Lead, Orpiment, Indico, Bise, Gold, Silver, Copper, Grass, blue Flowers, Violets, Bubbles of Water tinged with various Colours, Peacock's Feathers, the tincture of *Lignum Nephriticum*, and such like, in red homogeneal Light appeared totally red, in blue Light totally blue, in green Light totally green, and so of other Colours. In the homogeneal Light of of any Colour they all appeared totally of that same Colour, with this only difference, that some of them reflected that Light more strongly, others more faintly. I never yet found any Body which by reflecting homogeneal Light could sensibly change its Colour.

From all which it is manifeſt, that if the Sun's Light conſiſted of but one ſort of rays, there would be but one Colour in the whole World, nor would it be poſſible to produce any new Colour by reflexions and refractions, and by conſequence that the variety of Colours depends upon the compoſition of Light.

DEFINITION.

The homogeneal light and rays which appear red, or rather make Objects appear ſo, I call rubrific or red-makng ; thoſe which make Objects appear yellow, green, blue and violet, I call yellow-making, green-making, blue-making, violet-making, and ſo of the reſt. And if at any time I ſpeak of light and rays as coloured or endued with Colours, I would be underſtood to ſpeak not philoſophically and properly, but groſly, and according to ſuch conceptions as vulgar People in ſeeing all theſe Experiments would be apt to frame. For the rays to ſpeak properly are not coloured. In them there is nothing elſe than a certain power and diſpoſition to ſtir up a ſenſation of this or that Colour. For as ſound in a Bell or musical String, or other ſounding Body, is nothing but a trembling Motion, and in the Air nothing but that Motion propagated from the Object, and in the Senſorium 'tis a ſenſe of that Motion under the form of ſound ; ſo Colours in the Object are nothing but a diſpoſition to reflect this or that ſort of rays more copiouſly than the reſt ; in the rays they are nothing but their diſpoſitions to propagate

gate this or that Motion into the Sensorium, and in the Sensorium they are sensations of those Motions under the forms of Colours.

PROP. III. PROB. I.

To define the refrangibility of the several sorts of homogeneal Light answering to the several Colours.

For determining this Problem I made the following Experiment.

EXPER. VII.

When I had caused the rectilinear line sides AF, GM, of the Spectrum of Colours made by the Prism to be distinctly defined, as in the fifth Experiment of the first ~~Book~~ Part is described, there were found in it all the homogeneal Colours in the same order and situation one among another as in the Spectrum of simple Light, described in the fourth ~~Experiment~~ Proposition of that ~~Book~~ Part: For the Circles of which the Spectrum of compound Light PT is composed, and which in the middle parts of the Spectrum interfere and are intermixt with one another, are not intermixt in their outmost parts where they touch those rectilinear sides AF and GM. And therefore in those rectilinear sides when distinctly defined, there is no new Colour generated by refraction. I observed also, that if any where between the two outmost Circles TMF and PGA a right line, as $\gamma\delta$, was cross to the Spectrum, so as at both ends to fall perpendicularly upon its rectilinear sides, there appeared *Fig.* 4.

one and the ſame Colour and degree of Colour from one end of this line to the other. I delineated therefore in a Paper the perimeter of the Spectrum FAPGMT, and in trying the third Experiment of the firſt Book, I held the Paper ſo that the Spectrum might fall upon this delineated Figure, and agree with it exactly, whilſt an Aſſiſtant whoſe Eyes for diſtinguiſhing Colours were more critical than mine, did by right lines $\alpha\beta$, $\gamma\delta$, $\epsilon\zeta$, &c. drawn croſs the Spectrum, note the confines of the Colours that is of the red $M\alpha\beta F$ of the orange $\alpha\gamma\delta\beta$, of the yellow $\gamma\epsilon\zeta\delta$, of the green $\epsilon\eta\theta\zeta$, of the blue $\eta\iota\kappa\theta$, of the indico $\iota\lambda\mu\kappa$, and of the violet $\lambda GA\mu$. And this operation being divers times repeated both in the ſame and in ſeveral Papers, I found that the Obſervations agreed well enough with one another, and that the rectilinear ſides MG and FA were by the ſaid croſs lines divided after the manner of a muſical Chord. Let GM be produced to X, that MX may be equal to GM, and conceive GX, λX, ιX, ηX, ϵX, γX, αX, MX, to be in proportion to one another, as the numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, and ſo to repreſent the Chords of the Key, and of a Tone, a third Minor, a fourth, a fifth, a ſixth Major, a ſeventh, and an eighth above that Key: And the intervals $M\alpha$, $\alpha\gamma$, $\gamma\epsilon$, $\epsilon\eta$, $\eta\iota$, $\iota\lambda$, and λG, will be the ſpaces which the ſeveral Colours (red, orange, yellow, green, blue, indico, violet) take up.

Now theſe intervals or ſpaces ſubtending the differences of the refractions of the rays going to the limits of thoſe Colours, that is, to the points M, α, γ, ϵ, η, ι, λ, G, may without any ſenſible Error be accounted proportional to the differences of the ſines of refraction of thoſe

rays

rays having one common sine of incidence, and therefore since the common sine of incidence of the most and least refrangible rays out of Glass into Air was, (by a method described above) found in proportion to their sines of refraction, as 50 to 77 and 78, divide the difference between the sines of refraction 77 and 78, as the line G M is divided by those intervals, you will have 77, $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, 78, the sines of refraction of those rays out of Glass into Air, their common sine of incidence being 50. So then the sines of the incidences of all the red-making rays out of Glass into Air, were to the sines of their refractions, not greater than 50 to 77, nor less than 50 to $77\frac{1}{8}$, but they varied from one another according to all intermediate Proportions. And the sines of the incidences of the green-making rays were to the sines of their refractions in all proportions from that of 50 to $77\frac{1}{3}$, unto that of 50 to $77\frac{1}{2}$. And by the like limits above-mentioned were the refractions of the rays belonging to the rest of the Colours defined, the sines of the red-making rays extending from 77 to $77\frac{1}{8}$, those of the orange-making from $77\frac{1}{8}$ to $77\frac{1}{5}$, those of the yellow-making from $77\frac{1}{5}$ to $77\frac{1}{3}$, those of the green-making from $77\frac{1}{3}$ to $77\frac{1}{2}$, those of the blue-making from $77\frac{1}{2}$ to $77\frac{2}{3}$, those of the indico-making from $77\frac{2}{3}$ to $77\frac{7}{9}$, and those of the violet from $77\frac{7}{9}$ to 78.

These are the Laws of the refractions made out of Glass into Air, and thence by the third Axiom of the first part of the Book the Laws of the refractions made out of Air into Glass are easily derived.

EXPER.

EXPER. VIII.

I found moreover that when Light goes out of Air through ſeveral contiguous refracting Mediums as through Water and Glaſs, and thence goes out again into Air, whether the refracting ſuperficies be parallel or inclined to one another, that Light as often as by contrary refractions 'tis ſo corrected, that it emergeth in lines parallel to thoſe in which it was incident, continues ever after to be white. But if the emergent rays be inclined to the incident, the whiteneſs of the emerging Light will by degrees in paſſing on from the place of emergence, become tinged in its edges with Colours. This I tryed by refracting Light with Priſms of Glaſs within a priſmatick Veſſel of Water. Now thoſe Colours argue a diverging and ſeparation of the heterogeneous rays from one another by means of their unequal refractions, as in what follows will more fully appear. And, on the contrary, the permanent whiteneſs argues, that in like incidences of the rays there is no ſuch ſeparation of the emerging rays, and by conſequence no inequality of their whole refractions. Whence I ſeem to gether the two following Theorems.

1. The Exceſſes of the ſines of refraction of ſeveral ſorts of rays above their common ſine of incidence when the refractions are made out of divers denſer mediums immediately into one and the ſame rarer medium, of Air are to one another in a given Proportion.

2. The

2. The Proportion of the ſine of incidence to the ſine of refraction of one and the ſame ſort of rays out of one medium into another, is compoſed of the Proportion of the ſine of incidence to the ſine of refraction out of the firſt medium into any third medium, and of the Proportion of the ſine of incidence to the ſine of refraction out of that third medium into the ſecond medium.

By the firſt Theorem the refractions of the rays of every ſort made out of any medium into Air are known by having the refraction of the rays of any one ſort. As for inſtance, if the refractions of the rays of every ſort out of Rain-water into Air be deſired, let the common ſine of incidence out of Glaſs into Air be ſubducted from the ſines of refraction, and the Exceſſes will be 27, $27\frac{1}{8}$, $27\frac{1}{5}$, $27\frac{1}{3}$, $27\frac{1}{2}$, $27\frac{2}{3}$, $27\frac{7}{9}$, 28. Suppoſe now that the ſine of incidence of the leaſt refrangible rays be to their ſine of refraction out of Rain-water into Air as three to four, and ſay as 1 the difference of thoſe ſines is to 3 the ſine of incidence, ſo is 27 the leaſt of the Exceſſes above-mentioned to a fourth number 81; and 81 will be the common ſign of incidence out of Rain-water into Air, to which ſine if you add all the above-mentioned Exceſſes you will have the deſired ſines of the refractions 108, $108\frac{1}{8}$, $108\frac{1}{5}$, $108\frac{1}{3}$, $108\frac{1}{2}$, $108\frac{2}{3}$, $108\frac{7}{9}$, 109.

By the latter Theorem the refraction out of one medium into another is gathered as often as you have the refractions out of them both into any third medium. As if the ſine of incidence of any ray out of Glaſs into Air be to its ſine of refraction as 20 to 31, and the ſine of incidence of the ſame ray out of Air into Water, be

to

to its ſine of refraction as four to three; the ſine of incidence of that ray out of Glaſs into Water will be to its ſine of refraction as 20 to 31 and 4 to 3 joyntly, that is, as the Factum of 20 and 4 to the Factum of 31 and 3, or as 80 to 93.

And theſe Theorems being admitted into Opticks, there would be ſcope enough of handling that Science voluminouſly after a new manner; not only by teaching thoſe things which tend to the perfection of viſion, but alſo by determining mathematically all kinds of Phænomena of Colours which could be produced by refractions. For to do this, there is nothing elſe requiſite than to find out the ſeparations of heterogeneous rays, and their various mixtures and proportions in every mixture. By this way of arguing I invented almoſt all the Phænomena deſcribed in theſe Books, beſide ſome others leſs neceſſary to the Argument; and by the ſucceſſes I met with in the tryals, I dare promiſe, that to him who ſhall argue truly, and then try all things with good Glaſſes and ſufficient circumſpection, the expected event will not be wanting. But he is firſt to know what Colours will ariſe from any others mixt in any aſſigned Proportion.

PROP. IV. THEOR. III.

Colours may be produced by compoſition which ſhall be like to the Colours of homogeneal Light as to the appearance of Colour, but not as to the immutability of Colour and conſtitution of Light. And thoſe Colours by how much they are more compounded by ſo much are they leſs full and intenſe, and by too much compoſition they may be

diluted

diluted aud weakened till they cease. There may be also Colours produced by composition, which are not fully like any of the Colours of homogeneal Light.

For a mixture of homogeneal red and yellow compounds an orange, like in appearance of Colour to that orange which in the series of unmixed prismatick Colours lies between them; but the Light of one orange is homogeneal as to refrangibility, that of the other is heterogeneal, and the Colour of the one, if viewed through a Prism, remains unchanged, that of the other is changed and resolved into its component Colours red and yellow. And after the same manner other neighbouring homogeneal Colours may compound new Colours, like the intermediate homogeneal ones, as yellow and green, the Colour between them both, and afterwards, if blue be added, there will be made a green the middle Colour of the three which enter the composition. For the yellow and blue on either hand, if they are equal in quantity they draw the intermediate green equally towards themselves in composition, and so keep it as it were in equilibrio, that it verge not more to the yellow on the one hand, than to the blue on the other, but by their mixt actions remain still a middle Colour. To this mixed green there may be further added some red and violet, and yet the green will not presently cease but only grow less full and vivid, and by increasing the red and violet it will grow more and more dilute, until by the prevalence of the added Colours it be overcome and turned into whiteness, or some other Colour. So if to the Colour of any homogeneal Light, the Sun's white Light composed of all sorts of rays be added, that Colour will not vaniſh or change its ſpecies but be diluted, and by adding more and more white it will be diluted more and more perpetually. Laſtly, if red and violet be mingled, there will be generated according to their various Proportions various Purples, ſuch as are not like in appearance to the Colour of any homogeneal Light, and of theſe Purples mixt with yellow and blue may be made other new Colours.

PROP. V. THEOR. IV.

Whiteneſs and all grey Colours between white and black, may be compounded of Colours, and the whiteneſs of the Sun's Light is compounded of all the primary Colours mixt in a due proportion.

The Proof by Experiments.

EXPER. IX.

Fig. 5. The Sun ſhining into a dark Chamber through a little round Hole in the Window ſhut, and his Light being there refracted by a Priſm to caſt his coloured Image PT upon the oppoſite Wall: I held a white Paper V to that Image in ſuch manner that it might be illuminated by the coloured Light reflected from thence, and yet not intercept any part of that Light in its paſſage from the Priſm to the Spectrum. And I found that when the Paper was held nearer to any Colour than to the reſt, it appeared of that Colour to which it approached neareſt; but when it was equally or almoſt

equally

equally diſtant from all the Colours, ſo that it might be equally illuminated by them all it appeared white. And in this laſt ſituation of the Paper, if ſome Colours were intercepted, the Paper loſt its white Colour, and appeared of the Colour of the reſt of the Light which was not intercepted. So then the Paper was illuminated with Lights of various Colours, namely, red, yellow, green, blue and violet, and every part of the Light retained its proper Colour, until it was incident on the Paper, and became reflected thence to the Eye; ſo that if it had been either alone (the reſt of the Light being intercepted) or if it had abounded moſt and been predominant in the Light reflected from the Paper, it would have tinged the Paper with its own Colour; and yet being mixed with the reſt of the Colours in a due proportion, it made the Paper look white, and therefore by a compoſition with the reſt produced that Colour. The ſeveral parts of the coloured Light reflected from the Spectrum, whilſt they are propagated from thence thro' the Air, do perpetually retain their proper Colours, becauſe wherever they fall upon the Eyes of any Spectator, they make the ſeveral parts of the Spectrum to appear under their proper Colours. They retain therefore their proper Colours when they fall upon the Paper V, and ſo by the confuſion and perfect mixture of thoſe Colours compound the whiteneſs of the Light reflected from thence.

EXPER. X.

Let that Spectrum or ſolar Image PT fall now upon the Lens MN above four Inches broad, and about ſix *Fig.* 6.

Feet distant from the Prism ABC, and so figured that it may cause the coloured Light which divergeth from the Prism to converge and meet again at its Focus G, about six or eight Feet distant from the Lens, and there to fall perpendicularly upon a white Paper DE. And if you move this Paper to and fro, you will perceive that near the Lens, as at *de*, the whole solar Image (suppose at pt) will appear upon it intensly coloured after the manner above-explained, and that by receding from the Lens those Colours will perpetually come towards one another, and by mixing more and more dilute one another continually, until at length the Paper come to the Focus G, where by a perfect mixture they will wholly vanish and be converted into whiteness, the whole Light appearing now upon the Paper like a little white Circle. And afterwards by receding further from the Lens, the rays which before converged will now cross one another in the Focus G, and diverge from thence, and thereby make the Colours to appear again, but yet in a contrary order; suppose at *δε*, where the red t is now above which before was below, and the violet p is below which before was above.

Let us now stop the Paper at the Focus G where the Light appears totally white and circular, and let us consider its whiteness. I say, that this is composed of the converging Colours. For if any of those Colours be intercepted at the Lens, the whiteness will cease and degenerate into that Colour which ariseth from the composition of the other Colours which are not intercepted. And then if the intercepted Colours be let pass and fall upon that compound Colour, they mix with it, and by their mixture restore the whiteness.

So

So if the violet, blue and green be intercepted, the remaining yellow, orange and red will compound upon the Paper an orange, and then if the intercepted Colours be let pass they will fall upon this compounded orange, and together with it decompound a white. So also if the red and violet be intercepted, the remaining yellow, green and blue, will compound a green upon the Paper, and then the red and violet being let pass will fall upon this green, and together with it decompound a white. And that in this composition of white the several rays do not suffer any change in their colorific qualities by acting upon one another, but are only mixed, and by a mixture of their Colours produce white, may further appear by these Arguments.

If the Paper be placed beyond the Focus G, suppose at $\delta\varepsilon$, and then the red Colour at the Lens be alternately intercepted, and let pass again, the violet Colour on the Paper will not suffer any change thereby, as it ought to do if the several sorts of rays acted upon one another in the Focus G, where they cross. Neither will the red upon the Paper be changed by any alternate stopping, and letting pass the violet which crosseth it.

And if the Paper be placed at the Focus G, and the white round Image at G be viewed through the Prism HIK, and by the refraction of that Prism be translated to the place rv, and there appear tinged with various Colours, namely, the violet at v and red at r, and others between, and then the red Colour at the Lens be often stopt and let pass by turns, the red at r will accordingly disappear and return as often, but the violet at v will not thereby suffer any change. And so by stopping and letting pass alternately the blue at the

Lens,

v Lens, the blue at ~~r~~ v will accordingly diſappear and return, without any change made in the red at r. The red therefore depends on one ſort of rays, and the blue on another ſort, which in the Focus G where they are commixt do not act on one another. And there is the ſame reaſon of the other Colours.

I conſidered further, that when the moſt refrangible rays P p, and the leaſt refrangible ones T t, are by converging inclined to one another, the Paper, if held very oblique to thoſe rays in the Focus G, might reflect one ſort of them more copiouſly than the other ſort, and by that means the reflected Light would be tinged in that Focus with the Colour of the predominant rays, provided thoſe rays ſeverally retained their Colours or colorific qualities in the compoſition of white made by them in that Focus. But if they did not retain them in that white, but became all of them ſeverally endued there with a diſpoſition to ſtrike the ſenſe with the perception of white, then they could never loſe their whiteneſs by ſuch reflexions. I inclined therefore the Paper to the rays very obliquely, as in the ſecond Experiment of this Book, that the moſt refrangible rays might be more copiouſly reflected than the reſt, and the whiteneſs at length changed ſucceſſively into blue, indico and violet. Then I inclined it the contrary way, that least the ~~mo~~ leaſt refrangible rays might be more copious in the reflected Light than the reſt, and the whiteneſs turned ſucceſſively to yellow, orange and red.

Laſtly, I made an Inſtrument X Y in faſhion of a Comb, whoſe Teeth being in number ſixteen were about an Inch and an half broad, and the intervals of the Teeth about two Inches wide. Then by interpoſing

ſuc-

successively the Teeth of this Instrument near the Lens, I intercepted part of the Colours by the interposed Tooth, whilst the rest of them went on through the interval of the Teeth to the Paper D E, and there painted a round solar Image. But the Paper I had first placed so, that the Image might appear white as often as the Comb was taken away; and then the Comb being as was said interposed, that whiteness by reason of the intercepted part of the Colours at the Lens did always change into the Colour compounded of those Colours which were not intercepted, and that Colour was by the motion of the Comb perpetually varied so, that in the passing of every Tooth over the Lens all these Colours red, yellow, green, blue and purple, did always succeed one another. I caused therefore all the Teeth to pass successively over the Lens, and when the motion was slow, there appeared a perpetual succession of the Colours upon the Paper: But if I so much accelerated the motion, that the Colours by reason of their quick succession could not be distinguished from one another, the appearance of the single Colours ceased. There was no red, no yellow, no green, no blue, nor purple to be seen any longer, but from a confusion of them all there arose one uniform white Colour. Of the Light which now by the mixture of all the Colours appeared white, there was no part really white. One part was red, another yellow, a third green, a fourth blue, a fifth purple, and every part retains its proper Colour till it strike the Sensorium. If the impressions follow one another slowly, so that they may be severally perceived, there is made a distinct sensation of all the Colours one after another in a continual succession.

But

But if the impreſſions follow one another ſo quickly that they cannot be ſeverally perceived, there ariſeth out of them all one common ſenſation, which is neither of this Colour alone nor of that alone, but hath it ſelf indifferently to 'em all, and this is a ſenſation of whiteneſs. By the quickneſs of the ſucceſſions the impreſſions of the ſeveral Colours are confounded in the Senſorium, and out of that confuſion ariſeth a mixt ſenſation. If a burning Coal be nimbly moved round in a Circle with Gyrations continually repeated, the whole Circle will appear like fire; the reaſon of which is, that the ſenſation of the Coal in the ſeveral places of that Circle remains impreſt on the Senſorium, until the Coal return again to the ſame place. And ſo in a quick conſecution of the Colours the impreſſion of every Colour remains in the Senſorium, until a revolution of all the Colours be compleated, and that firſt Colour return again. The impreſſions therefore of all the ſucceſſive Colours are at once in the Senſorium, and joyntly ſtir up a ſenſation of them all; and ſo it is manifeſt by this Experiment, that the commixt impreſſions of all the Colours do ſtir up and beget a ſenſation of white, that is, that whiteneſs is compounded of all the Colours.

And if the Comb be now taken away, that all the Colours may at once paſs from the Lens to the Paper, and be there intermixed, and together reflected thence to the Spectators Eyes; their impreſſions on the Senſorium being now more ſubtily and perfectly commixed there, ought much more to ſtir up a ſenſation of whiteneſs.

You

You may instead of the Lens use two Prisms HIK and LMN, which by refracting the coloured Light the contrary way to that of the first refraction, may make the diverging rays converge and meet again in G, as you see it represented in the seventh Figure. For *Fig.* 7. where they meet and mix they will compose a white Light as when a Lens is used.

EXPER. XI.

Let the Sun's coloured Image PT fall upon the Wall *Fig.* 8. of a dark Chamber, as in the third Experiment of the first Book, and let the same be viewed through a Prism abc, held parallel to the Prism ABC, by whose refraction that Image was made, and let it now appear lower than before, suppose in the place S over against the red colour T. And if you go near to the Image PT, the Spectrum S will appear oblong and coloured like the Image PT; but if you recede from it, the Colours of the Spectrum S will be contracted more and more, and at length vanish, that Spectrum S becoming perfectly round and white; and if you recede yet further, the Colours will emerge again, but in a contrary order. Now that Spectrum S appears white in that case when the rays of several sorts which converge from the several parts of the Image PT, to the Prism abc, are so refracted unequally by it, that in their passage from the Prism to the Eye they may diverge from one and the same point of the Spectrum S, and so fall afterwards upon one and the same point in the bottom of the Eye, and there be mingled.

And further, if the Comb be here made uſe of, by whoſe Teeth the Colours at the Image PT may be ſucceſſively intercepted; the Spectrum S when the Comb is moved ſlowly will be perpetually tinged with ſucceſſive Colours: But when by accelerating the motion of the Comb, the ſucceſſion of the Colours is ſo quick that they cannot be ſeverally ſeen, that Spectrum S, by a confuſed and mixt ſenſation of them all, will appear white.

EXPER. XII.

Fig. 9. The Sun ſhining through a large Priſm ABC upon a Comb XY, placed immediately behind the Priſm, his Light which paſſed through the interſtices of the Teeth fell upon a white Paper DE. The breadths of the Teeth were equal to their interſtices, and ſeven Teeth together with their interſtices took up an Inch in breadth. Now when the Paper was about two or three Inches diſtant from the Comb, the Light which paſſed through its ſeveral interſtices painted ſo many ranges of Colours kl, mn, op, qr, *&c.* which were parallel to one another and contiguous, and without any mixture of white. And theſe ranges of Colours, if the Comb was moved continually up and down with a reciprocal motion, aſcended and deſcended in the Paper, and when the motion of the Comb was ſo quick, that the Colours could not be diſtinguiſhed from one another, the whole Paper by their confuſion and mixture in the Senſorium appeared white.

Let

Let the Comb now reſt, and let the Paper be removed further from the Priſm, and the ſeveral ranges of Colours will be dilated and expanded into one another more and more, and by mixing their Colours will dilute one another, and at length, when the diſtance of the Paper from the Comb is about a Foot, or a little more (ſuppoſe in the place 2 D 2 E) they will ſo far dilute one another as to become white.

With any Obſtacle let all the Light be now ſtopt which paſſes through any one interval of the Teeth, ſo that the range of Colours which comes from thence may be taken away, and you will ſee the Light of the reſt of the ranges to be expanded into the place of the range taken away, and there to be coloured. Let the intercepted range paſs on as before, and its Colours falling upon the Colours of the other ranges, and mixing with them, will reſtore the whiteneſs.

Let the Paper 2 D 2 E be now very much inclined to the rays, ſo that the moſt refrangible rays may be more copiouſly reflected than the reſt, and the white Colour of the Paper through the exceſs of thoſe rays will be changed into blue and violet. Let the Paper be as much inclined the contrary way, that the leaſt refrangible rays may be now more copiouſly reflected than the reſt, and by their exceſs the whiteneſs will be changed into yellow and red. The ſeveral rays therefore in that white Light do retain their colorific qualities, by which thoſe of any ſort, when-ever they become more copious than the reſt, do by their exceſs and predominance cauſe their proper Colour to appear.

And by the ſame way of arguing, applied to the third Experiment of this Book, it may be concluded, that the white Colour of all refracted Light at its very firſt emergence, where it appears as white as before its incidence, is compounded of various Colours.

EXPER. XIII.

In the foregoing Experiment the ſeveral intervals of the Teeth of the Comb do the office of ſo many Priſms, every interval producing the Phænomenon of one Priſm. Whence inſtead of thoſe intervals uſing ſeveral Priſms, I try'd to compound whiteneſs by mixing their Colours, and did it by uſing only three Priſms, as alſo by uſing only

Fig. 10. two as follows. Let two Priſms A B C and a b c, whoſe refracting Angles B and b are equal, be ſo placed parallel to one another, that the refracting Angle B of the one may touch the Angle c at the baſe of the other, and their planes C B and c b, at which the rays emerge, may lye in directum. Then let the Light trajected through them fall upon the Paper M N, diſtant about 8 or 12 Inches from the Priſms. And the Colours generated by the interior limits B and c of the two Priſms, will be mingled at P T, and there compound white. For if either Priſm be taken away, the Colours made by the other will appear in that place P T, and when the Priſm is reſtored to its place again, ſo that its Colours may there fall upon the Colours of the other, the mixture of them both will reſtore the whiteneſs.

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This Experiment ſucceeds alſo, as I have tryed, when the Angle b of the lower Priſm, is a little greater than the Angle B of the upper, and between the interior Angles B and c, there intercedes ſome ſpace B c, as is repreſented in the Figure, and the refracting planes BC and b c, are neither in directum, nor parallel to one another. For there is nothing more requiſite to the ſucceſs of this Experiment, than that the rays of all ſorts may be uniformly mixed upon the Paper in the place P T. If the moſt refrangible rays coming from the ſuperior Priſm take up all the ſpace from M to P, the rays of the ſame ſort which come from the inferior Priſm ought to begin at P, and take up all the reſt of the ſpace from thence towards N. If the leaſt refrangible rays coming from the ſuperior Priſm take up the ſpace M T, the rays of the ſame kind which come from the other Priſm ought to begin at T, and take up the remaining ſpace T N. If one ſort of the rays which have intermediate degrees of refrangibility, and come from the ſuperior Priſm be extended through the ſpace M Q, and another ſort of thoſe rays through the ſpace M R, and a third ſort of them through the ſpace M S, the ſame ſorts of rays coming from the lower Priſm, ought to illuminate the remaining ſpaces Q N, R N, S N reſpectively. And the ſame is to be underſtood of all the other ſorts of rays. For thus the rays of every ſort will be ſcattered uniformly and evenly through the whole ſpace M N, and ſo being every where mixt in the ſame proportion, they muſt every where produce the ſame Colour. And therefore ſince by this mixture they produce white in the exterior ſpaces M P and T N, they muſt alſo produce white in the interior ſpace P T. This

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is the reaſon of the compoſition by which whiteneſs was produced in this Experiment, and by what other way ſoever I made the like compoſition the reſult was whiteneſs.

Laſtly, If with the Teeth of a Comb of a due ſize, the coloured Lights of the two Priſms which fall upon the ſpace PT be alternately intercepted, that ſpace PT, when the motion of the Comb is ſlow, will always appear coloured, but by accelerating the motion of the Comb ſo much, that the ſucceſſive Colours cannot be diſtinguiſhed from one another, it will appear white.

EXPER. XIV.

Hitherto I have produced whiteneſs by mixing the Colours of Priſms. If now the Colours of natural Bodies are to be mingled, let Water a little thickned with Soap be agitated to raiſe a froth, and after that froth has ſtood a little, there will appear to one that ſhall view it intently various Colours every where in the ſurfaces of the ſeveral Bubbles; but to one that ſhall go ſo far off that he cannot diſtinguiſh the Colours from one another, the whole froth will grow white with a perfect whiteneſs.

EXPER. XV.

Laſtly, in attempting to compound a white by mixing the coloured Powders which Painters uſe, I conſidered that all coloured Powders do ſuppreſs and ſtop in them a very conſiderable part of the Light by which

they

they are illuminated. For they become coloured by reflecting the Light of their own Colours more copiously, and that of all other Colours more sparingly, and yet they do not reflect the Light of their own Colours so copiously as white Bodies do. If red Lead, for instance, and a white Paper, be placed in the red Light of the coloured Spectrum made in a dark Chamber by the refraction of a Prism, as is described in the third Eperiment of the first Book; the Paper will appear more lucid than the red Lead, and therefore reflects the red-making rays more copiously than red Lead doth. And if they be held in the Light of any other Colour, the Light reflected by the Paper will exceed the Light reflected by the red Lead in a much greater proportion. And the like happens in Powders of other Colours. And therefore by mixing such Powders we are not to expect a strong and full white, such as is that of Paper, but some dusky obscure one, such as might arise from a mixture of light and darkness, or from white and black, that is, a grey, or dun, or russet brown, such as are the Colours of a Man's Nail, of a Mouse, of Ashes, of ordinary Stones, of Mortar, of Dust and Dirt in Highways, and the like. And such a dark white I have often produced by mixing coloured Powders. For thus one part of red Lead, and five parts of *Viride Æris*, composed a dun Colour like that of a Mouse. For these two Colours were severally so compounded of others, that in both together were a mixture of all Colours; and there was less red Lead used than *Viride Æris*, because of the fulness of its Colour. Again, one part of red Lead, and four parts of blue Bise, composed a dun Colour verging a little to purple, and by adding to this a

certain

certain mixture of Orpiment and *Viridi Æris* in a due proportion, the mixture loſt its purple tincture, and became perfectly dun. But the Experiment ſucceeded beſt without Minium thus. To Orpiment I added by little and little a certain full bright purple, which Painters uſe until the Orpiment ceaſed to be yellow, and became of a pale red. Then I diluted that red by adding a little *Viride Æris*, and a little more blue Biſe than *Viridi Æris*, until it became of ſuch a grey or pale white, as verged to no one of the Colours more than to another. For thus it became of a Colour equal in whiteneſs to that of Aſhes or of Wood newly cut, or of a Man's Skin. The Orpiment reflected more Light than did any other of the Powders, and therefore conduced more to the whiteneſs of the compounded Colour than they. To aſſign the proportions accurately may be difficult, by reaſon of the different goodneſs of Powders of the ſame kind. Accordingly as the Colour of any Powder is more or leſs full and luminous, it ought to be uſed in a leſs or greater proportion.

Now conſidering that theſe grey and dun Colours may be alſo produced by mixing whites and blacks, and by conſequence differ from perfect whites not in Species of Colours but only in degree of luminouſneſs, it is manifeſt that there is nothing more requiſite to make them perfectly white than to increaſe their Light ſufficiently; and, on the contrary, if by increaſing their Light they can be brought to perfect whiteneſs, it will thence alſo follow, that they are of the ſame Species of Colour with the beſt whites, and differ from them only in the quantity of Light. And this I tryed as follows. I took the third of the above-mentioned grey mixtures

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(that which was compounded of Orpiment, Purple, Biſe and *Viride Æris)* and rubbed it thickly upon the floor of my Chamber, where the Sun ſhone upon it through the opened Caſement; and by it, in the ſhadow, I laid a piece of white Paper of the ſame bigneſs. Then going from them to the diſtance of 12 or 18 Feet, ſo that I could not diſcern the unevenneſs of the ſurface of the Powder, nor the little ſhadows let fall from the gritty particles thereof; the Powder appeared intenſly white, ſo as to tranſcend even the Paper it ſelf in whiteneſs, eſpecially if the Paper were a little ſhaded from the Light of the Clouds, and then the Paper compared with the Powder appeared of ſuch a grey Colour as the Powder had done before. But by laying the Paper where the Sun ſhines through the Glaſs of the Window, or by ſhutting the Window that the Sun might ſhine through the Glaſs upon the Powder, and by ſuch other fit means of increaſing or decreaſing the Lights wherewith the Powder and Paper were illuminated, the Light wherewith the Powder is illuminated may be made ſtronger in ſuch a due proportion than the Light wherewith the Paper is illuminated, that they ſhall both appear exactly alike in whiteneſs. For when I was trying this, a Friend coming to viſit me, I ſtopt him at the door, and before I told him what the Colours were, or what I was doing; I askt him, Which of the two whites were the beſt, and wherein they differed? And after he had at that diſtance viewed them well, he anſwered, That they were both good whites, and that he could not ſay which was beſt, nor wherein their Colours differed. Now if you conſider, that this white of the Powder in the Sun-ſhine was compounded of the

Colours which the component Powders (Orpiment, Purple, Bise, and *Viride Æris)* have in the same Sunshine, you must acknowledge by this Experiment, as well as by the former, that perfect whiteness may be compounded of Colours.

From what has been said it is also evident, that the whiteness of the Sun's Light is compounded of all the Colours wherewith the several sorts of rays whereof that Light consists, when by their several refrangibilities they are separated from one another, do tinge Paper or any other white Body whereon they fall. For those Colours by Prop. 2. are unchangeable, and whenever all those rays with those their Colours are mixt again, they reproduce the same white Light as before.

PROP. VI. PROB. II.

In a mixture of primary Colours, the quantity and quality of each being given, to know the Colour of the compound.

Fig. 11. With the Center O and Radius O D describe a Circle A D F, and distinguish its circumference into seven parts D E, E F, F G, G A, A B, B C, C D, proportional to the seven musical Tones or Intervals of the eight Sounds, *Sol*, *la*, *fa*, *sol*, *la*, *mi*, *fa*, *sol*, contained in an Eight, that is, proportional to the numbers $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{16}$, $\frac{1}{9}$. Let the first part D E represent a red Colour, the second E F orange, the third F G yellow, the fourth G H green, the fifth A B blue, the sixth B C indico, and the seventh C D violet. And conceive that these are all the Colours of uncompounded Light gradually

passing

paſſing into one another, as they do when made by Priſms; the circumference DEFGABCD, repreſenting the whole ſeries of Colours from one end of the Sun's coloured Image to the other, ſo that from D to E be all degrees of red, at E the mean Colour between red and orange, from E to F all degrees of orange, at F the mean between orange and yellow, from F to G all degrees of yellow, and ſo on. Let p be the center of gravity of the Arch DE, and q, r, s, t, v, x, the centers of gravity of the Arches EF, FG, GA, AB, BC and CD reſpectively, and about thoſe centers of gravity let Circles proportional to the number of rays of each Colour in the given mixture be deſcribed; that is, the circle p proportional to the number of the red-making rays in the mixture, the Circle q proportional to the number of the orange-making rays in the mixture, and ſo of the reſt. Find the common center of gravity of all thoſe Circles p, q, r, s, t, v, x. Let that center be Z; and from the center of the Circle ADF, through Z to the circumference, drawing the right line OY, the place of the point Y in the circumference ſhall ſhew the Colour ariſing from the compoſition of all the Colours in the given mixture, and the line OZ ſhall be proportional to the fulneſs or intenſeneſs of the Colour, that is, to its diſtance from whiteneſs. As if Y fall in the middle between F and G, the compounded Colour ſhall be the beſt yellow; if Y verge from the middle towards F or G, the compounded Colour ſhall accordingly be a yellow, verging towards orange or green. If Z fall upon the circumference the Colour ſhall be intenſe and florid in the higheſt degree; if it fall in the mid way between the circumference and center, it ſhall be but half ſo intenſe, that is, it ſhall be ſuch a Colour as would be made by diluting the intenſeſt yellow with an equal quantity of whiteneſs; and if it fall upon the center O, the Colour ſhall have loſt all its intenſeneſs, and become a white. But it is to be noted, That if the point Z fall in or near the line O D, the main ingredients being the red and violet, the Colour compounded ſhall not be any of the priſmatic Colours, but a purple, inclining to red or violet, accordingly as the point Z lieth on the ſide of the line D O towards E or towards C, and in general the compounded violet is more bright and more fiery than the uncompounded. Alſo if only two of the primary Colours which in the Circle are oppoſite to one another be mixed in an equal proportion, the point Z ſhall fall upon the center O, and yet the Colour compounded of thoſe two ſhall not be perfectly white, but ſome faint anonymous Colour. For I could never yet by mixing only two primary Colours produce a perfect white. Whether it may be compounded of a mixture of three taken at equal diſtances in the circumference I do not know, but of four or five I do not much queſtion but it may. But theſe are curioſities of little or no moment to the underſtanding the Phænomena of nature. For in all whites produced by nature, there uſes to be a mixture of all ſorts of rays, and by conſequence a compoſition of all Colours.

To give an inſtance of this Rule; ſuppoſe a Colour is compounded of theſe homogeneal Colours, of violet 1 part, of indico 1 part, of blue 2 parts, of green 3 parts, of yellow 5 parts, of orange 6 parts, and of red 10 parts. Proportional to theſe parts I deſcribe the Circles x, v, t, s, r, q, p reſpectively, that is, ſo that if the Circle x be be 1, the Circle v may be 1, the Circle t 2, the Circle s 3, and the Circles r, q and p, 5, 6 and 10. Then I find Z the common center of gravity of theſe Circles, and through Z drawing the line O Y, the point Y falls upon the circumference between E and F, ſome thing nearer to E than to F, and thence I conclude, that the Colour compounded of theſe ingredients will be an orange, verging a little more to red than to yellow. Alſo I find that O Z is a little leſs than one half of O Y, and thence I conclude, that this orange hath a little leſs than half the fulneſs or intenſeneſs of an uncompounded orange; that is to ſay, that it is ſuch an orange as may be made by mixing an homogeneal orange with a good white in the proportion of the line O Z to the line Z Y, this proportion being not of the quantities of mixed orange and white powders, but of the quantities of the lights reflected from them.

This Rule I conceive accurate enough for practiſe, though not mathematically accurate; and the truth of it may be ſufficiently proved to ſenſe, by ſtopping any of the Colours at the Lens in the tenth Experiment of this Book. For the reſt of the Colours which are not ſtopped, but paſs on to the Focus of the Lens, will there compound either accurately or very nearly ſuch a Colour as by this Rule ought to reſult from their mixture.

PROP.

PROP. VII. THEOR. V.

All the Colours in the Univerſe which are made by Light, and depend not on the power of imagination, are either the Colours of homogeneal Lights, or compounded of theſe and that either accurately or very nearly, according to the Rule of the foregoing Problem.

For it has been proved (in Prop. 1. ~~Lib.~~ Part .2.) that the changes of Colours made by refractions do not ariſe from any new modifications of the rays impreſt by thoſe refractions, and by the various terminations of light and ſhadow, as has been the conſtant and general opinion of Philoſophers. It has alſo been proved that the ſeveral Colours of the homogeneal rays do conſtantly anſwer to their degrees of refrangibility, (Prop. 1. ~~Lib.~~ Part. 1. and Prop. 2. ~~Lib.~~ Part. 2.) and that their degrees of refrangibility cannot be changed by refractions and reflexions, (Prop. 2. ~~Lib.~~ Part. 1.) and by conſequence that thoſe their Colours are likewiſe immutable. It has alſo been proved directly by refracting and reflecting homogeneal Lights apart, that their Colours cannot be changed, (Prop. 2. ~~Lib.~~ Part. 2.) It has been proved alſo, that when the ſeveral ſorts of rays are mixed, and in croſſing paſs through the ſame ſpace, they do not act on one another ſo as to change each others colorifick qualities, (Exper. 10. ~~Lib.~~ Part. 2.) but by mixing their actions in the Senſorium beget a ſenſation differing from what either would do apart, that is a ſenſation of a mean Colour between their proper Colours ; and particularly when by the concourſe and mixtures of all ſorts of rays, a white Colour

Colour is produced, the white is a mixture of all the Colours which the rays would have apart, (Prop. 5. ~~Lib.~~ Part. 2.) The rays in that mixture do not loſe or alter their ſeveral colorifick qualities, but by all their various kinds of actions mixt in the Senſorium, beget a ſenſation of a middling Colour between all their Colours which is whiteneſs. For whiteneſs is a mean between all Colours, having it ſelf indifferently to them all, ſo as with equal facility to be tinged with any of them. A red Powder mixed with a little blue, or a blue with a little red, doth not preſently loſe its Colour, but a white Powder mixed with any Colour is preſently tinged with that Colour, and is equally capable of being tinged with any Colour what-ever. It has been ſhewed alſo, that as the Sun's Light is mixed of all ſorts of rays, ſo its whiteneſs is a mixture of the Colours of all ſorts of rays ; thoſe rays having from the beginning their ſeveral colorific qualities as well as their ſeveral refrangibilities, and retaining them perpetually unchang'd notwithſtanding any refractions or reflexions they may at any time ſuffer, and that when-ever any ſort of the Sun's rays is by any means (as by reflexion in Exper. 9 and 10. ~~Lib.~~ Part. 1. or by refraction as happens in all refractions) ſeparated from the reſt, they then manifeſt their proper Colours. Theſe things have been proved, and the ſum of all this amounts to the Propoſition here to be proved. For if the Sun's Light is mixed of ſeveral ſorts of rays, each of which have originally their ſeveral refrangibilities and colorifick qualities, and notwithſtanding their refractions and reflections, and their various ſeparations or mixtures, keep thoſe their original properties perpetually the ſame without alteration ;

tion ; then all the Colours in the World muſt be ſuch as conſtantly ought to ariſe from the original colorific qualities of the rays whereof the Lights conſiſt by which thoſe Colours are ſeen. And therefore if the reaſon of any Colour what-ever be required, we have nothing elſe to do then to conſider how the rays in the Sun's Light have by reflexions or refractions, or other cauſes been parted from one another,or mixed together ; or otherwiſe to find out what ſorts of rays are in the Light by which that Colour is made, and in what proportion ; and then by the laſt Problem to learn the Colour which ought to ariſe by mixing thoſe rays (or their Colours) in that proportion. I ſpeak here of Colours ſo far as they ariſe from Light. For they appear ſometimes by other cauſes, as when by the power of phantaſy we ſee Colours in a Dream, or a mad Man ſees things before him which are not there ; or when we ſee Fire by ſtriking the Eye, or ſee Colours like the Eye of a Peacock's Feather, by preſſing our Eyes in either corner whilſt we look the other way. Where theſe and ſuch like cauſes interpoſe not, the Colour always anſwers to the ſort or ſorts of the rays whereof the Light conſiſts, as I have conſtantly found in what-ever Phænomena of Colours I have hitherto been able to examin. I ſhall in the following Propoſitions give inſtances of this in the Phænomena of chiefeſt note.

PROP.

PROP. VIII. PROB. III.

By the discovered Properties of Light to explain the Colours made by Prisms.

Let ABC represent a Prism refracting the Light of the Sun, which comes into a dark Chamber through a Hole F φ almost as broad as the Prism, and let MN represent a white Paper on which the refracted Light is cast, and suppose the most refrangible or deepest violet making rays fall upon the space P π, the least refrangible or deepest red-making rays upon the space T τ, the middle sort between the Indico-making aud blue-making rays upon the space Q χ, the middle sort of the green-making rays upon the space R ρ, the middle sort between the yellow-making and orange-making rays upon the space S σ, and other intermediate sorts upon intermediate spaces. For so the spaces upon which the several sorts adequately fall will by reason of the different refrangibility of those sorts be one lower than another. Now if the Paper MN be so near the Prism that the spaces PT and πτ do not interfere with one another, the distance between them T π will be illuminated by all the sorts of rays in that proportion to one another which they have at their very first coming out of the Prism, and consequently be white. But the spaces PT and πτ on either hand, will not be illuminated by them all, and therefore will appear coloured. And particularly at P, where the outmost violet-making rays fall alone, the Colour must be the deepest violet. At Q where the violet-making and indico-making rays are mixed, it Fig. 12.

muſt be a violet inclining much to indico. At R where the violet-making, indico-making, blue-making, and one half of the green-making rays are mixed, their Colours muſt (by the conſtruction of the ſecond Problem) compound a middle Colour between indico and blue. At S where all the rays are mixed except the red-making and orange-making, their Colours ought by the ſame Rule to compound a faint blue, verging more to green than indico. And in the progreſs from S to T, this blue will grow more and more faint and dilute, till at T, where all the Colours begin to be mixed, it end in whiteneſs.

So again, on the other ſide of the white at τ, where the leaſt refrangible or utmoſt red-making rays are alone the Colour muſt be the deepeſt red. At σ the mixture of red and orange will compound a red inclining to orange. At ρ the mixture of red, orange, yellow, and one half of the green muſt compound a middle Colour between orange and yellow. At χ the mixture of all Colours but violet and indico will compound a faint yellow, verging more to green than to orange. And this yellow will grow more faint and dilute continually in its progreſs from χ to π, where by a mixture of all ſorts of rays it will become white.

Theſe Colours ought to appear were the Sun's Light perfectly white: But becauſe it inclines to yellow, the exceſs of the yellow-making rays whereby 'tis tinged with that Colour, being mixed with the faint blue between S and T, will draw it to a faint green. And ſo the Colours in order from P to τ ought to be violet, indico, blue, very faint green, white, faint yellow, orange, red. Thus it is by the computation: And they that pleaſe to

view

view the Colours made by a Prisin will find it so in Nature.

These are the Colours on both sides the white when the Paper is held between the Prism, and the point X where the Colours meet, and the interjacent white vanishes. For if the Paper be held still farther off from the Prism, the most refrangible and least refrangible rays will be wanting in the middle of the Light, and the rest of the rays which are found there, will by mixture produce a fuller green than before. Also the yellow and blue will now become less compounded, and by consequence more intense than before. And this also agrees with experience.

And if one look through a Prism upon a white Object encompassed with blackness or darkness, the reason of the Colours arising on the edges is much the same, as will appear to one that shall a little consider it. If a black Object be encompassed with a white one, the Colours which appear through the Prism are to be derived from the Light of the white one, spreading into the Regions of the black, and therefore they appear in a contrary order to that, in which they appear when a white Object is surrounded with black. And the same is to be understood when an Object is viewed, whose parts are some of them less luminous than others. For in the Borders of the more and less luminous parts, Colours ought always by the same Principles to arise from the excess of the Light of the more luminous, and to be of the same kind as if the darker parts were black, but yet to be more faint and dilute.

What is ſaid of Colours made by Priſms may be eaſily applied to Colours made by the Glaſſes of Teleſcopes, or Microſcopes, or by the humours of the Eye. For if the Object-glaſs of a Teleſcope be thicker on one ſide than on the other, or if one half of the Glaſs, or one half of the Pupil of the Eye be covered with any opake ſubſtance: the Object-glaſs, or that part of it or of the Eye which is not covered, may be conſidered as a Wedge with crooked ſides, and every Wedge of Glaſs, or other pellucid ſubſtance, has the effect of a Priſm in refracting the Light which paſſes through it.

How the Colours in the 9th and 10th Experiments of the firſt Part ariſe from the different reflexibility of Light, is evident by what was there ſaid. But it is obſervable in the 9th Experiment, that whilſt the Sun's direct Light is yellow, the exceſs of the blue-making rays in the reflected Beam of Light MN, ſuffices only to bring that yellow to a pale white inclining to blue, and not to tinge it with a manifeſtly blue Colour. To obtain therefore a better blue, I uſed inſtead of the yellow Light of the Sun the white Light of the Clouds, by varying a little the Experiment as follows.

EXPER. XVI.

Fig. 13. Let HFG repreſent a Priſm in the open Air, and S the Eye of the Spectator, viewing the Clouds by their Light coming into the Priſm at the plane ſide FIGK, and reflected in it by its baſe HEIG, and thence going out through its plain ſide HEFK to the Eye. And when the Priſm and Eye are conveniently placed, ſo that the Angles of incidence and reflexion at the baſe

may

may be about 40 degrees, the Spectator will ſee a Bow MN of a blue Colour, running from one end of the baſe to the other, with the concave ſide towards him, and the part of the baſe IMNG beyond this Bow will be brighter than the other part EMNH on the other ſide of it. This blue Colour MN being made by nothing elſe than by reflexion of a ſpecular ſuperficies, ſeems ſo odd a Phænomenon, and ſo unaccountable for by the vulgar Hypotheſis of Philoſophers, that I could not but think it deſerved to be taken notice of. Now for underſtanding the reaſon of it, ſuppoſe the plane ABC to cut the plane ſides and baſe of the Priſm perpendicularly. From the Eye to the line BC, wherein that plane cuts the baſe, draw the lines Sp and St, in the Angles Spc 50 degr. $\frac{1}{9}$, and Stc 49 degr. $\frac{1}{28}$, and the point *p* will be the limit beyond which none of the moſt refrangible rays can paſs through the baſe of the Priſm, and be refracted, whoſe incidence is ſuch that they may be reflected to the Eye; and the point t will be the like limit for the leaſt refrangible rays, that is, beyond which none of them can paſs through the baſe, whoſe incidence is ſuch that by reflexion they may come to the Eye. And the point r taken in the middle way between p and t, will be the like limit for the meanly refrangible rays. And therefore all the least refrangible rays which fall upon the baſe beyond t, that is, between t and B, and can come from thence to the Eye will be reflected thither: But on this ſide t, that is, between t and c, many of theſe rays will be tranſmitted through the baſe. And all the moſt refrangible rays which fall upon the baſe beyond p, that is, between p and B, and can by reflexion come from thence to the Eye, will be reflected

thither,

thither, but every where between p and c, many of these rays will get through the base and be refracted; and the same is to be understood of the meanly refrangible rays on either side of the point r. Whence it follows, that the base of the Prism must every where between t and B, by a total reflexion of all sorts of rays to the Eye, look white and bright. And every where between p and C, by reason of the transmission of many rays of every sort, look more pale, obscure and dark. But at r, and in other places between p and t, where all the more refrangible rays are reflected to the Eye, and many of the less refrangible are transmitted, the excess of the most refrangible in the reflected Light will tinge that Light with their Colour, which is violet and blue. And this happens by taking the line C p r t B any where between the ends of the Prism H G and E I.

PROP. IX. PROB. IV.

By the discovered Properties of Light to explain the Colours of the Rain-bow.

This Bow never appears but where it Rains in the Sun-shine, and may be made artificially by spouting up Water which may break aloft, and scatter into Drops, and fall down like Rain. For the Sun shining upon these Drops certainly causes the Bow to appear to a Spectator standing in a due position to the Rain and Sun. And hence it is now agreed upon, that this Bow is made by refraction of the Sun's Light in Drops of falling Rain. This was understood by some of the Ancients, and of late more fully discovered and explained by the Famous

Antonius

Antonius de Dominis Archbishop of *Spalato*, in his Book *De Radiis Visus & Lucis*, published by his Friend *Bartolus* at *Venice*, in the Year 1611, and written above twenty Years before. For he teaches there how the interior Bow is made in round Drops of Rain by two refractions of the Sun's Light, and one reflexion between them, and the exterior by two refractions and two sorts of reflexions between them in each Drop of Water, and proves his Explications by Experiments made with a Phial full of Water, and with Globes of Glass filled with Water, and placed in the Sun to make the Colours of the two Bows appear in them. The same Explication *Des-Cartes* hath pursued in his Meteors, and mended that of the exterior Bow. But whilst they understood not the true origin of Colours, it's necessary to pursue it here a little further. For understanding therefore how the Bow is made, let a Drop of Rain or any other spherical transparent Body be represented by the Sphere BNFG, described with the Center C, and Semi-diameter CN. And let AN be one of the Sun's rays incident upon it at N, and thence refracted to F, where let it either go out of the Sphere by refraction towards V, or be reflected to G; and at G let it either go out by refraction to R, or be reflected to H; and at H let it go out by refraction towards S, cutting the incident ray in Y; produce AN and RG, till they meet in X, and upon AX and NF let fall the perpendiculars CD and CE, and produce CD till it fall upon the circumference at L. Parallel to the incident ray AN draw the Diameter BQ, and let the sine of incidence out of Air into Water be to the sine of refraction as I to R. Now if you suppose the point of incidence N to *Fig.* 14.

move

move from the point B, continually till it come to L, the Arch QF will first increase and then decrease, and so will the Angle AXR which the rays AN and GR contain; and the Arch QF and Angle AXR will be biggest when ND is to CN as $\sqrt{11-RR}$ to $\sqrt{3RR}$, in which case NE will be to ND as 2R to 1. Also the Angle AYS which the rays AN and HS contain will first decrease, and then increase and grow least when ND is to CN as $\sqrt{11-RR}$ to $\sqrt{8RR}$, in which case NE will be to ND as 3R to 1. And so the Angle which the next emergent ray (that is, the emergent ray after three reflexions) contains with the incident ray AN will come to its limit when ND is to CN as $\sqrt{11-RR}$ to $\sqrt{15RR}$, in which case NE will be to ND as 4R to 1, and the Angle which the ray next after that emergent, that is, the ray emergent after four reflexions, contains with the incident will come to its limit, when ND is to CN as $\sqrt{11-RR}$ to $\sqrt{24RR}$, in which case NE will be to ND as 5R to 1; and so on infinitely, the numbers 3, 8, 15, 24, &c. being gathered by continual addition of the terms of the arithmetical progression 3, 5, 7, 9, &c. The truth of all this Mathematicians will easily examine.

Now it is to be observed, that as when the Sun comes to his Tropicks, days increase and decrease but a very little for a great while together; so when by increasing the distance CD, these Angles come to their limits, they vary their quantity but very little for some time together, and therefore a far greater number of the rays which fall upon all the points N in the Quadrant BL, shall emerge in the limits of these Angles, then in any other inclinations. And further it is

to

to be obſerved, that the rays which differ in refrangibility will have different limits of their Angles of emergence, and by conſequence according to their different degrees of refrangibility emerge moſt copiouſly in different Angles, and being ſeparated from one another appear each in their proper Colours. And what thoſe Angles are may be eaſily gathered from the foregoing Theorem by computation.

For in the leaſt refrangible rays the ſines I and R (as was found above) are 108 and 81, and thence by computation the greateſt Angle AXR will be found 42 degrees and 2 minutes, and the leaſt Angle AYS, 50 degr. and 57 minutes. And in the moſt refrangible rays the ſines I and R are 109 and 81, and thence by computation the greateſt Angle AXR will be found 40 degrees and 17 minutes, and the leaſt Angle AYS 54 degrees and 7 minutes.

Suppoſe now that O is the Spectator's Eye, and OP a line *Fig.* 15. drawn parallel to the Sun's rays, and let POE, POF, POG, POH, be Angles of 40 degr. 17 min. 42 degr. 2 min. 50 degr. 57 min. and 54 degr. 7 min. reſpectively, and theſe Angles turned about their common ſide OP, ſhall with their other ſides OE, OF; OG, OH dedeſcribe the verges of two Rain-bows AFBE and CHDG. For if E, F, G, H, be Drops placed any where in the conical ſuperficies deſcribed by OE, OF, OG, OH, and be illuminated by the Sun's rays SE, SF, SG, SH; the Angle SEO being equal to the Angle POE or 40 degr. 17 min. ſhall be the greateſt Angle in which the moſt refrangible rays can after one reflexion be refracted to the Eye, and therefore all the Drops in the line OE ſhall ſend the moſt refrangible rays moſt copiouſly to the Eye, and thereby ſtrike the ſenſes with the deepeſt violet Colour in that region. And in like manner the Angle SFO being equal to the Angle POF, or 42 deg. 2 min. ſhall be the greateſt in which the leaſt refrangible rays after one reflexion can emerge out of the Drops, and therefore thoſe rays ſhall come moſt copiouſly to the Eye from the Drops in the line OF, and ſtrike the ſenſes with the deepeſt red Colour in that region. And by the ſame argument, the rays which have intermediate degrees of refrangibility ſhall come moſt copiouſly from Drops between E and F, and ſtrike the ſenſes with the intermediate Colours in the order which their degrees of refrangibility require, that is, in the progreſs from E to F, or from the inſide of the Bow to the outſide in this order, violet, indico, blue, green, yellow, orange, red. But the violet, by the mixture of the white Light of the Clouds, will appear faint and incline to purple.

Again, the Angle SGO being equal to the Angle POG, or 50 gr. 51 min. ſhall be the leaſt Angle in which the leaſt refrangible rays can after two reflexions emerge out of the Drops, and therefore the leaſt refrangible rays ſhall come moſt copiouſly to the Eye from the Drops in the line OG, and ſtrike the ſenſe with the deepeſt red in that region. And the Angle SHO being equal to the Angle POH or 54 gr. 7 min. ſhall be the leaſt Angle in which the moſt refrangible rays after two reflections can emerge out of the Drops, and therefore thoſe rays ſhall come moſt copiouſly to the Eye from the Drops in the line OH, and ſtrike the ſenſes with the deepeſt violet in that region. And by the ſame argument, the Drops in the regions between G and H ſhall ſtrike the ſenſe with the

the intermediate Colours in the order which their degrees of refrangibility require, that is, in the progreſs from G to H, or from the inſide of the Bow to the outſide in this order, red, orange, yellow, green, blue, indico, violet. And ſince theſe four lines O E, O F, O G. O H, may be ſituated any where in the above-mentioned conical ſuperficies, what is ſaid of the Drops and Colours in theſe lines is to be underſtood of the Drops and Colours every where in thoſe ſuperficies.

Thus ſhall there be made two Bows of Colours, an interior and ſtronger, by one reflexion in the Drops, and an exterior and fainter by two; for the Light becomes fainter by every reflexion. And their Colours ſhall ly in a contrary order to one another, the red of both Bows bordering upon the ſpace G F which is between the Bows. The breadth of the interior Bow E O F meaſured croſs the Colours ſhall be 1 degr. 45 min. and the breadth of the exterior G O H ſhall be 3 degr. 10 min. and the diſtance between them G O F ſhall be 8 gr. 55 min. the greateſt Semi-diameter of the innermoſt, that is, the Angle P O F being 42 gr. 2 min. and the leaſt Semi-diameter of the outermoſt P O G, being 50 gr. 57 min. Theſe are the meaſures of the Bows, as they would be were the Sun but a point; for by the breadth of his Body the breadth of the Bows will be increaſed and their diſtance decreaſed by half a degree, and ſo the breadth of the interior Iris will be 2 degr. 15 min. that of the exterior 3 degr. 40 min. their diſtance 8 degr. 25 min. the greateſt Semi-diameter of the interior Bow 42 degr. 17 min. and the leaſt of the exterior 50 degr. 42 min. And ſuch are the dimenſions of the Bows in the Heavens found to be very nearly, when their Colours appear ſtrong and perfect. For once, by ſuch means as I then had, I meaſured the greateſt Semi-diameter of the interior Iris about 42 degrees, the breadth of the red, yellow and green in that Iris 63 or 64 minutes, beſides the outmoſt faint red obſcured by ye brightneſs of the Clouds, for which we may allow 3 or 4 minutes more. The breadth of the blue was about 40 minutes more beſides the violet, which was ſo much obſcured by the brightneſs of the Clouds, that I could not meaſure its breadth. But ſuppoſing the breadth of the blue and violet together to equal that of the red, yellow and green together, the whole breadth of this Iris will be about $2\frac{1}{4}$ degrees as above. The leaſt diſtance between this Iris and the exterior Iris was about 8 degrees and 30 minutes. The exterior Iris was broader than the interior, but ſo faint, eſpecially on the blue ſide, that I could not meaſure its breadth diſtinctly. At another time when both Bows appeared more diſtinct, I meaſured the breadth of the interior Iris 2 gr. 10′, and the breadth of the red, yellow and green in the exterior Iris, was to the breadth of the ſame Colours in the interior as 3 to 2.

This Explication of the Rain-bow is yet further confirmed by the known Experiment (made by *Antonius de Dominis* and *Des-Cartes)* of hanging up any where in the Sun-ſhine a Glaſs-Globe filled with Water, and viewing it in ſuch a poſture that the rays which come from the Globe to the Eye may contain with the Sun's rays an Angle of either 42 or 50 degrees. For if the Angle be about 42 or 43 degrees, the Spectator (ſuppoſe at O) ſhall ſee a full red Colour in that ſide of the Globe oppoſed to the Sun as 'tis repreſented at F, and

if

if that Angle become less (suppose by depressing the Globe to E) there will appear other Colours, yellow, green and blue successively in the same side of the Globe. But if the Angle be made about 50 degrees (suppose by lifting up the Globe to G)there will appear a red Colour in that side of the Globe towards the Sun, and if the Angle be made greater (suppose by lifting up the Globe to H) the red will turn successively to the other Colours yellow, green and blue. The same thing I have tried by letting a Globe rest, and raising or depressing the Eye, or otherwise moving it to make the Angle of a just magnitude.

I have heard it represented, that if the Light of a Candle be refracted by a Prism to the Eye ; when the blue Colour falls upon the Eye the Spectator shall see red in the Prism, and when the red falls upon the Eye he shall see blue ; and if this were certain, the Colours of the Globe and Rain-bow ought to appear in a contrary order to what we find. But the Colours of the Candle being very faint, the mistake seems to arise from the difficulty of discerning what Colours fall on the Eye. For, on the contrary, I have sometimes had occasion to observe in the Sun's Light refracted by a Prism, that the Spectator always sees that Colour in the Prism which falls upon his Eye. And the same I have found true also in Candle-Light. For when the Prism is moved slowly from the line which is drawn directly from the Candle to the Eye, the red appears first in the Prism and then the blue, and therefore each of them is seen when it falls upon the Eye. For the red passes over the Eye first, and then the blue.

The

The Light which comes through Drops of Rain by two refractions without any reflexion, ought to appear ſtrongeſt at the diſtance of about 26 degrees from the Sun, and to decay gradually both ways as the diſtance from him increaſes and decreaſes. And the ſame is to be underſtood of Light tranſmitted through ſpherical Hail-ſtones. And if the Hail be a little flatted, as it often is, the Light tranſmitted may grow ſo ſtrong at a little leſs diſtance than that of 26 degrees, as to form a Halo about the Sun or Moon; which Halo, as often as the Hail-ſtones are duly figured may be coloured, and then it muſt be red within by the leaſt refrangible rays, and blue without by the moſt refrangible ones, eſpecially if the Hail-ſtones have opake Globules of Snow in their center to intercept the Light within the Halo (as *Hugenius* has obſerved) and make the inſide thereof more diſtinctly defined than it would otherwiſe be. For ſuch Hail-ſtones, though ſpherical, by terminating the Light by the Snow, may make a Halo red within and colourleſs without, and darker in the red than without, as Halos uſe to be. For of thoſe rays which paſs cloſe by the Snow the rubriform will be leaſt refracted, and ſo come to the Eye in the directeſt lines.

The Light which paſſes through a Drop of rain after two refractions, and three or more reflexions, is ſcarce ſtrong enough to cauſe a ſenſible Bow; but in thoſe Cylinders of Ice by which *Hugenius* explains the *Parhelia*, it may perhaps be ſenſible.

PROP.

PROP. X. PROB. V.

By the diſcovered Properties of Light to explain the permanent Colours of natural Bodies.

Theſe Colours ariſe from hence, that ſome natural Bodies reflect ſome ſorts of rays, others other ſorts more copiouſly than the reſt. Minium reflects the leaſt refrangible or red-making rays moſt copiouſly, and thence appears red. Violets reflect the moſt refrangible, moſt copiouſly, and thence have their Colour, and ſo of other Bodies. Every Body reflects the rays of its own Colour more copiouſly than the reſt, and from their exceſs and predominance in the reflected Light has its Colour.

EXPER. XVII.

For if in the homogeneal Lights obtained by the ſolution of the Problem propoſed in the 4th Propoſition of the firſt ~~Book~~ Part you place Bodies of ſeveral Colours, you will find, as I have done, that every Body looks moſt ſplendid and luminous in the Light of its own Colour. Cinnaber in the homogeneal red Light is moſt reſplendent, in the green Light it is manifeſtly leſs reſplendent, and in the blue Light ſtill leſs. Indico in the violet blue Light is moſt reſplendent, and its ſplendor is gradually diminiſhed as it is removed thence by degrees through the green and yellow Light to the red. By a Leek the green Light, and next that the blue and yellow which compound green, are more ſtrongly reflected

flected than the other Colours red and violet, and ſo of the reſt. But to make theſe Experiments the more manifeſt, ſuch Bodies ought to be choſen as have the fulleſt and moſt vivid Colours, and two of thoſe Bodies are to be compared together. Thus, for inſtance, if Cinnaber and *ultra* marine blue, or ſome other full blue be held together in the homogeneal Light, they will both appear red, but the Cinnaber will appear of a ſtrongly luminous and reſplendent red, and the *ultra* marine blue of a faint obſcure and dark red; and if they be held together in the blue homogeneal Light they will both appear blue, but the *ultra* marine will appear of a ſtrongly luminous and reſplendent blue, and the Cinnaber of a faint and dark blue. Which puts it out of diſpute, that the Cinnaber reflects the red Light much more copiouſly than the *ultra* marine doth, and the *ultra* marine reflects the blue Light much more copiouſly than the Cinnaber doth. The ſame Experiment may be tryed ſuccesfully with red Lead and Indico, or with any other two coloured Bodies, if due allowance be made for the different ſtrength or weakneſs of their Colour and Light.

And as the reaſon of the Colours of natural Bodies is evident by theſe Experimenrs, ſo it is further confirmed and put paſt diſpute by the two firſt Experiments of the firſt ~~Book~~, whereby 'twas proved in ſuch Bodies that the reflected Lights which differ in Colours do differ alſo in degrees of refrangibility. For thence it's certain, that ſome Bodies reflect the more refrangible, others the leſs refrangible rays more copiouſly.

And

And that this is not only a true reaſon of theſe Colours, but even the only reaſon may appear further from this conſideration, that the Colour of homogeneal Light cannot be changed by the reflexion of natural Bodies.

For if Bodies by reflexion cannot in the leaſt change the Colour of any one ſort of rays, they cannot appear coloured by any other means than by reflecting thoſe which either are of their own Colour, or which by mixture muſt produce it.

But in trying Experiments of this kind care muſt be had that the Light be ſufficiently homogeneal. For if Bodies be illuminated by the ordinary priſmatick Colours, they will appear neither of their own day-light Colours, nor of the Colour of the Light caſt on them, but of ſome middle Colour between both, as I have found by Experience. Thus red Lead (for inſtance) illuminated with the ordinary priſmatick green will not appear either red or green, but orange or yellow, or between yellow and green, accordingly, as the green Light by which 'tis illuminated is more or leſs compounded. For becauſe red Lead appears red when illuminated with white Light, wherein all ſorts of rays are equally mixed, and in the green Light all ſorts of rays are not equally mixed, the exceſs of the yellow-making, green-making and blue-making rays in the incident green Light, will cauſe thoſe rays to abound ſo much in the reflected Light as to draw the Colour from red towards their Colour. And becauſe the red Lead reflects the red-making rays moſt copiouſly in proportion to their number, and next after them the orange-making and yellow-making rays; theſe rays in the reflected Light will be more in proportion to the Light than they were in the incident green Light, and thereby will draw the reflected Light from green towards their Colour. And therefore the red Lead will appear neither red nor green, but of a Colour between both.

In tranſparently coloured Liquors 'tis obſervable, that their Colour uſes to vary with their thickneſs. Thus, for inſtance, a red Liquor in a conical Glaſs held between the Light and the Eye, looks of a pale and dilute yellow at the bottom where 'tis thin, and a little higher where 'tis thicker grows orange, and where 'tis ſtill thicker becomes red, and where 'tis thickeſt the red is deepeſt and darkeſt. For it is to be conceived that ſuch a Liquor ſtops the indico-making and violet-making rays moſt eaſily, the blue-making rays more difficultly, the green-making rays ſtill more difficultly, and the red-making moſt difficultly: And that if the thickneſs of the Liquor be only ſo much as ſuffices to ſtop a competent number of the violet-making and indico-making rays, without diminiſhing much the number of the reſt, the reſt muſt (by Prop. 6. ~~Lib.~~ Part. 2.) compound a pale yellow. But if the Liquor be ſo much thicker as to ſtop alſo a great number of the blue-making rays, and ſome of the green-making, the reſt muſt compound an orange; and where it is ſo thick as to ſtop alſo a great number of the green-making and a conſiderable number of the yellow-making, the reſt muſt begin to compound a red, and this red muſt grow deeper and darker as the yellow making and orange-making rays are more and more ſtopt by increaſing the thickneſs of the Liquor, ſo that few rays beſides the red-making can get through.

Of this kind is an Experiment lately related to me by Mr. *Halley*, who, in diving deep in a diving vessel into the Sea, found in a clear Sun-shine day, that when he was sunk many ~~the water above seen through a small glass window in the vessel~~ & Fathoms deep into the Water, the upper part of his Hand on which the Sun shone directly through the Water & through a small glass window in ye vessel looked of a red Colour,+ and the water below & the under part of his Hand illuminated by Light reflected from the Water below looked green. For thence it may be gathered, that the Sea-water reflects back the violet and blue-making rays most easily, and lets the red-making rays pass most freely and copiously to great depths. For thereby the Sun's direct Light at all great depths, by reason of the predominating red-making rays, must appear red; and the greater the depth is, the fuller and intenser must that red be. And at such depths as the violet-making rays scarce penetrate unto, the blue-making, green-making and yellow-making rays being reflected from below more copiously than the red-making ones, must compound a green. ~~And at less depths the reflected violet & blue being~~ ~~from below~~ ~~more copious may tinge~~ the water with the usual colour of ye Sea between green & blue; besides the reflection from the sky & clouds above.

+ like that of a Damask Rose

Now if there be two Liquors of full Colours, suppose a red and a blue, and both of them so thick as suffices to make their Colours sufficiently full; though either Liquor be sufficiently transparent apart, yet will you not be able to see through both together. For if only the red-making rays pass through one Liquor, and only the blue-making through the other, no rays can pass through both. This Mr. *Hook* tried casually with Glass-wedges filled with red and blue Liquors, and was surprized at the unexpected event, the reason of it being then unknown; which makes me trust the more to his Experiment, though I have not tryed it my self. But he that would repeat it, must take care the Liquors be of very good and full Colours.

Now whilst Bodies become coloured by reflecting or transmitting this or that sort of rays more copiously than the rest, it is to be conceived that they stop and stifle in themselves the rays which they do not reflect or transmit. For if Gold be foliated and held between your Eye and the Light, the Light looks of a greenish blue, and therefore massy Gold lets into its Body the blue-making rays to be reflected to and fro within it till they be stopt and stifled, whilst it reflects the yellow-making outwards, and thereby looks yellow. And much after the same manner that Leaf-gold is yellow by reflected, and blue by transmitted Light, and massy Gold is yellow in all positions of the Eye; there are some Liquors as the tincture of *Lignum Nephriticum*, and some sorts of Glass which transmit one sort of Light most copiously, and reflect another sort, and thereby look of several Colours, according to the position of the Eye to the Light. But if these Liquors or Glasses were so thick and massy that no Light could get through them, I question not but that they would like all other opake Bodies appear of one and the same Colour in all positions of the Eye, though this I cannot yet affirm by experience. For all coloured Bodies, so far as my Observation reaches, may be seen through if made sufficiently thin, and therefore are in some measure transparent, and differ only in degrees of transparency from tinged transparent Liquors; these Liquors, as well as those Bodies, by a sufficient thickness becoming opake. A transparent Body which looks of any Colour by transmitted Light, may also look of the same Colour by reflected Light, the Light of that Colour being reflected by the further surface of the Body, or by the Air beyond it. And then the reflected Colour will be diminished, and perhaps cease, by

making

making the Body very thick, and pitching it on the back-side to diminish the reflexion of its further surface, so that the Light reflected from the tinging particles may predominate. In such cases, the Colour of the reflected Light will be apt to vary from that of the Light transmitted. But whence it is that tinged Bodies and Liquors reflect some sort of rays, and intromit or transmit other sorts, shall be said in the next Book. In this Proposition I content my self to have put it past dispute, that Bodies have such Properties, and thence appear coloured.

PROP. XI. PROB. VI.

By mixing coloured Lights to compound a Beam of Light of the same Colour and Nature with a Beam of the Sun's direct Light, and therein to experience the truth of the foregoing Propositions.

Let A B C a b c represent a Prism by which the Sun's Light let into a dark Chamber through the Hole F, may be refracted towards the Lens M N, and paint upon it at p, q, r, s and t, the usual Colours violet, blue, green, yellow and red, and let the diverging rays by the refraction of this Lens converge again towards X, and there, by the mixture of all those their Colours, compound a white according to what was shewn above. Then let another Prism D E G d e g, parallel to the former, be placed at X, to refract that white Light upwards towards Y. Let the refracting Angles of the Prisms, and their distances from the Lens be equal, so that the rays which converged from the Lens towards X, and without refraction, would there have crossed and diverged again, may by the refraction of the second Prism be *Fig.* 16.

reduced

reduced into Parallelifm and diverge no more. For then thofe rays will recompofe a Beam of white Light XY. If the refracting Angle of either Prifm be the bigger, that Prifm muft be fo much the nearer to the Lens. You will know when the Prifms and the Lens are well fet together by obferving if the Beam of Light XY which comes out of the fecond Prifm be perfectly white to the very edges of the Light, and at all diftances from the Prifm continue perfectly and totally white like a Beam of the Sun's Light. For till this happens, the pofition of the Prifms and Lens to one another muft be corrected, and then if by the help of a long Beam of Wood, as is reprefented in the Figure, or by a Tube, or fome other fuch inftrument made for that purpofe, they be made faft in that fituation, you may try all the fame Experiments in this compounded Beam of Light XY, which ~~in the foregoing Experiments~~ have been made in the Sun's direct Light. For this compounded Beam of Light has the fame appearance, and is endowed with all the fame Properties with a direct Beam of the Sun's Light, fo far as my Obfervation reaches. And in trying Experiments in this Beam you may by ftopping any of the Colours p, q, r, s and t, at the Lens, fee how the Colours produced in the Experiments are no other than thofe which the rays had at the Lens before they entered the compofition of this Beam : And by confequence that they arife not from any new modifications of the Light by refractions and reflexions, but from the various feparations and mixtures of the rays originally endowed with their colour-making qualities.

So, for inftance, having with a Lens $4\frac{1}{4}$ Inches broad, and two Prifms on either Hand $6\frac{1}{4}$ Feet diftant from the Lens, made fuch a Beam of compounded Light : to

examin

examin the reaſon of the Colours made by Priſms, I refracted this compounded Beam of Light XY with another Priſm HIKkh, and thereby caſt the uſual priſmatick Colours PQRST upon the Paper LV placed behind. And then by ſtopping any of the Colours p, q, r, s, t, at the Lens, I found that the ſame Colour would vaniſh at the Paper. So if the purple P was ſtopped at the Lens, the purple P upon the Paper would vaniſh, and the reſt of the Colours would remain unaltered, unleſs perhaps the blue, ſo far as ſome purple latent in it at the Lens might be ſeparated from it by the following refractions. And ſo by intercepting the green upon the Lens, the green R upon the Paper would vaniſh, and ſo of the reſt; which plainly ſhews, that as the white Beam of Light XY was compounded of ſeveral Lights variouſly coloured at the Lens, ſo the Colours which afterwards emerge out of it by new refractions are no other than thoſe of which its whiteneſs was compounded. The refraction of the Priſm HIK kh generates the Colours PQRST upon the Paper, not by changing the colorific qualities of the rays, but by ſeparating the rays which had the very ſame colorific qualities before they entered the compoſition of the refracted Beam of white ~~of~~ Light XY. For otherwiſe the rays which were of one Colour at the Lens might be of another upon the Paper, contrary to what we find.

So again, to examin the reaſon of the Colours of natural Bodies, I placed ſuch Bodies in the Beam of Light XY, and found that they all appeared there of thoſe their own Colours which they have in Day-light, and that thoſe Colours depend upon the rays which had the ſame Colours at the Lens before they entred the compo-

ſition

ſition of that Beam. Thus, for inſtance, Cinnaber illuminated by this Beam appears of the ſame red Colour as in Day-light; and if at the Lens you intercept the green-making and blue-making rays, its redneſs will become more full and lively: But if you there intercept the red-making rays, it will not any longer appear red, but become yellow or green, or of ſome other Colour, according to the ſorts of rays which you do not intercept. So Gold in this Light X Y appears of the ſame yellow Colour as in Day-light, but by intercepting at the Lens a due quantity of the yellow-making rays it will appear white like Silver (as I have tryed) which ſhews that its yellowneſs ariſes from the exceſs of the intercepted rays tinging that whiteneſs with their Colour when they are let paſs. So the infuſion of *Lignum Nephriticum* (as I have alſo tryed) when held in this Beam of Light X Y, looks blue by the reflected part of the Light, and ~~yellow~~ by the tranſmitted part of it, as when 'tis viewed in Day-light, but if you intercept the blue at the Lens the infuſion will loſe its reflected blue Colour, whilſt its tranſmitted red remains perfect and by the loſs of ſome blue-making rays wherewith it was allayed becomes more intenſe and full. And, on the contrary, if the red and orange-making rays be intercepted at Lens, the infuſion will loſe its tranſmitted red, whilſt its blue will remain and become more full and perfect. Which ſhews, that the infuſion does not tinge the rays with blue and yellow, but only tranſmit thoſe moſt copiouſly which were red-making before, and reflects thoſe moſt copiouſly which were blue-making before. And after the ſame manner may the reaſons of other Phænomena be examined, by trying them in this artificial Beam of Light X Y.

THE

Fig. 1.

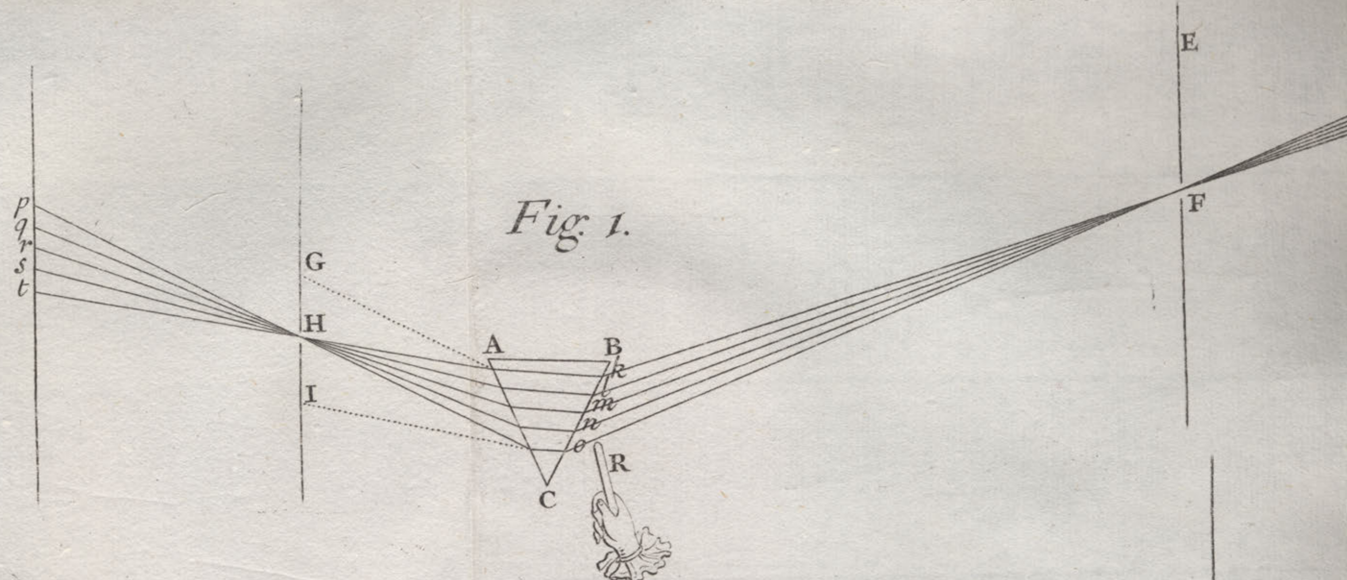


Fig. 2.

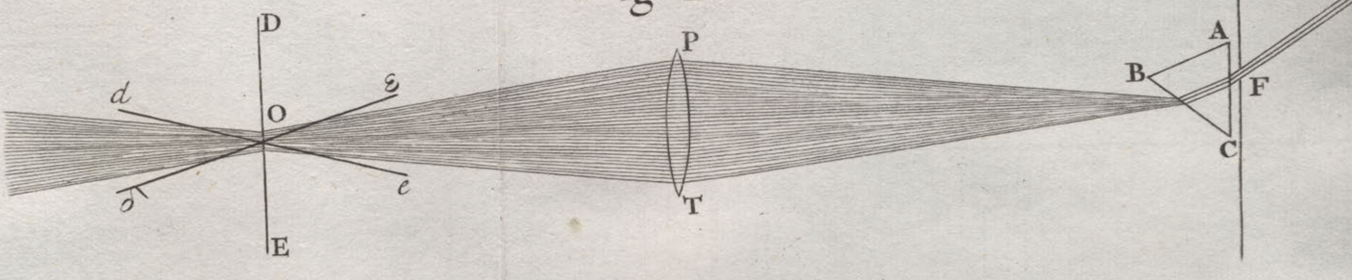


Fig. 3.

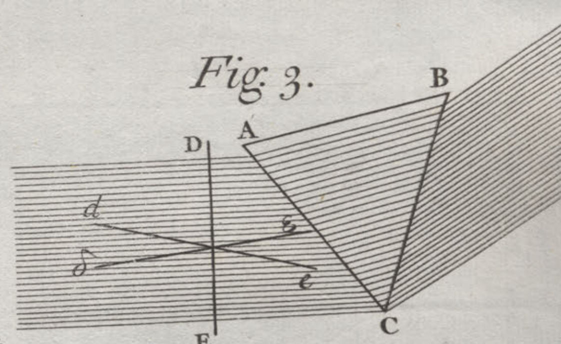


Fig. 4.

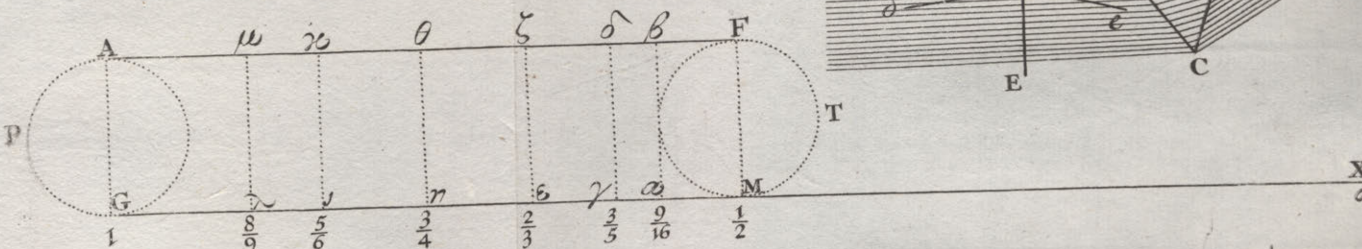
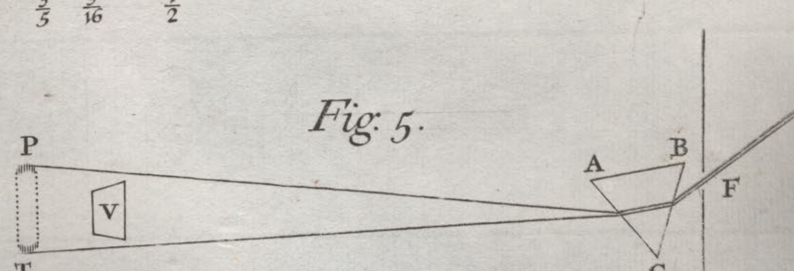


Fig. 5.



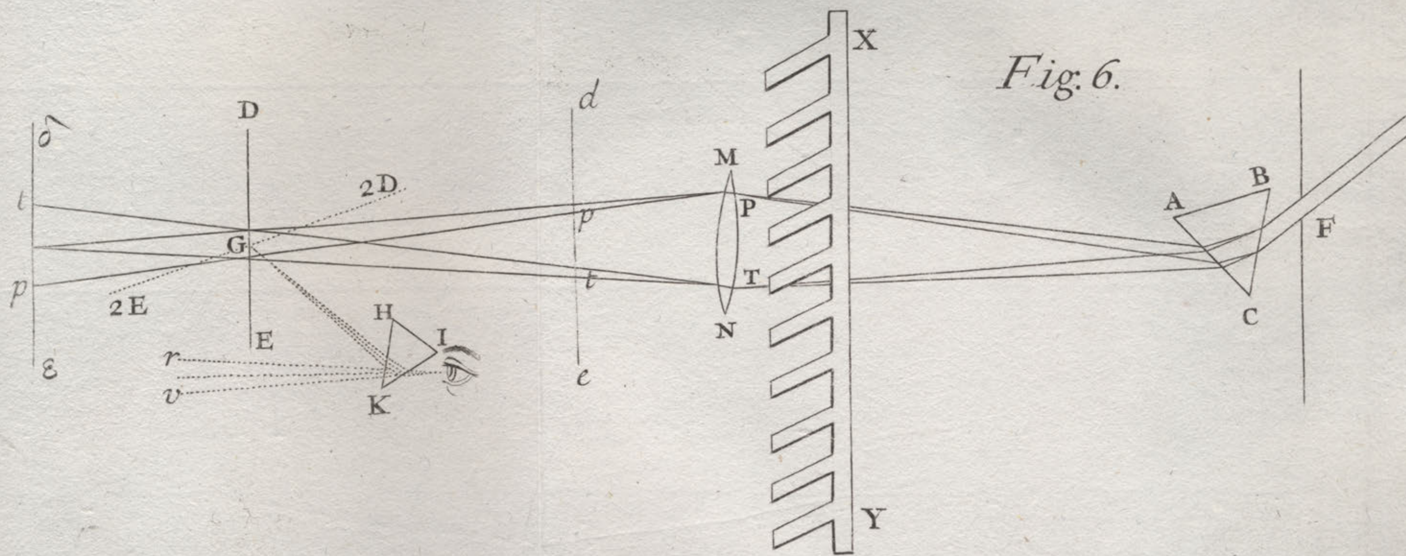


Fig. 6.

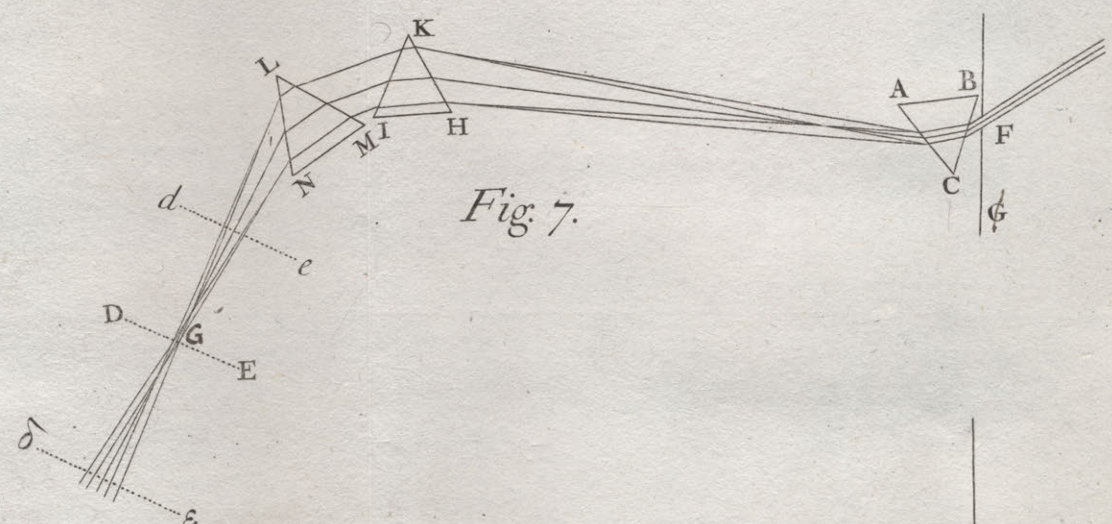


Fig. 7.

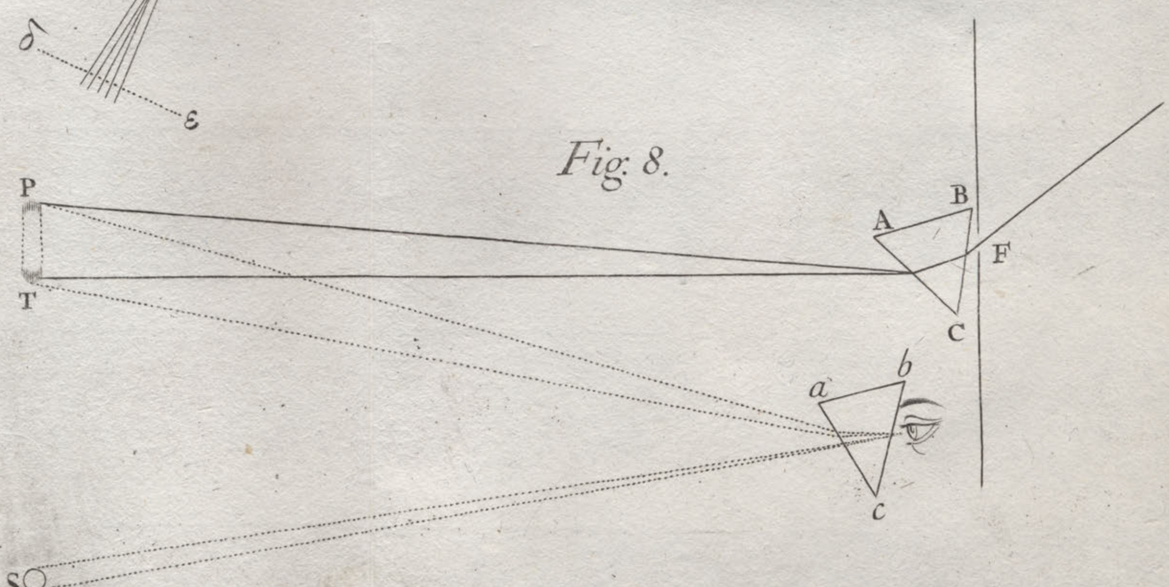


Fig. 8.

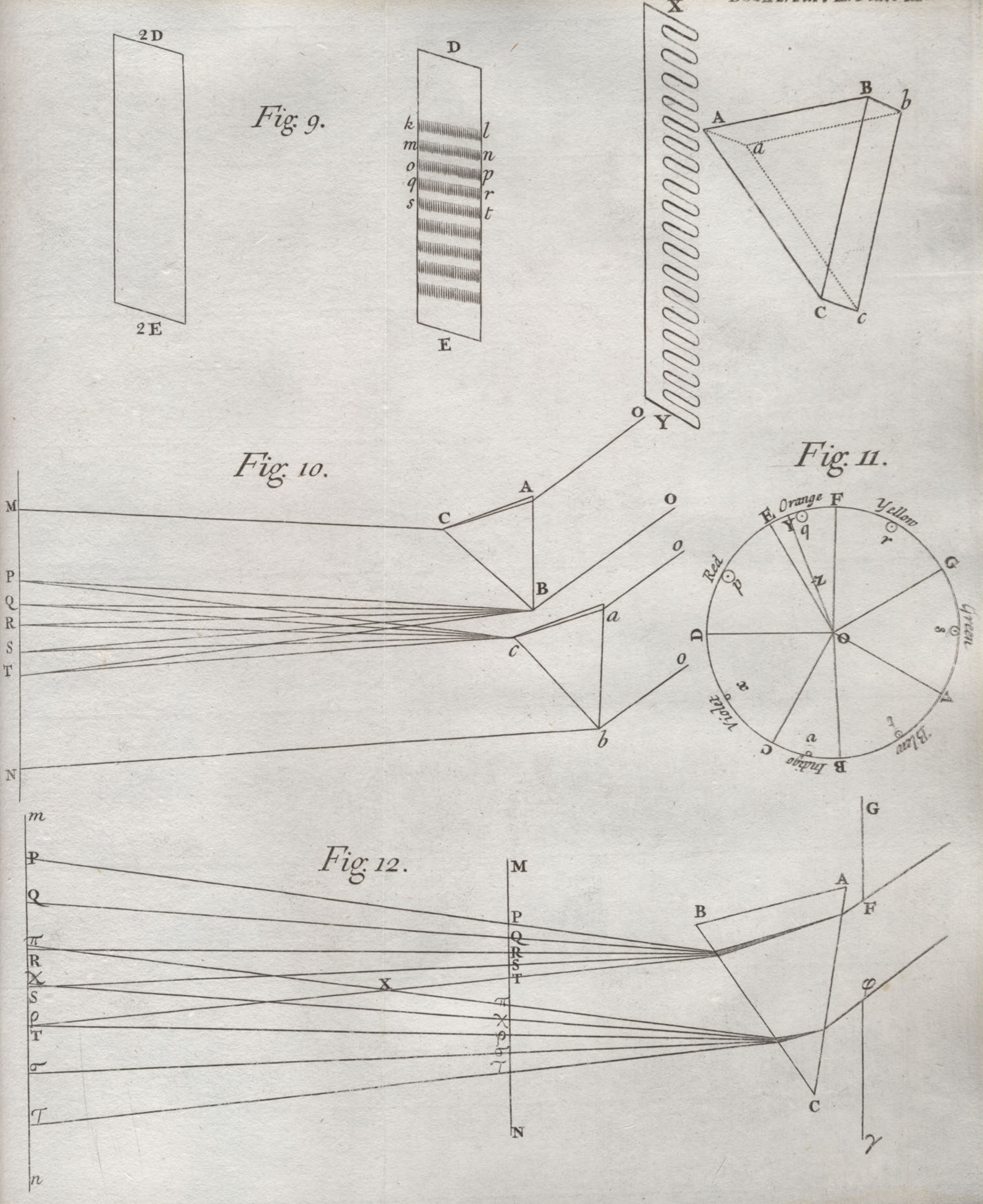


Fig. 9.

Fig. 10.

Fig. 11.

Fig. 12.

Fig. 13.

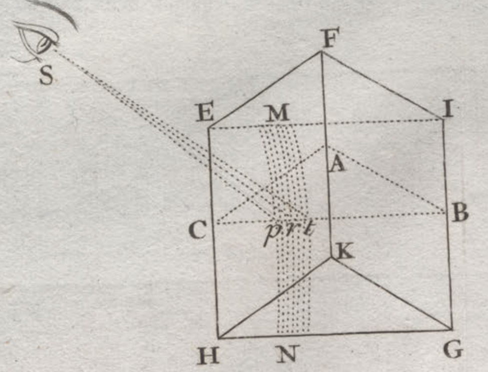
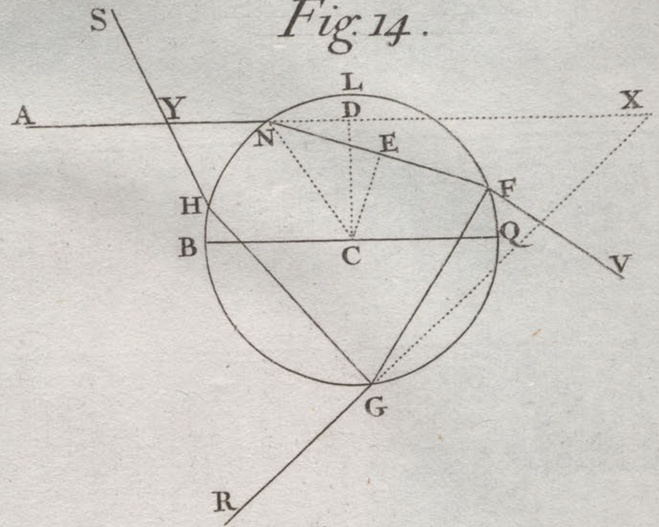


Fig. 14.



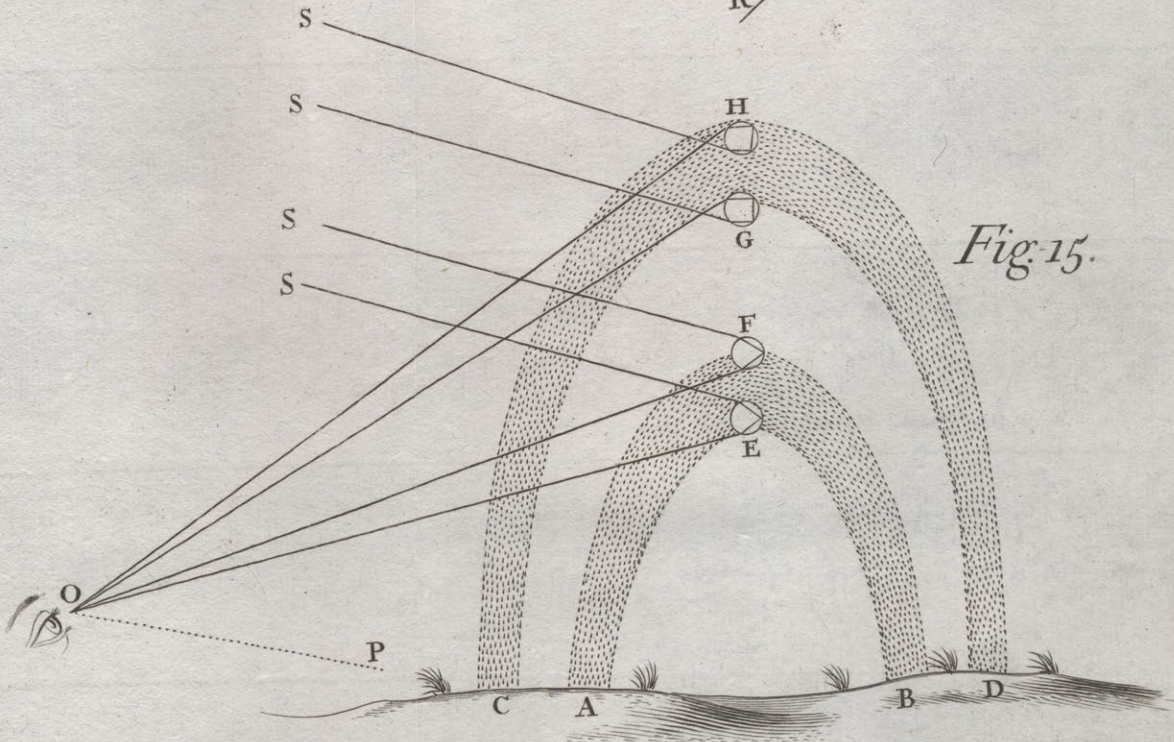


Fig. 15.

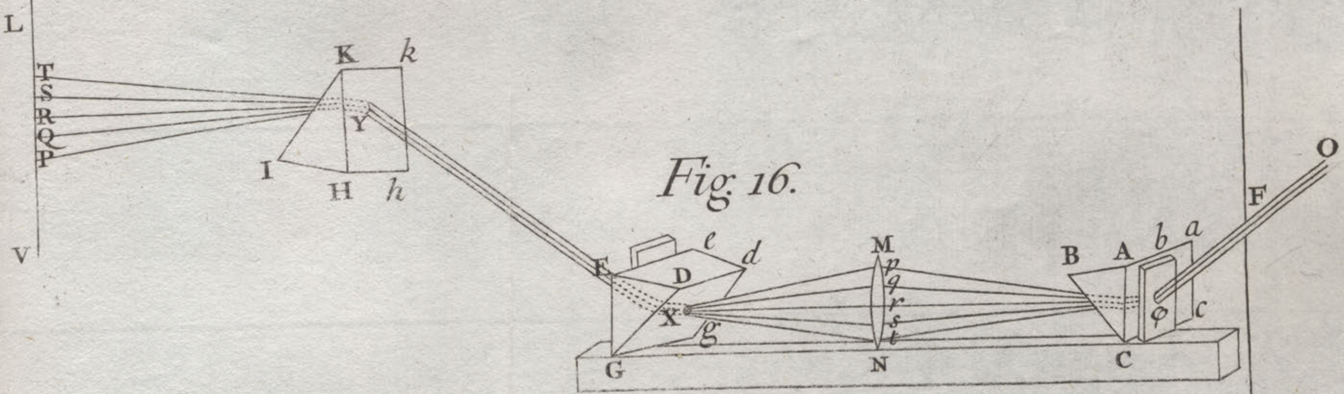


Fig. 16.

THE SECOND BOOK OF OPTICKS.

PART I.

Obſervations concerning the Reflexions, Refractions, and Colours of thin tranſparent Bodies.

IT has been obſerved by others that tranſparent Subſtances, as Glaſs, Water, Air, *&c.* when made very thin by being blown into Bubbles, or otherwiſe formed into Plates, do exhibit various Colours according to their various thinneſs, although at a greater thickneſs they appear very clear and colourleſs. In the former Book I forbore to treat of theſe Colours, becauſe they ſeemed of a more difficult conſideration, and were not neceſſary for eſtabliſhing the Properties of Light there diſcourſed of. But becauſe they may conduce to further diſcoveries for completing the Theory of Light, eſpecially as to the conſtitution of the parts of natural Bodies, on which their Colours or Tranſparency depend; I have here ſet down an account of them. To render this Diſcourſe ſhort and diſtinct, I have firſt deſcribed the principal of my

Obſervations, and then conſidered and made uſe of them. The Obſervations are theſe.

OBS. I.

Compreſſing two Priſms hard together that their Sides (which by chance were a very little convex) might ſomewhere touch one another: I found the place in which they touched to become abſolutely tranſparent, as if they had there been one continued piece of Glaſs. For when the Light fell ſo obliquely on the Air, which in other places was between them, as to be all reflected; it ſeemed in that place of contact to be wholly tranſmitted, inſomuch that when looked upon, it appeared like a black or dark Spot, by reaſon that little or no ſenſible Light was reflected from thence, as from other places; and when looked through it ſeemed (as it were) a hole in that Air which was formed into a thin Plate, by being compreſſed between the Glaſſes. And through this hole Objects that were beyond might be ſeen diſtinctly, which could not at all be ſeen through other parts of the Glaſſes where the Air was interjacent. Although the Glaſſes were a little convex, yet this tranſparent Spot was of a conſiderable breadth, which breadth ſeemed principally to proceed from the yielding inwards of the parts of the Glaſſes, by reaſon of their mutual preſſure. For by preſſing them very hard together it would become much broader than otherwiſe.

OBS. II.

When the Plate of Air, by turning the Prisms about their common Axis, became so little inclined to the incident Rays, that some of them began to be transmitted, there arose in it many slender Arcs of Colours which at first were shaped almost like the Conchoid, as you see them delineated in the first Figure. And by continuing the motion of the Prisms, these Arcs increased and bended more and more about the said transparent Spot, till they were completed into Circles or Rings incompassing it, and afterwards continually grew more and more contracted. *Fig.* 1.

These Arcs at their first appearance were of a violet and blue Colour, and between them were white Arcs of Circles, which presently by continuing the motion of the Prisms became a little tinged in their inward Limbs with red and yellow, and to their outward Limbs the blue was adjacent. So that the order of these Colours from the central dark Spot, was at that time white, blue, violet; black; red, orange, yellow, white, blue, violet, &c. But the yellow and red were much fainter than the blue and violet.

The motion of the Prisms about their Axis being continued, these Colours contracted more and more, shrinking towards the whiteness on either side of it, until they totally vanished into it. And then the Circles in those parts appeared black and white, without any other Colours intermixed. But by further moving the Prisms about, the Colours again emerged out of the whiteness, the violet and blue as its inward Limb, and at its outward Limb the red and yellow. So that now their order from the central Spot was white, yellow, red; black; violet, blue, white, yellow, red, &c. contrary to what it was before.

O B S. III.

When the Rings or ſome parts of them appeared only black and white, they were very diſtinct and well defined, and the backneſs ſeemed as intenſe as that of the central Spot. Alſo in the borders of the Rings, where the Colours began to emerge out of the whiteneſs, they were pretty diſtinct, which made them viſible to a very great Multitude. I have ſometimes numbred above thirty Succeſſions (reckoning every black and white Ring for one Succeſſion) and ſeen more of them, which by reaſon of their ſmalneſs I could not number. But in other Poſitions of the Priſms, at which the Rings appeared of many Colours, I could not diſtinguiſh above eight or nine of them, and the exterior of thoſe were very confuſed and dilute.

In theſe two Obſervations to ſee the Rings diſtinct, and without any other Colour than black and white, I found it neceſſary to hold my Eye at a good diſtance from them. For by approaching nearer, although in the ſame inclination of my Eye to the plane of the Rings, there emerged a blueiſh Colour out of the white, which by dilating it ſelf more and more into the black rendred the Circles leſs diſtinct, and left the white a little tinged with red and yellow. I found alſo by looking through a ſlit or oblong hole, which was narrower than the Pupil of my Eye, and held cloſe to

it

it parallel to the Priſms, I could ſee the Circles much diſtincter and viſible to a far greater number than otherwiſe.

OBS. IV.

To obſerve more nicely by the order of the Colours which aroſe out of the white Circles as the Rays became leſs and leſs inclined to the plate of Air; I took two Object Glaſſes, the one a Plano-convex for a fourteen-foot Teleſcope, and the other a large double convex for one of about fifty-foot; and upon this, laying the other with its its plane-ſide downwards, I preſſed them ſlowly together, to make the Colours ſucceſſively emerge in the middle of the Circles, and then ſlowly lifted the upper Glaſs from the lower to make them ſucceſſively vaniſh again in the ſame place. The Colour, which by preſſing the Glaſſes together emerged laſt in the middle of the other Colours, would upon its firſt appearance look like a Circle of a Colour almoſt uniform from the circumference to the center, and by compreſſing the Glaſſes ſtill more, grow continually broader until a new Colour emerged in its center, and thereby it became a Ring encompaſſing that new Colour. And by compreſſing the Glaſſes ſtill more, the Diameter of this Ring would encreaſe, and the breadth of its Orbit or Perimeter decreaſe until another new Colour emerged in the center of the laſt: And ſo on until a third, a fourth, a fifth, and other following new Colours ſucceſſively emerged there, and became Rings encompaſſing the innermoſt Colour, the laſt of which was the black Spot. And, on the contrary, by lifting

lifting up the upper Glaſs from the lower, the diameter of the Rings would decreaſe, and the breadth of their Orbit encreaſe, until their Colours reached ſucceſſively to the center ; and then they being of a conſiderable breadth, I could more eaſily diſcern and diſtinguiſh their Species than before. And by this means I obſerved their Succeſſion and Quantity to be as followeth.

Next, to the pellucid central Spot made by the contact of the Glaſſes ſucceeded blue, white, yellow, and red, the blue was ſo little in quantity that I could not diſcern it in the circles made by the Priſms, nor could I well diſtinguiſh any violet in it, but the yellow and red were pretty copious, and ſeemed about as much in extent as the white, and four or five times more than the blue. The next Circuit in order of Colours immediately encompaſſing theſe were violet, blue, green, yellow, and red, and theſe were all of them copious and vivid, excepting the green, which was very little in quantity, and ſeemed much more faint and dilute than the other Colours. Of the other four, the violet was the leaſt in extent, and the blue leſs than the yellow or red. The third Circuit or Order was purple, blue, green, yellow, and red ; in which the purple ſeemed more reddiſh than the violet in the former Circuit, and the green was much more conſpicuous, being as briſque and copious as any of the other Colours, except the yellow ; but the red began to be a little faded, inclining very much to purple. After this ſucceeded the fourth Circuit of green and red. The green was very copious and lively, inclining on the one ſide to blue, and on the other ſide to yellow. But in

this

this fourth Circuit there was neither violet, blue, nor yellow, and the red was very imperfect and dirty. Alſo the ſucceeding Colours became more and more imperfect and dilute, till after three or four Revolutions they ended in perfect whiteneſs. Their Form, when the Glaſſes were moſt compreſſed ſo as to make the black Spot appear in the Center, is delineated in the Second Figure; where *a*, *b*, *c*, *d*, *e* : *f*, *g*, *h*, *i*, *k* : *l*, *m*, *n*, *o*, *p* : *q*, *r* : *s*, *t* : *v*, *x* : *y* denote the Colours reck'ned in order from the center, black, blue, white, yellow, red : violet, blue, green, yellow, red : purple, blue, green, yellow, red : green, red : greeniſh blue, red : greeniſh blue, pale red : greeniſh blue, reddiſh white. *Fig.* 2.

OBS. V.

To determine the interval of the Glaſſes, or thickneſs of the interjacent Air, by which each Colour was produced, I meaſured the Diameters of the firſt ſix Rings at the moſt lucid part of their Orbits, and ſquaring them, I found their Squares to be in the Arithmetical Progreſſion of the odd Numbers, 1. 3. 5. 7. 9. 11. And ſince one of theſe Glaſſes was Plain, and the other Spherical, their Intervals at thoſe Rings muſt be in the ſame Progreſſion. I meaſured alſo the Diameters of the dark or faint Rings between the more lucid Colours, and found their Squares to be in the Arithmetical Progreſſion of the even Numbers, 2. 4. 6. 8. 10. 12. And it being very nice and difficult to take theſe meaſures exactly; I repeated them at divers times at divers parts of the Glaſſes, that by their Agreement I might be confirmed in them. And the ſame Method I uſed in deter-

determining ſome others of the following Obſervations.

OBS. VI.

The Diameter of the ſixth Ring at the moſt lucid part of its Orbit was $\frac{58}{100}$ parts of an Inch, and the Diameter of the Sphere on which the double convex Object-Glaſs was ground was about 102 Feet, and hence I gathered the thickneſs of the Air or Aereal Interval of the Glaſſes at that Ring. But ſome time after, ſuſpecting that in making this Obſervation I had not determined the Diameter of the Sphere with ſufficient accurateneſs, and being uncertain whether the Plano-convex Glaſs was truly plain, and not ſomething concave or convex on that ſide which I accounted plain; and whether I had not preſſed the Glaſſes together, as I often did, to make them touch (for by preſſing ſuch Glaſſes together their parts eaſily yield inwards, and the Rings thereby become ſenſibly broader than they would be, did the Glaſſes keep their Figures) I repeated the Experiment, and found the Diameter of the ſixth lucid Ring about $\frac{55}{100}$ parts of an Inch. I repeated the Experiment alſo with ſuch an Object-Glaſs of another Teleſcope as I had at hand. This was a double convex ground on both ſides to one and the ſame Sphere, and its Focus was diſtant from it $83\frac{2}{5}$ Inches. And thence, if the Sines of incidence and refraction of the bright yellow Light be aſſumed in proportion as 11 to 17, the Diameter of the Sphere to which the Glaſs was figured will by computation be found 182 Inches. This Glaſs I laid upon a flat one, ſo that the black

black Spot appeared in the middle of the Rings of Colours without any other preſſure than that of the weight of the Glaſs. And now meaſuring the Diameter of the fifth dark Circle as accurately as I could, I found it the fifth part of an Inch preciſely. This meaſure was taken with the points of a pair of Compaſſes on the upper ſurface on the upper Glaſs, and my Eye was about eight or nine Inches diſtance from the Glaſs, almoſt perpendicularly over it, and the Glaſs was $\frac{1}{6}$ of an Inch thick, and thence it is eaſy to collect that the true Diameter of the Ring between the Glaſſes was greater than its meaſured Diameter above the Glaſſes in the proportion of 80 to 79 or thereabouts, and by conſequence equal to $\frac{16}{79}$ parts of an Inch, and its true Semi-diameter equal to $\frac{8}{79}$ parts. Now as the Diameter of the Sphere (182 Inches) is to the Semi-diameter of this fifth dark Ring ($\frac{8}{79}$ parts of an Inch) ſo is this Semi-diameter to the thickneſs of the Air at this fifth dark Ring; which is therefore $\frac{32}{567931}$ or $\frac{100}{1774784}$ parts of an Inch, and the fifth part thereof; *viz.* the $\frac{1}{88739}$th part of an Inch, is the thickneſs of the Air at the firſt of theſe dark Rings. $\frac{100}{1774784}$

The ſame Experiment I repeated with another double convex Object-glaſs ground on both ſides to one and the ſame Sphere. Its Focus was diſtant from it $168\frac{1}{2}$ Inches, and therefore the Diameter of that Sphere was 184 Inches. This Glaſs being laid upon the ſame plain Glaſs, the Diameter of the fifth of the dark Rings, when the black Spot in their center appeared plainly without preſſing the Glaſſes, was by the meaſure of the Compaſſes upon the upper Glaſs $\frac{121}{600}$ parts of an Inch, and by conſequence between the Glaſſes it was $\frac{1222}{6000}$. For the upper Glaſs was $\frac{1}{8}$ of an Inch thick,

and

and my Eye was distant from it 8 Inches. And a third proportional to half this from the Diameter of the Sphere is $\frac{5}{88850}$ parts of an Inch. This is therefore the thickness of the Air at this Ring, and a fifth part thereof, *viz.* the $\frac{1}{88850}$th part of an Inch is the thickness thereof at the first of the Rings as above.

I tryed the same thing by laying these Object-Glasses upon flat pieces of a broken Looking-glass, and found the same measures of the Rings: Which makes me rely upon them till they can be determined more accurately by Glasses ground to larger Spheres, though in such Glasses greater care must be taken of a true plain.

These Dimensions were taken when my Eye was placed almost perpendicularly over the Glasses, being about an Inch, or an Inch and a quarter, distant from the incident rays, and eight Inches distant from the Glass; so that the rays were inclined to the Glass in an Angle of about 4 degrees. Whence by the following Observation you will understand, that had the rays been perpendicular to the Glasses, the thickness of the Air at these Rings would have been less in the proportion of the Radius to the secant of 4 degrees, that is of 10000 to 10024. Let the thicknesses found be therefore diminished in this proportion, and they will become $\frac{1}{88952}$ and $\frac{1}{89063}$, or (to use the nearest round number) the $\frac{1}{89000}$th part of an Inch. This is the thickness of the Air at the darkest part of the first dark Ring made by perpendicular rays, and half this thickness multiplied by the progression, 1, 3, 5, 7, 9, 11, &c. gives the thicknesses of the Air at the most luminous parts of all the brightest Rings, *viz.* $\frac{1}{178000}$, $\frac{3}{178000}$, $\frac{5}{178000}$, $\frac{7}{178000}$, &c. their arithmetical means

means $\frac{2}{178000}$, $\frac{4}{178000}$, $\frac{6}{178000}$, &c. being its thickneſſes at the darkeſt parts of all the dark ones.

OBS. VII.

The Rings were leaſt when my Eye was placed perpendicularly over the Glaſſes in the Axis of the Rings: And when I viewed them obliquely they became bigger, continually ſwelling as I removed my Eye further from the Axis. And partly by meaſuring the Diameter of the ſame Circle at ſeveral obliquities of my Eye, partly by other means, as alſo by making uſe of the two Priſms for very great obliquities. I found its Diameter, and conſequently the thickneſs of the Air at its perimeter in all thoſe obliquities to be very nearly in the proportions expreſſed in this Table.

Angle of Incidence on the Air.		*Angle of Refraction into the Air.*		*Diameter of the Ring.*	*Thickneſs of the Air.*
deg.	min.				
00	00	00	00	10	10
06	26	10	00	$10\frac{1}{13}$	$10\frac{2}{13}$
12	45	20	00	$10\frac{1}{3}$	$10\frac{2}{3}$
18	49	30	00	$10\frac{3}{4}$	$11\frac{1}{2}$
24	30	40	00	$11\frac{2}{5}$	13
29	37	50	00	$12\frac{1}{2}$	$15\frac{1}{2}$
33	58	60	00	14	20
35	47	65	00	$15\frac{1}{4}$	$23\frac{1}{4}$
37	19	70	00	$16\frac{4}{5}$	$28\frac{1}{4}$
38	33	75	00	$19\frac{1}{4}$	37
39	27	80	00	$22\frac{6}{7}$	$52\frac{1}{4}$
40	00	85	00	29	$84\frac{1}{10}$
40	11	90	00	35	$122\frac{1}{2}$

In the two firſt Columns are expreſſed the obliquities of the incident and emergent rays to the plate of the Air, that is, their angles of incidence and refraction. In the third Column the Diameter of any coloured Ring at thoſe obliquities is expreſſed in parts, of which ten conſtitute that Diameter when the rays are perpendicular. And in the fourth Column the thickneſs of the Air at the circumference of that Ring is expreſſed in parts of which alſo ten conſtitute ~~that~~ its thickneſs when the rays are perpendicular.

And from theſe meaſures I ſeem to gather this Rule: That the thickneſs of the Air is proportional to the ſecant of an angle, whoſe Sine is a certain mean proportional between the Sines of incidence and refraction. And that mean proportional, ſo far as by theſe meaſures I can determine it, is the firſt of an hundred and ſix arithmetical mean proportionals between thoſe Sines counted from the Sine of refraction when the refraction is made out of the Glaſs into the plate of Air, or from the Sine of incidence when the refraction is made out of the plate of Air into the Glaſs.

O B S. VIII.

The dark Spot in the middle of the Rings increaſed alſo by the obliquation of the Eye, although almoſt inſenſibly. But if inſtead of the Object-Glaſſes the Priſms were made uſe of, its increaſe was more manifeſt when viewed ſo obliquely that no Colours appeared about it. It was leaſt when the rays were incident moſt obliquely on the interjacent Air, and as the obliquity decreaſed it increaſed more and more until the coloured Rings ap-

peared,

peared, and then decreaſed again, but not ſo much as it increaſed before. And hence it is evident, that the tranſparency was not only at the abſolute contact of the Glaſſes, but alſo where they had ſome little interval. I have ſometimes obſerved the Diameter of that Spot to be between half and two fifth parts of the Diameter of the exterior circumference of the red in the firſt circuit or revolution of Colours when viewed almoſt perpendicularly; whereas when viewed obliquely it hath wholly vaniſhed and become opake and white like the other parts of the Glaſs; whence it may be collected that the Glaſſes did then ſcarcely, or not at all, touch one another, and that their interval at the perimeter of that Spot when viewed perpendicularly was about a fifth or ſixth part of their interval at the circumference of the ſaid red.

OBS. IX.

By looking through the two contiguous Object-Glaſſes, I found that the interjacent Air exhibited Rings of Colours, as well by tranſmitting Light as by reflecting it. The central Spot was now white, and from it the order of the Colours were yellowiſh red; black; violet, blue, white, yellow, red; violet, blue, green, yellow, red, *&c.* But theſe Colours were very faint and dilute unleſs when the Light was trajected very obliquely through the Glaſſes: For by that means they became pretty vivid. Only the firſt yellowiſh red, like the blue in the fourth Obſervation, was ſo little and faint as ſcarcely to be diſcerned. Comparing the coloured Rings made by reflexion, with theſe made by

tranſ-

transmission of the Light; I found that white was opposite to black, red to blue, yellow to violet, and green to a compound of red and violet. That is, those parts of the Glass were black when looked through, which when looked upon appeared white, and on the contrary. And so those which in one case exhibited blue, did in the other case exhibit red. And the like of the other Colours. The manner you have represented in the third Figure, where A B, C D, are the surfaces of the Glasses contiguous at E, and the black lines between them are their distances in arithmetical progression, and the Colours written above are seen by reflected Light, and those below by Light transmitted.

Fig. 3.

O B S. X.

Wetting the Object-Glasses a little at their edges, the water crept in slowly between them, and the Circles thereby became less and the Colours more faint: Insomuch that as the water crept along one half of them at which it first arrived would appear broken off from the other half, and contracted into a less room. By measuring them I found the proportions of their Diameters to the Diameters of the like Circles made by Air to be about seven to eight, and consequently the intervals of the Glasses at like Circles, caused by those two mediums Water and Air, are as about three to four. Perhaps it may be a general Rule, That if any other medium more or less dense than water be compressed between the Glasses, their intervals at the Rings caused thereby will be to their intervals caused by interjacent Air,

Air, as the Sines are which meaſure the refraction made out of that medium into Air.

O B S. XI.

When the water was between the Glaſſes, if I preſſed the upper Glaſs variouſly at its edges to make the Rings move nimbly from one place to another, a little white Spot would immediately follow the center of them, which upon creeping in of the ambient water into that place would preſently vaniſh. Its appearance was ſuch as interjacent Air would have cauſed, and it exhibited the ſame Colours. But it was not Air, for where any bubbles of Air were in the water they would not vaniſh. The reflexion muſt have rather been cauſed by a ſubtiler medium, which could recede through the Glaſſes at the creeping in of the water.

O B S. XII.

Theſe Obſervations were made in the open Air. But further to examin the effects of coloured Light falling on the Glaſſes, I darkened the Room, and viewed them by reflexion of the Colours of a Priſm caſt on a Sheet of white Paper, my Eye being ſo placed that I could ſee the coloured Paper by reflexion in the Glaſſes, as in a Looking-glaſs. And by this means the Rings became diſtincter and viſible to a far greater number than in the open Air. I have ſometimes ſeen more than twenty of them, whereas in the open Air I could not diſcern above eight or nine.

OBS. XIII.

Appointing an aſſiſtant to move the Priſm to and fro about its Axis, that all the Colours might ſucceſſively fall on that part of the Paper which I ſaw by reflexion from that part of the Glaſſes, where the Circles appeared, ſo that all the Colours might be ſucceſſively reflected from the Circles to my Eye whilſt I held it immovable, I found the Circles which the red Light made to be manifeſtly bigger than thoſe which were made by the blue and violet. And it was very pleaſant to ſee them gradually ſwell or contract according as the Colour of the Light was changed. The interval of the Glaſſes at any of the Rings when they were made by the utmoſt red Light, was to their interval at the ſame Ring when made bythe utmoſt violet, greater than as 3 to 2,and leſs than as 13 to 8,by the moſt of my Obſervations it was as 14 to 9. And this proportion ſeemed very nearly the ſame in all obliquities of my Eye; unleſs when two Priſms were made uſe of inſtead of the Object-Glaſſes. For then at a certain great obliquity of my Eye, the Rings made by the ſeveral Colours ſeemed equal, and at a greater obliquity thoſe made by the violet would be greater than the ſame Rings made by the red. The refraction of the Priſm in this caſe cauſing the moſt refrangible rays to fall more obliquely on that plate of the Air than the leaſt refrangible ones. Thus the Experiment ſucceeded in the coloured Light, which was ſufficiently ſtrong and copious to make the Rings ſenſible. And thence it may be gathered, that if the moſt refrangible and leaſt

refran-

refrangible rays had been copious enough to make the Rings ſenſible without the mixture of other rays, the proportion which here was 14 to 9 would have been a little greater, ſuppoſe $14\frac{1}{4}$ or $14\frac{1}{3}$ to 9.

O B S. XIV.

Whilſt the Priſm was turn'd about its Axis with an uniform motion, to make all the ſeveral Colours fall ſucceſſively upon the Object-Glaſſes, and thereby to make the Rings contract and dilate: The contraction or dilation of each Ring thus made by the variation of its Colour was ſwifteſt in the red, and ſloweſt in the violet, and in the intermediate Colours it had intermediate degrees of celerity. Comparing the quantity of contraction and dilation made by all the degrees of each Colour, I found that it was greateſt in the red; leſs in the yellow, ſtill leſs in the blue, and leaſt in the violet. And to make as juſt an eſtimation as I could of the proportions of their contractions or dilations, I obſerved that the whole contraction or dilation of the Diameter of any Ring made by all the degrees of red, was to that of the Diameter of the ſame Ring made by all the degrees of violet, as about four to three, or five to four, and that when the Light was of the middle Colour between yellow and green, the Diameter of the Ring was very nearly an arithmetical mean between the greateſt Diameter of the ſame Ring made by the outmoſt red, and the leaſt Diameter thereof made by the outmoſt violet: Contrary to what happens in the Colours of the oblong Spectrum made by the refraction of a Priſm, where the red is moſt contracted, the violet moſt expanded, and dilatation

 in

in the midſt of all the Colours is the confine of green and blue. And hence I ſeem to collect that the thickneſſes of the Air between the Glaſſes there, where the Ring is ſucceſſively made by the limits of the five principal Colours (red, yellow, green, blue, violet) in order (that is, by the extreme red, by the limit of red and yellow in the middle of the orange, by the limit of yellow and green, by the limit of green and blue, by the limit of blue and violet in the middle of the indigo, and by the extreme violet) are to one another very nearly as the ſix lengths of a Chord which ſound the notes in a ſixth Major, *ſol*, *la*, *mi*, *fa*, *ſol*, *la*. But it agrees ſomething better with the Obſervation to ſay, that the thickneſſes of the Air between the Glaſſes there, where the Rings are ſucceſſively made by the limits of the ſeven Colours, red, orange, yellow, green, blue, indigo, violet in order, are to one another as the Cube-roots of the Squares of the eight lengths of a Chord, which ſound the notes in an eighth, *ſol*, *la*, *fa*, *ſol*, *la*, *mi*, *fa*, *ſol*; that is, as the Cube-roots of the Squares of the Numbers, $1, \frac{8}{9}, \frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{9}{16}, \frac{1}{2}$.

OBS. XV.

These Rings were not of various Colours like thoſe made in the open Air, but appeared all over of that priſmatique Colour only with which they were illuminated. And by projecting the priſmatique Colours immediately upon the Glaſſes, I found that the Light which fell on the dark Spaces which were between the coloured Rings, was tranſmitted through the Glaſſes without any variation of Colour. For on a

white

white Paper placed behind, it would paint Rings of the ſame Colour with thoſe which were reflected, and of the bigneſs of their immediate Spaces. And from thence the origin of theſe Rings is manifeſt; namely, That the Air between the Glaſſes, according to its various thickneſs, is diſpoſed in ſome places to reflect, and in others to tranſmit the Light of any one Colour (as you may ſee repreſented in the fourth Figure) *Fig*. 4. and in the ſame place to reflect that of one Colour where it tranſmits that of another.

OBS. XVI.

The Squares of the Diameters of theſe Rings made by any priſmatique Colour were in arithmetical progreſſion as in the fifth Obſervation. And the Diameter of the ſixth Circle, when made by the citrine yellow, and viewed almoſt perpendicularly, was about $\frac{58}{100}$ parts of an Inch, or a little leſs, agreeable to the ſixth Obſervation.

The precedent Obſervations were made with a rarer thin medium, terminated by a denſer, ſuch as was Air or Water compreſſed between two Glaſſes. In thoſe that follow are ſet down the appearances of a denſer medium thin'd within a rarer, ſuch as are plates of Muſcovy-glaſs, Bubbles of Water, and ſome other thin ſubſtances terminated on all ſides with Air.

OBS. XVII.

If a Bubble be blown with Water firſt made tenacious by diſſolving a little Soap in it, 'tis a common Obſervation, that after a while it will appear tinged with a great variety of Colours. To defend theſe Bubbles from being agitated by the external Air (whereby their Colours are irregularly moved one among another, ſo that no accurate Obſervation can be made of them,) as ſoon as I had blown any of them I covered it with a clear Glaſs, and by that means its Colours emerged in a very regular order, like ſo many concentrick Rings incompaſſing the top of the Bubble. And as the Bubble grew thinner by the continual ſubſiding of the Water, theſe Rings dilated ſlowly and over-ſpread the whole Bubble, deſcending in order to the bottom of it, where they vaniſhed ſucceſſively. In the mean while, after all the Colours were emerged at the top, there grew in the Center of the Rings a ſmall round black Spot, like that in the firſt Obſervation, which continually dilated it ſelf till it became ſometimes more than $\frac{1}{2}$ or $\frac{3}{4}$ of an Inch in breadth before the Bubble broke. At firſt I thought there had been no Light reflected from the Water in that place, but obſerving it more curiouſly, I ſaw within it ſeveral ſmaller round Spots, which appeared much blacker and darker than the reſt, whereby I knew that there was ſome reflexion at the other places which were not ſo dark as thoſe Spots. And by further tryal I fouud that I could ſee the Images of ſome things (as of a Candle or the Sun) very faintly reflected, not only from the great black Spot, but

alſo

alſo from the little darker Spots which were within it.

Beſides the aforeſaid coloured Rings there would often appear ſmall Spots of Colours, aſcending and deſcending up and down the ſides of the Bubble, by reaſon of ſome inequalities in the ſubſiding of the Water. And ſometimes ſmall black Spots generated at the ſides would aſcend up to the larger black Spot at the top of the Bubble, and unite with it.

OBS. XVIII.

Becauſe the Colours of theſe Bubbles were more extended and lively than thoſe of the Air thin'd between two Glaſſes, and ſo more eaſy to to diſtinguiſhed, I ſhall here give you a further deſcription of their order, as they were obſerved in viewing them by reflexion of the Skies when of a white Colour, whilſt a black Subſtance was placed behind the Bubble. And they were theſe, red, blue; red, blue; red, blue; red, green; red, yellow, green, blue, purple; red, yellow, green, blue, violet; red, yellow, white, blue, black.

The three firſt Succeſſions of red and blue were very dilute and dirty, eſpecially the firſt, where the red ſeemed in a manner to be white. Among theſe there was ſcarce any other Colour ſenſible beſides red and blue, only the blues (and principally the ſecond blue) inclined a little to green.

The fourth red was alſo dilute and dirty, but not ſo much as the former three; after that ſucceeded little or no yellow, but a copious green, which at firſt inclined a little to yellow, and then became a pretty briſque and good willow green, and afterwards changed to a bluiſh Colour; but there ſucceeded neither blue nor violet.

The fifth red at firſt inclined very much to purple, and afterwards became more bright and briſque, but yet not very pure. This was ſucceeded with a very bright and intenſe yellow, which was but little in quantity, and ſoon changed to green: But that green was copious and ſomething more pure, deep and lively, than the former green. After that followed an excellent blue of a bright sky-colour, and then a purple, which was leſs in quantity than the blue, and much inclined to red.

The ſixth Red was at firſt of a very fair and lively Scarlet, and ſoon after of a brighter Colour, being very pure and briſque, and the beſt of all the reds. Then after a lively orange followed an intenſe bright and copious yellow, which was alſo the beſt of all the yellows, and this changed firſt to a greeniſh yellow, and then to a greeniſh blue; but the green between the yellow and the blue, was very little and dilute, ſeeming rather a greeniſh white than a green. The blue which ſucceeded became very good, and of a very fair bright sky-colour, but yet ſomething inferior to the former blue; and the violet was intenſe and deep with little or no redneſs in it. And leſs in quantity than the blue.

In the laſt red appeared a tincture of ſcarlet next to violet, which ſoon changed to a brighter Colour, inclining to an orange; and the yellow which followed was at firſt pretty good and lively, but afterwards it grew more dilute, until by degrees it ended in perfect white-

whitenefs. And this whitenefs, if the Water was very tenacious and well-tempered, would flowly fpread and dilate it felf over the greater part of the Bubble; continually growing paler at the top, where at length it would crack in many places, and thofe cracks, as they dilated, would appear of a pretty good, but yet obfcure and dark sky-colour; the white between the blue Spots diminifhing, until it refembled the threds of an irregular Net-work, and foon after vanifhed and left all the upper part of the Bubble of the faid dark blue Colour. And this Colour, after the aforefaid manner, dilated it felf downwards, until fometimes it hath overfpread the whole Bubble. In the mean while at the top, which was of a darker blue than the bottom, and appeared alfo full of many round blue Spots, fomething darker than the reft, there would emerge one or more very black Spots, and within thofe other Spots of an intenfer blacknefs, which I mentioned in the former Obfervation; and thefe continually dilated themfelves until the Bubble broke.

If the Water was not very tenacious the black Spots would break forth in the white, without any fenfible intervention of the blue. And fometimes they would break forth within the precedent yellow, or red, or perhaps within the blue of the fecond order, before the intermediate Colours had time to difplay themfelves.

By this defcription you may perceive how great an affinity thefe Colours have with thofe of Air defcribed in the fourth Obfervation, although fet down in a contrary order, by reafon that they begin to appear when the Bubble is thickeft, and are moft conve-

niently

niently reckoned from the loweſt and thickeſt part of the Bubble upwards.

O B S. XIX.

Viewing in ſeveral oblique poſitions of my Eye the Rings of Colours emerging on the top of the Bubble, I found that they were ſenſibly dilated by increaſing the obliquity, but yet not ſo much by far as thoſe made by thin'd Air in the ſeventh Obſervation. For there they were dilated ſo much as, when viewed moſt obliquely, to arrive at a part of the plate more than twelve times thicker than that where they appeared when viewed perpendicularly; whereas in this caſe the thickneſs of the Water, at which they arrived when viewed moſt obliquely, was to that thickneſs which exhibited them by perpendicular rays, ſomething leſs than as 8 to 5. By the beſt of my Obſervations it was between 15 and $15\frac{1}{2}$ to 10, an increaſe about 24 times leſs than in the other caſe.

Sometimes the Bubble would become of an uniform thickneſs all over, except at the top of it near the black Spot, as I knew, becauſe it would exhibit the ſame appearance of Colours in all poſitions of the Eye. And then the Colours which were ſeen at its apparent circumference by the obliqueſt rays, would be different from thoſe that were ſeen in other places, by rays leſs oblique to it. And divers Spectators might ſee the ſame part of it of differing Colours, by viewing it at very differing obliquities. Now obſerving how much the Colours at the ſame places of the Bubble, or at divers places of equal thickneſs, were varied by the

ſeveral

ſeveral obliquities of the rays; by the aſſiſtance of the 4th, 14th, 16th and 18th Obſervations, as they are hereafter explained, I collect the thickneſs of the Water requiſite to exhibit any one and the ſame Colour, at ſeveral obliquities, to be very nearly in the proportion expreſſed in this Table.

Incidence on the Water.		*Refraction into the Water.*		*Thickneſs of the Water.*
deg.	min.	deg.	min.	
00	00	00	00	10
15	00	11	11	$10\frac{1}{4}$
30	00	22	1	$10\frac{4}{5}$
45	00	32	2	$11\frac{4}{5}$
60	00	40	30	13
75	00	46	25	$14\frac{1}{2}$
90	00	48	35	$15\frac{1}{5}$

In the two firſt Columns are expreſſed the obliquities of the rays to the ſuperficies of the Water, that is, their Angles of incidence and refraction. Where I ſuppoſe that the Sines which meaſure them are in round numbers as 3 to 4, though probably the diſſolution of Soap in the Water, may a little alter its refractive Vertue. In the third Column the thickneſs of the Bubble, at which any one Colour is exhibited in thoſe ſeveral obliquities, is expreſt in parts, of which ten conſtitute ~~that~~ its thickneſs when the rays are perpendicular. And the Rule found by the 7th Observation agrees well with these measures if duly applied; namely, that the thickness of a plate of water requisite to exhibit one & the same colour at several obliquities of the eye, is proportional to ye secant of an angle whose sine is the first of an 106 arithmetical mean proportionals between the sines of incidence & refraction counted from the lesser sine, that is, from the sine of refraction when the refraction is made out of Air into Water, otherwise from the sine of incidence.

I have ſometimes obſerved, that the Colours which ariſe on poliſhed Steel by heating it, or on Bell-metal, and ſome other metalline ſubſtances, when melted and poured on the ground, where they may cool in the open Air, have, like the Colours of Water-bubbles, been a little changed by viewing them at divers obliquities, and particularly that a deep blue, or violet, when viewed very obliquely, hath been changed to a deep red. But the changes of these Colours are not so great and sensible as of those made by Water. For the Scoria or vitrified part of the Metal, which most Metals when heated or melted do continually protrude, and send out to their surface, and which by covering the Metals in form of a thin glassy skin, causes these Colours, is much denser than Water; and I find that the change made by the obliquation of the Eye is least in Colours of the densest thin substances.

OBS. XX.

As in the ninth Observation, so here, the Bubble, by transmitted Light, appeared of a contrary Colour to that which it exhibited by reflexion. Thus when the Bubble being looked on by the Light of the Clouds reflected from it, seemed red at its apparent circumference, if the Clouds at the same time, or immediately after, were viewed through it, the Colour at its circumference would be blue. And, on the contrary, when by reflected Light it appeared blue, it would appear red by transmitted Light.

OBS. XXI.

By wetting very thin plates of Muscovy-glass, whose thinness made the like Colours appear, the Colours

became

became more faint and languid; eſpecially by wetting the plates on that ſide oppoſite to the Eye: But I could not perceive any variation of their ſpecies. So then the thickneſs of a plate requiſite to produce any Colour, depends only on the denſity of the plate, and not on that of the ambient medium: And hence, by the 10th and 16th Obſervations, may be known the thickneſs which Bubbles of Water, or Plates of Muſcovy-glaſs, or other ſubſtances, have at any Colour produced by them.

OBS. XXII.

A thin tranſparent Body, which is denſer than its ambient medium, exhibits more briſque and vivid Colours than that which is ſo much rarer; as I have particularly obſerved in the Air and Glaſs. For blowing Glaſs very thin at a Lamp-furnace, thoſe plates incompaſſed with Air did exhibit Colours much more vivid than thoſe of Air made thin between two Glaſſes.

OBS. XXIII.

Comparing the quantity of Light reflected from the ſeveral Rings, I found that it was moſt copious from the firſt or inmoſt, and in the exterior Rings became gradually leſs and leſs. Alſo the whiteneſs of the firſt Ring was ſtronger than that reflected from thoſe parts of the thinner medium which were without the Rings; as I could manifeſtly perceive by viewing at a diſtance the Rings made by the two Object-

Glaſſes; or by comparing two Bubbles of Water blown at diſtant times, in the firſt of which the whiteneſs appeared, which ſucceeded all the Colours, and in the other, the whiteneſs which preceded them all.

OBS. XXIV.

When the two Object-Glaſſes were lay'd upon one another, ſo as to make the Rings of the Colours appear, though with my naked Eye I could not diſcern above 8 or 9 of thoſe Rings, yet by viewing them through a Priſm I have ſeen a far greater multitude, inſomuch that I could number more than forty, beſides many others, that were ſo very ſmall and cloſe together, that I could not keep my Eye ſteddy on them ſeverally ſo as to number them, but by their extent I have ſometimes eſtimated them to be more than a hundred. And I believe the Experiment may be improved to the diſcovery of far greater numbers. For they ſeem to be really unlimited, though viſible only ſo far as they can be ſeparated by the refraction, as I ſhall hereafter explain.

But it was but one ſide of theſe Rings, namely, that towards which the refraction was made, which by that refraction was rendered diſtinct, and the other ſide became more confuſed than when viewed by the naked Eye, inſomuch that there I could not diſcern above one or two, and ſometimes none of thoſe Rings, of which I could diſcern eight or nine with my naked Eye. And their Segments or Arcs, which on the other ſide appeared ſo numerous, for the moſt part exceeded

exceeded not the third part of a Circle. If the Refraction was very great, or the Prism very distant from the Object-Glasses, the middle part of those Arcs became also confused, so as to disappear and constitute an even whiteness, whilst on either side their ends, as also the whole Arcs furthest from the center, became distincter than before, appearing in the form as you see them designed in the fifth Figure. *Fig.* 5.

The Arcs, where they seemed distinctest, were only white and black successively, without any other Colours intermixed. But in other places there appeared Colours, whose order was inverted by the refraction in such manner, that if I first held the Prism very near the Object-Glasses, and then gradually removed it further off towards my Eye, the Colours of the 2d, 3d, 4th, and following Rings shrunk towards the white that emerged between them, until they wholly vanished into it at the middle of the Arcs, and afterwards emerged again in a contrary order. But at the ends of the Arcs they retained their order unchanged.

I have sometimes so lay'd one Object-Glass upon the other, that to the naked Eye they have all over seemed uniformly white, without the least appearance of any of the coloured Rings; and yet by viewing them through a Prism, great multitudes of those Rings have discovered themselves. And in like manner plates of Muscovy-glass, and Bubbles of Glass blown at a Lamp-furnace, which were not so thin as to exhibit any Colours to the naked Eye, have through the Prism exhibited a great variety of them ranged irregularly up and down in the form of waves. And so

Bubbles

Bubbles of Water, before they began to exhibit their Colours to the naked Eye of a By-ſtander, have appeared through a Priſm, girded about with many parallel and horizontal Rings; to produce which effect, it was neceſſary to hold the Priſm parallel, or very nearly parallel to the Horizon, and to diſpoſe it ſo that the rays might be refracted upwards.

THE

THE SECOND BOOK OF OPTICKS.

PART II.

Remarks upon the foregoing Observations.

HAving given my Observations of these Colours, before I make use of them to unfold the Causes of the Colours of natural Bodies, it is convenient that by the simplest of them, such as are the 2d, 3d, 4th, 9th, 12th, 18th, 20th, and 24th, I first explain the more expounded. And first to shew how the Colours in the fourth and eighteenth Observations are produced, let there be taken in any right line from the point Y, the lengths YA, YB, YC, YD, YE, YF, YG, *Fig.* 6. YH, in proportion to one another, as the Cube-roots of the Squares of the numbers, $\frac{1}{2}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, 1, whereby the lengths of a musical Chord to sound all the Notes in an Eighth are represented; that is, in the proportion of the numbers 6300, 6814, 7114, 7631, 8255, 8855, 9243, 10000. And at the points A, B, C, D,

E, F,

E, F, G, H, let perpendiculars A α, B β, &c. be erected, by whoſe intervals the extent of the ſeveral Colours ſet underneath againſt them, is to be repreſented. Then divide the line A α in ſuch proportion as the numbers 1, 2, 3, 5, 6, 7, 9, 10, 11, &c. ſet at the points of diviſion denote. And through thoſe diviſions from Y draw lines 1 I, 2 K, 3 L, 5 M, 6 N, 7 O, &c.

Now if A 2 be ſuppoſed to repreſent the thickneſs of any thin tranſparent Body, at which the outmoſt violet is moſt copiouſly reflected in the firſt Ring, or Series of Colours, then by the 13th Obſervation H K, will repreſent its thickneſs, at which the utmoſt red is moſt copiouſly reflected in the ſame Series. Alſo by the 5th and 16th Obſervations, A 6 and H N will denote the thickneſſes at which thoſe extreme Colours are moſt copiouſly reflected in the ſecond Series, and A 10 and H Q the thickneſſes, at which they are moſt copiouſly reflected in the third Series, and ſo on. And the thickneſs at which any of the intermediate Colours are reflected moſt copiouſly, will, according to the 14th Obſervation, be defined by the diſtance of the line A H from the intermediate parts of the lines 2 K, 6 N, 10 Q, &c. againſt which the names of thoſe Colours are written below.

But further, to define the latitude of theſe Colours in each Ring or Series, let A 1 deſign the leaſt thickneſs, and A 3 the greateſt thickneſs, at which the extreme violet in the firſt Series is reflected, and let H I, and H L, deſign the like limits for the extreme red, and let the intermediate Colours be limited by the intermediate parts of the lines 1 I, and 3 L, againſt which the names of thoſe Colours are written, and ſo on: But yet

yet with this caution, that the reflections be ſuppoſed ſtrongeſt at the intermediate Spaces, 2 K, 6 N, 10 Q, &c. and from thence to decreaſe gradually towards theſe limits, 1 I, 3 L, 5 M, 7 O, &c. on either ſide; where you muſt not conceive them to be preciſely limited, but to decay indefinitely. And whereas I have aſſigned the ſame latitude to every Series, I did it, becauſe although the Colours in the firſt Series ſeem to be a little broader than the reſt, by reaſon of a ſtronger reflexion there, yet that inequality is ſo inſenſible as ſcarcely to be determined by Obſervation.

Now according to this deſcription, conceiving that the rays originally of ſeveral Colours are by turns reflected at the Spaces 1 I L 3, 5 M O 7, 9 P R 11, &c. and tranſmitted at the Spaces A H I 1, 3 L M 5, 7 O P 9, &c. it is eaſy to know what Colour muſt in the open Air be exhibited at any thickneſs of a tranſparent thin body. For if a Ruler be applied parallel to A H, at that diſtance from it by which the thickneſs of the body is repreſented, the alternate Spaces 1 I L 3, 5 M O 7, &c. which it croſſeth will denote the reflected original Colours, of which the Colour exhibited in the open Air is compounded. Thus if the conſtitution of the green in the third Series of Colours be deſired, apply the Ruler as you ſee at $\pi \varrho \sigma \varphi$, and by its paſſing through ſome of the blue at π and yellow at σ, as well as through the green at ϱ, you may conclude that the green exhibited at that thickneſs of the body is principally conſtituted of original green, but not without a mixture of ſome blue and yellow.

By this means you may know how the Colours from the center of the Rings outward ought to ſucceed in order as they were deſcribed in the 4th and 18th Obſervations. For if you move the Ruler gradually from AH through all diſtances, having paſt over the firſt ſpace which denotes little or no reflexion to be made by thinneſt ſubſtances, it will firſt arrive at 1 the violet, and then very quickly at the blue and green, which together with that violet compound blue, and then at the yellow and red, by whoſe further addition that blue is converted into whiteneſs, which whiteneſs continues during the tranſit of the edge of the Ruler from I to 3, and after that by the ſucceſſive deficience of its component Colours, turns firſt to compound yellow, and then to red, and laſt of all the red ceaſeth at L. Then begin the Colours of the ſecond Series, which ſucceed in order during the tranſit of the edge of the Ruler from 5 to O, and are more lively than before, becauſe more expanded and ſevered. And for the ſame reaſon, inſtead of the former white there intercedes between the blue and yellow a mixture of orange, yellow, green, blue and indico, all which together ought to exhibit a dilute and imperfect green. So the Colours of the third Series all ſucceed in order; firſt, the violet, which a little interferes with the red of the ſecond order, and is thereby inclined to a reddiſh purple; then the blue and green, which are leſs mixed with other Colours, and conſequently more lively than before, eſpecially the green: Then follows the yellow, ſome of which towards the green is diſtinct and good, but that part of it towards the ſucceeding red, as alſo that red is mixed with the violet and blue of the fourth Series,

ries, whereby various degrees of red very much inclining to purple are compounded. This violet and blue, which ſhould ſucceed this red, being mixed with, and hidden in it, there ſucceeds a green. And this at firſt is much inclined to blue, but ſoon becomes a good green, the only unmixed and lively Colour in this fourth Series. For as it verges towards the yellow, it begins to interfere with the Colours of the fifth Series, by whoſe mixture the ſucceeding yellow and red are very much diluted and made dirty, eſpecially the yellow, which being the weaker Colour is ſcarce able to ſhew it ſelf. After this the ſeveral Series interfere more and more, and their Colours become more and more intermixed, till after three or four more revolutions (in which the red and blue predominate by turns) all ſorts of Colours are in all places pretty equally bended, and compound an even whiteneſs.

And ſince by the 15th Obſervation the rays indued with one Colour are tranſmitted, where thoſe of another Colour are reflected, the reaſon of the Colours made by the tranſmitted Light in the 9th and 20th Obſervations is from hence evident.

If not only the order and ſpecies of theſe Colours, but alſo the preciſe thickneſs of the plate, or thin body at which they are exhibited, be deſired in parts of an Inch, that may be alſo obtained by aſſiſtance of the 6th or 16th Obſervations. For according to thoſe Obſervations the thickneſs of the thinned Air, which between two Glaſſes exhibited the moſt luminous parts of the firſt ſix Rings were $\frac{1}{178000}$, $\frac{3}{178000}$, $\frac{5}{178000}$, $\frac{7}{178000}$, $\frac{9}{178000}$, $\frac{11}{178000}$ parts of an Inch. Suppoſe the Light reflected moſt copiouſly at theſe thickneſſes be the bright citrine yellow, or confine of yellow and orange, and theſe thickneſſes will Fμ, Fν, Fξ, Fο, Fτ be $G\mu$, $G\nu$, $G\xi$, $G o$, $G\tau$. And this being known, it is eaſy to determine what thickneſs of Air is repreſented by $G\varphi$, or by any other diſtance of the ruler from A H.

But further, ſince by the 10th Obſervation the thickneſs of Air was to the thickneſs of Water, which between the ſame Glaſſes exhibited the ſame Colour, as 4 to 3, and by the 21th Obſervation the Colours of thin bodies are not varied by varying the ambient medium; the thickneſs of a Bubble of Water, exhibiting any Colour, will be $\frac{3}{4}$ of the thickneſs of Air producing the ſame Colour. And ſo according to the ſame 10th and 21th Obſervations the thickneſs of a plate of Glaſs, whoſe refraction of the mean refrangible ray, is meaſured by the proportion of the Sines 31 to 20, may be $\frac{20}{31}$ of the thickneſs of Air producing the ſame Colours; and the like of other mediums. I do not affirm, that this proportion of 20 to 31, holds in all the rays; for the Sines of other ſorts of rays have other proportions. But the differences of thoſe proportions are ſo little that I do not here conſider them. On theſe Grounds I have compoſed the following Table, wherein the thickneſs of Air, Water, and Glaſs, at which each Colour is moſt intenſe and ſpecifick, is expreſſed in parts of an Inch divided into Ten hundred thouſand equal parts.

The thickness of coloured Plates and Particles of

		Air.	Water.	Glass.
Their Colours of the first Order,	Very Black	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$
	Black	1	$\frac{3}{4}$	$\frac{20}{31}$
	Beginning of Black	2	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{11}{20}$
	White	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	8	6	$5\frac{1}{6}$
	Red	9	$6\frac{3}{4}$	$5\frac{4}{5}$
Of the second Order,	Violet	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$
	Indico	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue	14	$10\frac{1}{2}$	9
	Green	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright Red	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$
Of the third Order,	Purple	21	$15\frac{3}{4}$	$13\frac{11}{20}$
	Indico	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$
	Blue	$23\frac{2}{5}$	$17\frac{11}{20}$	$15\frac{1}{10}$
	Green	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$
	Yellow	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$
	Red	29	$21\frac{3}{4}$	$18\frac{5}{7}$
	Bluish Red	32	24	$20\frac{2}{3}$
Of the fourth Order,	Bluish Green	34	$25\frac{1}{2}$	22
	Green	$35\frac{2}{7}$	$26\frac{1}{2}$	$22\frac{3}{4}$
	Yellowish Green	36	27	$23\frac{2}{9}$
	Red	$40\frac{1}{3}$	$30\frac{1}{4}$	26
Of the fifth Order,	Greenish Blue	46	$34\frac{1}{2}$	$29\frac{2}{3}$
	Red	$52\frac{1}{2}$	$39\frac{3}{8}$	34
Of the sixth Order,	Greenish Blue	$58\frac{3}{4}$	44	38
	Red	65	$48\frac{3}{4}$	42
Of the seventh Order,	Greenish Blue	71	$53\frac{1}{4}$	$45\frac{4}{5}$
	Ruddy White	77	$57\frac{3}{4}$	$49\frac{2}{3}$

Now

Now if this Table be compared with the 6th Scheme, you will there ſee the conſtitution of each Colour, as to its Ingredients, or the original Colours of which it is compounded, and thence be enabled to judge of its intenſeneſs or imperfection; which may ſuffice in explication of the 4th and 18th Obſervations, unleſs it be further deſired to delineate the manner how the Colours appear, when the two Object-Glaſſes are lay'd upon one another. To do which, let there be deſcribed a large Arc of a Circle, and a ſtreight Line which may touch that Arc, and parallel to that Tangent ſeveral occult Lines, at ſuch diſtances from it, as the numbers ſet againſt the ſeveral Colours in the Table denote. For the Arc, and its Tangent, will repreſent the ſuperficies of the Glaſſes terminating the interjacent Air; and the places where the occult Lines cut the Arc will ſhow at what diſtances from the Center, or Point of contact, each Colour is reflected.

There are alſo other uſes of this Table: For by its aſſiſtance the thickneſs of the Bubble in the 19th Obſervation was determined by the Colours which it exhibited. And ſo the bigneſs of the parts of natural Bodies may be conjectured by their Colours, as ſhall be hereafter ſhewn. Alſo, if two or more very thin plates be lay'd one upon another, ſo as to compoſe one plate equalling them all in thickneſs, the reſulting Colour may be hereby determined. For inſtance, Mr. *Hook* in his *Miſcrographia* obſerves, that a faint yellow plate of Muſcovy-glaſs lay'd upon a blue one, conſtituted a very deep purple. The yellow of the firſt Order is a faint one, and the thickneſs of the plate exhibiting it, according to the Table is $4\frac{3}{5}$, to which add 9, the thickneſs

neſs exhibiting blue of the ſecond Order, and the ſum will be $13\frac{3}{5}$, which is the thickneſs exhibiting the purple of the third Order.

To explain, in the next place, the Circumſtances of the 2d and 3d Obſervations; that is, how the Rings of the Colours may (by turning the Priſms about their common Axis the contrary way to that expreſſed in thoſe Obſervations) be converted into white and black Rings, and afterwards into Rings of Colours again, the Colours of each Ring lying now in an inverted order; it muſt be remembred, that thoſe Rings of Colours are dilated by the obliquation of the rays to the Air which intercedes the Glaſſes, and that according to the Table in the 7th Obſervation, their dilatation or increaſe of their Diameter is moſt manifeſt and ſpeedy when they are obliqueſt. Now the rays of yellow being more refracted by the firſt ſuperficies of the ſaid Air than thoſe of red, are thereby made more oblique to the ſecond ſuperficies, at which they are reflected to produce the coloured Rings, and conſequently the yellow Circle in each Ring will be more dilated than the red; and the exceſs of its dilatation will be ſo much the greater, by how much the greater is the obliquity of the rays, until at laſt it become of equal extent with the red of the ſame Ring. And for the ſame reaſon the green, blue and violet, will be alſo ſo much dilated by the ſtill greater obliquity of their rays, as to become all very nearly of equal extent with the red, that is, equally diſtant from the center of the Rings. And then all the Colours of the ſame Ring muſt be coincident, and by their mixture exhibit a white Ring. And theſe white Rings muſt have black and dark Rings between them, becauſe they do not

ſpread

ſpread and interfere with one another as before. And for that reaſon alſo they muſt become diſtincter and viſible to far greater Numbers. But yet the violet being obliqueſt will be ſomething more dilated in proportion to its extent then the other Colours, and ſo very apt to appear at the exterior verges of the white.

Afterwards, by a greater obliquity of the rays, the violet and blue become more ſenſibly dilated than the red and yellow, and ſo being further removed from the center of the Rings, the Colours muſt emerge out of the white in an order contrary to that which they had before, the violet and blue at the exterior limbs of each Ring, and the red and yellow at the interior. And the violet, by reaſon of the greateſt obliquity of its rays, being in proportion moſt of all expanded, will ſooneſt appear at the exterior limb of each white Ring, and become more conſpicuous than the reſt. And the ſeveral Series of Colours belonging to the ſeveral Rings, will, by their unfolding and ſpreading, begin again to interfere, and thereby render the Rings leſs diſtinct, and not viſible to ſo great numbers.

If inſtead of the Priſms the Object-glaſſes be made uſe of, the Rings which they exhibit become not white and diſtinct by the obliquity of the Eye, by reaſon that the rays in their paſſage through that Air which intercedes the Glaſſes are very nearly parallel to thoſe Lines in which they were firſt incident on the Glaſſes, and conſequently the rays indued with ſeveral Colours are not inclined one more than another to that Air, as it happens in the Priſms.

There is yet another circumſtance of theſe Experiments to be conſidered, and that is why the black and white Rings

Rings which when viewed at a diſtance appear diſtinct, ſhould not only become confuſed by viewing them near at hand, but alſo yield a violet Colour at both the edges of every white Ring. And the reaſon is, that the rays which enter the Eye at ſeveral parts of the Pupil, have ſeveral obliquities to the Glaſſes, and thoſe which are moſt oblique, if conſidered apart, would repreſent the Rings bigger than thoſe which are the leaſt oblique. Whence the breadth of the perimeter of every white Ring is expanded outwards by the obliqueſt rays, and inwards by the leaſt oblique. And this expanſion is ſo much the greater by how much the greater is the difference of the obliquity; that is, by how much the Pupil is wider, or the Eye nearer to the Glaſſes. And the breadth of the violet muſt be moſt expanded, becauſe the rays apt to excite a ſenſation of that Colour are moſt oblique to a ſecond, or further ſuperficies of the thin'd Air at which they are reflected, and have alſo the greateſt variation of obliquity, which makes that Colour ſooneſt emerge out of the edges of the white. And as the breadth of every Ring is thus augmented, the dark intervals muſt be diminiſhed, until the neighbouring Rings become continuous, and are blended, the exterior firſt, and then thoſe nearer the Center, ſo that they can no longer be diſtinguiſh'd apart, but ſeem to conſtitute an even and uniform whiteneſs.

Among all the Obſervations there is none accompanied with ſo odd circumſtances as the 24th. Of thoſe the principal are, that in thin plates, which to the naked Eye ſeem of an even and uniform tranſparent whitenefs, without any terminations of fhadows, the refraction of a Prifm fhould make Rings of Colours appear, whereas it ufually makes Objects appear coloured only there where they are terminated with fhadows, or have parts unequally luminous; and that it fhould make thofe Rings exceedingly diftinct and white, although it ufually renders Objects confufed and coloured. The caufe of thefe things you will underftand by confidering, that all the Rings of Colours are really in the plate, when viewed with the naked Eye, although by reafon of the great breadth of their circumferences they fo much interfere and are blended together, that they feem to conftitute an even whitenefs. But when the rays pafs through the Prifm to the Eye, the orbits of the feveral Colours in every Ring are refracted, fome more than others, according to their degrees of refrangibility: By which means the Colours on one fide of the Ring (that is on one fide of its Center) become more unfolded and dilated, and thofe on the other fide more complicated and contracted. And where by a due refraction they are fo much contracted, that the fevral Rings become narrower than to interfere with one another, they muft appear diftinct, and alfo white, if the conftituent Colours be fo much contracted as to be wholly coincident. But, on the other fide, where the orbit of every Ring is made broader by the further unfolding of its Colours, it muft interfere more with other Rings than before, and fo become lefs diftinct.

To explain this a little further, fuppofe the concen-
Fig. 7. trick Circles A V, and B X, reprefent the red and violet of any order, which, together with the intermediate Colours,

Colours, constitute any one of these Rings. Now these being viewed through a Prism, the violet Circle B X, will by a greater refraction be further translated from its place than the red A V, and so approach nearer to it on that side, towards which the refractions are made. For instance, if the red be translated to *av*, the violet may be translated to *b* x, so as to approach nearer to it at x than before, and if the red be further translated to a v, the violet may be so much further translated to b x as to convene with it at x, and if the red be yet further translated to α Υ, the violet may be still so much further translated to $\beta\xi$ as to pass beyond it at ξ, and convene with it at *e* and *f*. And this being understood not only of the red and violet, but of all the other intermediate Colours, and also of every revolution of those Colours, you will easily perceive how those of the same revolution or order, by their nearness at *xv* and Υ ξ, and their coincidence at x v, *e* and *f*, ought to constitute pretty distinct Arcs of Circles, especially at x v, or at *e* and *f*, and that they will appear severally at *x v*, and at x v exhibit whiteness by their coincidence, and again appear several at Υ ξ, but yet in a contrary order to that which they had before, and still retain beyond *e* and *f*. But, on the other side, at *ab*, a b, or α β, these Colours must become much more confused by being dilated and spread so, as to interfere with those of other Orders. And the same confusion will happen at Υ ξ between *e* and *f*, if the refraction be very great, or the Prism very distant from the Object-Glasses: In which case no parts of the Rings will be seen, save only two little Arcs at *e* and *f*, whose distance from one

 another,

another will be augmented by removing the Priſm ſtill further from the Object-Glaſſes: And theſe little Arcs muſt be diſtincteſt and whiteſt at their middle, and at their ends, where they begin to grow confuſed they muſt be coloured. And the Colours at one end of every Arc muſt be in a contrary order to thoſe at the other end, by reaſon that they croſs in the intermediate white; namely their ends, which verge towards ϒξ, will be red and yellow on that ſide next the Center, and blue and violet on the other ſide. But their other ends which verge from ϒξ will on the contrary be blue and violet on that ſide towards the Center, and on the other ſide red and yellow.

Now as all theſe things follow from the Properties of Light by a mathematical way of reaſoning, ſo the truth of them may be manifeſted by Experiments. For in a dark room, by viewing theſe Rings through a Priſm, by reflexion of the ſeveral priſmatique Colours, which an aſſiſtant cauſes to move to and fro upon a Wall or Paper from whence they are reflected, whilſt the Spectator's Eye, the Priſm and the Object-Glaſſes (as in the 13th Obſervation) are placed ſteddy: the poſition of the Circles made ſucceſſively by the ſeveral Colours, will be found ſuch, in reſpect of one another, as I have deſcribed in the Figures *abxv*, or abxv, or αβξϒ. And by the ſame method the truth of the Explications of other Obſervations may be examined.

By what hath been ſaid the like Phænomina of Water, and thin plates of Glaſs may be underſtood. But in ſmall fragments of thoſe plates, there is this

further

further obſervable, that where they lye flat upon a Table and are turned about their Centers whilſt they are viewed through a Priſm, they will in ſome poſtures exhibit waves of various Colours, and ſome of them exhibit theſe waves in one or two poſitions only, but the moſt of them do in all poſitions exhibit them, and make them for the moſt part appear almoſt all over the plates. The reaſon is, that the ſuperficies of ſuch plates are not even, but have many cavities and ſwellings, which how ſhallow ſoever do a little vary the thickneſs of the plate. For at the ſeveral ſides of thoſe cavities, for the reaſons newly deſcribed, there ought to be produced waves in ſeveral poſtures of the Priſm. Now though it be but ſome very ſmall, and narrower parts of the Glaſs, by which theſe waves for the moſt part are cauſed, yet they may ſeem to extend themſelves over the whole Glaſs, becauſe from the narroweſt of thoſe parts there are Colours of ſeveral Orders that is of ſeveral Rings, confuſedly reflected, which by refraction of the Priſm are unfolded, ſeparated, and according to their degrees of refraction, diſperſed to ſeveral places, ſo as to conſtitute ſo many ſeveral waves, as there were divers orders of Colours promiſcuouſly reflected from that part of the Glaſs.

Theſe are the principal Phænomena of thin Plates or Bubbles, whoſe explications depend on the properties of Light, which I have heretofore delivered. And theſe you ſee do neceſſarily follow from them, and agree with them, even to their very leaſt circumſtances; and not only ſo, but do very much tend to their proof. Thus, by the 24th Obſervation, it appears, that the

rays

rays of ſeveral Colours made as well by thin Plates or Bubbles, as by refractions of a Priſm, have ſeveral degrees of refrangibility, whereby thoſe of each order, which at their reflexion from the Plate or Bubble are intermixed with thoſe of other orders, are ſeparated from them by refraction, and aſſociated together ſo as to become viſible by themſelves like Arcs of Circles. For if the rays were all alike refrangible, 'tis impoſſible that the whiteneſs, which to the naked ſence appears uniform, ſhould by refraction have its parts tranſpoſed and ranged into thoſe black and white Arcs.

It appears alſo that the unequal refractions of difform rays proceed not from any contingent irregularities; ſuch as are veins, an uneven poliſh, or fortuitous poſition of the pores of Glaſs; unequal and caſual motions in the Air or Æther; the ſpreading, breaking, or dividing the ſame ray into many diverging parts, or the like. For, admitting any ſuch irregularities, it would be impoſſible for refractions to render thoſe Rings ſo very diſtinct, and well defined, as they do in the 24th Obſervation. It is neceſſary therefore that every ray have its proper and conſtant degree of refrangibility connate with it, according to which its refraction is ever juſtly and regularly performed, and that ſeveral rays have ſeveral of thoſe degrees.

And what is ſaid of their refrangibility may be alſo underſtood of their reflexibility, that is of their diſpoſitions to be reflected ſome at a greater, and others at a leſs thickneſs, of thin Plates or Bubbles, namely, that thoſe diſpoſitions are alſo connate with the rays, and immutable; as may appear by the 13th, 14th, and

15th

15th Obſervations compared with the fourth and eight~~h~~eenth.

By the precedent Obſervations it appears alſo, that whteneſs is a diſſimilar mixture of all Colours, and that Light is a mixture of rays indued with all thoſe Colours. For conſidering the multitude of the Rings of Colours, in the 3d, 12th and 24th Obſervations, it is manifeſt that although in the 4th and 18th Obſervations there appear no more than eight or nine of thoſe Rings, yet there are really a far greater number, which ſo much interfere and mingle with one another, as after thoſe eight or nine revolutions to dilute one another wholly, and conſtitute an even and ſenſibly uniform whiteneſs. And conſequently that whiteneſs muſt be allowed a mixture of all Colours, and the Light which conveys it to the Eye muſt be a mixture of rays indued with all thoſe Colours.

But further, by the 24th Obſervation, it appears, that there is a conſtant relation between Colours and Refrangibility, the moſt refrangible rays being violet, the leaſt refrangible red, and thoſe of intermediate Colours having proportionably intermediate degrees of refrangibility. And by the 13th, 14th and 15th Obſervations, compared with the 4th or 18th, there appears to be the ſame conſtant relation between Colour and Reflexibility, the violet being in like circumſtances reflected at leaſt thickneſſes of any thin Plate or Bubble, the red at greateſt thickneſſes, and the intermediate Colours at intermediate thickneſſes. Whence it follows, that the colorifique diſpoſitions of rays are alſo connate with them and immutable, and by conſequence

that

that all the productions and appearances of Colours in the World are derived not from any physical change caused in Light by refraction or reflexion, but only from the various mixtures or separations of rays, by virtue of their different Refrangibility or Reflexibility. And in this respect the Science of Colours becomes a Speculation as truly mathematical as any other part of Optiques. I mean so far as they depend on the nature of Light, and are not produced or altered by the power of imagination, or by striking or pressing the Eyes.

TH

Fig. 1.

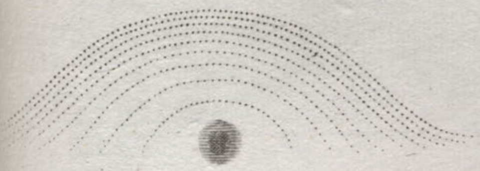


Fig. 2.

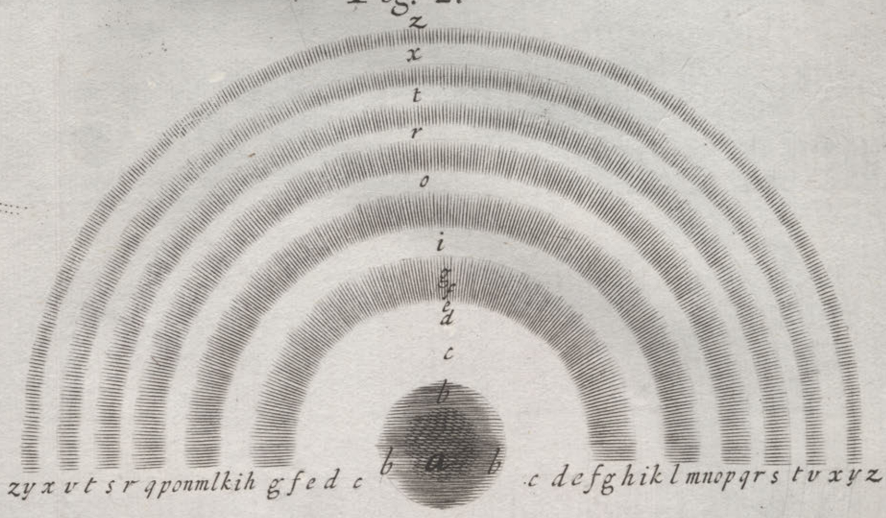


Fig. 3.

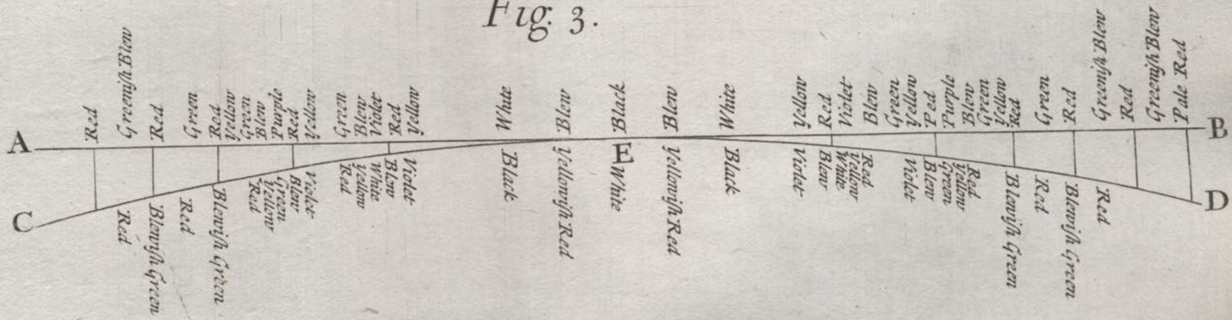


Fig. 4.

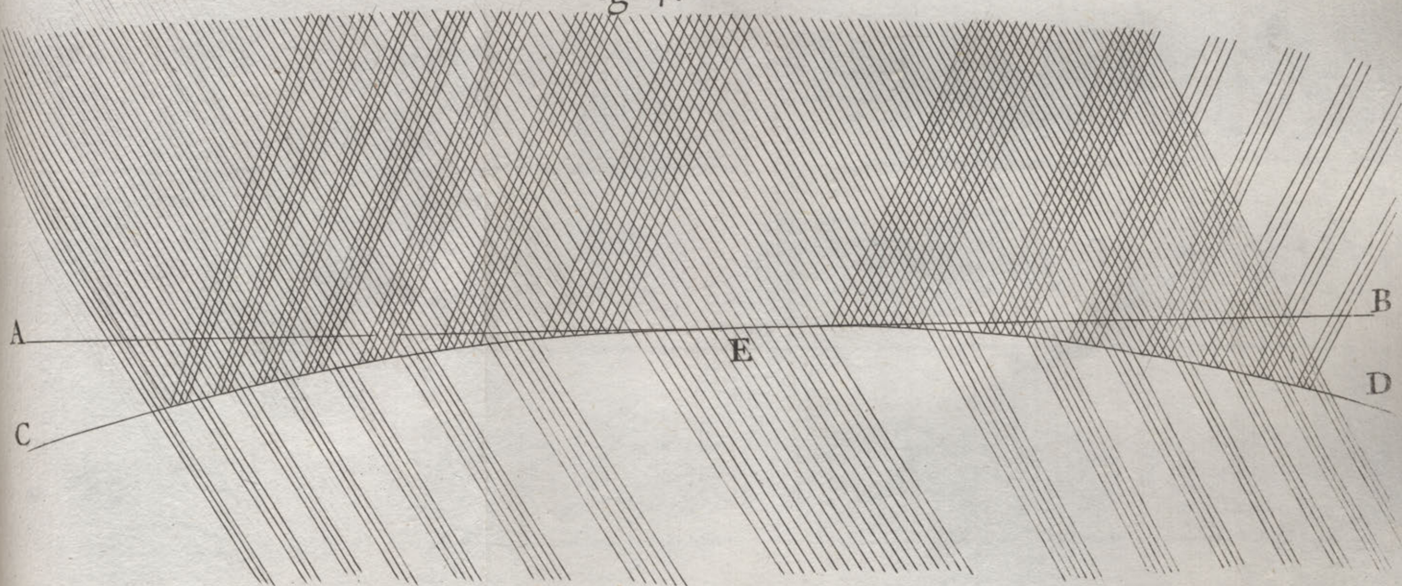
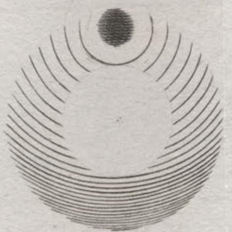


Fig. 5.



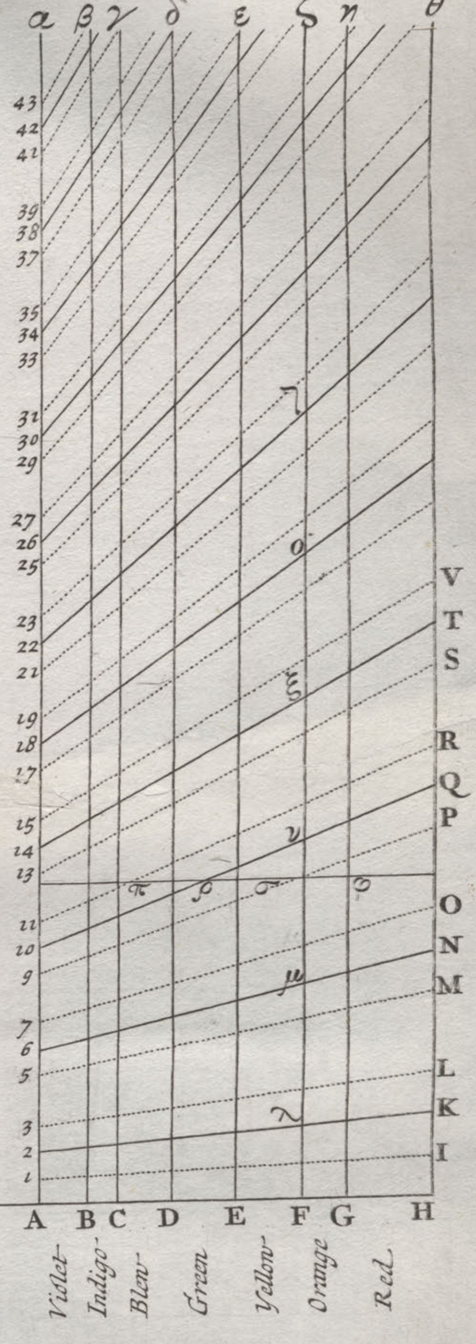


Fig. 6.

Fig. 7.

THE SECOND BOOK OF OPTICKS.

PART III.

Of the permanent Colours of natural Bodies, and the Analogy between them and the Colours of thin transparent Plates.

I Am now come to another part of this Deſign, which is to conſider how the Phænomena of thin tranſparent Plates ſtand related to thoſe of all other natural Bodies. Of theſe Bodies I have already told you that they appear of divers Colours, accordingly as they are diſpoſed to reflect moſt copiouſly the rays originally indued with thoſe Colours. But their Conſtitutions, whereby they reflect ſome rays more copiouſly than others, remains to be diſcovered, and theſe I ſhall endeavour to manifeſt in the following Propoſitions.

PROP. I.

Those superficies of transparent Bodies reflect the greatest quantity of Light, which have the greatest refracting power; that is, which intercede mediums that differ most in their refractive densities. And in the confines of equally refracting mediums there is no reflexion.

The Analogy between reflexion and refraction will appear by considering, that when Light passeth obliquely out of one medium into another which refracts from the perpendicular, the greater is the difference of their refractive density, the less obliquity is requisite to cause a total reflexion. For as the Sines are which measure the refraction, so is the Sine of incidence at which the total reflexion begins, to the radius of the Circle, and consequently that angle of incidence is least where there is the greatest difference of the Sines. Thus in the passing of Light out of Water into Air, where the refraction is measured by the Ratio of the Sines 3 to 4, the total reflexion begins when the Angle of incidence is about 48 degrees 35 minutes. In passing out of Glass into Air, where the refraction is measured by the Ratio of the Sines 20 to 31, the total reflexion begins when the Angle of incidence is 40 deg. 10 min. and so in passing out of crystal, or more strongly refracting mediums into Air, there is still a less obliquity requisite to cause a total reflexion. Superficies therefore which refract most do soonest reflect all the Light which is incident on them, and so must be allowed most strongly reflexive.

But

But the truth of this Propoſition will further appear by obſerving, that in the ſuperficies interceding two tranſparent mediums, ſuch as are (Air, Water, Oyl, Common-Glaſs, Cryſtal, Metalline-Glaſſes, Iſland-Glaſſes, white tranſparent Arſnick, Diamonds, &c.) the reflexion is ſtronger or weaker accordingly, as the ſuperficies hath a greater or leſs refracting power. For in the confine of Air and Sal-gemm 'tis ſtronger than in the confine of Air and Water, and ſtill ſtronger in the confine of Air and Common-Glaſs or Cryſtal, and ſtronger in the confine of Air and a Diamond. If any of theſe, and ſuch like tranſparent Solids, be immerged in Water, its reflexion becomes much weaker than before, and ſtill weaker if they be immerged in the more ſtrongly refracting Liquors of well-rectified oyl of Vitriol or ſpirit of Turpentine. If Water be diſtinguiſhed into two parts, by any imaginary ſurface, the reflexion in the confine of thoſe two parts is none at all. In the confine of Water and Ice 'tis very little, in that of Water and Oyl 'tis ſomething greater, in that of Water and Sal-gemm ſtill greater, and in that of Water and Glaſs, or Cryſtal, or other denſer ſubſtances ſtill greater, accordingly as thoſe mediums differ more or leſs in their refracting powers. Hence in the confine of Common-Glaſs and Cryſtal, there ought to be a weak reflexion, and a ſtronger reflexion in the confine of Common and Metalline-Glaſs, though I have not yet tried this. But, in the confine of two Glaſſes of equal denſity, there is not any ſenſible reflexion, as was ſhewn in the firſt Obſervation. And the ſame may be underſtood of the ſuperficies interceding two Cryſtals, or two Liquors, or any other Subſtances in which no refraction is cauſed. So then the reason why uniform pellucid mediums, (such as Water, Glass, or Crystal) have no sensible reflexion but in their external superficies, where they are adjacent to other mediums of a different density, is because all their contiguous parts have one and the same degree of density.

PROP. II.

The least parts of almost all natural Bodies are in some measure transparent: And the opacity of those Bodies ariseth from the multitude of reflexions caused in their internal Parts.

That this is so has been observed by others, and will easily be granted by them that have been conversant with Miscroscopes. And it may be also tryed by applying any substance to a Hole through which some Light is immitted into a dark room. For how opake soever that substance may seem in the open Air, it will by that means appear very manifestly transparent, if it be of a sufficient thinness. Only white metalline Bodies must be excepted, which by reason of their excessive density seem to reflect almost all the Light incident on their first superficies, unless by solution in menstruums they be reduced into very small particles, and then they become transparent.

PROP. III.

Between the parts of opake and coloured Bodies are many spaces, either empty or replenished, with mediums of other densities; as Water between the tinging corpuscles wherewith any Liquor is impregnated, Air between the

aqueous

aqueous globules that constitute Clouds or Mists; and for the most part spaces void of both Air and Water, but yet perhaps not wholly void of all substance, between the parts of hard Bodies.

The truth of this is evinced by the two precedent Propositions: For by the second Proposition there are many reflexions made by the internal parts of Bodies, which, by the first Proposition, would not happen if the parts of those Bodies were continued without any such interstices between them, because reflexions are caused only in superficies, which intercede mediums of a differing density by Prop. 1.

But further, that this discontinuity of parts is the principal cause of the opacity of Bodies, will appear by considering, that opake substances become transparent by filling their pores with any substance of equal or almost equal density with their parts. Thus Paper dipped in Water or Oyl, the *Oculus mundi* Stone steep'd in Water, Linnen-cloth oyled or varnished, and many other substances soaked in such Liquors as will intimately pervade their little pores, become by that means more transparent than otherwise; so, on the contrary, the most transparent substances may by evacuating their pores, or separating their parts, be rendred sufficiently opake, as Salts or wet Paper, or the *Oculus mundi* Stone by being dried, Horn by being scraped, Glass by being reduced to powder, or otherwise flawed, Turpentine by being stirred about with Water till they mix imperfectly, and Water by being formed into many small Bubbles, either alone in the form of froth, or by shaking it together with Oyl of Turpentine, or with some other convenient Liquor, with which it will

not

not perfectly incorporate. And to the increaſe of the opacity of theſe Bodies it conduces ſomething, that by the 23th Obſervation the reflexions of very thin tranſparent ſubſtances are conſiderably ſtronger than thoſe made by the ſame ſubſtances of a greater thickneſs.

PROP. IV.

The parts of Bodies and their Interſtices muſt not be leſs than of ſome definite bigneſs, to render them opake and coloured.

For the opakeſt Bodies, if their parts be ſubtily divided, (as Metals by being diſſolved in acid menſtruums, *&c.*) become perfectly tranſparent. And you may alſo remember, that in the eighth Obſervation there was no ſenſible reflexion at the ſuperficies of the Object-Glaſſes where they were very near one another, though they did not abſolutely touch. And in the 17th Obſervation the reflexion of the Water-bubble where it became thinneſt was almoſt inſenſible, ſo as to cauſe very black Spots to appear on the top of the Bubble by the want of reflected Light.

On theſe grounds I perceive it is that Water, Salt, Glaſs, Stones, and ſuch like ſubſtances, are tranſparent. For, upon divers conſiderations, they ſeem to be as full of pores or interſtices between their parts as other Bodies are, but yet their parts and interſtices to be too ſmall to cauſe reflexions in their common ſurfaces.

PROP.

PROP. V.

The transparent parts of Bodies according to their several sizes must reflect rays of one Colour, and transmit those of another, on the same grounds that thin Plates or Bubbles do reflect or transmit those rays. And this I take to be the ground of all their Colours.

For if a thin'd or plated Body, which being of an even thickness, appears all over of one uniform Colour, should be slit into threds, or broken into fragments, of the same thickness with the plate; I see no reason why every thred or fragment should not keep its Colour, and by consequence why a heap of those threds or fragments should not constitute a mass or powder of the same Colour, which the plate exhibited before it was broken. And the parts of all natural Bodies being like so many fragments of a Plate, must on the same grounds exhibit the same Colours.

Now that they do so, will appear by the affinity of their properties. The finely coloured Feathers of some Birds, and particularly those of Peacocks Tails, do in the very same part of the Feather appear of several Colours in several positions of the Eye, after the very same manner that thin Plates were found to do in the 7th and 19th Observations, and therefore arise from the thinness of the transparent parts of the Feathers; that is, from the slenderness of the very fine Hairs, or *Capillamenta*, which grow out of the sides of the grosser lateral branches or fibres of those Feathers. And to the same purpose it is, that the Webs of some Spiders by being

being ſpun very fine have appeared coloured, as ſome have obſerved, and that the coloured fibres of ſome ſilks by varying the poſition of the Eye do vary their Colour. Alſo the Colours of ſilks, cloths, and other ſubſtances, which Water or Oyl can intimately penetrate, become more faint and obſcure by being immerged in thoſe liquors, and recover their vigor again by being dried, much after the manner declared of thin Bodies in the 10th and 21th Obſervations. Leaf-gold, ſome ſorts of painted Glaſs, the infuſion of *Lignum Nephriticum*, and ſome other ſubſtances reflect one Colour, and tranſmit another, like thin Bodies in the 9th and 20th Obſervations. And ſome of thoſe coloured powders which Painters uſe, may have their Colours a little changed, by being very elaborately and finely ground. Where I ſee not what can be juſtly pretended for thoſe changes, beſides the breaking of their parts into leſs parts by that contrition after the ſame manner that the Colour of a thin Plate is changed by varying its thickneſs. For which reaſon alſo it is that the coloured flowers of Plants and Vegitables by being bruiſed uſually become more tranſparent than before, or at leaſt in ſome degree or other change their Colours. Nor is it much leſs to my purpoſe, that by mixing divers liquors very odd and remarquable productions and changes of Colours may be effected, of which no cauſe can be more obvious and rational than that the ſaline corpuſcles of one liquor do variouſly act upon or unite with the tinging corpuſcles of another, ſo as to make them ſwell, or ſhrink (whereby not only their bulk but their denſity alſo may be changed) or to divide them into ſmaller corpuſcles, (whereby a coloured liquor may become

come tranſparent) or to make many of them aſſociate into one cluſter, whereby two tranſparent liquors may compoſe a coloured one. For we ſee how apt thoſe ſaline menſtruums are to penetrate and diſſolve ſubſtances to which they are applied, and ſome of them to precipitate what others diſſolve. In like manner, if we conſider the various Phænomena of the Atmoſphære, we may obſerve, that when Vapors are firſt raiſed, they hinder not the tranſparency of the Air, being divided into parts too ſmall to cauſe any reflexion in their ſuperficies. But when in order to compoſe drops of rain they begin to coaleſce and conſtitute globules of all intermediate ſizes, thoſe globules when they become of a convenient ſize to reflect ſome Colours and tranſmit others, may conſtitute Clouds of various Colours according to their ſizes. And I ſee not what can be rationally conceived in ſo tranſparent a ſubſtance as Water for the production of theſe Colours, beſides the various ſizes of its fluid and globuler parcels.

PROP. VI.

The parts of Bodies on which their Colours depend, are denſer than the medium, which pervades their interſtices.

This will appear by conſidering, that the Colour of a Body depends not only on the rays which are incident perpendicularly on its parts, but on thoſe alſo which are incident at all other Angles. And that according to the 7th Obſervation, a very little variation of obliquity will change the reflected Colour where the thin body or ſmall particle is rarer than the ambient medium, insomuch that such a small particle will at diversly oblique incidences reflect all sorts of Colours, in so great a variety that the Colour resulting from them all, confusedly reflected from a heap of such particles, must rather be a white or grey than any other Colour, or at best it must be but a very imperfect and dirty Colour. Whereas if the thin body or small particle be much denser than the ambient medium, the Colours according to the 19th Observation are so little changed by the variation of obliquity, that the rays which are reflected least obliquely may predominate over the rest so much as to cause a heap of such particles to appear very intensly of their Colour.

It conduces also something to the confirmation of this Proposition, that, according to the 22th Observation, the Colours exhibited by the denser thin body within the rarer, are more brisque than those exhibited by the rarer within the denser.

PROP. VII.

The bigness of the component parts of natural Bodies may be conjectured by their Colours.

For since the parts of these Bodies by Prop. 5. do most probably exhibit the same Colours with a Plate of equal thickness, provided they have the same refractive density; and since their parts seem for the most part to have much the same density with Water or Glass, as by many circumstances is obvious to collect; to determine the sizes of those parts you need only have recourse to the precedent Tables, in which the thickness of Water or Glass exhibiting any Colour is expressed. Thus

if

if it be desired to know the Diameter of a corpuscle, which being of equal density with Glass shall reflect green of the third order; the number $16\frac{1}{4}$ shews it to be $\frac{16\frac{1}{4}}{100000}$ parts of an Inch.

The greatest difficulty is here to know of what order the Colour of any Body is. And for this end we must have recourse to the 4th and 18th Observations, from whence may be collected these particulars.

Scarlets, and other *reds*, *oranges* and *yellows*, if they be pure and intense are most probably of the second order. Those of the first and third order also may be pretty good, only the yellow of the first order is faint, and the orange and red of the third order have a great mixture of violet and blue.

There may be good *greens* of the fourth order, but the purest are of the third. And of this order the green of all vegitables seem to be, partly by reason of the intensenes of their Colours, and partly because when they wither some of them turn to a greenish yellow, and others to a more perfect yellow or orange, or perhaps to red, passing first through all the aforesaid intermediate Colours. Which changes seem to be effected by the exhaling of the moisture which may leave the tinging corpuscles more dense, and something augmented by the accretion of the oyly and earthy part of that moisture. Now the green without doubt is of the same order with those Colours into which it changeth, because the changes are gradual, and those Colours, though usually not very full, yet are often too full and lively to be of the fourth order.

Blues and *purples* may be either of the ſecond or third order, but the beſt are of the third. Thus the Colour of violets ſeems to be of that order, becauſe their Syrup by acid Liquors turns red, and by urinous and alcalizale turns green. For ſince it is of the nature of Acids to diſſolve or attenuate, and of Alcalies to precipitate or incraſſate, if the purple Colour of the Syrup was of the ſecond order, an acid Liquor by attenuating its tinging corpuſcles would change it to a red of the firſt order, and an Alcaly by incraſſating them would change it to a green of the ſecond order; which red and green, eſpecially the green, ſeem too imperfect to be the Colours produced by theſe changes. But if the ſaid purple be ſuppoſed of the third order, its change to red of the ſecond, and green of the third, may without any inconvenience be allowed.

If there be found any Body of a deeper and leſs reddiſh purple than that of the violets, its Colour moſt probably is of the ſecond order. But yet their being no Body commonly known whoſe Colour is conſtantly more deep than theirs, I have made uſe of their name to denote the deepeſt and leaſt reddiſh purples, ſuch as manifeſtly tranſcend their Colour in purity.

The *blue* of the firſt order, though very faint and little, may poſſibly be the Colour of ſome ſubſtances; and particularly the azure Colour of the Skys ſeems to be of this order. For all vapours when they begin to condenſe and coaleſce into ſmall parcels, become firſt of that bigneſs whereby ſuch an Azure muſt be reflected before they can conſtitute Clouds of other Colours. And ſo this being the firſt Colour which vapors begin to reflect, it ought to be the Colour of the fineſt and moſt tranſparent Skys in which vapors are not arrived to that groſneſs requiſite to reflect other Colours, as we find it is by experience.

Whiteneſs, if moſt intenſe and luminous, is that of the firſt order, if leſs ſtrong and luminous a mixture of the Colours of ſeveral orders. Of this laſt kind is the whiteneſs of Froth, Paper, Linnen, and moſt white ſubſtances; of the former I reckon that of white metals to be. For whilſt the denſeſt of metals, Gold, if foliated is tranſparent, and all metals become tranſparent if diſſolved in menſtruums or vitrified, the opacity of white metals ariſeth not from their denſity alone. They being leſs denſe than Gold would be more tranſparent than it, did not ſome other cauſe concur with their denſity to make them opake. And this cauſe I take to be ſuch a bigneſs of their particles as fits them to reflect the white of the firſt order. For if they be of other thickneſſes they may reflect other Colours, as is manifeſt by the Colours which appear upon hot Steel in tempering it, and ſometimes upon the ſurface of melted metals in the Skin or Scoria which ariſes upon them in their cooling. And as the white of the firſt order is the ſtrongeſt which can be made by Plates of tranſparent ſubſtances, ſo it ought to be ſtronger in the denſer ſubſtances of metals than in the rarer of Air, Water and Glaſs. Nor do I ſee but that metallic ſubſtances of ſuch a thickneſs as may fit them to reflect the white of the firſt order, may, by reaſon of their great denſity (according to the tenour of the firſt of theſe Propoſitions) reflect all the Light incident upon them, and ſo be as opake and ſplendent as its poſſible for any Body to be. Gold, or Copper mixed with leſs than half their weight

of

of Silver, or Tin, or Regulus of Antimony, in fusion or amalgamed with a very little Mercury become white; which shews both that the particles of white metals have much more superficies, and so are smaller, than those of Gold and Copper, and also that they are so opake as not to suffer the particles of Gold or Copper to shine through them. Now it is scarce to be doubted, but that the Colours of Gold and Copper are of the second or third order, and therefore the particles of white metals cannot be much bigger than is requisite to make them reflect the white of the first order. The volatility of Mercury argues that they are not much bigger, nor may they be much less, least they lose their opacity, and become either transparent as they do when attenuated by vitrification, or by solution in menstruums, or black as they do when ground smaller, by rubbing Silver,or Tin, or Lead, upon other substances to draw black Lines. The first and only Colour which white metals take by grinding their particles smaller is black, and therefore their white ought to be that which borders upon the black Spot in the center of the Rings of Colours, that is, the white of the first order. But if you would hence gather the bigness of metallic particles, you must allow for their density. For were Mercury transparent, its density is such that the Sine of incidence upon it (by my computation) would be to the sine of its refraction, as 71 to 20, or 7 to 2. And therefore the thickness of its particles, that they may exhibit the same Colours with those of Bubbles of Water, ought to be less than the thickness of the Skin of those Bubbles in the proportion of 2 to 7. Whence its possible that the particles of Mercury may be as little

as

as the particles of ſome tranſparent and volatile fluids, and yet reflect the white of the firſt order.

Laſtly, for the production of *black*, the corpuſcles muſt be leſs than any of thoſe which exhibit Colours. For at all greater ſizes there is too much Light reflected to conſtitute this Colour. But if they be ſuppoſed a little leſs than is requiſite to reflect the white and very faint blue of the firſt order, they will, according to the 4th, 8th, 17th and 18th Obſervations, reflect ſo very little as to appear intenſly black, and yet may perhaps variouſly refract it to and fro within themſelves ſo long, until it happen to be ſtifled and loſt, by which means they will appear black in all poſitions of the Eye without any tranſparency. And from hence may be underſtood why Fire, and the more ſubtile diſſolver Putrefaction, by dividing the particles of ſubſtances, turn them to black, why ſmall quantities of black ſubſtances impart their Colour very freely and intenſly to other ſubſtances to which they are applied; the minute particles of theſe, by reaſon of their very great number, eaſily overſpreading the groſs particles of others; why Glaſs ground very elaborately with Sand on a copper Plate, 'till it be well poliſhed, makes the Sand, together with what is worn off from the Glaſs and Copper, become very black: why black ſubſtances do ſooneſt of all others become hot in the Sun's Light and burn, (which effect may proceed partly from the multitude of refractions in a little room, and partly from the eaſy commotion of ſo very ſmall corpuſcles;) and why blacks are uſually a little inclined to a bluiſh Colour. For that they are ſo may be ſeen by illuminating white Paper by Light reflected from black ſub-

ſtances.

ſtances. For the Paper will uſually appear of a bluiſh white; and the reaſon is, that black borders on the obſcure blue of the firſt order deſcribed in the 18th Obſervation, and therefore reflects more rays of that Colour than of any other.

In theſe Deſcriptions I have been the more particular, becauſe it is not impoſſible but that Miſcroſcopes may at length be improved to the diſcovery of the particles of Bodies on which their Colours depend, if they are not already in ſome meaſure arrived to that degree of perfection. For if thoſe Inſtruments are or can be ſo far improved as with ſufficient diſtinctneſs to repreſent Objects five or ſix hundred times bigger than at a Foot diſtance they appear to our naked Eyes, I ſhould hope that we might be able to diſcover ſome of the greateſt of thoſe corpuſcles. And by one that would magnify three or four thouſand times perhaps they might all be diſcovered, but thoſe which produce blackneſs. In the mean while I ſee nothing material in this Diſcourſe that may rationally be doubted of excepting this Poſition, That tranſparent corpuſcles of the ſame thickneſs and denſity with a Plate, do exhibit the ſame Colour. And this I would have underſtood not without ſome latitude, as well becauſe thoſe corpuſcles may be of irregular Figures, and many rays muſt be obliquely incident on them, and ſo have a ſhorter way through them than the length of their Diameters, as becauſe the ſtraitneſs of the medium pent in on all ſides within ſuch corpuſcles may a little alter its motions or other qualities on which the reflexion depends. But yet I cannot much ſuſpect the laſt, becauſe I have obſerved of ſome ſmall Plates of Muſcovy-Glaſs which were of an

even

even thickneſs, that through a Miſcroſcope they have appeared of the ſame Colour at their edges and corners where the included medium was terminated, which they appeared of in other places. However it will add much to our ſatisfaction, if thoſe corpuſcles could be diſcovered with Miſcroſcopes; which if we ſhall at length attain to, I fear it will be the utmoſt improvement of this ſenſe. For it ſeems impoſſible to ſee the more ſecret and noble works of nature within the corpuſcles by reaſon of their tranſparency.

PROP. VIII.

The cauſe of Reflexion is not the impinging of Light on the ſolid or impervious parts of Bodies, as is commonly believed.

This will appear by the following Conſiderations. Firſt, That in the paſſage of Light out of Glaſs into Air there is a reflexion as ſtrong as in its paſſage out of Air into Glaſs, or rather a little ſtronger, and by many degrees ſtronger than in its paſſage out of Glaſs into Water. And it ſeems not probable that Air ſhould have more reflecting parts than Water or Glaſs. But if that ſhould poſſibly be ſuppoſed, yet it will avail nothing; for the reflexion is as ſtrong or ſtronger when the Air is drawn away from the Glaſs, (ſuppoſe in the Air-pump invented by Mr. *Boyle*) as when it is adjacent to it. Secondly, If Light in its paſſage out of Glaſs into Air be incident more obliquely than at an Angle of 40 or 41 degrees it is wholly reflected, if leſs obliquely it is in great meaſure tranſmitted. Now it is not to be imagined that Light at one degree of obliquity ſhould meet with pores enough in the Air to tranſmit the greater part of it, and at another degree of obliquity ſhould meet with nothing but parts to reflect it wholly, eſpecially conſidering that in its paſſage out of Air into Glaſs, how oblique ſoever be its incidence, it finds pores enough in the Glaſs to tranſmit ~~the~~ a greateſt part of it. If any Man ſuppoſe that it is not reflected by the Air, but by the outmoſt ſuperficial parts of the Glaſs, there is ſtill the ſame difficulty: Beſides, that ſuch a Suppoſition is unintelligible, and will alſo appear to be falſe by applying Water behind ſome part of the Glaſs inſtead of Air. For ſo in a convenient obliquity of the rays ſuppoſe of 45 or 46 degrees, at which they are all reflected where the Air is adjacent to the Glaſs, they ſhall be in great meaſure tranſmitted where the Water is adjacent to it; which argues, that their reflexion or tranſmiſſion depends on the conſtitution of the Air and Water behind the Glaſs, and not on the ſtriking off the rays upon the parts of the Glaſs. Thirdly, If the Colours made by a Priſm placed at the entrance of a beam of Light into a darkened room be ſucceſſively caſt on a ſecond Priſm placed at a greater diſtance from the former, in ſuch manner that they are all alike incident upon it, the ſecond Priſm may be ſo inclined to the incident rays, that thoſe which are of a blue Colour ſhall be all reflected by it, and yet thoſe of a red Colour pretty copiouſly tranſmitted. Now if the reflexion be cauſed by the parts of Air or Glaſs, I would aſk, why at the ſame obliquity of incidence the blue ſhould wholly impinge on thoſe parts ſo as to be all reflected, and yet the red find pores enough to be in great meaſure tranſmitted. Fourthly, where two Glaſſes touch one another,

another, there is no ſenſible reflexion as was declared in the firſt Obſervation; and yet I ſee no reaſon why the rays ſhould not impinge on the parts of Glaſs as much when contiguous to other Glaſs as when contiguous to Air. Fifthly, When the top of a Water-bubble (in the 17th Obſervation) by the continual ſubſiding and exhaling of the Water grew very thin, there was ſuch a little and almoſt inſenſible quantity of Light reflected from it, that it appeared intenſly black; whereas round about that black Spot, where the Water was thicker, the reflexion was ſo ſtrong as to make the Water ſeem very white. Nor is it only at the leaſt thickneſs of thin Plates or Bubbles, that there is no manifeſt reflexion, but at many other thickneſſes continually greater and greater. For in the 15th Obſervation the rays of the ſame Colour were by turns tranſmitted at one thickneſs, and reflected at another thickneſs, for an indeterminate number of ſucceſſions. And yet in the ſuperficies of the thinned Body, where it is of any one thickneſs, there are as many parts for the rays to impinge on, as where it is of any other thickneſs. Sixthly, If reflexion were cauſed by the parts of reflecting Bodies, it would be impoſſible for thin Plates or Bubbles at the ſame place to reflect the rays of one Colour and tranſmit thoſe of another, as they do according to the 13th and 15th Obſervations. For it is not to be imagined that at one place the rays which for inſtance exhibit a blue Colour, ſhould have the fortune to daſh upon the parts, and thoſe which exhibit a red to hit upon the pores of the Body; and then at another place, where the Body is either a little thicker, or a little thinner, that on the contrary the blue ſhould hit upon its pores, and the red upon its parts. Laſtly, were the rays of Light reflected by impinging on the ſolid parts of Bodies, their reflexions from poliſhed Bodies could not be ſo regular as they are. For in poliſhing Glaſs with Sand, Putty or Tripoly, it is not to be imagined that thoſe ſubſtances can by grating and fretting the Glaſs bring all its leaſt particles to an accurate poliſh ; ſo that all their ſurfaces ſhall be truly plain or truly ſpherical, and look all the ſame way, ſo as together to compoſe one even ſurface. The ſmaller the particles of thoſe ſubſtances are, the ſmaller will be the ſcratches by which they continually fret and wear away the Glaſs until it be poliſhed, but be they never ſo ſmall they can wear away the Glaſs no otherwiſe than by grating and ſcratching it, and breaking the proturberances , and therefore poliſh it no otherwiſe than by bringing its roughneſs to a very fine Grain, ſo that the ſcratches and frettings of the ſurface become too ſmall to be viſible. And therefore if Light were reflected by impinging upon the ſolid parts of the Glaſs, it would be ſcattered as much by the moſt poliſhed Glaſs as by the rougheſt. So then it remains a Problem, how Glaſs poliſhed by fretting ſubſtances can reflect Light ſo regularly as it does. And this Problem is ſcarce otherwiſe to be ſolved than by ſaying, that the reflexion of a ray is effected, not by a ſingle point of the reflecting Body, but by ſome power of the Body which is evenly diffuſed all over its ſurface, and by which it acts upon the ray without immediate contact. For that the parts of Bodies do act upon Light at a diſtance ſhall be ſhewn hereafter.

Now if Light be reflected not by impinging on the ſolid parts of Bodies, but by ſome other principle ; its probable that as many of its rays as impinge on the ſolid parts of Bodies are not reflected but ſtifled and loſt in the Bodies. For otherwiſe we muſt allow two ſorts of reflexions. Should all the rays be reflected which impinge on the internal parts of clear Water or Cryſtal, thoſe ſubſtances would rather have a cloudy Colour than a clear tranſparency. To make Bodies look black, its neceſſary that many rays be ſtopt, retained and loſt in them, and it ſeems not probable that any rays can be ſtopt and ſtifled in them which do not impinge on their parts.

And hence we may underſtand that Bodies are much more rare and porous than is commonly believed. Water is 19 times lighter, and by conſequence 19 times rarer than Gold, and Gold is ſo rare as very readily and without the leaſt oppoſition to tranſmit the magnetick Effluvia, and eaſily to admit Quick-ſilver into its pores, and to let Water paſs through it. For a concave Sphere of Gold filled with Water, and ſodered up, has upon preſſing the Sphere with great force, let the Water ſqueeze through it, and ſtand all over its outſide in multitudes of ſmall Drops, like dew, without burſting or cracking the Body of the Gold as I have been informed by an Eye-witneſs. From all which we may conclude, that Gold has more pores than ſolid parts, and by conſequence that Water has above forty-times more pores than parts. And he that ſhall find out an Hypotheſis, by which Water may be ſo rare, and yet not be capable of compreſſion by force, may doubtleſs by the ſame Hypotheſis make Gold and Water, and all

other

other Bodies as much rarer as he pleases, so that Light may find a ready passage through transparent substances.

PROP. IX.

Bodies reflect and refract Light by one and the same power variously exercised in various circumstances.

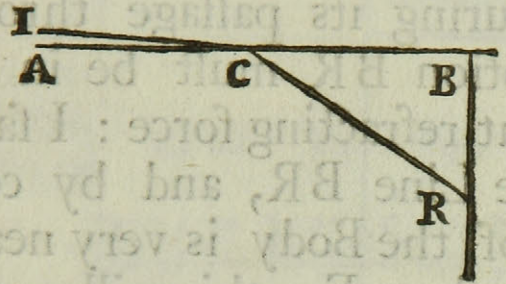
This appears by several Considerations. First, Because when Light goes out of Glass into Air, as obliquely as it can possibly do, if its incidence be made still more oblique, it becomes totally reflected. For the power of the Glass after it has refracted the Light as obliquely as is possible if the incidence be still made more oblique, becomes too strong to let any of its rays go through, and by consequence causes total reflexions. Secondly, Because Light is alternately reflected and transmitted by thin Plates of Glass for many successions accordingly, as the thickness of the Plate increases in an arithmetical Progression. For here the thickness of the Glass determines whether that power by which Glass acts upon Light shall cause it to be reflected, or suffer it to be transmitted. And, Thirdly, because those surfaces of transparent Bodies which have the greatest refracting power, reflect the greatest quantity of Light, as was shewed in the first Proposition.

PROP. X.

If Light be swifter in Bodies than in Vacuo in the proportion of the Sines which measure the refraction of the Bodies, the forces of the Bodies to reflect and refract Light, are

are very nearly proportional to the densities of the same Bodies, excepting that unctuous and sulphureous Bodies refract more than others of this same density.

Let A B represent the refracting plane surface of any Body, and I C a ray incident very obliquely upon the



Body in C, so that the Angle A C I may be infinitely little, and let C R be the refracted ray. From a given point B perpendicular to the refracting surface erect B R meeting with the refracted ray C R in R, and if C R represent the motion of the refracted ray, and this motion be distinguished into two motions C B and B R, whereof C B is a parallel to the refracting plane, and B R perpendicular to it: C B shall represent the motion of the incident ray, and B R the motion generated by the refraction, as Opticians have of late explained.

Now if any body or thing in moving through any space of a giving breadth terminated on both sides by two parallel plains, be urged forward in all parts of that space by forces tending directly forwards towards the last plain, and before its incidence on the first plane, had no motion towards it, or but an infinitly little one; and if the forces in all parts of that space, between the planes be at equal distances from the planes equal to one another, but at several distances be bigger or less in any given proportion, the motion generated by the forces in the whole passage of the body or thing through

through that ſpace ſhall be in a ſubduplicate proportion of the forces, as Mathematicians will eaſily underſtand. And therefore if the ſpace of activity of the refracting ſuperficies of the Body be conſidered as ſuch a ſpace, the motion of the ray generated by the refracting force of the Body, during its paſſage through that ſpace that is the motion BR muſt be in a ſubduplicate proportion of that refracting force: I ſay therefore that the ſquare of the Line BR, and by conſequence the refracting force of the Body is very nearly as the denſity of the ſame Body. For this will appear by the following Table, wherein the proportion of the Sines which meaſure the refraxions of ſeveral Bodies, the ſquare of BR ſuppoſing CB an unite, the denſities of the Bodies eſtimated by their ſpecifick gravities, and their refractive power in reſpect of their denſities are ſet down in ſeveral Columns.

The

The refracting Bodies.	*The Proportion of the Sines of incidence and refraction of yellow Light.*	*The Square of* B R, *to which the refracting force of the Body is proportionate.*	*The density and specific gravity of the Body.*	*The refractive power of the Body in respect of its density.*
A Pseudo-Topazius, being a natural, pellucid, brittle, hairy Stone, of a yellow Colour	23 to 14	1'699	4'27	3979
Air	3101 to 3100 ~~3851 to 3850~~	0'000646 ~~0'00052~~	0'0012 ~~0'00125~~	5383 ~~4160~~
Glass of Antimony	17 to 9	2'568	5'28	4864
A Selenitis	61 to 41	1'213	2'252	5386
Glass vulgar	31 to 20	1'4025	2'58	5436
Crystal of the Rock	25 to 16	1'445	2'65	5450
Island Crystal	5 to 3	1'778	2'72	6536
Sal Gemmæ	17 to 11	1'388	2'143	6477
Alume	35 to 24	1'1267	1'714	6570
Borax	22 to 15	1'1511	1'714	6716
Niter	32 to 21	1'345	1'9	7079
Dantzick Vitriol	303 to 200	1'295	1'715	7551
Oyl of Vitriol	10 to 7	1'041	1'7	6124
Rain Water	529 to 396	0'7845	1.	7845
Gumm Arabic	31 to 21	1'179	1'375	8574
Spirit of Wine well rectified	100 to 73	0'8765	0'866	10121
Camphire	3 to 2	1'25	0'996	12551
Oyl Olive	22 to 15	1'1511	0'913	12607
Lintseed Oyl	40 to 27	1'1948	0'932	12819
Spirit of Turpentine	25 to 17	1'1626	0'874	13222
Ambar	14 to 9	1'42	1'04	13654
A Diamond	100 to 41	4'949	3'4	14556

The refraction of the Air in this Table is determined by that of the Atmosphere observed by Astronomers. For if Light pass through many refracting substances or mediums gradually denser and denser, and terminated with parallel ſurfaces, the ſumm of all the refractions will be equal to the ſingle refraction which it would have ſuffered in paſſing immediately out of the firſt medium into the laſt. And this holds true, though the number of the refracting ſubſtances be increaſed to infinity, and the diſtances from one another as much decreaſed, ſo that the Light may be refracted in every point of its paſſage, and by continual refractions bent into a curve Line. And therefore the whole refraction of Light in paſſing through the Atmoſphere from the higheſt and rareſt part thereof down to the loweſt and denſeſt part, muſt be equal to the refraction which it would ſuffer in paſſing at like obliquity out of a Vacuum immediately into Air of equal denſity with that in the loweſt part of the Atmoſphere.

Now, by this Table, ~~the refractions of~~ a Pſeudo-Topaz, a Selenitis, Rock Cryſtal, Iſland Cryſtal, Vulgar Glaſs (that is, Sand melted together) and Glaſs of Antimony, which are terreſtrial ſtony alcalizate concretes, and Air which probably ariſes from ſuch ſubſtances by fermentation, though theſe be ſubſtances very differing from one another in denſity, yet they have their refractive powers almoſt in the ſame proportion to one another as their denſities are, excepting that the refraction of that ſtrange ſubſtance Iſland-Cryſtal is a little bigger than the reſt. And particularly Air, which is 3400 times rarer than the Pſeudo-Topaz, and 4200 times rarer than Glaſs of Antimony, has notwithſtanding its rarity the ſame refractive power in reſpect of its denſity which thoſe two very denſe ſubſtances have in reſpect of theirs, excepting ſo far as thoſe two differ from one another.

Again,

Again, ~~the refraction of~~ Camphire, Oyl-Olive, Lint-seed Oyl, Spirit of Turpentine and Amber, which are fat ſulphureous unctuous Bodies, and a Diamond, which probably is an unctuous ſubſtance coagulated, have their refractive powers in proportion to one another as their denſities without any conſiderable variation. But the refractive power of theſe unctuous ſubſtances is two or three times greater in reſpect of their denſities than the refractive powers of the former ſubſtances in reſpect of theirs.

Water has a refractive power in a middle degree between thoſe two ſorts of ſubſtances, and probably is of a middle nature. For out of it grow all vegetable and animal ſubſtances, which conſiſt as well of ſulphureous fat and inflamable parts, as of earthy lean and alcalizate ones.

Salts and Vitriols have refractive powers in a middle degree between thoſe of earthy ſubſtances and Water, and accordingly are compoſed of thoſe two ſorts of ſubſtances. For by diſtillation and rectification of their Spirits a great part of them goes into Water, and a great part remains behind in the form of a dry fixt earth capable of vitrification.

Spirit of Wine has a refractive power in a middle degree between thoſe of Water and oyly ſubſtances, and accordingly ſeems to be compoſed of both, united by fermentation ; the Water, by means of ſome ſaline Spirits with which 'tis impregnated, diſſolving the Oyl, and volatizing it by the action. For Spirit of Wine is inflamable by means of its oyly parts, and being diſtilled often from Salt of Tartar, grows by every diſtillation more and more aqueous and flegmatick. And

Chymists observe, that Vegitables (as Lavender, Rue, Marjoram, *&c.*) distilled *per se*, before fermentation yield Oyls without any burning Spirits, but after fermentation yield ardent Spirits without Oyls: Which shews, that their Oyl is by fermentation converted into Spirit. They find also, that if Oyls be poured in small quantity upon fermentating Vegetables, they distil over after fermentation in the form of Spirits.

So then, by the foregoing Table, all Bodies seem to have their refractive powers proportional to their densities, (or very nearly;) excepting so far as they partake more or less of sulphurous oyly particles, and thereby have their refractive power made greater or less. Whence it seems rational to attribute the refractive power of all Bodies chiefly, if not wholly, to the sulphurous parts with which they abound. For it's probable that all Bodies abound more or less with Sulphurs. And as Light congregated by a Burning-glass acts most upon sulphurous Bodies, to turn them into fire and flame; so, since all action is mutual, Sulphurs ought to act most upon Light. For that the action between Light and Bodies is mutual, may appear from this Consideration, That the densest Bodies which refract and reflect Light most strongly grow hottest in the Summer-Sun, by the action of the refracted or reflected Light.

I have hitherto explained the power of Bodies to reflect and refract, and shewed, that thin transparent plates, fibres and particles do, according to their several thicknesses and densities, reflect several sorts of rays, and thereby appear of several Colours, and by consequence that nothing more is requisite for producing all the

the Colours of natural Bodies than the ſeveral ſizes and denſities of their tranſparent particles. But whence it is that theſe plates, fibres and particles do, according to their ſeveral thickneſſes and denſities, reflect ſeveral ſorts of rays, I have not yet explained. To give ſome inſight into this matter, and make way for underſtanding the next Part of this Book, I ſhall conclude this Part with a few more Propoſitions. Thoſe which preceded reſpect the nature of Bodies, theſe the nature of Light: For both muſt be underſtood before the reaſon of their actions upon one another can be known. And becauſe the laſt Propoſition depended upon the velocity of Light, I will begin with a Propoſition of that kind.

PROP. XI.

Light is propagated from luminous Bodies in time, and ſpends about ſeven or eight minutes of an hour in paſſing from the Sun to the Earth.

This was obſerved firſt by *Romer*, and then by others, by means of the Eclipſes of the Satellites of *Jupiter*. For theſe Eclipſes, when the Earth is between the Sun and *Jupiter*, happen about ſeven or eight minutes ſooner than they ought to do by the Tables, and when the Earth is beyond the Sun they happen about ſeven or eight minutes later than they ought to do; the reaſon being, that the Light of the Satellites has farther to go in the latter caſe than in the former by the Diameter of the Earth's Orbit. Some inequalities of time may ariſe from the excentricities of the Orbs of the Satellites; but thoſe cannot anſwer in all the Satellites, and at all times to the position and distance of the Earth from the Sun. The mean motions of *Jupiter*'s Satellites is also swifter in his descent from his Aphelium to his Perihelium, than in his ascent in the other half of his Orb : But this inequality has no respect to the position of the Earth, and in the three interior Satellites is insensible, as I find by computation from the Theory of their gravity.

PROP. XII.

Every ray of Light in its passage through any refracting surface is put into a certain transient constitution or state, which in the progress of the ray returns at equal intervals, and disposes the ray at every return to be easily transmitted through the next refracting surface, and between the returns to be easily reflected by it.

This is manifest by the 5th, 9th, 12th and 15th Observations. For by those Observations it appears, that one and the same sort of rays at equal Angles of incidence on any thin transparent plate, is alternately reflected and transmitted for many successions accordingly, as the thickness of the plate increases in arithmetical progression of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, *&c.* so that if the first reflexion (that which makes the first or innermost of the Rings of Colours there described) be made at the thickness 1, the rays shall be transmitted at the thicknesses 0, 2, 4, 6, 8, 10, 12, *&c.* and thereby make the central Spot and Rings of Light, which appear by transmission, and be reflected at the thickness 1, 3, 5, 7, 9, 11, *&c.* and thereby make the Rings which appear

appear by reflexion. And this alternate reflexion and tranſmiſſion, as I gather by the 24th Obſervation, continues for above an hundred viciſſitudes, and by the the Obſervations in the next part of this Book, for many thouſands, being propagated from one ſurface of a Glaſs-plate to the other, though the thickneſs of the plate be a quarter of an Inch or above: So that this alternation ſeems to be propagated from every refracting ſurface to all diſtances without end or limitation.

This alternate reflexion and refraction depends on both the ſurfaces of every thin plate, becauſe it depends on their diſtance. By the 21th Obſervation, if either ſurface of a thin plate of Muſcovy-Glaſs be wetted, the Colours cauſed by the alternate reflexion and refraction grow faint, and therefore it depends on them both.

It is therefore performed at the ſecond ſurface, for if it were performed at the firſt, before the rays arrive at the ſecond, it would not depend on the ſecond.

It is alſo influenced by ſome action or diſpoſition, propagated from the firſt to the ſecond, becauſe otherwiſe at the ſecond it would not depend on the firſt. And this action or diſpoſition, in its propagation, intermits and returns by equal intervals, becauſe in all its progreſs it inclines the ray at one diſtance from the firſt ſurface to be reflected by the ſecond, at another to be tranſmitted by it, and that by equal intervals for innumerable viciſſitudes. And becauſe the ray is diſpoſed to reflexion at the diſtances 1, 3, 5, 7, 9, *&c.* and to tranſmiſſion at the diſtances 0, 2, 4, 6, 8, 10, *&c*, (for its tranſmiſſion through the firſt ſurface, is at the di-

ſtance

ſtance o, and it is tranſmitted through both together, if their diſtance be infinitely little or much leſs than 1) the diſpoſition to be tranſmitted at the diſtances 2, 4, 6, 8, 10, &c. is to be accounted a return of the ſame diſpoſition which the ray firſt had at the diſtance o, that is at its tranſmiſſion through the firſt refracting surface. All which is the thing I would prove.

What kind of action or diſpoſition this is? Whether it conſiſt in a circulating or a vibrating motion of the ray, or of the medium, or ſomething elſe? I do not here enquire. Thoſe that are averſe from aſſenting to any new diſcoveries, but ſuch as they can explain by an Hypotheſis, may for the preſent ſuppoſe, that as Stones by falling upon Water put the Water into an undulating motion, and all Bodies by percuſſion excite vibrations in the Air; ſo the rays of Light, by impinging on any refracting or reflecting ſurface, excite vibrations in the refracting or reflecting medium or ſubſtance, and by exciting them agitate the ſolid parts of the refracting or reflecting Body, and by agitating them cauſe the Body to grow warm or hot; that the vibrations thus excited are propagated in the refracting or reflecting medium or ſubſtance, much after the manner that vibrations are propagated in the Air for cauſing ſound, and move faſter than the rays ſo as to overtake them; and that when any ray is in that part of the vibration which conſpires with its motion, it eaſily breaks through a refracting ſurface, but when it is in the contrary part of the vibration which impedes its motion, it is eaſily reflected; and, by conſequence, that every ray is ſucceſſively diſpoſed to be eaſily reflected, or eaſily tranſmitted, by every vibration which overtakes it. But

whether

whether this Hypothesis be true or false I do not here consider. I content my self with the bare discovery, that the rays of Light are by some cause or other alternately disposed to be reflected or refracted for many vicissitudes.

DEFINITION.

The returns of the disposition of any ray to be reflected I will call its Fits of easy reflexion, *and those of its disposition to be transmitted its* Fits of easy transmission, *and the space it passes between every return and the next return*, *the* Interval of its Fits.

PROP. XIII.

The reason why the surfaces of all thick transparent Bodies reflect part of the Light incident on them, and refract the rest, is, that some rays at their incidence are in Fits of easy reflexion, and others in Fits of easy transmission.

This may be gathered from the 24th Observation, where the Light reflected by thin plates of Air and Glass, which to the naked Eye appeared evenly white all over the plate, did through a Prism appear waved with many successions of Light and Darkness made by alternate fits of easy reflexion and easy transmission, the Prism severing and distinguishing the waves of which the white reflected Light was composed, as was explained above.

And hence Light is in fits of eaſy reflexion and eaſy tranſmiſſion, before its incidence on tranſparent Bodies. And probably it is put into ſuch fits at its firſt emiſſion from luminous Bodies, and continues in them during all its progreſs. For theſe fits are of a laſting Nature, as will appear by the next part of this Book.

In this Propoſition I ſuppoſe the tranſparent Bodies to be thick, becauſe if the thickneſs of the Body be much leſs than the interval of the fits of eaſy reflexion and tranſmiſſion of the rays, the Body loſeth its reflecting power. For if the rays, which at their entering into the Body are put into fits of eaſy tranſmiſſion, arrive at the furtheſt ſurface of the Body before they be out of thoſe fits they muſt be tranſmitted. And this is the reaſon why Bubbles of Water loſe their reflecting power when they grow very thin, and why all opake Bodies when reduced into very ſmall parts become tranſparent.

PROP. XIV.

Thoſe ſurfaces of tranſparent Bodies, which if the ray be in a fit of refraction do refract it moſt ſtrongly, if the ray be in a fit of reflexion do reflect it moſt eaſily.

For we ſhewed above in Prop. 8. that the cauſe of reflexion is not the impinging of Light on the ſolid impervious parts of Bodies, but ſome other power by which thoſe ſolid parts act on Light at a diſtance. We ſhewed alſo in Prop. 9. that Bodies reflect and refract Light by one and the ſame power variouſly exerciſed in various circumſtances, and in Prop. 1. that the moſt ſtrongly refracting ſurfaces reflect the moſt Light: All which compared together evince and ratify both this and the last Proposition.

PROP. XV.

In any one and the same sort of rays emerging in any Angle out of any refracting surface into one and the same medium, the interval of the following fits of easy reflexion and transmission are either accurately or very nearly, as the Rectangle of the secant of the Angle of refraction, and of the secant of another Angle, whose sine is the first of 106 *arithmetical mean proportionals, between the sines of incidence and refraction counted from the sine of refraction.*

This is manifest by the 7th Observation.

PROP. XVI.

In several sorts of rays emerging in equal Angles out of any refracting surface into the same medium, the intervals of the following fits of easy reflexion and easy transmission are either accurately, or very nearly, as the Cube-roots of the Squares of the lengths of a Chord, which sound the notes in an Eight, sol, la, fa, sol, la, mi, fa, sol, *with all their intermediate degrees answering to the Colours of those rays, according to the Analogy described in the seventh Experiment of the second* part of the first *Book.*

This is manifest by the 13th and 14th Observations.

PROP. XVII.

If rays of any one ſort paſs perpendicularly into ſeveral mediums, the intervals of the fits of eaſy reflexion and tranſmiſſion in any one medium, is to thoſe intervals in any other as the ſine of incidence to the ſine of refraction, when the rays paſs out of the firſt of thoſe two mediums into the ſecond.

This is manifeſt by the 10th Obſervation.

PROP. XVIII.

If the rays which paint the Colour in the confine of yellow and orange paſs perpendicularly out of any medium into Air, the intervals of their fits of eaſy reflexion are the $\frac{1}{89000}$th part of an Inch. And of the ſame length are the intervals of their fits of eaſy tranſmiſſion.

This is manifeſt by the 6th Obſervation.

From theſe Propoſitions it is eaſy to collect the intervals of the fits of eaſy reflexion and eaſy tranſmiſſion of any ſort of rays refracted in any Angle into any medium, and thence to know, whether the rays ſhall be reflected or tranſmitted at their ſubſequent incidence upon any other pellucid medium. Which thing being uſeful for underſtanding, the next part of this Book was here to be ſet down. And for the ſame reaſon I add the two following Propoſitions.

PROP.

PROP. XIX.

If any ſort of rays falling on the polite ſurface of any pellucid medium be reflected back, the fits of eaſy reflexion which they have at the point of reflexion, ſhall ſtill continue to return, and the returns ſhall be at diſtances from the point of reflexion in the arithmetical progreſſion of the numbers 2, 4, 6, 8, 10, 12,&c. *and between theſe fits the rays ſhall be in fits of eaſy tranſmiſſion.*

For ſince the fits of eaſy reflexion and eaſy tranſmiſſion are of a returning nature, there is no reaſon why theſe fits, which continued till the ray arrived at the reflecting medium, and there inclined the ray to reflexion, ſhould there ceaſe. And if the ray at the point of reflexion was in a fit of eaſy reflexion, the progreſſion of the diſtances of theſe fits from that point muſt begin from 0, and ſo be of the numbers 0, 2, 4, 6, 8, *&c.* And therefore the progreſſion of the diſtances of the intermediate fits of eaſy tranſmiſſion reckoned from the ſame point, muſt be in the progreſſion of the odd numbers 1, 3, 5, 7, 9, *&c.* contrary to what happens when the fits are propagated from points of refraction.

PROP. XX.

The intervals of the fits of eaſy reflexion and eaſy tranſmiſſion, propagated from points of reflexion into any medium, are equal to the intervals of the like fits which the ſame rays would have, if refracted into the ſame medium

medium in Angles of refraction equal to their Angles of reflexion.

For when Light is reflected by the ſecond ſurface of thin plates, it goes out afterwards freely at the firſt ſurface to make the Rings of Colours which appear by reflexion, and by the freedom of its egreſs, makes the Colours of theſe Rings more vivid and ſtrong than thoſe which appear on the other ſide of the plates by the tranſmitted Light. The reflected rays are therefore in fits of eaſy tranſmiſſion at their egreſs; which would not always happen, if the intervals of the fits within the plate after reflexion were not equal both in length and number to their intervals before it. And this confirms alſo the proportions ſet down in the former Propoſition. For if the rays both in going in and out at the firſt ſurface be in fits of eaſy tranſmiſſion, and the intervals and numbers of thoſe fits between the firſt and ſecond ſurface, before and after reflexion, be equal; the diſtances of the fits of eaſy tranſmiſſion from either ſurface, muſt be in the ſame progreſſion after reflexion as before; that is, from the firſt ſurface which tranſmitted them, in the progreſſion of the even numbers 0, 2, 4, 6, 8, *&c.* and from the ſecond which reflected them, in that of the odd numbers 1, 3, 5, 7, *&c.* But theſe two Propoſitions will become much more evident by the Obſervations in the following part of this Book.

THE

THE SECOND BOOK OF OPTICKS.

PART IV.

Obſervations concerning the Reflexions and Colours of thick tranſparent poliſhed Plates.

THere is no Glaſs or Speculum how well ſoever poliſhed, but, beſides the Light which it refracts or reflects regularly, ſcatters every way irregularly a faint Light, by means of which the poliſhed ſurface, when illuminated in a dark Room by a beam of the Sun's Light, may be eaſily ſeen in all poſitions of the Eye. There are certain Phænomena of this ſcattered Light, which when I firſt obſerved them, ſeemed very ſtrange and ſurpriſing to me. My Obſervations were as follows.

OBS. I.

The Sun ſhining into my darkened Chamber through a Hole $\frac{1}{3}$ of an Inch wide, I let the intromitted beam of Light fall perpendicularly upon a Glaſs Speculum ground concave on one ſide and convex on the other, to a Sphere of five Feet and eleven Inches Radius, and quick-ſilvered over on the convex ſide. And holding a white opake Chart, or a Quire of Paper at the Center of the Spheres to which the Speculum was ground, that is, at the diſtance of about five Feet and eleven Inches from the Speculum, in ſuch manner, that the beam of Light might paſs through a little Hole made in the middle of the Chart to the Speculum, and thence be reflected back to the ſame Hole: I obſerved upon the Chart four or five concentric Iriſes or Rings of Colours, like Rain-bows, encompaſſing the Hole much after the manner that thoſe, which in the fourth and following Obſervations of the firſt part of this third Book appeared between the Object-Glaſſes, encompaſſed the black Spot, but yet larger and fainter than thoſe. Theſe Rings as they grew larger and larger became diluter and fainter, ſo that the fifth was ſcarce viſible. Yet ſometimes, when the Sun ſhone very clear, there appeared faint Lineaments of a ſixth and ſeventh. If the diſtance of the Chart from the Speculum was much greater or much leſs than that of ſix Feet, the Rings became dilute and vaniſhed. And if the diſtance of the Speculum from the Window was much greater than that of ſix Feet, the reflected beam of Light would be ſo broad at the diſtance of ſix Feet from the Speculum where the Rings appeared,

appeared, as to obſcure one or two of the innermoſt Rings. And therefore I uſually placed the Speculum at about ſix Feet from the Window; ſo that its Focus might there fall in with the center of its concavity at the Rings upon the Chart. And this poſture is always to be underſtood in the following Obſervations where no other is expreſt.

OBS. II.

The Colours of theſe Rain-bows ſucceeded one another from the center outwards, in the ſame form and order with thoſe which were made in the ninth Obſervation of the firſt Part of this Book by Light not reflected, but tranſmitted through the two Object-Glaſſes. For, firſt, there was in their common center a white round Spot of faint Light, ſomething broader than the reflected beam of Light; which beam ſometimes fell upon the middle of the Spot, and ſometimes by a little inclination of the Speculum receded from the middle, and left the Spot white to the center.

This white Spot was immediately encompaſſed with a dark grey or ruſſet, and that darkneſs with the Colours of the firſt Iris, which were on the inſide next the darkneſs a little violet and indico, and next to that a blue, which on the outſide grew pale, and then ſucceeded a little greeniſh yellow, and after that a brighter yellow, and then on the outward edge of the Iris a red which on the outſide inclined to purple.

This Iris was immediately encompaſſed with a ſecond, whoſe Colours were in order from the inſide outwards, purple, blue, green, yellow, light red, a red mixed with purple.

Then immediately followed the Colours of the third Iris, which were in order outwards a green inclining to purple, a good green, and a red more bright than that of the former Iris.

The fourth and fifth Iris ſeemed of a bluiſh green within, and red without, but ſo faintly that it was difficult to diſcern the Colours.

OBS. III.

Meaſuring the Diameters of theſe Rings upon the Chart as accurately as I could, I found them alſo in the ſame proportion to one another with the Rings made by Light tranſmitted through the two Object-Glaſſes. For the Diameters of the four firſt of the bright Rings meaſured between the brighteſt parts of their orbits, at the diſtance of ſix Feet from the Speculum were $1\frac{11}{16}$, $2\frac{3}{8}$, $2\frac{11}{12}$, $3\frac{3}{8}$ Inches, whoſe ſquares are in arithmetical progreſſion of the numbers 1, 2, 3, 4. If the white circular Spot in the middle be reckoned amongſt the Rings, and its central Light, where it ſeems to be moſt luminous, be put equipollent to an infinitely little Ring; the ſquares of the Diameters of the Rings will be in the progreſſion 0, 1, 2, 3, 4, *&c.* I meaſured alſo the Diameters of the dark Circles between theſe luminous ones, and found their ſquares in the progreſſion of the numbers $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, *&c.* the Diameters of the firſt four at the diſtance of ſix Feet from the Speculum, being $1\frac{3}{16}$, $2\frac{1}{16}$, $2\frac{2}{3}$, $3\frac{3}{20}$ Inches. If the diſtance of the Chart from the Speculum was increaſed

creaſed or diminiſhed, the Diameters of the Circles were increaſed or diminiſhed proportionally.

O B S. IV.

By the analogy between theſe Rings and thoſe deſcribed in the Obſervations of the firſt Part of this Book, I ſuſpected that there were many more of them which ſpread into one another, and by interfering mixed their Colours, and diluted one another ſo that they could not be ſeen apart. I viewed them therefore through a Priſm, as I did thoſe in the 24th Obſervation of the firſt Part of this Book. And when the Priſm was ſo placed as by refracting the Light of their mixed Colours to ſeparate them, and diſtinguiſh the Rings from one another, as it did thoſe in that Obſervation, I could then ſee them diſtincter than before, and eaſily number eight or nine of them, and ſometimes twelve or thirteen. And had not their Light been ſo very faint, I queſtion not but that I might have ſeen many more.

O B S. V.

Placing a Priſm at the Window to refract the intromitted beam of Light, and caſt the oblong Spectrum of Colours on the Speculum: I covered the Speculum with a black Paper which had in the middle of it a Hole to let any one of the Colours paſs through to the Speculum, whilſt the reſt were intercepted by the Paper. And now I found Rings of that Colour only which fell upon the Speculum. If the Speculum was illuminated with red the Rings were totally red with dark inter-

 vals,

vals, if with blue they were totally blue, and ſo of the other Colours. And when they were illuminated with any one Colour, the Squares of their Diameters meaſured between their moſt luminous parts, were in the arithmetical progreſſion of the numbers 0, 1, 2, 3, 4, and the Squares of the Diameters of their dark intervals in the progreſſion of the intermediate numbers $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$: But if the Colour was varied they varied their magnitude. In the red they were largeſt, in the indico and violet leaſt, and in the intermediate Colours yellow, green and blue; they were of ſeveral intermediate bigneſſes anſwering to the Colour, that is, greater in yellow than in green, and greater in green than in blue. And hence I knew that when the Speculum was illuminated with white Light, the red and yellow on the outſide of the Rings were produced by the leaſt refrangible rays, and the blue and violet by the moſt refrangible, and that the Colours of each Ring ſpread into the Colours of the neighbouring Rings on either ſide, after the manner explained in the firſt and ſecond Part of this Book, and by mixing diluted one another ſo that they could not be diſtinguiſhed, unleſs near the center where they were leaſt mixed. For in this Obſervation I could ſee the Rings more diſtinctly, and to a greater number than before, being able in the yellow Light to number eight or nine of them, beſides a faint ſhadow of a tenth. To ſatisfy my ſelf how much the Colours of the ſeveral Rings ſpread into one another, I meaſured the Diameters of the ſecond and third Rings, and found them when made by the confine of the red and orange to be to the ſame Diameters when made by the confine of blue and indico, as 9 to 8, or thereabouts. For it was hard

to

to determine this proportion accurately. Alſo the Circles made ſucceſſively by the red, yellow and green, differed more from one another than thoſe made ſucceſſively by the green, blue and indico. For the Circle made by the violet was too dark to be ſeen. To carry on the computation, Let us therefore ſuppoſe that the differences of the Diameters of the Circles made by the outmoſt red, the confine of red and orange, the confine of orange and yellow, the confine of yellow and green, the confine of green and blue, the confine of blue and indico, the confine of indico and violet, and outmoſt violet, are in proportion as the differences of the lengths of a Monochord which ſound the tones in an Eight; *ſol*, *la*, *fa*, *ſol*, *la*, *mi*, *fa*, *ſol*, that is, as the numbers $\frac{1}{9}$, $\frac{1}{18}$, $\frac{1}{12}$, $\frac{1}{12}$, $\frac{2}{27}$, $\frac{1}{27}$, $\frac{1}{18}$. And if the Diameter of the Circle made by the confine of red and orange be 9 A, and that of the Circle made by the confine of blue and indico be 8 A as above, their difference 9 A ---- 8 A will be to the difference of the Diameters of the Circles made by the outmoſt red, and by the confine of red and orange, as $\frac{1}{18} + \frac{1}{12} + \frac{1}{12} + \frac{2}{27}$ to $\frac{1}{9}$, that is as $\frac{8}{27}$ to $\frac{1}{9}$ or 8 to 3, and to the difference of the Circles made by the outmoſt violet, and by the confine of blue and indico, as $\frac{1}{18} + \frac{1}{12} + \frac{1}{12} + \frac{2}{27}$ to $\frac{1}{27} + \frac{1}{18}$, that is, as $\frac{8}{27}$ to $\frac{5}{54}$, or as 16 to 5. And therefore theſe differences will be $\frac{3}{8}$ A and $\frac{5}{16}$ A. Add the firſt to 9 A and ſubduct the laſt from 8 A, and you will have the Diameters of the Circles made by the leaſt and moſt refrangible rays $\frac{75}{8}$ A and $\frac{61\frac{1}{2}}{8}$ A. Theſe Diameters are therefore to one another as 75 to $61\frac{1}{2}$ or 50 to 41, and their Squares as 2500 to 1681, that is, as 3 to 2 very nearly. Which proportion differs not much from the proportion of the Diameters of the

Circles

Circles made by the outmoſt red and outmoſt violet in the 13th Obſervation of the firſt part of this Book.

O B S. VI.

Placing my Eye where theſe Rings appeared plaineſt, I ſaw the Speculum tinged all over with waves of Colours (red, yellow, green, blue ;) like thoſe which in the Obſervations of the firſt Part of this Book appeared between the Object-Glaſſes and upon Bubbles of Water, but much larger. And after the manner of thoſe, they were of various magnitudes in various poſitions of the Eye, ſwelling and ſhrinking as I moved my Eye this way and that way. They were formed like Arcs of concentrick Circles as thoſe were, and when my Eye was over againſt the center of the concavity of the Speculum (that is, 5 Feet and 10 Inches diſtance from the Speculum) their common center was in a right Line with that center of concavity, and with the Hole in the Window. But in other poſtures of my Eye their center had other poſitions. They appeared by the Light of the Clouds propagated to the Speculum through the Hole in the Window, and when the Sun ſhone through that Hole upon the Speculum, his Light upon it was of the Colour of the Ring whereon it fell, but by its ſplendor obſcured the Rings made by the Light of the Clouds, unleſs when the Speculum was removed to a great diſtance from the Window, ſo that his Light upon it might be broad and faint. By varying the poſition of my Eye, and moving it nearer to or farther from the direct beam of the Sun's Light, the Colour of the Sun's reflected Light conſtantly varied upon the Speculum,

as

as it did upon my Eye, the ſame Colour always appearing to a By-ſtander upon my Eye which to me appeared upon the Speculum. And thence I knew that the Rings of Colours upon the Chart were made by theſe reflected Colours propagated thither from the Speculum in ſeveral Angles, and that their production depended not upon the termination of Light and Shaddow.

O B S. VII.

By the Analogy of all theſe Phænomena with thoſe of the like Rings of Colours deſcribed in the firſt Part of this Book, it ſeemed to me that theſe Colours were produced by this thick plate of Glaſs, much after the manner that thoſe were produced by very thin plates. For, upon tryal, I found that if the Quick-ſilver were rubbed off from the back-ſide of the Speculum, the Glaſs alone would cauſe the ſame Rings of Colours, but much more faint than before; and therefore the Phænomenon depends not upon the Quick-ſilver, unleſs ſo far as the Quick-ſilver, by ~~the~~ increaſing the reflexion of the back-ſide of the Glaſs, increaſes the Light of the Rings of Colours. I found alſo that a Speculum of metal without Glaſs made ſome years ſince for optical uſes, and very well wrought, produced none of thoſe Rings; and thence I underſtood that theſe Rings ariſe not from one ſpecular ſurface alone, but depend upon the two ſurfaces of the plate of Glaſs whereof the Speculum was made, and upon the thickneſs of the Glaſs between them. For as in the 7th and 19th Obſervations of the firſt Part of this Book a thin plate of Air, Water, or Glaſs of an even thickneſs appeared of one Colour when the rays were perpendicular to it, of another when they were a little oblique, of another when more oblique, of another when ſtill more oblique, and ſo on ; ſo here, in the ſixth Obſervation, the Light which emerged out of the Glaſs in ſeveral obliquities, made the Glaſs appear of ſeveral Colours, and being propagated in thoſe obliquities to the Chart, there painted Rings of thoſe Colours. And as the reaſon why a thin plate appeared of ſeveral Colours in ſeveral obliquities of the rays, was, that the rays of one and the ſame ſort are reflected by the thin plate at one obliquity and tranſmitted at another, and thoſe of other ſorts tranſmitted where theſe are reflected, and reflected where theſe are tranſmitted : So the reaſon why the thick plate of Glaſs whereof the Speculum was made did appear of various Colours in various obliquities, and in thoſe obliquities propagated thoſe Colours to the Chart, was, that the rays of one and the ſame ſort did at one obliquity emerge out of the Glaſs, at another did not emerge but were reflected back towards the Quick-ſilver by the hither ſurface of the Glaſs, and accordingly as the obliquity became greater and greater emerged and were reflected alternately for many ſucceſſions, and that in one and the ſame obliquity the rays of one ſort were reflected, and thoſe of another tranſmitted. This fifteenth is manifeſt by the ~~firſt~~ fifteenth Obſervation of this part of this Book : For in that Obſervation, when the Speculum was illuminated by any one of the priſmatick Colours, that Light made many Rings of the ſame Colour upon the Chart with dark intervals, and therefore at its emergence out of the Speculum was alternately tranſmitted, and not tranſ-

tranſmitted from the Speculum to the Chart for many ſucceſſions, according to the various obliquities of its emergence. And when the Colour caſt on the Speculum by the Priſm was varied, the Rings became of the Colour caſt on it, and varied their bigneſs with their Colour, and therefore the Light was now alternately tranſmitted and not tranſmitted from the Speculum to the ~~Lens~~ Chart at other obliquities than before. It ſeemed to me therefore that theſe Rings were of one and the ſame original with thoſe of thin plates, but yet with this difference that thoſe of thin plates are made by the alternate reflexions and tranſmiſſions of the rays at the ſecond ſurface of the plate after one paſſage through it: But here the rays go twice through the plate before they are alternately reflected and tranſmitted; firſt, they go through it from the firſt ſurface to the Quick-ſilver, and then return through it from the Quick-ſilver to the firſt ſurface, and there are either tranſmitted to the Chart or reflected back to the Quick-ſilver, accordingly as they are in their fits of eaſie reflexion or tranſmiſſion when they arrive at that ſurface. For the intervals of the fits of the rays which fall perpendicularly on the Speculum, and are reflected back in the ſame perpendicular Lines, by reaſon of the equality of theſe Angles and Lines, are of the ſame length and number within the Glaſs after reflexion as before by the 19th Propoſition of the third Part of this Book. And therefore ſince all the rays that enter through the firſt ſurface are in their fits of eaſy tranſmiſſion at their entrance, and as many of theſe as are reflected by the ſecond are in their fits of eaſy reflexion there, all theſe muſt be again in their fits of eaſy tranſmiſſion at their return to the firſt, and by conſequence there go out of the Glaſs to the Chart, and form upon it the white Spot of Light in the center of the Rings. For the reaſon holds good in all ſorts of rays, and therefore all ſorts muſt go out promiſcuouſly to that Spot, and by their mixture cauſe it to be white. But the intervals of the fits of thoſe rays which are reflected more obliquely than they enter, muſt be greater after reflexion than before by the 15th and 20th Prop. And thence it may happen that the rays at their return to the firſt ſurface, may in certain obliquities be in fits of eaſy reflexion, and return back to the Quick-ſilver, and in other intermediate obliquities be again in fits of eaſy tranſmiſſion, and ſo go out to the Chart, and paint on it the Rings of Colours about the white Spot. And becauſe the intervals of the fits at equal obliquities are greater and fewer in the leſs refrangible rays, and leſs and more numerous in the more refrangible, therefore the leſs refrangible at equal obliquities ſhall make fewer Rings than the more refrangible, and the Rings made by thoſe ſhall be larger than the like number of Rings made by theſe; that is, the red Rings ſhall be larger than the yellow, the yellow than the green, the green than the blue, and the blue than the violet, as they were really found to be in the 5th Obſervation. And therefore the firſt Ring of all Colours incompaſſing the white Spot of Light ſhall be red without and violet within, and yellow, and green, and blue in the middle, as it was found in the ſecond Obſervation; and theſe Colours in the ſecond Ring, and thoſe that follow ſhall be more expanded till they ſpread into one another, and blend one another by interfering.

Theſe

These seem to be the reasons of these Rings in general, and this put me upon observing the thickness of the Glass, and considering whether the dimensions and proportions of the Rings may be truly derived from it by computation.

OBS. VIII.

I measured therefore the thickness of this concavo-convex plate of Glass, and found it every-where $\frac{1}{4}$ of an Inch precisely. Now, by the 6th Observation of the first Part of this Book, a thin plate of Air transmits the brightest Light of the first Ring, that is the bright yellow, when its thickness is the $\frac{1}{89000}$th part of an Inch, and by the 10th Observation of the same part, a thin plate of Glass transmits the same Light of the same Ring when its thickness is less in proportion of the sine of refraction to the sine of incidence, that is, when its thickness is the $\frac{11}{1513000}$th or $\frac{1}{137545}$th part of an Inch, supposing the sines are as 11 to 17. And if this thickness be doubled it transmits the same bright Light of the second Ring, if tripled it transmits that of the third, and so on, the bright yellow Light in all these cases being in its fits of transmission. And therefore if its thickness be multiplied 34386 times so as to become $\frac{1}{4}$ of an Inch it transmits the same bright Light of the 34386th Ring. Suppose this be the bright yellow Light transmitted perpendicularly from the reflecting convex side of the Glass through the concave side to the white Spot in the center of the Rings of Colours on the Chart: And by a rule in the seventh & nineteenth Observations in the first Part of this ~~first~~ Book, and by the 15th and 20th Propositions of the third Part of this Book, if the rays be made oblique to the Glaſs, the thickneſs of the Glaſs requiſite to tranſmit the ſame bright Light of the ſame Ring in any obliquity is to this thickneſs of $\frac{1}{4}$ of an Inch, as the ſecant of an Angle whoſe ſine is the firſt of an hundred and ſix arithmetical means between the ſines of incidence and refraction, counted from the ſine of incidence when the refraction is made out of any plated Body into any medium incompaſſing it, that is, in this caſe, out of Glaſs into Air. Now if the thickneſs of the Glaſs be increaſed by degrees, ſo as to bear to its firſt thickneſs, (*viz.* that of a quarter of an Inch) the proportions which 34386 (the number of fits of the perpendicular rays in going through the Glaſs towards the white Spot in the center of the Rings,) hath to 34385, 34384, 34383 and 34382 (the numbers of the fits of the oblique rays in going through the Glaſs towards the firſt, ſecond, third and fourth Rings of Colours,) and if the firſt thickneſs be divided into 100000000 equal parts, the increaſed thickneſſes will be 100002908, 100005816, 100008725 and 100011633, and the Angles of which theſe thickneſſes are ſecants will be 26′ 13″, 37′ 5″, 45′ 6″ and 52′ 26″, the Radius being 100000000; and the ſines of theſe Angles are 762, 1079, 1321 and 1525, and the proportional ſines of refraction 1172, 1659, 2031 and 2345, the Radius being 100000. For ſince the ſines of incidence out of Glaſs into Air are to the ſines of refraction as 11 to 17, and to the above-mentioned ſecants as 11 to the firſt of 106 arithmetical means between 11 and 17, that is as 11 to $11\frac{6}{106}$, thoſe ſecants will be to the ſines of refraction as $11\frac{6}{106}$ to 17, and by this Analogy will give theſe ſines. So then

if

if the obliquities of the rays to the concave ſurface of the Glaſs be ſuch that the ſines of their refraction in paſſing out of the Glaſs through that ſurface into the Air be 1172, 1659, 2031, 2345, the bright Light of the 34386th Ring ſhall emerge at the thickneſſes of the Glaſs which are to $\frac{1}{4}$ of an Inch as 34386 to 34385, 34384, 34383, 34382, reſpectively. And therefore if the thickneſs in all theſe caſes be $\frac{1}{4}$ of an Inch (as it is in the Glaſs of which the Speculum was made) the bright Light of the 34385th Ring ſhall emerge where the ſine of refraction is 1172, and that of the 34384th, 384383th and 34382th Ring where the ſine is 1659, 2031, and 2345 reſpectively. And in theſe Angles of refraction the Light of theſe Rings ſhall be propagated from the Speculum to the Chart, and there paint Rings about the white central round Spot of Light which we ſaid was the Light of the 34386th Ring. And the Semidiameters of theſe Rings ſhall ſubtend the Angles of refraction made at the concave ſurface of the Speculum, and by conſequence their Diameters ſhall be to the diſtance of the Chart from the Speculum as thoſe ſines of refraction doubled are to the Radius that is as 1172, 1659, 2031, and 2345, doubled are to 100000. And therefore if the diſtance of the Chart from the concave ſurface of the Speculum be ſix Feet (as it was in the third of theſe Obſervations) the Diameters of the Rings of this bright yellow Light upon the Chart ſhall be 1'688, 2'389, 2'925, 3'375 Inches: For theſe Diameters are to 6 Feet as the above-mentioned ſines doubled are to the Radius. Now theſe Diameters of the bright yellow Rings, thus found by computation are the very ſame with thoſe found in the third of theſe Obſervations by meaſuring them,

them, (*viz.* with $1\frac{11}{16}$, $2\frac{3}{8}$, $2\frac{11}{12}$, and $3\frac{3}{8}$ Inches, and therefore the Theory of deriving these Rings from the thickness of the plate of Glass of which the Speculum was made, and from the obliquity of the emerging rays agrees with the Observation. In this computation I have equalled the Diameters of the bright Rings made by Light of all Colours, to the Diameters of the Rings made by the bright yellow. For this yellow makes the brightest part of the Rings of all Colours. If you desire the Diameters of the Rings made by the Light of any other unmixed Colour, you may find them readily by putting them to the Diameters of the bright yellow ones in a subduplicate proportion of the intervals of the fits of the rays of those Colours when equally inclined to the refracting or reflecting surface which caused those fits, that is, by putting the Diameters of the Rings made by the rays in the extremities and limits of the seven Colours, red, orange, yellow, green, blue, indico, violet, proportional to the Cube-roots of the numbers, 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, which express the lengths of a Monochard sounding the notes in an Eight : For by this means the Diameter of the Rings of these Colours will be found pretty nearly in the same proportion to one another, which they ought to have by the fifth of these Observations.

And thus I satisfied my self that these Rings were of the same kind and original with those of thin plates, and by consequence that the fits or alternate dispositions of the rays to be reflected and transmitted are propagated to great distances from every reflecting and refracting surface. But yet to put the matter out of doubt I added the following Observation.

OBS.

OBS. IX.

If theſe Rings thus depend on the thickneſs of the plate of Glaſs their Diameters at equal diſtances from ſeveral Speculums made of ſuch concavo-convex plates of Glaſs as are ground on the ſame Sphere, ought to be reciprocally in a ſubduplicate proportion of the thickneſſes of the plates of Glaſs. And if this proportion be found true by experience it will amount to a demonſtration that theſe Rings (like thoſe formed in thin plates) do depend on the thickneſs of the Glaſs. I procured therefore another concavo-convex plate of Glaſs ground on both ſides to the ſame Sphere with the former plate: Its thickneſs was $\frac{5}{62}$ parts of an Inch; and the Diameters of the three firſt bright Rings meaſured between the brighteſt parts of their orbits at the diſtance of 6 Feet from the Glaſs were 3. $4\frac{1}{6}$. $5\frac{1}{8}$. Inches. Now the thickneſs of the other Glaſs being $\frac{1}{4}$ of an Inch was to thickneſs of this Glaſs as $\frac{1}{4}$ to $\frac{5}{62}$, that is as 31 to 10, or 310000000 to 100000000, and the roots of theſe numbers are 17607 and 10000, & in the proportion of the firſt of theſe roots to the ſecond are the Diameters of the bright Rings made in this Obſervation by the thinner Glaſs, 3. $4\frac{1}{6}$. $5\frac{1}{8}$ to the Diameters of the ſame Rings made in the third of theſe Obſervations by the thicker Glaſs $1\frac{11}{16}$. $2\frac{3}{8}$ $2\frac{11}{12}$, that is, the Diameters of the Rings are reciprocally in a ſubduplicate proportion of thickneſſes of the plates of Glaſs.

So then in plates of Glaſs which are alike concave on one ſide, and alike convex on the other ſide, and alike quick-ſilvered on the convex ſides, and differ in nothing

but

but their thickneſs, the Diameters of the Rings are reciprocally in a ſubduplicate proportion of the thickneſſes of the plates. And this ſhews ſufficiently that the Rings depend on both the ſurfaces of the Glaſs. They depend on the convex ſurface becauſe they are more luminous when that ſurface is quick-ſilvered over than when it is without Quick-ſilver. They depend alſo upon the concave ſurface, becauſe without that ſurface a Speculum makes them not. They depend on both ſurfaces and on the diſtances between them, becauſe their bigneſs is varied by varying only that diſtance. And this dependance is of the ſame kind with that which the Colours of thin plates have on the diſtance of the ſurfaces of thoſe plates, becauſe the bigneſs of the Rings and their proportion to one another, and the variation of their bigneſs ariſing from the variation of the thickneſs of the Glaſs, and the orders of their Colours, is ſuch as ought to reſult from the Propoſitions in the end of the third Part of this Book, derived from the the Phænomena of the Colours of thin plates ſet down in the firſt Part.

There are yet other Phænomena of theſe Rings of Colours but ſuch as follow from the ſame Propoſitions, and therefore confirm both the truth of thoſe Propoſitions, and the Analogy between theſe Rings and the Rings of Colours made by very thin plates. I ſhall ſubjoyn ſome of them.

OBS. X.

When the beam of the Sun's Light was reflected back from the Speculum not directly to the Hole in the Window, but to a place a little diſtant from it, the common center of that Spot, and of all the Rings of Colours fell in the middle way between the beam of the incident Light, and the beam of the reflected Light, and by conſequence in the center of the ſpherical concavity of the Speculum, whenever the Chart on which the Rings of Colours fell was placed at that center. And as the beam of reflected Light by inclining the Speculum receded more and more from the beam of incident Light and from the common center of the coloured Rings between them, thoſe Rings grew bigger and bigger, and ſo alſo did the white round Spot, and new Rings of Colours emerged ſucceſſively out of their common center, and the white Spot became a white Ring encompaſſing them; and the incident and reflected beams of Light always fell upon the oppoſite parts of this Ring, illuminating its perimeter like two mock Suns in the oppoſite parts of an Iris. So then the Diameter of this Ring, meaſured from the middle of its Light on one ſide to the middle of its Light on the other ſide, was always equal to the diſtance between the middle of the incident beam of Light, and the middle of the reflected beam meaſured at the Chart on which the Rings appeared: And the rays which formed this Ring were reflected by the Speculum in Angles equal to their Angles of incidence, and by conſequence to their Angles of refraction at their entrance into the Glaſs, but yet their Angles of reflexion were not in the ſame planes with their Angles of incidence.

OBS. XI.

The Colours of the new Rings were in a contrary order to thoſe of the former, and aroſe after this manner. The white round Spot of Light in the middle of the Rings continued white to the center till the diſtance of the incident and reflected beams at the chart was about $\frac{7}{8}$ parts of an Inch, and then it began to grow dark in the middle. And when that diſtance was about $1\frac{3}{16}$ of an Inch, the white Spot was become a Ring encompaſſing a dark round Spot which in the middle inclined to violet and indico. And the luminous Rings incompaſſing it were grown equal to thoſe dark ones which in the four firſt Obſervations encompaſſed them, that is to ſay, the white Spot was grown a white Ring equal to the firſt of thoſe dark Rings, and the firſt of thoſe luminous Rings was now grown equal to the ſecond of thoſe dark ones, and the ſecond of thoſe luminous ones to the third of thoſe dark ones, and ſo on. For the Diameters of the luminous Rings were now $1\frac{3}{16}$, $2\frac{1}{16}$, $2\frac{2}{3}$, $3\frac{3}{20}$, *&c.* Inches.

When the diſtance between the incident and reflected beams of Light became a little bigger, there emerged out of the middle of the dark Spot after the indico a blue, and then out of that blue a pale green, and ſoon after a yellow and red. And when the Colour at the center was brighteſt, being between yellow and red, the bright Rings were grown equal to thoſe Rings which in the four firſt Obſervations next encompaſſed them; that

that is to ſay, the white Spot in the middle of thoſe Rings was now become a white Ring equal to the firſt of thoſe bright Rings, and the firſt of thoſe bright ones was now become equal to the ſecond of thoſe, and ſo on. For the Diameters of the white Ring, and of the other luminous Rings incompaſſing it, were now $1\frac{11}{16}$, $2\frac{3}{8}$, $2\frac{11}{12}$, $3\frac{3}{8}$, &c. or thereabouts.

When the diſtance of the two beams of Light at the Chart was a little more increaſed, there emerged out of the middle in order after the red, a purple, a blue, a green, a yellow, and a red inclining much to purple, and when the Colour was brighteſt being between yellow and red, the former indico, blue, green, yellow and red, were become an Iris or Ring of Colours equal to the firſt of thoſe luminous Rings which appeared in the four firſt Obſervations, and the white Ring which was now become the ſecond of the luminous Rings was grown equal to the ſecond of thoſe, and the firſt of thoſe which was now become the third Ring was become the third of thoſe, and ſo on. For their Diameters were $1\frac{11}{16}$, $2\frac{3}{8}$, $2\frac{11}{12}$, $3\frac{3}{8}$ Inches, the diſtance of the two beams of Light, and the Diameter of the white Ring being $2\frac{3}{8}$ Inches.

When theſe two beams became more diſtant there emerged out of the middle of the purpliſh red, firſt a darker round Spot, and then out of the middle of that Spot a brighter. And now the former Colours (purple, blue, green, yellow, and purpliſh red) were become a Ring equal to the firſt of the bright Rings mentioned in the four firſt Obſervations, and the Rings about this Rings Ring were grown equal to the Rings about that reſpectively; the diſtance between the two beams of

Light and the Diameter of the white Ring (which was now become the third Ring) being about 3 Inches.

The Colours of the Rings in the middle began now to grow very dilute, and if the diſtance between the two beams was increaſed half an Inch, or an Inch more, they vaniſhed whilſt the white Ring, with one or two of the Rings next it on either ſide, continued ſtill viſible. But if the diſtance of the two beams of Light was ſtill more increaſed theſe alſo vaniſhed: For the Light which coming from ſeveral parts of the Hole in the Window fell upon the Speculum in ſeveral Angles of incidence made Rings of ſeveral bigneſſes, which diluted and blotted out one another, as I knew by intercepting ſome part of that Light. For if I intercepted that part which was neareſt to the Axis of the Speculum the Rings would be leſs, if the other part which was remoteſt from it they would be bigger.

OBS. XII.

When the Colours of the Priſm were caſt ſucceſſively on the Speculum, that Ring which in the two laſt Obſervations was white, was of the ſame bigneſs in all the Colours, but the Rings without it were greater in the green than in the blue, and ſtill greater in the yellow, and greateſt in the red. And, on the contrary, the Rings within that white Circle were leſs in the green than in the blue, and ſtill leſs in the yellow, and leaſt in the red. For the Angles of reflexion of thoſe rays which made this Ring being equal to their Angles of incidence, the fits of every reflected ray within the Glaſs after

after reflexion are equal in length and number to the fits of the ſame ray within the Glaſs before its incidence on the reflecting ſurface; and therefore ſince all the rays of all ſorts at their entrance into the Glaſs were in a fit of tranſmiſſion, they were alſo in a fit of tranſmiſſion at their returning to the ſame ſurface after reflexion; and by conſequence were tranſmitted and went out to the white Ring on the Chart. This is the reaſon why that Ring was of the ſame bigneſs in all the Colours, and why in a mixture of all it appears white. But in rays which are reflected in other Angles, the intervals of the fits of the leaſt refrangible being greateſt, make the Rings of their Colour in their progreſs from this white Ring, either outwards or inwards, increaſe or decreaſe by the greateſt ſteps; ſo that the Rings of this Colour without are greateſt, and within leaſt. And this is the reaſon why in the laſt Obſervation, when the Speculum was illuminated with white Light, the exterior Rings made by all Colours appeared red without and blue within, and the interior blue without and red within.

Theſe are the Phænomena of thick convexo-concave plates of Glaſs, which are every where of the ſame thickneſs. There are yet other Phænomena when theſe plates are a little thicker on one ſide than on the other, and others when the plates are more or leſs concave than convex, or plano-convex, or double-convex. For in all theſe caſes the plates make Rings of Colours, but after various manners; all which, ſo far as I have yet obſerved, follow from the Propoſitions in the end of the third part of this Book, and ſo conſpire to confirm the truth of thoſe Propoſitions. But the Phæno-

mena

mena are too various, and the Calculations whereby they follow from thoſe Propoſitions too intricate to be here proſecuted. I content my ſelf with having proſecuted this kind of Phænomena ſo far as to diſcover their cauſe, and by diſcovering it to ratify the Propoſitions in the third Part of this Book.

OBS. XIII.

As Light reflected by a Lens quick-ſilvered on the back-ſide makes the Rings of Colours above deſcribed, ſo it ought to make the like Rings of Colours in paſſing through a drop of Water. At the firſt reflexion of the rays within the drop, ſome Colours ought to be tranſmitted, as in the caſe of a Lens, and others to be reflected back to the Eye. For inſtance, if the Diameter of a ſmall drop or globule of Water be about the 500th part of an Inch, ſo that a red-making ray in paſſing through the middle of this globule has 250 fits of eaſy tranſmiſſion within the globule, and that all the red-making rays which are at a certain diſtance from this middle ray round about it have 249 fits within the globule, and all the like rays at a certain further diſtance round about it have 248 fits, and all thoſe at a certain further diſtance 247 fits, and ſo on; theſe concentrick Circles of rays after their tranſmiſſion, falling on a white Paper, will make concentrick rings of red upon the Paper, ſuppoſing the Light which paſſes through one ſingle globule ſtrong enough to be ſenſible. And, in like manner, the rays of other Colours will make Rings of other Colours. Suppoſe now that in a fair day the Sun ſhines through a thin Cloud of ſuch

globules

globules of Water or Hail, and that the globules are all of the ſame bigneſs, and the Sun ſeen through this Cloud ſhall appear incompaſſed with the like concentrick Rings of Colours, and the Diameter of the firſt Ring of red ſhall be $7\frac{1}{4}$ degrees, that of the ſecond $10\frac{1}{4}$ degrees, that of the third 12 degrees 33 minutes. And accordingly as the globules of Water are bigger or leſs, the Rings ſhall be leſs or bigger. This is the Theory, and experience anſwers it. For in *June* 1692. I ſaw by reflexion in a Veſſel of ſtagnating Water three Halos Crowns or Rings of Colours about the Sun, like three little Rainbows, concentrick to his Body. The Colours of the firſt or innermoſt Crown were blue next the Sun, red without, and white in the middle between the blue and red. Thoſe of the ſecond Crown were purple and blue within, and pale red without, and green in the middle. And thoſe of the third were pale blue within, and pale red without; theſe Crowns incloſed one another immediately, ſo that their Colours proceeded in this continual order from the Sun outward: blue, white, red; purple, blue, green, pale yellow and red; pale blue, pale red. The Diameter of the ſecond Crown meaſured from the middle of the yellow and red on one ſide of the Sun, to the middle of the ſame Colour on the other ſide was $9\frac{1}{3}$ degrees, or thereabouts. The Diameters of the firſt and third I had not time to meaſure, but that of the firſt ſeemed to be about five or ſix degrees, and that of the third about twelve. The like Crowns appear ſometimes about the Moon; for in the beginning of the year 1664, *Febr.* 19th at night, I ſaw two ſuch Crowns about her. The Diameter of the firſt or innermoſt was about three degrees, and that of the

ſecond

ſecond about five degrees and an half. Next about the Moon was a Circle of white, and next about that the inner Crown which was of a bluiſh green within next the white, and of a yellow and red without, and next about theſe Colours were blue and green on the inſide of the outward Crown, and red on the outſide of it. At the ſame time there appeared a Halo about 22 degrees 35′ diſtant from the center of the Moon. It was Elliptical, and its long Diameter was perpendicular to the Horizon verging below fartheſt from the Moon. I am told that the Moon has ſometimes three or more concentrick Crowns of Colours incompaſſing one another next about her Body. The more equal the globules of Water or Ice are to one another, the more Crowns of Colours will appear, and the Colours will be the more lively. The Halo at the diſtance of $22\frac{1}{2}$ degrees from the Moon is of another ſort. By its being oval and remoter from the Moon below than above, I conclude, that it was made by refraction in ſome ſort of Hail or Snow floating in the Air in an horizontal Poſture, the refracting Angle being about 58 or 60 degrees.

THE

THE THIRD BOOK OF OPTICKS.

Obſervations concerning the Inflexions of the rays of Light, and the Colours made thereby.

GRimaldo has informed us, that if a beam of the Sun's Light be let into a dark Room through a very ſmall Hole, the ſhadows of things in this Light will be larger than they ought to be if the rays went on by the Bodies in ſtreight Lines, and that theſe ſhadows have three parallel fringes, bands or ranks of coloured Light adjacent to them. But if the Hole be enlarged the fringes grow broad and run into one another, ſo that they cannot be diſtinguiſhed. Theſe broad ſhadows and fringes have been reckoned by ſome to proceed from the ordinary refraction of the Air, but without due examination of the matter. For the circumſtances of the Phænomenon, ſo far as I have obſerved them, are as follows.

OBS. I.

I made in a piece of Lead a ſmall Hole with a Pin, whoſe breadth was the 42th part of an Inch. For 21 of thoſe Pins laid together took up the breadth of half an Inch. Through this Hole I let into my darkened Chamber a beam of the Sun's Light, and found that the ſhadows of Hairs, Thred, Pins, Straws, and ſuch like ſlender ſubſtances placed in this beam of Light, were conſiderably broader than they ought to be, if the rays of Light paſſed on by theſe Bodies in right Lines. And particularly a Hair of a Man's Head, whoſe breadth was but the 280th part of an Inch, being held in this Light, at the diſtance of about twelve Feet from the Hole, did caſt a ſhadow which at the diſtance of four Inches from the Hair was the ſixtieth part of an Inch broad, that is, above four times broader than the Hair, and at the diſtance of two Feet from the Hair was about the eight and twentieth part of an Inch broad, that is, ten times broader than the Hair, and at the diſtance of ten Feet was the eighth part of an Inch broad, that is 35 times broader.

Nor is it material whether the Hair be incompaſſed with Air, or with any other pellucid ſubſtance. For I wetted a poliſhed plate of Glaſs, and laid the Hair in the Water upon the Glaſs, and then laying another poliſhed plate of Glaſs upon it, ſo that the Water might fill up the ſpace between the Glaſſes, I held them in the aforeſaid beam of Light, ſo that the Light might paſs through them perpendicularly, and the ſhadow of the Hair was at the ſame diſtances as big as before.

The

The ſhadows of ſcratches made in poliſhed plates of Glaſs were alſo much broader than they ought to be, and the Veins in poliſhed plates of Glaſs did alſo caſt the like broad ſhadows. And therefore the great breadth of theſe ſhadows proceeds from ſome other cauſe than the refraction of the Air.

Let the Circle X repreſent the middle of the Hair; *Fig.* 1. ADG, BEH, CFI, three rays paſſing by one ſide of the Hair at ſeveral diſtances; KNQ, LOR, MPS, three other rays paſſing by the other ſide of the Hair at the like diſtances; D, E, F and N, O, P, the places where the rays are bent in their paſſage by the Hair; G, H, I and Q, R, S, the places where the rays fall on a Paper GQ; IS the breadth of the ſhadow of the Hair caſt on the Paper, and TI, VS, two rays paſſing to the points I and S without bending when the Hair is taken away. And it's manifeſt that all the Light between theſe two rays TI and VS is bent in paſſing by the Hair, and turned aſide from the ſhadow IS, becauſe if any part of this Light were not bent it would fall on the Paper within the ſhadow, and there illuminate the Paper contrary to experience. And becauſe when the Paper is at a great diſtance from the Hair, the ſhadow is broad, and therefore the rays TI and VS are at a great diſtance from one another, it follows that the Hair acts upon the rays of Light at a good diſtance in their paſſing by it. But the action is ſtrongeſt on the rays which paſs by at leaſt diſtances, and grows weaker and weaker accordingly as the rays paſs by at diſtances greater and greater, as is repreſented in the Scheme: For thence it comes to paſs, that the ſhadow of the Hair is much broader in proportion to the diſtance of the Paper from the Hair, when the Paper is nearer the Hair than when it is at a great diſtance from it.

O B S. II.

The ſhadows of all Bodies (Metals, Stones, Glaſs, Wood, Horn, Ice, *&c.*) in this Light were bordered with three parallel fringes or bands of coloured Light, whereof that which was contiguous to the ſhadow was broadeſt and moſt luminous, and that which was remoteſt from it was narroweſt, and ſo faint, as not eaſily to be viſible. It was difficult to diſtinguiſh the Colours unleſs when the Light fell very obliquely upon a ſmooth Paper, or ſome other ſmooth vvhite Body, ſo as to make them appear much broader than they vvould otherwiſe do. And then the Colours were plainly viſible in this order: The firſt or innermoſt fringe was violet and deep blue next the ſhadovv, and then light blue, green and yellovv in the middle, and red vvithout. The ſecond fringe vvas almoſt contiguous to the firſt, and the third to the ſecond, and both vvere blue vvithin and yellovv and red vvithout, but their Colours vvere very faint eſpecially thoſe of the third. The Colours therefore proceeded in this order from the ſhadovv, violet, indico, pale blue, green, yellovv, red; blue, yellovv, red; pale blue, pale yellovv and red. The ſhadows made by ſcratches and bubbles in poliſhed plates of Glaſs vvere bordered vvith the like fringes of coloured Light. And if plates of Looking-glaſs ſloop'd off near the edges vvith a Diamond cut, be held in the ſame beam of Light, the Light which paſſes through the parallel planes of the Glaſs will be be bordered with the like fringes of Colours

lours where those Planes meet with the Diamond cut, and by this means there will sometimes appear four or five fringes of Colours. Let A B, C D represent the parallel planes of a Looking-glass, and B D the plane of the Diamond-cut, making at B a very obtuse Angle with the plane A B. And let all the Light between the rays E N I and F B M pass directly through the parallel planes of the Glass, and fall upon the Paper between I and M, and all the Light between the rays G O and H D be refracted by the oblique plane of the Diamond cut B D, and fall upon the Paper between K and L; and the Light which passes directly through the parallel planes of the Glass, and falls upon the Paper between I and M, will be bordered with three or more fringes at M. *Fig.* 2.

So by looking on the Sun through a feather or black Ribban held close to ye eye, several Rainbows will appear, the shadows wch the fibres or threds cast on the Tunica Retina, being bordered with the like fringes of colours.

O B S. III.

When the Hair was twelve Feet distant from the Hole, and its shadow fell obliquely upon a flat vvhite scale of Inches and parts of an Inch placed half a Foot beyond it, and also when the shadow fell perpendicularly upon the same scale placed nine Feet beyond it; I measured the breadth of the shadow and fringes as accurately as I could, and found them in parts of an Inch as follows.

The

At the diſtance of	half a Foot.	nine Feet.
The breadth of the Shadow	$\frac{1}{54}$	$\frac{1}{9}$
The breadth between the middles of the brighteſt Light of the innermoſt fringes on either ſide the ſhadow	$\frac{1}{38}$ or $\frac{1}{39}$	$\frac{7}{50}$
The breadth between the middles of the brighteſt Light of the middlemoſt fringes on either ſide the ſhadow	$\frac{1}{23\frac{1}{2}}$	$\frac{4}{17}$
The breadth between the middles of the brighteſt Light of the outmoſt fringes on either ſide the ſhadow	$\frac{1}{18}$ or $\frac{1}{18\frac{1}{2}}$	$\frac{3}{10}$
The diſtance between the middles of the brighteſt Light of the firſt and ſecond fringes	$\frac{1}{120}$	$\frac{1}{21}$
The diſtance between the middles of the brighteſt Light of the ſecond and third fringes	$\frac{1}{170}$	$\frac{1}{31}$
The breadth of the luminous part (green, white, yellow and red) of the firſt fringe	$\frac{1}{170}$	$\frac{1}{32}$
The breadth of the darker ſpace between the firſt and ſecond fringes.	$\frac{1}{240}$	$\frac{1}{45}$
The breadth of the luminous part of the ſecond fringe	$\frac{1}{290}$	$\frac{1}{55}$
The breadth of the darker ſpace between the ſecond and third fringes.	$\frac{1}{340}$	$\frac{1}{63}$

These

These measures I took by letting the shadow of the Hair at half a Foot distance fall so obliquely on the scale as to appear twelve times broader than when it fell perpendicularly on it at the same distance, and setting down in this Table the twelfth part of the measures I then took.

OBS. IV.

When the shadow and fringes were cast obliquely upon a smooth white Body, and that Body was removed further and further from the Hair, the first fringe began to appear and look brighter than the rest of the Light at the distance of less than a quarter of an Inch from the Hair, and the dark line or shadow between that and the second fringe began to appear at a less distance from the Hair than that of the third part of an Inch. The second fringe began to appear at a distance from the Hair of less than half an Inch, and the shadow between that and the third fringe at a distance less than an Inch, and the third fringe at a distance less than three Inches. At greater distances they became much more sensible, but kept very nearly the same proportion of their breadths and intervals which they had at their first appearing. For the distance between the middle of the first and middle of the second fringe, was to the distance between the middle of the second and middle of the third fringe, as three to two, or ten to seven. And the last of these two distances was equal to the breadth of the bright Light or luminous part of the first fringe. And this breadth was to the breadth of the bright Light of the second fringe as seven to four, and to the dark interval

interval of the firſt and ſecond fringe as three to two, and to the like dark interval between the ſecond and third as two to one. For the breadths of the fringes ſeemed to be in the progreſſion of the numbers 1, $\sqrt{\frac{1}{3}}$, $\sqrt{\frac{1}{5}}$ and their intervals to be in the ſame progreſſion vvith them; that is, the fringes and their intervals together to be in the continual progreſſion of the numbers 1, $\sqrt{\frac{1}{2}}$, $\sqrt{\frac{1}{3}}$, $\sqrt{\frac{1}{4}}$, $\sqrt{\frac{1}{5}}$, or thereabouts. And theſe proportions held the ſame very nearly at all diſtances from the Hair; the dark Intervals of the fringes being as broad in proportion to the fringes at their firſt appearance as afterwards at great diſtances from the Hair, though not ſo dark and diſtinct.

OBS. V.

The Sun ſhining into my darkened Chamber through a Hole a quarter of an Inch broad; I placed at the diſtance of two or three Feet from the Hole a Sheet of Paſt-board, vvhich vvas black'd all over on both ſides, and in the middle of it had a Hole about three quarters of an Inch ſquare for the Light to paſs through. And behind the Hole I faſtened to the Paſt-board vvith Pitch the blade of a ſharp Knife, to intercept ſome part of the Light vvhich paſſed through the Hole. The planes of the Paſt-board and blade of the Knife vvere parallel to one another, and perpendicular to the rays. And vvhen they vvere ſo placed that none of the Sun's Light fell on the Paſt-board, but all of it paſſed through the Hole to the Knife, and there part of it fell upon the blade of the Knife, and part of it paſſed by its edge: I let this part of the Light vvhich paſſed by, fall on a vvhite white Paper two or three Feet beyond the Knife, and there saw two streams of faint Light shoot out both ways from the beam of Light into the shadow like the tails of Comets. But because the Sun's direct Light by its brightness upon the Paper obscured these faint streams, so that I could scarce see them, I made a little Hole in the midst of the Paper for that Light to pass through and fall on a black cloth behind it; and then I saw the two streams plainly. They were like one another, and pretty nearly equal in length and breadth, and quantity of Light. Their Light at that end next the Sun's direct Light was pretty strong for the space of about a quarter of an Inch, or half an Inch, and in all its progress from that direct Light decreased gradually till it became insensible. The whole length of either of these streams measured upon the Paper at the distance of three Feet from the Knife was about six or eight Inches; so that it subtended an Angle at the edge of the Knife of about 10 or 12, or at most 14 degrees. Yet sometimes I thought I saw it shoot three or four degrees further, but with a Light so very faint that I could scarce perceive it, and suspected it might (in some measure at least) arise from some other cause than the two streams did. For placing my Eye in that Light beyond the end of that stream which was behind the Knife, and looking towards the Knife, I could see a line of Light upon its edge, and that not only when my Eye was in the line of the streams, but also when it was without that line either towards the point of the Knife, or towards the handle. This line of Light appeared contiguous to the edge of the Knife, and was narrower than the Light of the innermost fringe, and narroweſt when my Eye was furtheſt from the direct Light, and therefore ſeemed to paſs between the Light of that fringe and the edge of the Knife, and that which paſſed neareſt the edge to be moſt bent, though not all of it.

O B S. VI.

I placed another Knife by this ſo that their edges might be parallel and look towards one another, and that the beam of Light might fall upon both the Knives, and ſome part of it paſs between their edges. And when the diſtance of their edges was about the 400th part of an Inch the ſtream parted in the middle, and left a ſhadow between the two parts. This ſhadow was ſo black and dark that all the Light which paſſed between the Knives ſeemed to be bent, and turned aſide to the one hand or to the other. And as the Knives ſtill approached one another the ſhadow grew broader, and the ſtreams ſhorter at their inward ends which were next the ſhadow, until upon the contact of the Knives the whole Light vaniſhed leaving its place to the ſhadow.

And hence I gather that the Light which is leaſt bent, and goes to the inward ends of the ſtreams, paſſes by the edges of the Knives at the greateſt diſtance, and this diſtance when the ſhadow begins to appear between the ſtreams is about the eight-hundredth part of an Inch. And the Light which paſſes by the edges of the Knives at diſtances ſtill leſs and leſs is more and more bent, and goes to thoſe parts of the ſtreams which are further and further from the direct Light, becauſe when

when the Knives approach one another till they touch, thoſe parts of the ſtreams vaniſh laſt which are furtheſt from the direct Light.

OBS. VII.

In the fifth Obſervation the fringes did not appear, but by reaſon of the breadth of the Hole in the Window became ſo broad as to run into one another, and by joyning make one continued Light in the beginning of the ſtreams. But in the ſixth, as the Knives approached one another, a little before the ſhadow appeared between the two ſtreams, the fringes began to appear on the inner ends of the ſtreams on either ſide of the direct Light, three on one ſide made by the edge of one Knife, and three on the other ſide made by the edge of the other Knife. They were diſtincteſt when the Knives were placed at the greateſt diſtance from the Hole in the Window, and ſtill became more diſtinct by making the Hole leſs, inſomuch that I could ſometimes ſee a faint lineament of a fourth fringe beyond the three above-mentioned. And as the Knives continually approached one another, the fringes grew diſtincter and larger until they vaniſhed. The outmoſt fringe vaniſhed firſt, and the middlemoſt next, and the innermoſt laſt. And after they were all vaniſhed, and the line of Light which was in the middle between them was grown very broad, enlarging it ſelf on both ſides into the ſtreams of Light deſcribed in the fifth Obſervation, the above-mentioned ſhadow began to appear in the middle of this line, and divide it along the middle into two lines of Light, and increaſed until the whole

Light vaniſhed. This inlargement of the fringes was ſo great that the rays which go to the innermoſt fringe ſeemed to be bent above twenty times more when this fringe was ready to vaniſh, than when one of the Knives was taken away.

And from this and the former Obſervation compared, I gather, that the Light of the firſt fringe paſſed by the edge of the Knife at a diſtance greater than the eight-hundredth part of an Inch, and the Light of the ſecond fringe paſſed by the edge of the Knife at a greater diſtance than the Light of the firſt fringe did, and that of the third at a greater diſtance than that of the ſecond, and that of the ſtreams of Light deſcribed in the fifth and ſixth Obſervations paſſed by the edges of the Knives at leſs diſtances than that of any of the fringes.

O B S. VIII.

I cauſed the edges of two Knives to be ground truly ſtreight, and pricking their points into a board ſo that their edges might look towards one another, and meeting near their points contain a rectilinear Angle, I faſtned their handles together with Pitch to make this Angle invariable. The diſtance of the edges of the Knives from one another at the diſtance of four Inches from the angular point, where the edges of the Knives met, was the eighth part of an Inch, and therefore the Angle contained by the edges was about 1 degr. 54′. The Knives thus fixed together I placed in a beam of the Sun's Light, let into my darkened Chamber through a Hole the 42th part of an Inch wide, at the diſtance

of

of ten or fifteen Feet from the Hole, and let the Light which passed between their edges fall very obliquely upon a smooth white Ruler at the distance of half an Inch, or an Inch from the Knives, and there saw the fringes made by the two edges of the Knives run along the edges of the shadows of the Knives in lines parallel to those edges without growing sensibly broader, till they met in Angles equal to the Angle contained by the edges of the Knives, and where they met and joyned they ended without crossing one another. But if the Ruler was held at a much greater distance from the ~~Paper~~ Knives, the fringes became something broader and broader as they approached one another, and after they met they crossed one another, and then became much broader than before.

Whence I gather that the distances at which the fringes pass by the Knives are not increased nor altered by the approach of the Knives, but the Angles in which the rays are there bent are much increased by that approach; and that the Knife which is nearest any ray determines which way the ray shall be bent, and the other Knife increases the bent.

OBS. IX.

When the rays fell very obliquely upon the Ruler at the distance of the third part of an Inch from the Knives, the dark line between the first and second fringe of the shadow of one Knife, and the dark line between the first and second fringe of the shadow of the other Knife met with one another, at the distance of the fifth part of an Inch from the end of the Light which passed be-

tween

tween the Knives at the concourſe of their edges. And therefore the diſtance of the edges of the Knives at the meeting of theſe dark lines was the 160th part of an Inch. For as four Inches to the eighth part of an Inch, ſo is any length of the edges of the Knives meaſured from the point of their concourſe to the diſtance of the edges of the Knives at the end of that length, and ſo is the fifth part of an Inch to the 160th part. So then the dark lines above-mentioned meet in the middle of the Light which paſſes between the Knives where they are diſtant the 160th part of an Inch, and the one half of that Light paſſes by the edge of one Knife at a diſtance not greater than the 320th part of an Inch, and falling upon the Paper makes the fringes of the ſhadow of that Knife, and the other half paſſes by the edge of the other Knife, at a diſtance not greater than the 320th part of an Inch, and falling upon the Paper makes the fringes of the ſhadow of the other Knife. But if the Paper be held at a diſtance from the Knives greater than the third part of an Inch, the dark lines above-mentioned meet at a greater diſtance than the fifth part of an Inch from the end of the Light which paſſed between the Knives at the concourſe of their edges; and therefore the Light which falls upon the Paper where thoſe dark lines meet paſſes between the Knives where their edges are diſtant above the 160th part of an Inch.

For at another time when the two Knives were diſtant eight Feet and five Inches from the little Hole in the Window, made with a ſmall Pin as above, the Light which fell upon the Paper where the aforeſaid dark lines met, paſſed between the Knives, where the diſtance

ſtance between their edges was as in the following Table, when the diſtance of the Paper from the Knives was alſo as follows.

Diſtances of the Paper from the Knives in Inches.	*Diſtances between the edges of the Knives in milleſimal parts of an Inch.*
$1\frac{1}{2}$.	0'012.
$3\frac{1}{3}$.	0'020.
$8\frac{3}{5}$.	0'034.
32.	0'057.
96.	0'081.
131.	0'087.

And hence I gather that the Light which makes the fringes upon the Paper is not the ſame Light at all diſtances of the Paper from the Knives, but when the Paper is held near the Knives, the fringes are made by Light which paſſes by the edges of the Knives at a leſs diſtance, and is more bent than when the Paper is held at a greater diſtance from the Knives.

OBS. X.

When the fringes of the ſhadows of the Knives fell perpendicularly upon a Paper at a great diſtance from the Knives, they were in the form of Hyperbolas, and their dimenſions were as follows. Let C A, C B repreſent lines drawn upon the Paper parallel to the edges of the Knives, and between which all the Light would fall, if it paſſed between the edges of the Knives without inflexion; D E a right line drawn through C making Fig. 3.

the

the Angles ACD, BCE, equal to one another, and terminating all the Light whith falls upon the Paper from the point where the edges of the Knives meet; eis, fkt, and glv, three hyperbolical lines repreſenting the terminus of the ſhadow of one of the Knives, the dark line between the firſt and ſecond fringes of that ſhadow, and the dark line between the ſecond and third fringes of the ſame ſhadow; xip, ykq and zlr, three other Hyperbolical lines repreſenting the terminus of the ſhadow of the other Knife, the dark line between the firſt and ſecond fringes of that ſhadow, and the dark line between the ſecond and third fringes of the ſame ſhadow. And conceive that theſe three Hyperbolas are like and equal to the former three, and croſs them in the points i, k and l, and that the ſhadows of the Knives are terminated and diſtinguiſhed from the firſt luminous fringes by the lines eis and xip, until the meeting and croſſing of the fringes, and then thoſe lines croſs the fringes in the form of dark lines, terminating the firſt luminous fringes within ſide, and diſtinguiſhing them from another Light which begins to appear at i, and illuminates all the triangular ſpace ipDEs comprehended by theſe dark lines, and the right line DE. Of theſe Hyperbolas one Aſymptote is the line DE, and their other Aſymptotes are parallel to the lines CA and CB. Let rv repreſent a line drawn any where upon the Paper parallel to the Aſymptote DE, and let this line croſs the right lines AC in m and BC in n, and the ſix dark hyperbolical lines in p, q, r; s, t, v; and by meaſuring the diſtances ps, qt, rv, and thence collecting the the lengths of the ordinates np, nq, nr or ms, mt, mv, and doing this at ſeveral diſtances of the line rv,

from

from the Asymptote DE you may find as many points of these Hyperbolas as you please, and thereby know that these curve lines are Hyperbolas differing little from the conical Hyperbola. And by measuring the lines Ci, Ck, Cl, you may find other points of these Curves.

For instance, when the Knives were distant from the Hole in the Window ten Feet, and the Paper from the Knives 9 Feet, and the Angle contained by the edges of the Knives to which the Angle ACB is equal, was subtended by a chord which was to the Radius as 1 to 32, and the distance of the line rv from the Asymptote DE was half an Inch: I measured the lines ps, qt, rv, and found them 0'35, 0'65, 0'98 Inches respectively, and by adding to their halfs the line $\frac{1}{2}$ mn (which here was the 128th part of an Inch, or 0'0078 Inches) the sums np, nq, nr, were 0'1828, 0'3328, 0'4978 Inches. I measured also the distances of the brightest parts of the fringes which run between pq and st, qr and tv, and next beyond r and v, and found them 0'5, 0'8, and 1'17 Inches.

OBS. XI.

The Sun shining into my darkened Room through a small round Hole made in a plate of Lead with a slender Pin as above; I placed at the Hole a Prism to refract the Light, and form on the opposite Wall the Spectrum of Colours, described in the third Experiment of the first Book. And then I found that the shadows of all Bodies held in the coloured Light between the Prism and the Wall, were bordered with fringes of the Colour of that Light in which they were held. In the full red Light they were totally red without any ſenſible blue or violet, and in the deep blue Light they were totally blue without any ſenſible red or yellow ; and ſo in the green Light they were totally green, excepting a little yellow and blue, which were mixed in the green Light of the Priſm. And comparing the fringes made in the ſeveral coloured Lights, I found that thoſe made in the red Light were largeſt, thoſe made in the violet were leaſt, and thoſe made in the green were of a middle bigneſs. For the fringes with which the ſhadow of a Man's Hair were bordered, being meaſured croſs the ſhadow at the diſtance of ſix Inches from the Hair ; the diſtance between the middle and moſt luminous part of the firſt or innermoſt fringe on one ſide of the ſhadow, and that of the like fringe on the other ſide of the ſhadow, was in the full red Light $\frac{1}{37\frac{1}{2}}$ of an Inch, and in the full violet $\frac{1}{46}$. And the like diſtance between the middle and moſt luminous parts of the ſecond fringes on either ſide the ſhadow was in the full red Light $\frac{1}{22}$, and in the violet $\frac{1}{27}$ of an Inch. And theſe diſtances of the fringes held the ſame proportion at all diſtances from the Hair without any ſenſible variation.

So then the rays which made theſe fringes in the red Light paſſed by the Hair at a greater diſtance than thoſe did which made the like fringes in the violet ; and therefore the Hair in cauſing theſe fringes acted alike upon the red Light or leaſt refrangible rays at a greater diſtance, and upon the violet or moſt refrangible rays at a leſs diſtance, and by thoſe actions diſpoſed the red Light into larger fringes, and the violet into ſmaller, and the Lights of intermediate Colours into fringes of

inter-

intermediate bigneſſes without changing the Colour of of any ſort of Light.

When therefore the Hair in the firſt and ſecond of theſe Obſervations was held in the white beam of the Sun's Light, and caſt a ſhadow which was bordered with three fringes of coloured Light, thoſe Colours aroſe not from any new modifications impreſt upon the rays of Light by the Hair, but only from the various inflections whereby the ſeveral ſorts of rays were ſeparated from one another, which before ſeparation by the mixture of all their Colours, compoſed the white beam of the Sun's Light, but whenever ſeparated compoſe Lights of the ſeveral Colours which they are originally diſpoſed to exhibit. In this ~~13~~th Obſervation, where the 11th
Colours are ſeparated before the Light paſſes by the Hair, the leaſt refrangible rays, which when ſeparated from the reſt make red, were inflected at a greater diſtance from the Hair, ſo as to make three red fringes at a greater diſtance from the middle of the ſhadow of the Hair; and the moſt refrangible rays which when ſeparated make violet, were inflected at a leſs diſtance from the Hair, ſo as to make three violet fringes at a leſs diſtance from the middle of the ſhadow of the Hair. And other rays of intermediate degrees of refrangibility were inflected at intermediate diſtances from the Hair, ſo as to make fringes of intermediate Colours at intermediate diſtances from the middle of the ſhadow of the Hair. And in the ſecond Obſervation, where all the Colours are mixed in the white Light which paſſes by the Hair, theſe Colours are ſeparated by the various inflexions of the rays, and the fringes which they make appear all together, and the innermoſt

fringes being contiguous make one broad fringe compoſed of all the Colours in due order, the violet lying on the inſide of the fringe next the ſhadow, the red on the outſide furtheſt from the ſhadow, and the blue, green and yellow, in the middle. And, in like manner, the middlemoſt fringes of all the Colours lying in order, and being contiguous, make another broad fringe compoſed of all the Colours; and the outmoſt fringes of all the Colours lying in order, and being contiguous, make a third broad fringe compoſed of all the Colours. Theſe are the three fringes of coloured Light with which the ſhadows of all Bodies are bordered in the ſecond Obſervation.

When I made the foregoing Obſervations, I deſigned to repeat moſt of them with more care and exactneſs, and to make ſome new ones for determining the manner how the rays of Light are bent in their paſſage by Bodies for making the fringes of Colours with the dark lines between them. But I was then interrupted, and cannot now think of taking theſe things into further conſideration. And ſince I have not finiſhed this part of my Deſign, I ſhall conclude, with propoſing only ſome Queries in order to a further ſearch to be made by others.

Query 1. Do not Bodies act upon Light at a diſtance, and by their action bend its rays, and is not this action (*cæteris paribus*) ſtrongeſt at the leaſt diſtance?

Qu. 2. Do not the rays which differ in refrangibility differ alſo in flexibility, and are they not by their different inflexions ſeparated from one another, ſo as after ſeparation to make the Colours in the three fringes

above

above deſcribed? And after what manner are they inflected to make thoſe fringes?

Qu. 3. Are not the rays of Light in paſſing by the edges and ſides of Bodies, bent ſeveral times backwards and forwards, with a motion like that of an Eel? And do not the three fringes of coloured Light above-mentioned, ariſe from three ſuch bendings?

Qu. 4. Do not the rays of Light which fall upon Bodies, and are reflected or refracted, begin to bend before they arrive at the Bodies; and are they not reflected, refracted and inflected by one and the ſame Principle, acting variouſly in various circumſtances?

Qu. 5. Do not Bodies and Light act mutually upon one another, that is to ſay, Bodies upon Light in emitting, reflecting, refracting and inflecting it, and Light upon Bodies for heating them, and putting their parts into a vibrating motion wherein heat conſiſts?

Qu. 6. Do not black Bodies conceive heat more eaſily from Light than thoſe of other Colours do, by reaſon that the Light falling on them is not reflected outwards, but enters the Bodies, and is often reflected and refracted within them, until it be ſtifled and loſt?

Qu. 7. Is not the ſtrength and vigor of the action between Light and ſulphureous Bodies obſerved above, one reaſon why ſulphureous Bodies take fire more readily, and burn more vehemently, then other Bodies do?

Qu. 8. Do not all fixt Bodies when heated beyond a certain degree, emit Light and ſhine, and is not this emiſſion performed by the vibrating motions of their parts?

Qu. 9.

Qu. 9. Is not fire a Body heated ſo hot as to emit Light copiouſly? For what elſe is a red hot Iron than fire? And what elſe is a burning Coal than red hot Wood?

Qu. 10. Is not flame a vapour, fume or exhalation heated red hot, that is, ſo hot as to ſhine? For Bodies do not flame without emitting a copious fume, and this fume burns in the flame. The *Ignis Fatuus* is a vapour ſhining without heat, and is there not the ſame difference between this vapour and flame, as between rotten Wood ſhining without heat and burning Coals of fire? In diſtilling hot Spirits, if the head of the ſtill be taken off, the vapour which aſcends out of the Still will take fire at the flame of a Candle, and turn into flame, and the flame will run along the vapour from the Candle to the Still. Some Bodies heated by motion or fermentation, if the heat grow intenſe fume copiouſly, and if the heat be great enough the fumes will ſhine and become flame. Metals in fuſion do not flame for want of a copious fume, except Spelter which fumes copiouſly, and thereby flames. All flaming Bodies, as Oyl, Tallow, Wax, Wood, foſſil Coals, Pitch, Sulphur, by flaming waſte and vaniſh into burning ſmoke, which ſmoke, if the flame be put out, is very thick and viſible, and ſometimes ſmells ſtrongly, but in the flame loſes its ſmell by burning, and according to the nature of the ſmoke the flame is of ſeveral Colours, as that of Sulphur blue, that of Copper opened with Sublimate green, that of Tallow yellow, that of Camphire white. Smoke paſſing through flame cannot but grow red hot, and red hot ſmoke can have no other appearance than that of flame. Gunpowder in taking fire vanishes into flaming smoak. The charcoal & brimstone take fire readily & set the Nitre on fire, & the spirit of the Nitre being thereby rarefied into vapour rushes out with a blast as the vapor of water doth out of an Æolipile. The brimstone also being volatile turns into vapour wch increases the blast, & the acid vapour of ye brimstone (that vapour wch under a Bell destills into oyle of Sulphur) entring into the fixt body of the Nitre, sets the spirit of the Nitre more at liberty then before & causes a violent fermentation whereby the heat is increased & the fixt body of the Nitre is rarefied into smoak & makes the blast more violent & smart. For if salt of Tartar be mixed with Gunpowder & the compound be heated gradually till it takes fire, the explosion will be more smart & violent then otherwise, which can arise from nothing else then the action of the vapour of the Gunpowder upon the salt of Tartar. The explosion therefore of the Gunpowder arises from a quick & violent action whereby the compound being suddenly & vehemently heated rarefies & turns into smoak or vapour, & this vapour by the violence of the action being at the same time heated red hot appears like flame.

Qu. 11.

Qu. 11. Do not great Bodies conserve their heat the longest, their parts heating one another, and may not great dense and fix'd Bodies, when heated beyond a certain degree, emit Light so copiously, as by the emission and reaction of ~~its~~ their Light, and the reflexions and refractions of ~~its~~ their rays within ~~its~~ their pores, to grow still hotter, ~~till it~~ being heated faster by those causes then they are cooled by other causes, untill they comes to a certain period of heat, such as is that of the Sun? And are not the Sun and fix'd Stars great Earths vehemently hot, whose heat is conserved by the greatness of the Bodies, and the mutual action and reaction between them, and the Light which they emit, and whose parts are kept from fuming away, not only by their fixity, but also by the vast weight and density of the Atmospheres incumbent upon them, and very strongly compressing them, and condensing the vapours and exhalations which arise from them? For if water be made blood warm in any pellucid vessel, & the air be then drawn out of the vessel: the water will boyle in vacuo as vehemently as it doth with a much greater heat in a pot over a fire in the open air. For the weight of the incumbent Atmosphere keeps down the vapors & hinders the water from boyling till the heat be much greater then is requisite to make it boyle in vacuo. And so the vast weight of the Suns atmosphere may there hinder bodies from evaporating & fuming away without a much greater heat then would suffice to make them evaporate & fume away on the surface of the earth, & may recondense the vapors & exhalations as fast as they rise from the Sun & make them fall down again upon his body & encrease his heat by their action as the Air with us encreases the heat of a culinary fire, & by recondensing the vapors preserve that great body from wasting otherwise than perhaps by his emission of light.

Qu. 12. Do not the rays of Light in falling upon the bottom of the Eye excite vibrations in the *Tunica retina*? Which vibrations, being propagated along the solid fibres of the optick Nerves into the Brain, cause the sense of seeing. For because dense Bodies conserve their heat a long time, and the densest Bodies conserve their heat the longest, the vibrations of their parts are of a lasting nature, and therefore may be propagated along solid fibres of uniform dense matter to a great distance, for conveying into the Brain the impressions made upon all the Organs of sense. For that motion which can continue long in one and the same part of a Body, can be propagated a long way from one part to another, supposing the Body homogeneal, so that the motion may not be reflected, refracted, interrupted or disordered by any unevenness of the Body.

Qu. 13.

Qu. 13. Do not ſeveral ſort of rays make vibrations of ſeveral bigneſſes, which according to their bigneſſes excite ſenſations of ſeveral Colours, much after the manner that the vibrations of the Air, according to their ſeveral bigneſſes excite ſenſations of ſeveral ſounds? And particularly do not the moſt refrangible rays excite the ſhorteſt vibrations for making a ſenſation of deep violet, the leaſt refrangible the largeſt for making a ſenſation of deep red, and the ſeveral intermediate ſorts of rays, vibrations of ſeveral intermediate bigneſſes to make ſenſations of the ſeveral intermediate Colours?

Qu. 14. May not the harmony and diſcord of Colours ariſe from the proportions of the vibrations propagated through the fibres of the optick Nerves into the Brain, as the harmony and diſcord of ſounds ariſes from the proportions of the vibrations of the Air? For ſome Colours are agreeable, as thoſe of Gold and Indico, and others diſagree.

Qu. 15. Are not the Species of Objects ſeen with both Eyes united where the optick Nerves meet before they come into the Brain, the fibres on the right ſide of both Nerves uniting there, and after union going thence into the Brain in the Nerve which is on the right ſide of the Head, and the fibres on the left ſide of both Nerves uniting in the ſame place, and after union going into the Brain in the Nerve which is on the left ſide of the Head, and theſe two Nerves meeting in the Brain in ſuch a manner that their fibres make but one entire Species or Picture, half of which on the right ſide of the Senſorium comes from the right ſide of both Eyes through the right ſide of both

both optick Nerves to the place where the Nerves meet, and from thence on the right ſide of the Head into the Brain, and the other half on the left ſide of the Senſorium comes in like manner from the left ſide of both Eyes. For the optick Nerves of ſuch Animals as look the ſame way with both Eyes (as of Men, Dogs, Sheep, Oxen, *&c.*) meet before they come into the Brain, but the optick Nerves of ſuch Animals as do not look the ſame way with both Eyes (as of Fiſhes and of the Chameleon) do not meet, if I am rightly informed.

Qu. 16. When a Man in the dark preſſes either corner of his Eye with his Finger, and turns his Eye away from his Finger, he will ſee a Circle of Colours like thoſe in the Feather of a Peacock's Tail? Do not theſe Colours ariſe from ſuch motions excited in the bottom of the Eye by the preſſure of the Finger, as at other times are excited there by Light for cauſing Viſion? And when a Man by a ſtroke upon his Eye ſees a Flaſh of Light, are not the like Motions excited in the *Retina* by the ſtroke?

Fig. 1.

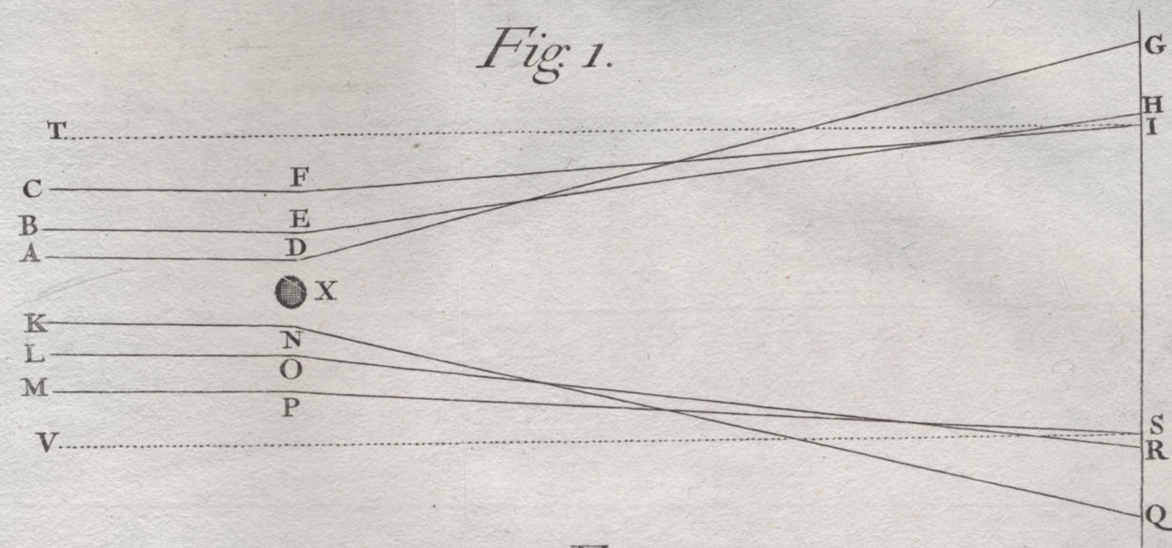


Fig. 2.

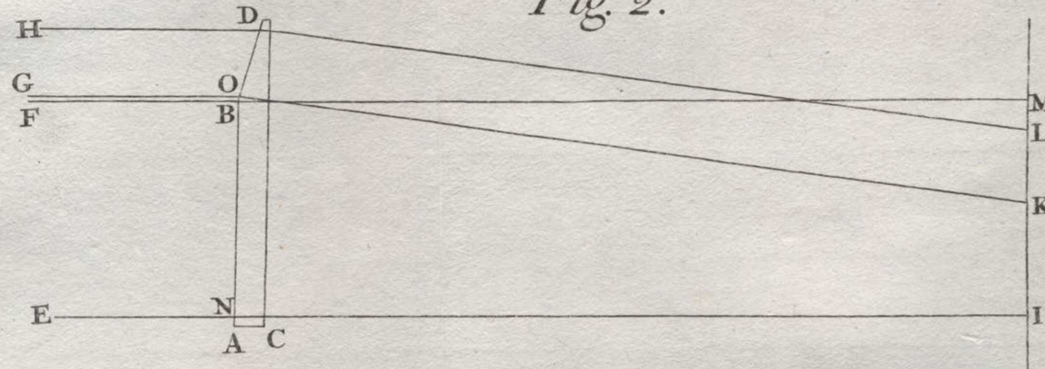
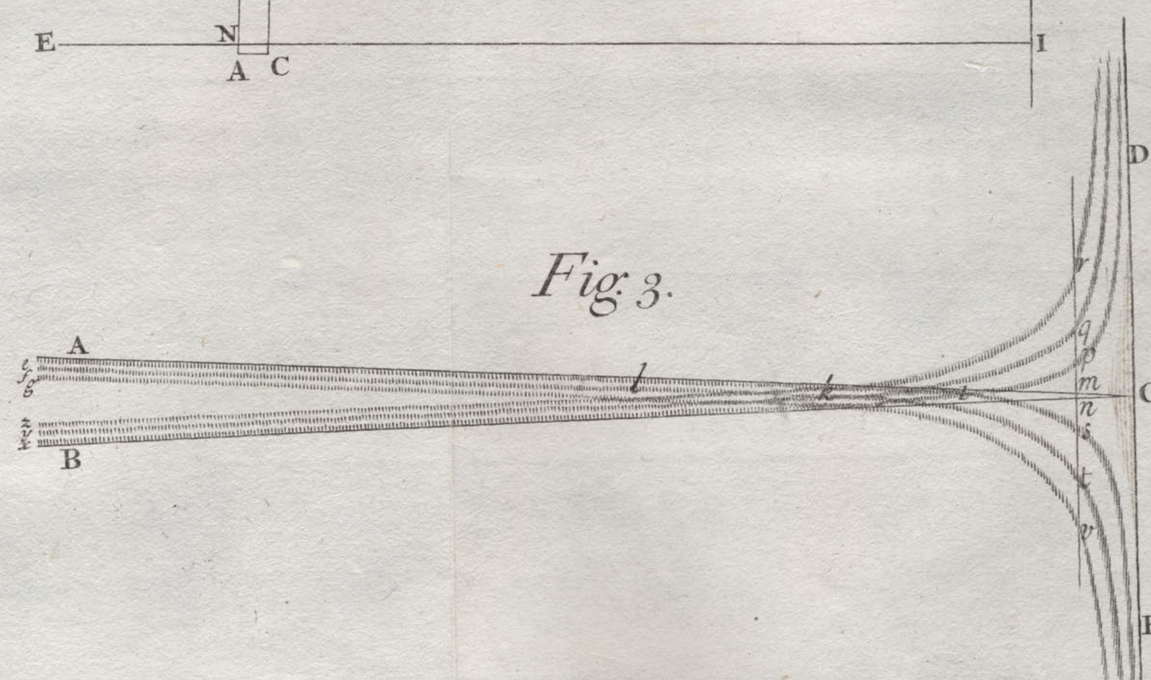


Fig. 3.



ENUMERATIO

LINEARUM

TERTII ORDINIS.

ENUMERATIO LINEARUM TERTII ORDINIS.

Lineæ Geometricæ secundum numerum dimensionum æquationis qua relatio inter Ordinatas & Abscissas definitur, vel (quod perinde est) secundùm numerum punctorum in quibus a linea recta secari possunt, optimè distinguuntur in Ordines. Qua ratione linea primi Ordinis erit Recta sola, eæ secundi sive quadratici ordinis erunt sectiones Conicæ & Circulus, & eæ tertii sive cubici Ordinis Parabola Cubica, Parabola Neiliana, Cissois veterum & reliquæ quas hic enumerare suscepimus. Curva autem primi generis, (siquidem recta inter Curvas non est numeranda) eadem est cum Linea secundi Ordinis, & Curva secundi generis eadem cum Linea Ordinis tertii. Et Linea Ordinis infinitesimi ea est quam recta in punctis infinitis secare potest, qualis est Spiralis, Cyclois, Quadratrix & linea omnis quæ per radii vel rotæ revolutiones infinitas generatur. I. *Linearum Ordines.*

II. *Proprietates Sectionem Conicarum competunt curvis superiorum generum.*

Sectionum Conicarum proprietates præcipuæ a Geometris passim traduntur. Et consimiles sunt proprietates Curvarum secundi generis & reliquarum, ut ex sequenti proprietatum præcipuarum enumeratione constabit.

III. *Curvarum secundi generis Ordinatæ, Diametri, Vertices, Centra, Axes.*

Nam si rectæ plures parallelæ & ad conicam sectionem utrinq; terminatæ ducantur, recta duas earum bisecans bisecabit alias omnes, ideoq; dicitur *Diameter* figuræ & rectæ bisectæ dicuntur *Ordinatim applicatæ* ad Diametrum, & concursus omnium Diametrorum est *Centrum* figuræ, & intersectio Curvæ & diametri *Vertex* nominatur, & diameter illa *Axis* est cui ordinatim applicatæ insistunt ad angulos rectos. Et ad eundem modum in Curvis secundi generis, si rectæ duæ quævis parallelæ ducantur occurrentes Curvæ in tribus punctis: recta quæ ita secat has parallelas ut summa duarum partium ex uno secantis latere ad curvam terminatarum æquetur parti tertiæ ex altero latere ad curvam terminatæ, eodem modo secabit omnes alias his parallelas curvæq; in tribus punctis occurrentes rectas, hoc est, ita ut summa partium duarum ex uno ipsius latere semper æquetur parti tertiæ ex altero latere. Has itaq; tres partes quæ hinc inde æquantur, *Ordinatim applicatas* & rectam secantem cui ordinatim applicantur *Diametrum* & intersectionem diametri & curvæ *Verticem* & concursum duarum diametrorum *Centrum* nominare licet. Diameter autem ad Ordinatas rectangula si modo aliqua sit, etiam *Axis* dici potest, & ubi omnes diametri in eodem puncto concurrunt istud erit *Centrum generale*.

Hyper-

Hyperbola primi generis duas *Aſymptotos*, ea ſecundi tres, ea tertii quatuor & non plures habere poteſt, & ſic in reliquis. Et quemadmodum partes lineæ cujuſvis rectæ inter Hyperbolam Conicam & duas ejus Aſymptotos ſunt hinc inde æquales: ſic in Hyperbolis ſecundi generis ſi ducatur recta quævis ſecans tam Curvam quàm tres ejus Aſymptotos in tribus punctis, ſumma duarum partium iſtius rectæ quæ a duobus quibuſvis Aſymptotis in eandem plagam ad duo puncta Curvæ extenduntur æqualis erit parti tertiæ quæ a tertia Aſymptoto in plagam contrariam ad tertium Curvæ punctum extenditur. IV. *Aſymptoti & earum proprietates.*

Et quemadmodum in Conicis ſectionibus non Parabolicis quadratum Ordinatim applicatæ, hoc eſt, rectangulum Ordinatarum quæ ad contrarias partes Diametri ducuntur, eſt ad rectangulum partium Diametri quæ ad Vertices Ellipſeos vel Hyperbolæ terminantur, ut data quædam linea quæ dicitur *Latus rectum*, ad partem diametri quæ inter Vertices jacet & dicitur *Latus transverſum*: ſic in Curvis non Parabolicis ſecundi generis Parallelepipedum ſub tribus Ordinatim applicatis eſt ad Parallelepipedum ſub partibus Diametri ad Ordinatas & tres Vertices figuræ abſciſſis, in ratione quadam data: in qua ratione ſi ſumantur tres rectæ ad tres partes diametri inter vertices figuræ ſitas ſingulæ ad ſingulas, tunc illæ tres rectæ dici poſſunt *Latera recta* figuræ, & illæ partes Diametri inter Vertices *Latera transverſa*. Et ſicut in Parabola Conica quæ ad unam & eandem diametrum unicum tantum habet Verticem, rectangulum ſub Ordinatis æquatur rectangulo ſub parte Diametri quæ ad Ordinatas & Verticem abſcinditur & recta V. *Latera recta & transverſa.*

quadam

quadam data quæ Latus rectum dicitur, sic in Curvis secundi generis quæ non nisi duos habent Vertices ad eandem Diametrum, Parallelepipedum sub Ordinatis tribus æquatur Parallelepipedo sub duabus partibus Diametri ad Ordinatas & Vertices illos duos abscissis, & recta quadam data quæ proinde *Latus rectum* dici potest.

VI. *Ratio contentorum sub Parallelarum segmentis.*

Deniq; sicut in Conicis sectionibus ubi duæ parallelæ ad Curvam utrinq; terminatæ secantur a duabus parallelis ad Curvam utrinq; terminatis, prima a tertia & secunda a quarta, rectangulum partium primæ est ad rectangulum partium tertiæ ut rectangulum partium secundæ ad rectangulum partium quartæ: sic ubi quatuor tales rectæ occurrunt Curvæ secundi generis singulæ in tribus punctis, parallelepipedum partium primæ rectæ erit ad parallelepidedum partium tertiæ, ut parallelepipedum partium secundæ ad parallelepipedum partium quartæ.

VII. *Crura Hyperbolica & Parabolica & eorum plaga.*

Curvarum secundi & superiorum generum æque atq; primi crura omnia in infinitum progredientia vel *Hyperbolici* sunt generis vel *Parabolici*. Crus *Hyperbolicum* voco quod ad Asymptoton aliquam in infinitum appropinquat, *Parabolicum* quod Asymptoto destituitur. Hæc crura ex tangentibus optime dignoscuntur. Nam si punctum contactus in infinitum abeat tangens cruris Hyperbolici cum Asymptoto coincidet & tangens cruris Parabolici in infinitum recedet, evanescet & nullibi reperietur. Invenitur igitur Asymptotos cruris cujusvis quærendo tangentem cruris illius ad punctum infinite distans. Plaga autem cruris infiniti invenitur quærendo positionem rectæ cujusvis quæ tangenti parallela est ubi punctum

ctum contactus in infinitum abit. Nam hæc recta in eandem plagam cum crure infinito dirigitur.

VIII. *Reductio Curvarum omnium generis secundi ad æquationum casus quatuor. Casus primus.*

Lineæ omnes Ordinis primi, tertii, quinti, septimi & imparis cujusq; duo habent ad minimum crura in infinitum versus plagas oppositas progredientia. Et lineæ omnes tertii Ordinis duo habent ejusmodi crura in plagas oppositas progredientia in quas nulla alia earum crura infinita (præterquam in Parabola Cartesiana) tendunt. Crura autem vel Hyperbolica sunt vel Parabolica. Et recta infinita cruribus parallela Curvam vel in duobus punctis vel in unico tantum secare potest. Casus non sunt plures. Si crura illa sint Hyperbolici generis, sit GAS eorum Asymptotos & huic parallela agatur recta quævis CBc ad Curvam utrinque (si fieri potest) terminata eademq; bisecetur in puncto X, & locus puncti illius X erit Hyperbola Conica (puta XΦ) cujus una Asymptotos est AS. Sit ejus altera Asymptotos AB, & æquatio qua relatio inter Ordinatam BC & Abscissam AB definitur, si AB dicatur x & BC y, semper induet hanc formam $xyy + ey = ax^3 + bxx + cx + d$. Ubi termini e, a, b, c, d, designant quantitates datas ~~eum~~ signis suis + & − affectas, quarum quælibet deesse possunt modo ex earum defectu figura in sectionem conicam non vertatur. Potest autem Hyberbola illa Conica cum asymptotis suis coincidere, id est punctum X in recta AB locari: & tunc terminus $+ey$ deest. *Fig. 1.*

IX. *Casus secundus.*

At si recta illa CBc non potest utrinq; ad Curvam terminari sed Curvæ in unico tantum puncto occurrit: age quamvis positione datam rectam AB asymptoto AS occurrentem in A, ut & aliam quamvis BC asymptoto illi parallelam Curvæque occurrentem in puncto C, & æquatio qua relatio inter Ordinatam

BC

B C & Abſciſſam A B definitur, ſemper induet hanc formam $xy = ax^3 + bxx + cx + d$.

X. *Caſus tertius.* Quod ſi crura illa oppoſita Parabolici ſint generis, recta C B c ad Curvam utrinque, ſi fieri poteſt, terminata in plagam crurum ducatur & biſecetur in B, & locus puncti B erit linea recta. Sit iſta A B, terminata ad datum quodvis punctum A, & æquatio qua relatio inter Ordinatam B C & Abſciſſam A B definitur, ſemper induet hanc formam, $yy = ax^3 + bxx + cx + d$.

XI. *Caſus quartus.* At vero ſi recta illa C B c in unico tantum puncto occurrat Curvæ, ideoq; ad Curvam utrinq; terminari non poſſit: ſit punctum illud C, & incidat recta illa ad punctum B in rectam quamvis aliam poſitione datam & ad datum quodvis punctum A terminatam A B: & æquatio qua relatio inter Ordinatam B C & Abſciſſam A C definitur ſemper induet hanc formam, $y = ax^3 + bxx + cx + d$.

XII. *Nomina formarum.* Enumerando curvas horum caſuum, Hyperbolam vocabimus *inſcriptam* quæ tota jacet in Aſymptotôn angulo ad inſtar Hyperbolæ conicæ, *circumſcriptam* quæ Aſymptotos ſecat & partes abſciſſas in ſinu ſuo amplectitur, *ambigenam* quæ uno crure infinito inſcribitur & altero circumſcribitur, *convergentem* cujus crura concavitate ſua ſeinvicem reſpiciunt & in plagam eandem diriguntur, *divergentem* cujus crura convexitate ſua ſeinvicem reſpiciunt & in plagas contrarias diriguntur, *cruribus contrariis præditam* cujus crura in partes contrarias convexa ſunt & in plagas contrarias infinita, *Conchoidalem* quæ vertice concavo & cruribus divergentibus ad aſymptoton applicatur, *anguineam* quæ flexibus contrariis aſymptoton ſecat

&

& utrinq; in crura contraria producitur, *cruciformem* quæ conjugatam decussat, *nodatam* quæ seipsam decussat in orbem redeundo, *cuspidatam* cujus partes duæ in angulo contactus concurrunt & ibi terminantur, *punctatam* quæ conjugatam habet Ovalem infinite parvam id est punctum, & *puram* quæ per impossibilitatem duarum radicum Ovali, Nodo, Cuspide & Puncto conjugato privatur. Eodem sensu Parabolam quoq; *convergentem*, *divergentem*, *cruribus contrariis præditam*, *cruciformem*, *nodatam*, *cuspidatam*, *punctatam* & *puram* nominabimus.

XIII. *De Hyperbola redundante & ejus tribus Asymptotis.*

In casu primo si terminus ax^3 affirmativus est Figura erit Hyperbola triplex cum sex cruribus Hyperbolicis quæ juxta tres Asymptotos quarum nullæ sunt parallelæ in infinitum progrediuntur, binæ juxta unamquamq; in plagas contrarias. Et hæ Asymptoti si terminus bxx non deest se mutuo secabunt in tribus punctis triangulum ($Dd\delta$) inter se continentes, sin terminus bxx deest convergent omnes ad idem punctum. In priori casu cape $AD = \frac{-b}{2a}$, & $Ad = A\delta = \frac{b}{2\sqrt{a}}$, ac junge Dd, $D\delta$, & erunt A~~D~~, Dd, $D\delta$ tres Asymptoti. In posteriori duc ordinatam quamvis BC, & in ea utrinq; producta cape hine inde BF & Bf sibi mutuo æquales & in ea ratione ad A B quam habet $\sqrt{a}$ ad 1, jungeq; AF, Af, & erunt A~~β~~, AF, Af tres Asympoti. Hanc autem Hyperbolam vocamus redundantem quia numero crurum Hyperbolicorum Sectiones Conicas superat.

Fig. 1. $\frac{b}{2\sqrt{a}}$ Ad

Ordinatæ principali AG parallelam

Fig 30

$\sqrt{a}$ ad 1, AG

XIV. *De hujus Hyperbolæ diametris & situ crurum infinitorum.*

In Hyperbola omni redundante si neq; terminus ey desit neq; sit $bb - 4ac$ æquale $\pm ae\sqrt{a}$, curva nullam habebit diametrum, sin eorum alterutrum accidat curva habebit unicam diametrum, & tres si utrumque. Diameter autem semper transit per intersectionem duarum Asymptoton & bisecat rectas omnes quæ ad Asymptotos illas utrinq; terminantur & parallelæ sunt Asymptoto tertiæ. Estq; abscissa AB diameter Figuræ quoties terminus ey deest. Diametrum vero absolute dictam hic & in sequentibus in vulgari significatu usurpo, nempe pro abscissa quæ passim habet ordinatas binas æquales ad idem punctum hinc inde insistentes.

XV.
Hyperbolæ novem redundantes quæ diametro destituuntur & tres habent Asymptotos triangulum capientes.

Si Hyperbola redundans nullam habet diametrum quærantur Æquationis hujus $ax^4 + bx^3 + cxx + dx + \frac{1}{4} = o$ radices quatuor seu valores ipsius x. Eæ sunto AP, Aϖ, Aπ, Ap. Erigantur ordinatæ PT, $\varpi\tau$, π⅂, pt, & hæ tangent Curvam in punctis totidem T, τ, ⅂, t, & tangendo dabunt limites Curvæ per quos species ejus innotescet.

Fig. 1, 2.

Nam si radices omnes AP, Aϖ, Aπ, Ap sunt reales, ejusdem signi & inæquales, Curva constat ex tribus Hyperbolis, (inscripta circumscripta & ambigena) *cum Ovali*. Hyperbolarum una jacet versus D, altera versus d, tertia versus δ, & Ovalis semper jacet intra triangulam D d δ, atq; etiam inter medios limites ⅂ & τ, in quibus utiq; tangitur ab ordinatis π⅂ & $\varpi\tau$. Et hæc est species prima.

Fig. 3, 4.

Si e radicibus duæ maximæ Aπ, A *p*, vel duæ minimæ AP, Aϖ æquantur inter se, & ejusdem sunt signi cum alteris duobus, Ovalis & Hyperbola circumscripta sibi inxicem junguntur coeuntibus earum punctis contactus ⅂ & t vel T & τ, & crura Hyperbolæ sese decussando in Ovalem continuantur, figuram *nodatam* efficientia. Quæ species est secunda.

Si

Si e radicibus tres maximæ $A\rho$, $A\pi$, $A\varpi$, vel tres minimæ $A\pi$, $A\varpi$, AP æquentur inter se, Nodus in *cuspidem* acutissimum convertetur. Nam crura duo Hyperbolæ circumscriptæ ibi in angulo contactus concurrent & non ultra producentur. Et hæc est species tertia. *Fig.* 5, 6.

Si e radicibus duæ mediæ $A\varpi$ & $A\pi$ æquentur inter se, puncta contactus τ & $\daleth$ coincidunt, & propterea Ovalis interjecta in punctum evanuit, & constat figura ex tribus Hyperbolis, inscripta, circumscripta & ambigena cum *puncto* conjugato. Quæ est species quarta. *Fig.* 7.

Si duæ ex radicibus sunt impossibiles & reliquæ duæ inæquales & ejusdem signi (nam signa contraria habere nequeunt,) *puræ* habebuntur Hyperbolæ tres sine Ovali vel Nodo vel cuspide vel puncto conjugato, & hæ Hyperbolæ vel ad latera trianguli ab Asymptotis comprehensi vel ad angulos ejus jacebunt & perinde speciem vel quintam vel sextam constituent. *Fig.* 7, 8, 13, 14.

Si e radicibus duæ sunt æquales & alteræ duæ vel impossibiles sunt vel reales cum signis quæ a signis æqualium radicum diversa sunt, figura *cruciformis* habebitur, nempe duæ ex Hyperbolis seinvicem decussabunt idq; vel ad verticem trianguli ab Asymptotis comprehensi, vel ad ejus basem. Quæ duæ species sunt septima & octava. *Fig.* 9, 10, 15, 16.

Si deniq; radices omnes sunt impossibiles vel si omnes sunt reales & inæquales & earum duæ sunt affirmativæ & alteræ duæ negativæ, tunc duæ habebuntur Hyperbolæ ad angulos oppositos duarum *Fig.* 11, 12.

Aſymptotôn cum Hyperbola *anguinea* circa Aſymptoton tertiam. Quæ ſpecies eſt nona.

Et hi ſunt omnes radicum caſus poſſibiles. Nam ſi duæ radices ſunt æquales inter ſe, & aliæ duæ ſunt etiam inter ſe æquales, Figura evadet Sectio Conica cum linea recta.

XVI. *Hyperbolæ duodecim redundantes cum unica tantum Diametro.* Si Hyperbola redundans habet unicam tantum Diametrum, ſit ejus Diameter Abſciſſa AB, & æquationis hujus $ax^3+bxx+cx+d=0$ quære tres radices ſeu valores x.

Fig. 17. Si radices illæ ſunt omnes reales & ejuſdem ſigni, Figura conſtabit ex *Ovali* intra triangulum $Dd\delta$ jacente & tribus Hyperbolis ad angulos ejus, nempe circumſcripta ad angulum D & inſcriptis duabus ad angulos d & δ. Et hæc eſt ſpecies decima.

Fig. 18. Si radices duæ majores ſunt æquales & tertia ejuſdem ſigni, crura Hyperbolæ jacentis verſus D ſeſe decuſſabunt in forma *Nodi* propter contactum Ovalis. Quæ ſpecies eſt undecima.

Fig. 19. Si tres radices ſunt æquales, Hyperbola iſta fit *cuſpidata* ſine Ovali. Quæ ſpecies eſt duodecima.

Fig. 20. Si radices duæ minores ſunt æquales & tertia ejuſdem ſigni, Ovalis in *punctum* evanuit. Quæ ſpecies eſt decima tertia. In ſpeciebus quatuor noviſſimis Hyperbola quæ jacet verſus D Aſymptotos in ſinu ſuo amplectitur, reliquæ duæ in ſinu Aſymptotôn jacent.

Fig. 20. *Fig.* 21. *Fig.* 22. *Fig.* 23. Si duæ ex radicibus ſunt impoſſibiles habebuntur tres Hyperbolæ *puræ* ſine Ovali decuſſatione vel cuſpide. Et hujus caſus ſpecies ſunt quatuor, nempe decima quarta ſi Hyperbola circumſcripta jacet verſus D &

decima

decima quinta ſi Hyperbola inſcripta jacet verſus D, decima ſexta ſi Hyperbola circumſcripta jacet ſub baſi dδ trianguli Ddδ, & decima ſeptima ſi Hyperbola inſcripta jacet ſub eadem baſi.

Si duæ radices ſunt æquales & tertia ſigni diverſi *Fig.* 24.
figura erit *cruciformis*. Nempe duæ ex tribus Hy- *Fig.* 25.
perbolis ſeinvicem decuſſabunt idq; vel ad verticem trianguli ab Aſymptotis comprehenſi vel ad ejus baſem. Quæ duæ ſpecies ſunt decima octava & decima nona.

Si duæ radices ſunt inæquales & ejuſdem ſigni & tertia eſt ſigni diverſi, duæ habebuntur Hyperbolæ in oppoſitis angulis duarum aſymptotôn cum *Conchoidali* intermedia. Conchoidalis autem vel jace- *Fig.* 27.
bit ad eaſdem partes aſymptoti ſuæ cum triangulo *Fig.* 26.
ab aſymptotis conſtituto, vel ad partes contrarias; & hi duo caſus conſtituunt ſpeciem vigeſimam & vigeſimam primam.

XVII. *Hyperbolæ duæ redundantes cum tribus Diametris.*

Hyperbola redundans quæ habet tres diametros conſtat ex tribus Hyperbolis in ſinubus aſymptotôn jacentibus, idq; vel ad angulos trianguli ab aſympto- *Fig.* 28.
tis comprehenſi vel ad ejus latera. Caſus prior dat *Fig.* 29.
ſpeciem vigeſimam ſecundam,& poſterior ſpeciem vigeſimam tertiam.

XVIII. *Hyperbolæ novem redundantes cum Aſymptotis tribus ad commune punctum convergentibus.*

Si tres aſymptoti in puncto communi ſe mutuo decuſſant, vertuntur ſpecies quinta & ſexta in vigeſimam quartam, ſeptima & octava in vigeſimam quintam, & nona in vigeſimam ſextam ubi Anguinea non tranſit per concurſum aſymptoton, & in vigeſimam ſeptimam ubi tranſit per concurſum illum, quo *Fig.* 30.
caſu termini b ac d deſunt, & concurſus aſympto- *Fig.* 31. *Fig.* 32.
tôn eſt centrum figuræ ab omnibus ejus partibus *Fig.* 33.
oppoſitis

oppositis æqualiter distans. Et hæ quatuor species Diametrum non habent.

Fig. 34. *Fig.* 35. *Fig.* 36. *Fig.* 37. Vertuntur etiam species decima quarta ac decima sexta in vigesimam octavam, decima quinta ac decima septima in vigesimam nonam, decima octava & decima nona in tricesimam, & vigesima cum vigesima prima in tricesimam primam. Et hæ species unicam habent diametrum.

Fig. 38. Ac deniq; species vigesima secunda & vigesima tertia vertuntur in speciem tricesimam secundam cujus tres sunt Diametri per concursum asymptotôn transeuntes. Quæ omnes conversiones facillime intelliguntur faciendo ut triangulum ab asymptotis comprehensum diminuatur donec in punctum evanescat.

XIX. *Hyperbolæ sex defectivæ diametrum non habentes.*

Si in primo æquationum casu terminus ax^3 negativus est, Figura erit Hyberbola defectiva unicam habens asymptoton & duo tantum crura Hyperbolica juxta asymptoton illam in plagas contrarias infinite progredientia. Et asymptotos illa est Ordinata prima & principalis AG. Si terminus ey non deest figura nullam habebit Diametrum, si deest habebit unicam. In priori casu species sic enumerantur.

Fig. 39. Si æquationis hujus $ax^4 = bx^3 + cxx + dx + \frac{1}{4}ee$, radices omnes $A\pi$, AP, Ap, $A\varpi$, sunt reales & inæquales, Figura erit Hyperbola anguinea asymptoton flexu contrario amplexa, cum *Ovali* conjugata. Quæ species est tricesima tertia.

Fig. 40. Si radices duæ mediæ AP & Ap æquentur inter se, Ovalis & Anguinea junguntur sese decussantes in forma *Nodi*. Quæ est species tricesima quarta.

Si

Si tres radices sunt æquales, Nodus vertetur in *cuspidem* acutissimum in vertice anguineæ. Et hæc est species tricesima quinta. *Fig.* 41.

Si e tribus radicibus ejusdem signi duæ maximæ Ap & $A\varpi$ sibi mutuo æquantur, Ovalis in *punctum* evanuit. Quæ species est tricesima sexta. *Fig.* 43.

Si radices duæ quævis imaginariæ sunt, sola manebit Anguinea *pura* sine Ovali, decussatione, cuspide vel puncto conjugato. Si Anguinea illa non transit per punctum A species est tricesima septima, *Fig.* 42. sin transit per punctum illud A (id quod contingit ubi termini b ac d desunt,) punctum illud A erit centrum figuræ rectas omnes per ipsum ductas & ad Curvam utrinq; terminatas bisecans. Et hæc est species tricesima octava. *Fig.* 43.

XX. *Hyperbolæ septem defectivæ, diametrum habentes.*

In altero casu ubi terminus ey deest & propterea figura Diametrum habet, si æquationis hujus $ax^3 = bxx + cx + d$ radices omnes AT, At, $A\tau$, sunt reales, inæquales & ejusdem signi, figura erit Hyperbola Conchoidalis cum *Ovali* ad convexitatem. Quæ est species tricesima nona. *Fig.* 45.

Si duæ radices sunt inæquales & ejusdem signi & tertia est signi contrarii, *Ovalis* jacebit ad concavitatem Conchoidalis. Estq; species quadragesima. *Fig.* 44.

Si radices duæ minores AT, At, sunt æquales & tertia $A\tau$ est ejusdem signi, Ovalis & Conchoidalis jungentur sese decussando in modum *Nodi*. Quæ species est quadragesima prima. *Fig.* 46.

Si tres radices sunt æquales, Nodus mutabitur in *Cuspidem* & figura erit *Cissois Veterum*. Et hæc est species quadragesima secunda. *Fig.* 47.

Si

Fig. 49. Si radices duæ majores ſunt æquales, & tertia eſt ejuſdem ſigni, Conchoidalis habebit *punctum* conjugatum ad convexitatem ſuam, eſtq; ſpecies quadrageſima tertia.

Fig. 49. Si radices duæ ſunt æquales & tertia eſt ſigni contrarii Conchoidalis habebit *punctum* conjugatum ad concavitatem ſuam, eſtq; ſpecies quadrageſima quarta.

Fig. 48, 49. Si radices duæ ſunt impoſſibiles habebitur Conchoidalis *pura* ſine Ovali, Nodo, Cuſpide vel puncto conjugato. Quæ ſpecies eſt quadrageſima quinta.

XXI. *Hyperbolæ ſeptem Parabolicæ Diametrum non habentes.*

Siquando in primo æquationum caſu terminus ax^3 deeſt & terminus bxx non deeſt, Figura erit Hyperbola Parabolica duo habens crura Hyperbolica ad unam Aſymptoton SAG & duo Parabolica in plagam unam & eandem convergentia. Si terminus ey non deeſt figura nullam habebit diametrum, ſin deeſt habebit unicam. In priori caſu ſpecies ſunt hæ.

Fig. 50. Si tres radices AP, Aϖ, Aπ æquationis hujus $bx^3 + cx + dx + \frac{1}{4}ee = 0$ ſunt inæquales & ejuſdem ſigni, figura conſtabit ex *Ovali* & aliis duabus Curvis quæ partim Hyperbolicæ ſunt & partim Parabolicæ. Nempe crura Parabolica continuo ductu junguntur cruribus Hyperbolicis ſibi proximis. Et hæc eſt ſpecies quadregeſima ſexta.

Fig. 51. Si radices duæ minores ſunt æquales & tertia eſt ejuſdem ſigni, Ovalis & una Curvarum illarum Hyperbolo-Parabolicarum junguntur & ſe decuſſant in formam *Nodi*. Quæ ſpecies eſt quadrageſima ſeptima.

Si

Si tres radices ſunt æquales, Nodus ille in Cuſpidem vertitur. Eſtq; ſpecies quadrageſima octava. *Fig.* 52.

Si radices duæ majores ſunt æquales & tertia eſt ejuſdem ſigni, Ovalis in *punctum* conjugatum evanuit. Quæ ſpecies eſt quadrageſima nona. *Fig.* 53.

Si duæ radices ſunt impoſſibiles, manebunt *puræ* illæ duæ curvæ Hyperbolo-parabolicæ ſine Ovali, decuſſation e, cuſpide vel puncto conjugato, & ſpeciem quinquageſimam conſtituent. *Fig.* 53, 54.

Si radices duæ ſunt æquales & tertia eſt ſigni contrarii, Curvæ illæ hyperbolo-parabolicæ junguntur ſeſe decuſſando in morem crucis. Eſtq; ſpecies quinquageſima prima. *Fig.* 55.

Si radices duæ ſunt inæquales & ejuſdem ſigni & tertia eſt ſigni contrarii, figura evadet Hyperbola anguinea circa Aſymptoton AG, cum Parabola conjugata. Et hæc eſt ſpecies quinquageſima ſecunda. *Fig.* 56.

XXII. *Hyperbolæ quatuor Parabolicæ Diametrum habentes.*

In altero caſu ubi terminus ey deeſt & figura Diametrum habet, ſi duæ radices æquationis hujus $bxx + cx + d = 0$ ſunt impoſſibiles, duæ habentur figuræ hyperbolo-parabolicæ a Diametro AB hinc inde æqualiter diſtantes. Quæ ſpecies eſt quinquageſima tertia. *Fig.* 57.

Si æquationis illius radices duæ ſunt impoſſibiles, Figuræ hyperbolo-parabolicæ junguntur ſeſe decuſſantes in morem crucis, & ſpeciem quinquageſimam quartam conſtituunt. *Fig.* 58.

Si radices illæ ſunt inæquales & ejuſdem ſigni, habetur Hyperbola Conchoidalis cum Parabola ex eodem latere Aſymptoti. Eſtq; ſpecies quinquageſima quinta. *Fig.* 59.

Fig. 60. Si radices illæ sunt signi contrarii, habetur Conchoidalis cum Parabola ad alteras partes Asymptoti. Quæ species est quinquagesima sexta.

XXIII. *Quatuor Hyperbolismi Hyperbolæ.* Siquando in primo æquationum casu terminus uterq; ax^3 & bxx deest, figura erit Hyperbolismus sectionis alicujus Conicæ. Hyperbolismum figuræ voco cujus Ordinata prodit applicando contentum sub Ordinata figuræ illius & recta data ad Abscissam communem. Hac ratione linea recta vertitur in hyperbolam Conicam, & sectio omnis Conica vertitur in aliquam figurarum quas hic Hyperbolismos sectionum Conicarum voco. Nam æquatio ad figuras de quibus agimus, nempe $xyy + ey = cx + d$, dat ~~seu~~ Ordinatam $y = \frac{-e \pm \sqrt{ee + 4dx + 4cxx}}{2x}$ quæ generatur applicando contentum sub Ordinata sectionis Conicæ $\frac{-e \pm \sqrt{ee + 4dx + 4cxx}}{2m}$ & recta data m, ad curvarum Abscissam communem x. Unde liquet quod figura genita Hyperbolismus erit Hyperbolæ, Ellipseos vel Parabolæ, perinde ut terminus cx affirmativus est vel negativus vel nullus.

Hyperbolismus Hyperbolæ tres habet asymptotos quarum una est Ordinata prima & principalis Ad, alteræ duæ sunt parallelæ Abscissæ AB & ab eadem hinc inde æqualiter distant. In Ordinata principali Ad cape Ad, $A\delta$ hinc inde æquales quantitati $\sqrt{c}$ & per puncta d ac δ age dg, $\delta\gamma$ Asymptotos Abscissæ AB parallelas.

Ubi terminus ey non deest figura nullam habet diametrum. In hoc casu si æquationis hujus $cxx + dx + \frac{1}{4}ee = 0$ radices duæ AP, Ap sunt reales

&

& inæquales (nam æquales eſſe nequeunt niſi figura ſit Conica ſectio) figura conſtabit ex tribus Hyperbolis ſibi oppoſitis quarum una jacet inter aſymptotos parallelas & alteræ duæ jacent extra. Et hæc eſt ſpecies quinquageſima ſeptima. *Fig.* 61.

Si radices illæ duæ ſunt impoſſibiles, habentur Hyperbolæ duæ oppoſitæ extra aſymptotos parallelas & Anguinea hyperbolica intra eaſdem. Hæc figura duarum eſt ſpecierum. Nam centrum non habet ubi terminus d non deeſt ; ſed ſi terminus ille deeſt punctum A eſt ejus centrum. Prior ſpecies eſt quinquageſima octava, poſterior quinquageſima nona. *Fig.* 62. *Fig.* 63.

Quod ſi terminus ey deeſt, figura conſtabit ex tribus hyperbolis oppoſitis quarum una jacet inter aſymptotos parallelas & alteræ duæ jacent extra ut in ſpecie quinquageſima quarta, & præterea diametrum habet quæ eſt abſciſſa AB. Et hæc eſt ſpecies ſexageſima. *Fig.* 64.

XXIV.
Tres Hyperboliſmi Ellipſeos.

Hyperboliſmus Ellipſeos per hanc æquationem definitur $xyy + ey = cx + d$, termino ex existente negativo ut supra, & unicam habet aſymptoton quæ eſt Ordinata principalis Ad. Si terminus ey non deeſt, figura eſt Hyperbola anguinea ſine diametro atq; etiam ſine centro ſi terminus d non deeſt. Quæ ſpecies eſt ſexageſima prima. *Fig.* 65.

At ſi terminus d deeſt, figura habet centrum ſine diametro & centrum ejus eſt punctum A. Species vero eſt ſexageſima ſecunda. *Fig.* 66.

Et ſi terminus ey deeſt & terminus d non deeſt, figura eſt Conchoidalis ad aſymptoton AG, habetq; diametrum ſine centro, & diameter ejus eſt Abſciſſa AB. Quæ ſpecies eſt ſexageſima tertia. *Fig.* 67.

XXV. *Duo Hyperbolismi Parabolæ.* Hyperbolismus Parabolæ per hanc æquationem definitur $xyy + ey = d$; & duas habet asymptotos, Abscissam AB & Ordinatam primam & principalem AG. Hyperbolæ vero in hac figura sunt duæ, non in asymptotôn angulis oppositis sed in angulis qui

Fig. 68. sunt deinceps jacentes, idq; ad utrumq; latus abscissæ AB, & vel sine diametro si terminus ey ha-

Fig. 69. betur, vel cum diametro si terminus ille deest. Quæ duæ species sunt sexagesima quarta & sexagesima quinta.

XXVI. *Tridens.* In secundo æquationum casu habebatur æquatio $xy = ax^3 + bxx + cx + d$. Et figura in hoc casu habet quatuor crura infinita quorum duo sunt hyperbolica circa asymptoton AG in contrarias partes tendentia & duo Parabolica convergentia & cum prioribus speciem Tridentis fere efformantia. Estq;

Fig. 76. hæc Figura Parabola illa per quam Cartesius æquationes sex dimensionum construxit. Hæc est igitur species sexagesima sexta.

XXVII. *Parabolæ quinque divergentes.* In tertio casu æquatio erat $yy = ax^3 + bxx + cx + d$, & Parabolum designat cujus crura divergunt ab invicem & in contrarias partes infinite progrediuntur. Abscissa AB est ejus diameter & species ejus sunt quinq; sequentes.

Fig. 70, 71. Si æquationis $ax^3 + bx^2 + cx + d = 0$ radices omnes $A\tau$, AT, At sunt reales & inæquales, figura est Parabola divergens campaniformis cum *Ovali* ad verticem. Et species est sexagesima septima.

Fig. 72. *Fig.* 73. Si radices duæ sunt æquales, Parabola prodit vel *nodata* contingendo Ovalem, vel *punctata* ob Ovalem infinite parvam. Quæ duæ species sunt sexagesima octava & sexagesima nona.

Si

Si tres radices ſunt æquales Parabola erit *cuſpidata* in vertice. Et hæc eſt Parabola Neiliana quæ vulgo ſemicubica dicitur. *Fig.* 75.

Si radices duæ ſunt impoſſibiles, habetur Parabola *pura* campaniformis ſpeciem ſeptuageſimam primam conſtituens. *Fig.* 73, 74.

XXVIII. *Parabola cubica.* *Fig.* 77.

In quarto caſu æquato erat $y = ax + bxx + cx + d$, & hæc æquatio Parabolam illam *Walliſianam* deſignat quæ crura habet contraria & *cubica* dici ſolet. Et ſic ſpecies omnino ſunt ſeptuaginta duæ.

XXIX. *Geneſis Curvarum per Umbras.*

Si in planum infinitum a puncto lucido illuminatum umbræ figurarum projiciantur, umbræ ſectionum Conicarum ſemper erunt ſectiones Conicæ, eæ Curvarum ſecundi generis ſemper erunt Curvæ ſecundi generis, eæ curvarum tertii generis ſemper erunt Curvæ tertii generis, & ſic deinceps in infinitum. Et quemadmodum Circulus umbram projiciendo generat ſectiones omnes conicas, ſic Parabolæ quinq; divergentes umbris ſuis generant & exhibent alias omnes ſecundi generis curvas, & ſic Curvæ quædam ſimpliciores aliorum generum inveniri poſſunt quæ alias omnes eorundem generum curvas umbris ſuis a puncto lucido in planum projectis formabunt.

XXX. *Curvarum puncta duplicia.*

Diximus Curvas ſecundi generis a linea recta in punctis tribus ſecari poſſe. Horum duo nonnunquam coincidunt. Ut cum recta per Ovalem infinite parvam tranſit vel per concurſum duarum partium Curvæ ſe mutuo ſecantium vel in cuſpidem coeuntium ducitur. Et ſiquando rectæ omnes in

plagam

plagam cruris alicujus infiniti tendentes Curvam in unico tantum puncto secant (ut fit in ordinatis Parabolæ Cartesianæ & Parabolæ cubicæ, nec non in rectis Abscissæ Hyperbolismorum Hyberbolæ & Parabolæ parallelis) concipiendum est quod rectæ illæ per alia duo Curvæ puncta ad infinitam distantiam sita (ut ita dicam) transeunt. Hujusmodi intersectiones duas coincidentes sive ad finitam sint distantiam sive ad infinitam, vocabimus punctum duplex. Curvæ autem quæ habent punctum duplex describi possunt per sequentia Theoremata.

XXXI.
Theoremata de Curvarum descriptione organica.
Fig. 78.

1. Si anguli duo magnitudine dati PAD, PBD circa polos positione datos A, B rotentur, & eorum crura AP, BP concursu suo P percurrant lineam rectam; crura duo reliqua AD, BD concursu suo D describent sectionem Conicam per polos A, B transeuntem: præterquam ubi linea illa recta transit per polorum alterutrum A vel B, vel anguli BAD, ABD simul evanescunt, quibus in casibus punctum D describet lineam rectam.

2. Si crura prima AP, BP concursu suo P percurrant sectionem Conicam per polum alterutrum A transeuntem, crura duo reliqua AD, BD concursu suo D describent Curvam secundi generis per polum alterum B transeuntem & punctum duplex habentem in polo primo A per quem sectio Conica transit: præterquam ubi anguli BAD, ABD simul evanescunt, quo casu punctum

ctum D deſcribet aliam ſectionem Conicam per polum A tranſeuntem.

3. At ſi ſectio Conica quam punctum P percurrit tranſeat per neutrum polorum A, B, punctum D deſcribet curvam ſecundi vel tertii generis punctum duplex habentem. Et punctum illud duplex in concurſu crurum deſcribentium, A D, B D invenietur ubi anguli B A P, A B P ſimul evaneſcunt. Curva autem deſcripta ſecundi erit generis ſi anguli B A D, A B D ſimul evaneſcunt, alias erit tertii generis & alia duo habebit puncta duplicia in polis A & B.

XXXII. *Sectionum Conicarum deſcriptio per data quinque puncta.*

Jam ſectio Conica determinatur ex datis ejus punctis quinq; & per eadem ſic deſcribi poteſt. Dentur ejus puncta quinq; A, B, C, D, E. Jungantur eorum tria quævis A, B, C & trianguli A B C rotentur anguli duo quivis C A B, C B A circa vertices ſuos A & B, & ubi crurum A C, B C interſectio C ſucceſſive applicatur ad puncta duo reliqua D, E, incidat interſectio crurum reliquorum A B & B A in puncta P & Q. Agatur & infinite producatur recta P Q, & anguli mobiles ita rotentur ut interſectio crurum A B, B A percurrat rectam P Q, & crurum reliquorum interſectio C deſcribet propoſitam ſectionem Conicam per Theorema primum.

XXXIII. *Curvarum ſecundi generis punctum duplex habentium deſcriptio per data ſeptem puncta.*

Curvæ omnes ſecundi generis punctum duplex habentes determinantur ex datis earum punctis ſeptem, quorum unum eſt punctum illud duplex, &

& per eadem puncta sic describi possunt. Dentur Curvæ describendæ puncta quælibet septem A, B, C, D, E, F, G quorum A est punctum duplex. Jungantur punctum A & alia duo quævis e punctis puta B & C; & trianguli ABC rotetur tum angulus CAB circa verticem suum A, tum angulorum reliquorum alteruter ABC circa verticem suum B. Et ubi crurum AC, BC concursus C successive applicatur ad puncta quatuor reliqua D, E, F, G incidat concursus crurum reliquorum AB & BA in puncta quatuor P, Q, R, S. Per puncta illa quatuor & quintum A describatur sectio Conica, & anguli præfati CAB, CBA ita rotentur ut crurum AB, BA concursus percurrat sectionem illam Conicam, & concursus reliquorum crurum AC, BC describet Curvam propositam per Theorema secundum.

Si vice puncti C datur positione recta BC quæ Curvam describendam tangit in B, lineæ AD, AP coincident, & vice anguli DAP habebitur linea recta circa polum A rotanda.

Si punctum duplex A infinite distat debebit Recta ad plagam puncti illius perpetuo dirigi & motu parallelo ferri interea dum angulus ABC circa polum B rotatur.

Describi etiam possunt hæ curvæ paulo aliter per Theorema tertium, sed descriptionem simpliciorem posuisse sufficit.

Eadem methodo Curvas tertii, quarti & superiorum generum describere licet, non omnes quidem sed quotquot ratione aliqua commoda per motum localem describi possunt. Nam curvam aliquam

secundi

ſecundi vel ſuperioris generis punctum duplex non habentem commode deſcribere Problema eſt inter difficiliora numerandum.

XXXIV. *Conſtructio æquationum per deſcriptionem Curvarum.*

Curvarum uſus in Geometria eſt ut per earum interſectiones Problemata ſolvantur. Proponatur æquatio conſtruenda dimenſionum novem $x^9 * + bx^7 + cx^6 + dx^5 + ex^4 + \begin{matrix} f \\ +m \end{matrix} x^3 + gxx + hx + k = 0$. Ubi b, c, d, *&c.* ſignificant quantitates quaſvis datas ſignis ſuis + & − affectas. Aſſumatur æquatio ad Parabolam cubicam $x^3 = y$, & æquatio prior, ſcribendo y pro x^3, evadet $y^3 + bxyy + cyy + dxxy + exy + my + fx^3 + gxx + hx + k = 0$, æquatio ad Curvam aliam ſecundi generis. Ubi m vel f deeſſe poteſt vel pro lubitu aſſumi. Et per harum Curvarum deſcriptiones & interſectiones dabuntur radices æquationis conſtruendæ. Parabolam cubicam ſemel deſcribere ſufficit.

Si æquatio conſtruenda per defectum duorum terminorum ultimorum hx & k reducatur ad ſeptem dimenſiones, Curva altera delendo m, habebit punctum duplex in principio abſciſſæ, & inde facile deſcribi poteſt ut ſupra.

Si æquatio conſtruenda per defectum terminorum trium ultimorum $gxx + hx + k$ reducatur ad ſex dimenſiones, Curva altera delendo f evadet ſectio Conica.

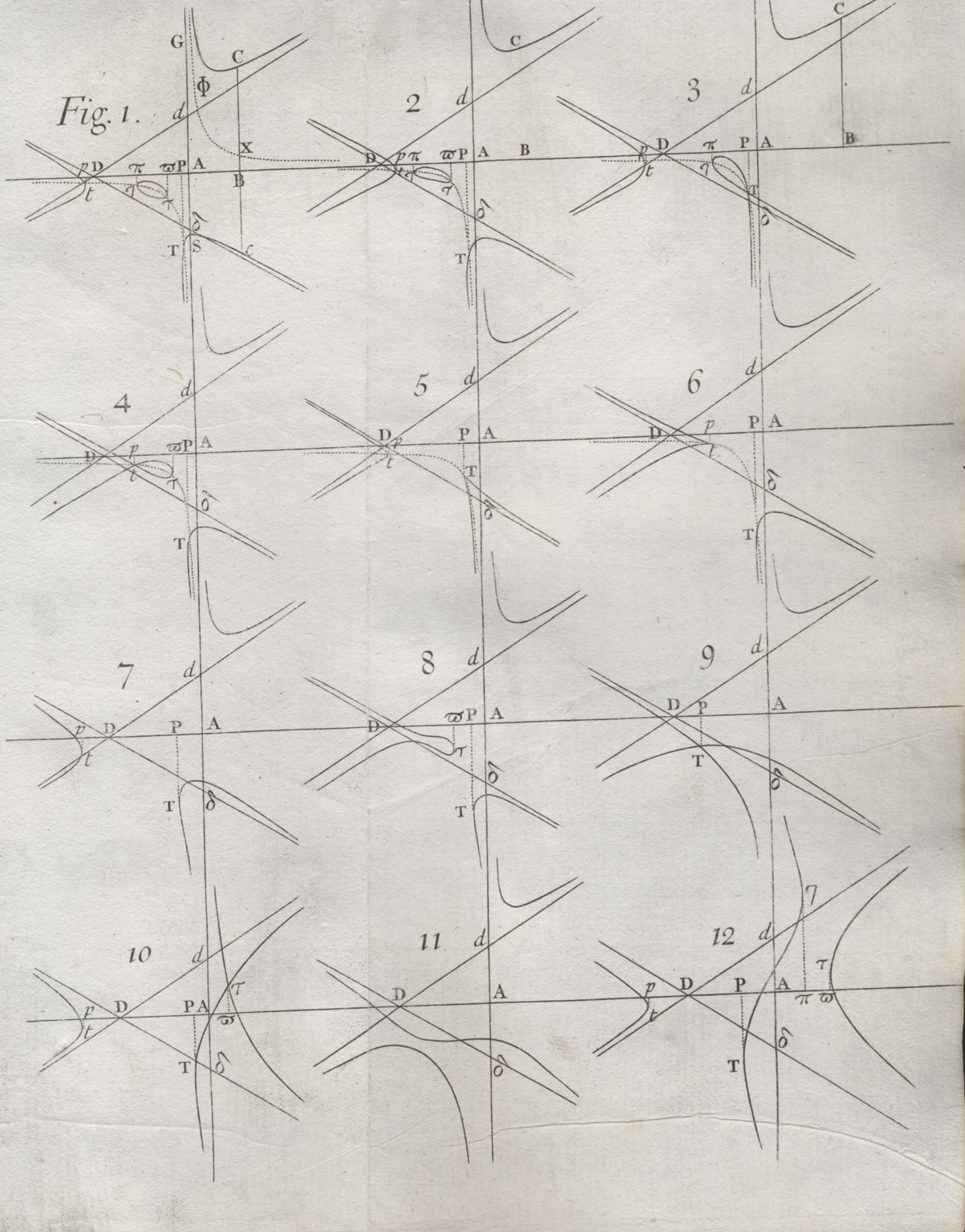
Et ſi per defectum ſex ultimorum terminorum æquatio conſtruenda reducatur ad tres dimenſiones, incidetur in conſtructionem *Walliſianam* per Parabolam cubicam & lineam rectam.

Conſtrui etiam poſſunt æquationes per Hyperboliſmum Parabolæ cum diametro. Ut ſi conſtruenda ſit hæc æquatio dimenſionum novem termino penultimo carens, $\begin{array}{l} a + cxx + dx^3 + ex^4 + fx + gx^6 + hx^7 \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad + m \end{array}$ $+ kx^8 + lx^9 = 0$; aſſumatur æquatio ad Hyperboliſmum illum $xxy = 1$, & ſcribendo y pro $\frac{1}{xx}$, æquatio conſtruenda vertetur in hanc $ay^3 + cyy + dxyy + ey + fxy + mxxy + g + hx + kxx + lx^3 = 0$, quæ curvam ſecundi generis deſignat cujus deſcriptione Problema ſolvetur. Et quantitatum m ac g alterutra hic deeſſe poteſt, vel pro lubitu aſſumi.

Per Parabolam cubicam & Curvas tertii generis conſtruuntur etiam æquationes omnes dimenſionum non pluſquam duodecim, & per eandem Parabolam & curvas quarti generis conſtruuntur omnes dimenſionum non pluſquam quindecim, Et ſic deinceps in infinitum. Et curvæ illæ tertii quarti & ſuperiorum generum deſcribi ſemper poſſunt inveniendo eorum puncta per Geometriam planam. Ut ſi conſtruenda ſit æquatio $x^{12} * + ax^{10} + bx^9 + cx^8 + dx^7 + ex^6 + fx^5 + gx^4 + hx^3 + ixx + kx + l = 0$, & deſcripta habeatur Parabola Cubica; ſit æquatio ad Parabolam illam cubicam $x^3 = y$, & ſcribendo y pro x^3 æquatio conſtruenda vertetur in hanc

$$\begin{array}{lllll} y^4 & + axy^3 & + cxxyy & + fxxy & + ixx = 0, \\ & + b & + dx & + gx & + kx \\ & & + e & + h & + l \end{array}$$

quæ eſt æquatio ad Curvam tertii generis cujus deſcriptione Problema ſolvetur. Deſcribi autem poteſt hæc Curva inveniendo ejus puncta per Geometriam planam, propterea quod indeterminata quantitas x non niſi ad duas dimenſiones aſcendit.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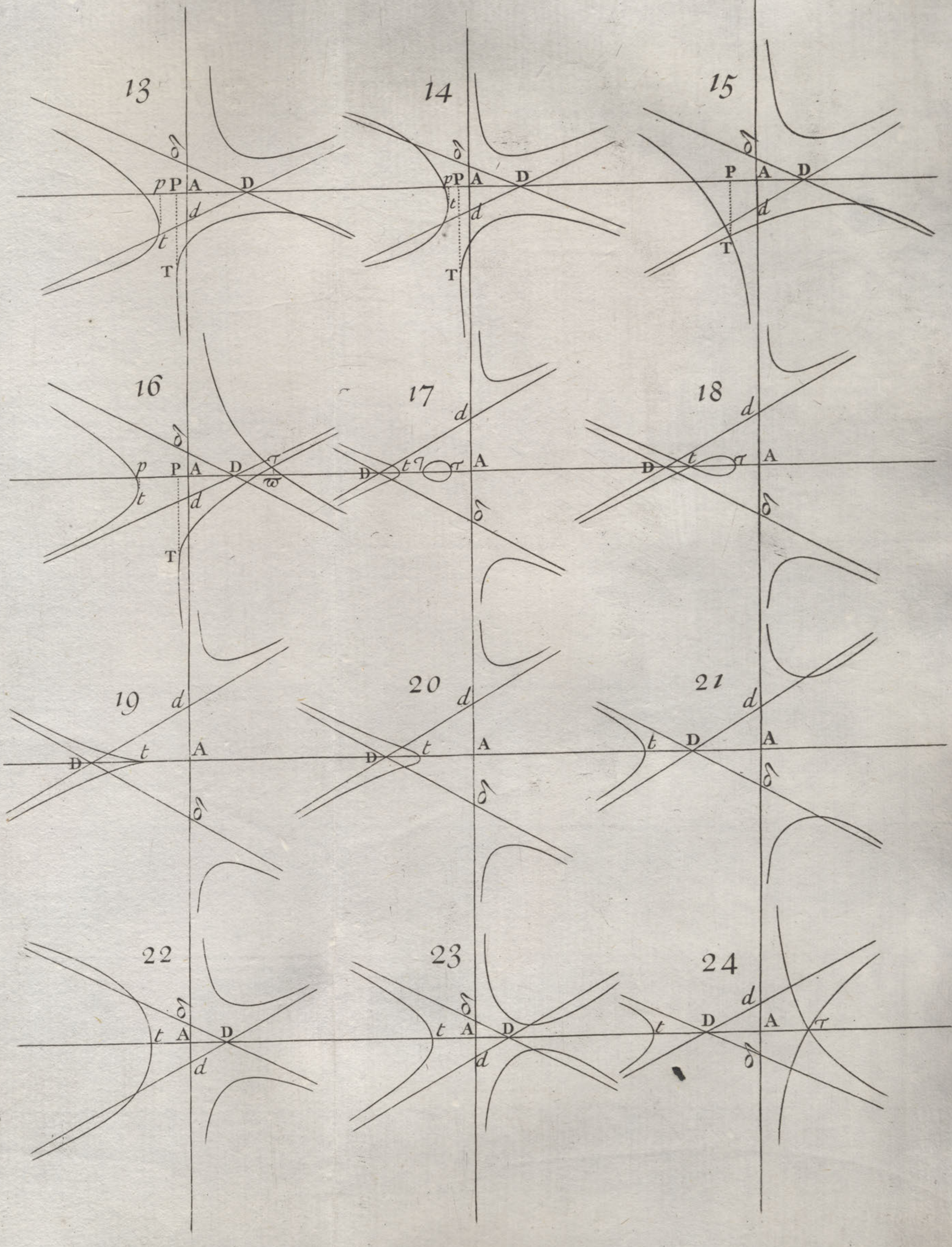
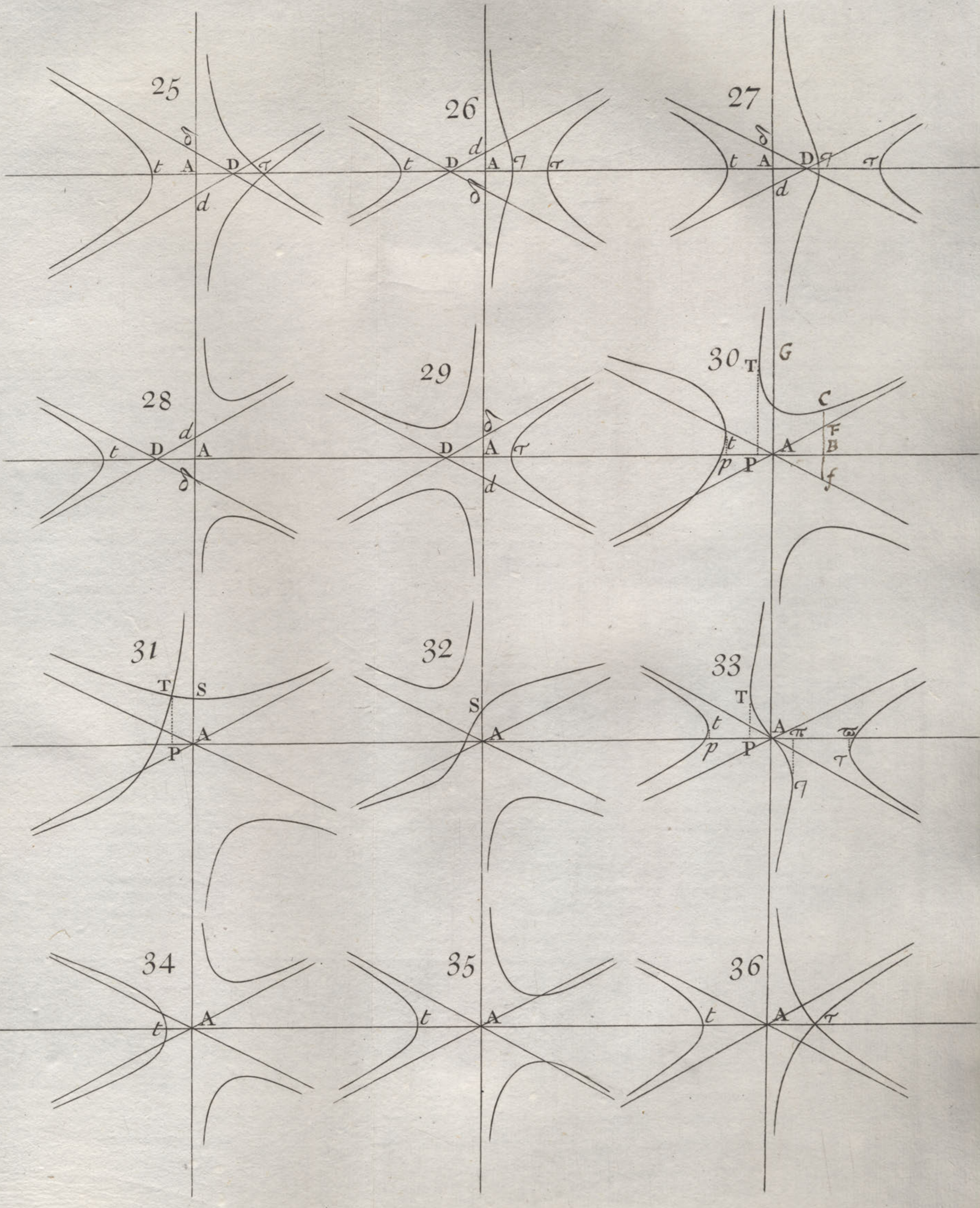
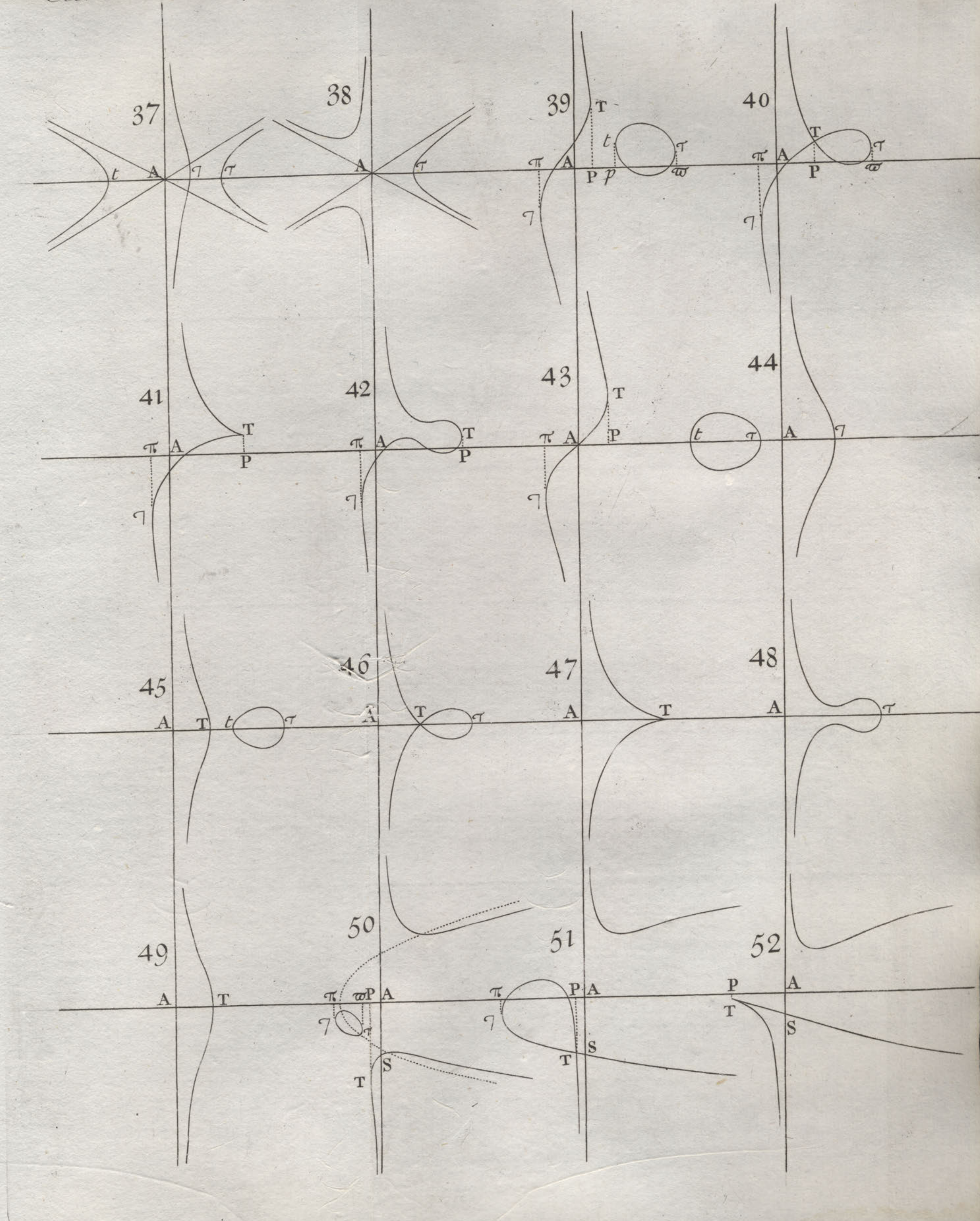
Curvarum Tab. II.



Curvarum Tab. III.

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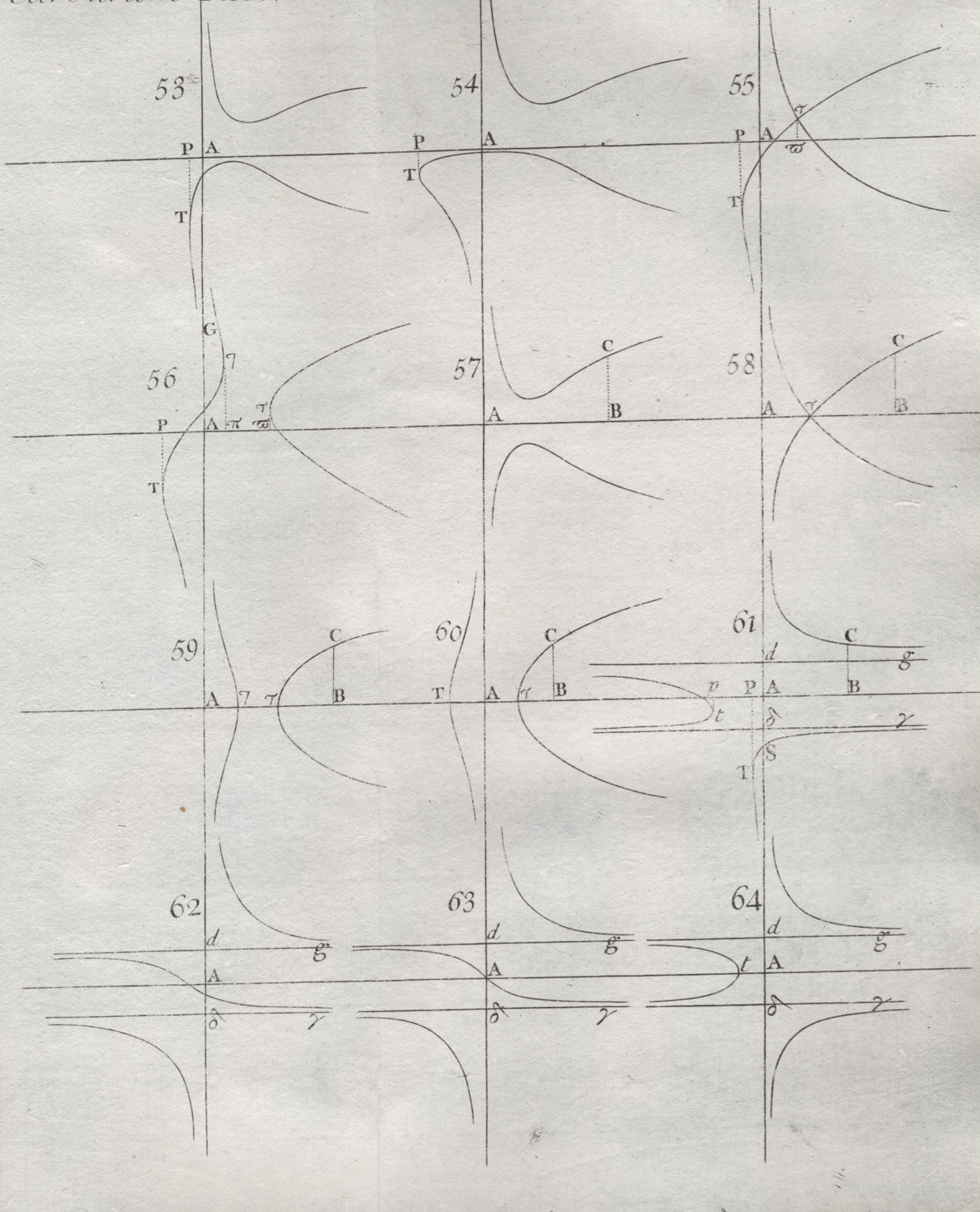
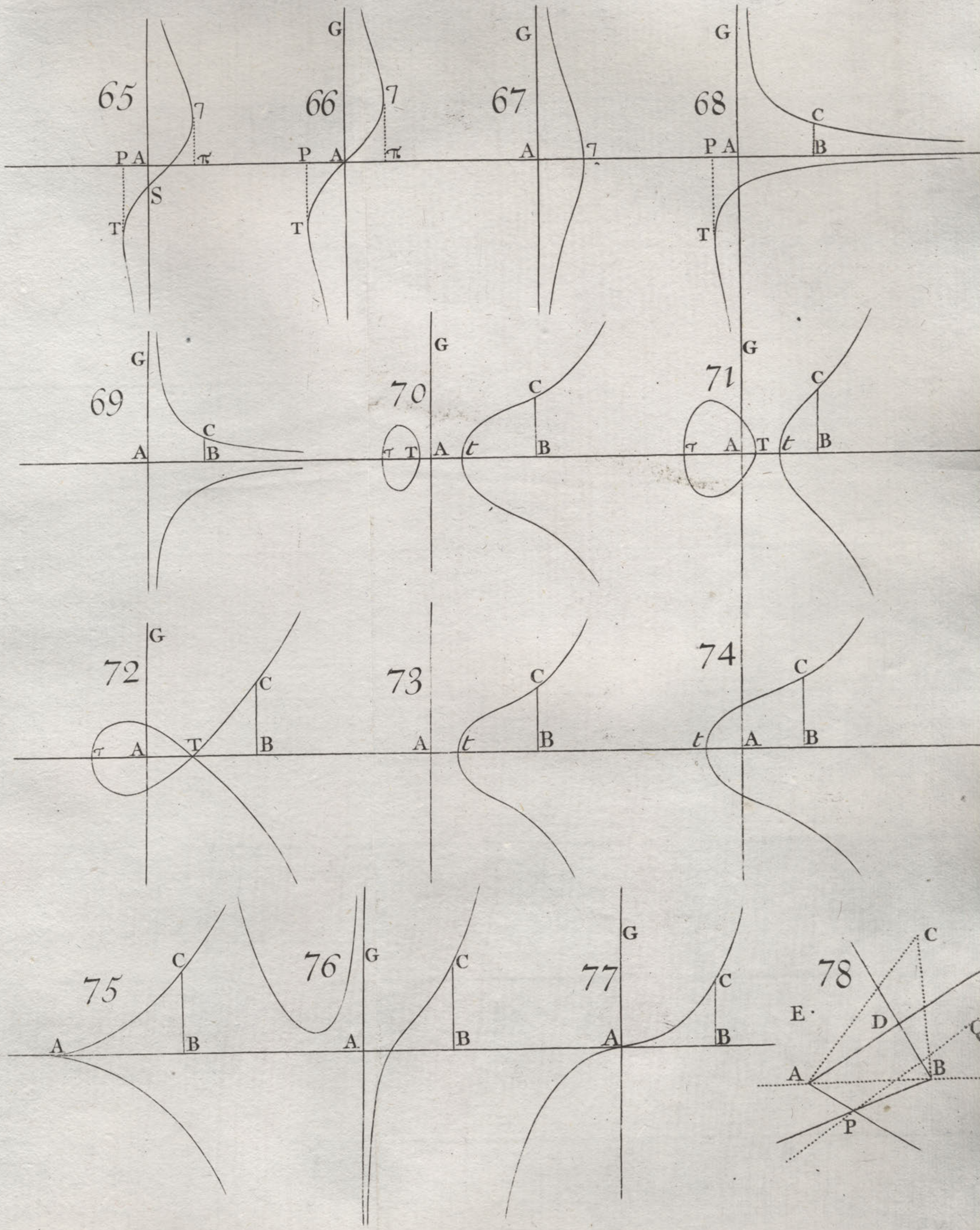
Curvarum Tab. V.

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TRACTATUS

DE

Quadratura Curvarum.

INTRODUCTIO.

Quantitates Mathematicas non ut ex partibus quam minimis constantes, sed ut motu continuo descriptas hic considero. Lineæ describuntur ac describendo generantur non per appositionem partium sed per motum continuum punctorum, superficies per motum linearum, solida per motum superficierum, anguli per rotationem laterum, tempora per fluxum continuum, & sic in cæteris. Hæ Geneses in rerum natura locum vere habent & in motu corporum quotidie cernuntur. Et ad hunc modum Veteres ducendo rectas mobiles in longitudinem rectarum immobilium genesin docuerunt rectangulorum.

Considerando igitur quod quantitates æqualibus temporibus crescentes & crescendo genitæ, pro velocitate majori vel minori qua crescunt ac generantur, evadunt majores vel minores; methodum quærebam deter-

determinandi quantitates ex velocitalibus motuum vel incrementorum quibus generantur; & has motuum vel incrementorum velocitates nominando *Fluxiones* & quantitates genitas nominando *Fluentes*, incidi paulatim *Annis* 1665 & 1666 in Methodum Fluxionum qua hic uſus ſum in Quadratura Curvarum.

Fluxiones ſunt quam proxime ut Fluentium augmenta æqualibus temporis particulis quam minimis genita, & ut accurate loquar, ſunt in prima ratione augmentorum naſcentium; exponi autem poſſunt per lineas quaſcunq; quæ ſunt ipſis proportionales. Ut
Fig. 1. ſi areæ ABC, ABDG Ordinatis BC, BD ſuper baſi AB uniformi cum motu progredientibus deſcribantur, harum arearum fluxiones erunt inter ſe ut Ordinatæ deſcribentes BC & BD, & per Ordinatas illas exponi poſſunt, propterea quod Ordinatæ illæ ſunt ut arearum augmenta naſcentia. Progrediatur Ordinata BC de loco ſuo BC in locum quemvis novum bc. Compleatur parallelogrammum BCEb, ac ducatur recta VTH quæ Curvam tangat in C ipſiſq; bc & BA productis occurrat in T & V: & Abſciſſæ AB, Ordinatæ BC, & Lineæ Curvæ ACc augmenta modo genita erunt Bb, Ec & Cc; & in horum augmentorum naſcentium ratione prima ſunt latera trianguli CET, ideoq; fluxiones ipſarum AB, BC & AC ſunt ut trianguli illius CET latera CE, ET & CT & per eadem latera exponi poſſunt, vel quod perinde eſt per latera trianguli conſimilis VBC.

Eodem recidit ſi ſumantur fluxiones in ultima ratione partium evaneſcentium. Agatur recta Cc & producatur eadem ad K. Redeat Ordinata bc

in

in locum suum priorem BC, & coeuntibus punctis C & c, recta CK coincidet cum tangente CH, & triangulum evanescens CEc in ultima sua forma evadet simile triangulo CET, & ejus latera evanescentia CE, Ec & Cc erunt ultimo inter se ut sunt trianguli alterius CET latera CE, ET & CT, & propterea in hac ratione sunt fluxiones linearum AB, BC & AC. Si puncta C & c parvo quovis intervallo ab invicem distant recta CK parvo intervallo a tangente CH distabit. Ut recta CK cum tangente CH coincidat & rationes ultimæ linearum CE, Ec & Cc inveniantur, debent puncta C & c coire & omnino coincidere. Errores quam minimi in rebus mathematicis non sunt contemnendi.

Simili argumento si circulus centro B radio BC descriptus in longitudinem Abscissæ AB ad angulos rectos uniformi cum motu ducatur, fluxio solidi geniti ABC erit ut circulus ille generans, & fluxio superficiei ejus erit ut perimeter Circuli illius & fluxio lineæ curvæ AC conjunctim. Nam quo tempore solidum ABC generatur ducendo circulum illum in longitudinem Abscissæ AB, eodem superficies ejus generatur ducendo perimetrum circuli illius in longitudinem Curvæ AC.

Recta PB circa polum datum P revolvens secet aliam positione datam rectam AB: quæritur proportio fluxionum rectarum illarum AB & PB. Fig. 2. Progrediatur recta PB de loco suo PB in locum novum Pb. In Pb capiatur PC ipsi PB æqualis, & ad AB ducatur PD sic, ut angulus bPD æqualis sit angulo bBC; & ob similitudinem triangulorum bBC, bPD erit augmentum Bb ad augmentum Cb ut Pb ad Db.

Redeat

Redeat jam P b in locum ſuum priorem P B ut augmenta illa evaneſcant, & evaneſcentium ratio ultima, id eſt ratio ultima P b ad D b, ea erit quæ eſt P B ad D B, exiſtente angulo P D B recto, & propterea in hac ratione eſt fluxio ipſius A B ad fluxionem ipſius P B.

Fig. 3. *Recta P B circa datum Polum P revolvens ſecet alias duas poſitione datas rectas A B & A E in B & E: quæritur proportio fluxionum rectarum illarum A B & A E.* Progrediatur recta revolvens P B de loco ſuo P B in locum novum P b rectas A B, A E in punctis b & e ſecantem, & rectæ A E parallela B C ducatur ipſi P b occurrens in C, & erit B b ad B C ut A b ad A e, & B C ad E e ut P B ad P E, & conjunctis rationibus B b ad E e ut $Ab \times PB$ ad $Ae \times PE$. Redeat jam linea P b in locum ſuum priorem P B, & augmentum evaneſcens B b erit ad augmentum evaneſcens E e ut $AB \times PB$ ad $AE \times PE$, ideoq; in hac ratione eſt fluxio rectæ A B ad fluxionem rectæ A E.

Hinc ſi recta revolvens P B lineas quaſvis Curvas poſitione datas ſecet in punctis B & E, & rectæ jam mobiles A B, A E Curvas illas tangant in Sectionum punctis B & E: erit fluxio Curvæ quam recta A B tangit ad fluxionem Curvæ quam recta A E tangit ut $AB \times PB$ ad $AE \times PE$. Id quod etiam eveniet ſi recta P B Curvam aliquam poſitione datam perpetuo tangat in puncto mobili P.

Fluat quantitas x uniformiter & invenienda ſit fluxio quantitatis x^n. Quo tempore quantitas x fluendo evadit $x+o$, quantitas x^n evadet $\overline{x+o}|^n$, id eſt per methodum ſerierum infinitarum, x^n+nox^{n-1}

+

$+\frac{nn-n}{2}oox^{n-2}+$ &c. Et augmenta o & $nox^{n-1}+\frac{nn-n}{2}oox^{n-2}$ $+$ &c. ſunt ad invicem ut 1 & $nx^{n-1}+\frac{nn-n}{2}ox^{n-2}+$ &c. Evaneſcant jam augmenta illa, & eorum ratio ultima erit 1 ad nx^{n-1} : ideoq; fluxio quantitatis x eſt ad fluxionem quantitatis x^n ut 1 ad nx^{n-1}.

Similibus argumentis per methodum rationum primarum & ultimarum colligi poſſunt fluxiones linearum ſeu rectarum ſeu curvarum in caſibus quibuſcunque, ut & fluxiones ſuperficierum, angulorum & aliarum quantitatum. In finitis autem quantitatibus Analyſin ſic inſtituere, & finitarum naſcentium vel evaneſcentium rationes primas vel ultimas inveſtigare, conſonum eſt Geometriæ Veterum : & volui oſtendere quod in Methodo Fluxionum non opus ſit figuras infinite parvas in Geometriam introducere. Peragi tamen poteſt Analyſis in figuris quibuſcunq; ſeu finitis ſeu infinite parvis quæ figuris evaneſcentibus finguntur ſimiles, ut & in figuris quæ pro infinite parvis haberi ſolent, modo caute procedas.

Ex Fluxionibus invenire Fluentes Problema difficilius eſt, & ſolutionis primus gradus æquipollet Quadraturæ Curvarum ; de qua ſequentia olim ſcripſi.

TRACTATUS
DE
Quadratura Curvarum.

QUantitates indeterminatas ut motu perpetuo creſcentes vel decreſcentes, id eſt ut fluentes vel defluentes in ſequentibus conſidero, deſignoq; literis $z, y, x, v,$ & earum fluxiones ſeu celeritates creſcendi noto iiſdem literis punctatis $\dot{z}, \dot{y}, \dot{x}, \dot{v}$. Sunt & harum fluxionum fluxiones ſeu mutationes magis aut minus celeres quas ipſarum z, y, x, v fluxiones ſecundas nominare licet & ſic dignare $\ddot{z}, \ddot{y}, \ddot{x}, \ddot{v},$ & harum fluxiones primas ſeu ipſarum z, y, x, v fluxiones tertias ſic $\dddot{z}, \dddot{y}, \dddot{x}, \dddot{v},$ & quartas ſic $\ddddot{z}, \ddddot{y}, \ddddot{x}, \ddddot{v}$. Et quemadmodum $\dddot{z}, \dddot{y}, \dddot{x}, \dddot{v}$ ſunt fluxiones quantitatum $\ddot{z}, \ddot{y}, \ddot{x}, \ddot{v},$ & hæ ſunt fluxiones quantitatum $\dot{z}, \dot{y}, \dot{x}, \dot{v}$ & hæ ſunt fluxiones quantitatum primarum z, y, x, v : ſic hæ quantitates conſiderari poſſunt ut fluxiones aliarum quas ſic deſignabo,

$\acute{z},$

$\overset{\prime}{z}, \overset{\prime}{y}, \overset{\prime}{x}, \overset{\prime}{v}$, & hæ ut fluxiones aliarum $\overset{\prime\prime}{z}, \overset{\prime\prime}{y}, \overset{\prime\prime}{x}, \overset{\prime\prime}{v}$, & hæ ut fluxiones aliarum $\overset{\prime\prime\prime}{z}, \overset{\prime\prime\prime}{y}, \overset{\prime\prime\prime}{x}, \overset{\prime\prime\prime}{v}$. Designant igitur $\overset{\prime\prime}{z}, \overset{\prime}{z}, z, \dot{z}, \ddot{z}, \dddot{z}, \ddddot{z}, \overset{.::}{z}$ *&c.* seriem quantitatum quarum quælibet posterior est fluxio præcedentis & quælibet prior est fluens quantitas fluxionem habens subsequentem. Similis est series $\overset{\prime\prime}{\overline{\sqrt{az-zz}}}$, $\overset{\prime}{\overline{\sqrt{az-zz}}}$, $\overline{\sqrt{az-zz}}$, $\overset{\cdot}{\overline{\sqrt{az-zz}}}$, $\overset{\cdot\cdot}{\overline{\sqrt{az-zz}}}$, $\overset{\cdot\cdot\cdot}{\overline{\sqrt{az-zz}}}$, ut & series $\frac{az+z^2}{\overset{\prime\prime}{a-z}}$, $\frac{az+z^2}{\overset{\prime}{a-z}}$, $\frac{az+z^2}{a-z}$, $\frac{az+z^2}{\overset{\cdot}{a-z}}$, $\frac{az+z^2}{\overset{\cdot\cdot}{a-z}}$, $\frac{az+z^2}{\overset{\cdot\cdot\cdot}{a-z}}$. Et notandum est quod quantitas quælibet prior in his seriebus est ut area figuræ curviliniæ cujus ordinatim applicata rectangula est quantitas posterior & abscissa est z : uti $\overset{\prime}{\overline{\sqrt{az-zz}}}$ area curvæ cujus ordinata est $\overline{\sqrt{az-zz}}$ & abscissa z. Quo autem spectant hæc omnia patebit in Propositionibus quæ sequuntur.

 PROP.

PROP. I. PROB. I.

Data æquatione quotcunq; fluentes quantitates involvente, invenire fluxiones.

Solutio.

Multiplicetur omnis æquationis terminus per indicem dignitatis quantitatis cujusq; fluentis quam involvit, & in singulis multiplicationibus mutetur dignitatis latus in fluxionem suam, & aggregatum factorum omnium sub propriis signis erit æquatio nova.

Explicatio.

Sunto a, b, c, d *&c.* quantitates determinatæ & immutabiles, & proponatur æquatio quævis quantitates fluentes z, y, x *&c.* involvens, uti $x^3 - xyy + aaz - b^3 = 0$. Multiplicentur termini primo per indices dignitatum x, & in singulis multiplicationibus pro dignitatis latere, seu x unius dimensionis, scribatur $\dot{x}$, & summa factorum erit $3\dot{x}x^2 - \dot{x}yy$. Idem fiat in y & prodibit $-2xy\dot{y}$. Idem fiat in z & prodibit $aa\dot{z}$. Ponatur summa factorum æqualis nihilo, & habebitur æquatio $3\dot{x}x^2 - \dot{x}yy - 2xy\dot{y} + aa\dot{z} = 0$. Dico quod hac æquatione definitur relatio fluxionum.

De-

Demonstratio.

Nam sit o quantitas admodum parva & sunto $o\dot{z}$, $o\dot{y}$, $o\dot{x}$, quantitatum z, y, x momenta id est incrementa momentanea synchrona. Et si quantitates fluentes jam sunt z, y & x, hæ post momentum temporis incrementis suis $o\dot{z}$, $o\dot{y}$, $o\dot{x}$ auctæ, evadent $z+o\dot{z}$, $y+o\dot{y}$, $x+o\dot{x}$, quæ in æquatione prima pro z, y & x scriptæ dant æquationem $x^3+3xxo\dot{x}+3xoo\dot{x}\dot{x}+o^3\dot{x}^3-xyy-o\dot{x}yy-2xo\dot{y}y-2xoo\dot{x}\dot{y}y-xoo\dot{y}\dot{y}-xo^3\dot{x}\dot{y}\dot{y}+aaz+aao\dot{z}-b3=o$. Subducatur æquatio prior, & residuum divisum per o erit $3\dot{x}x^2+3\dot{x}\dot{x}ox+\dot{x}^3oo-\dot{x}yy-2x\dot{y}y-2\dot{x}o\dot{y}y-xo\dot{y}\dot{y}-xoo\dot{y}\dot{y}+aa\dot{z}=o$. Minuatur quantitas o in infinitum, & neglectis terminis evanescentibus restabit $3\dot{x}x^2-\dot{x}yy-2x\dot{y}y+aa\dot{z}=o$. Q. E. D.

Explicatio plenior.

Ad eundem modum si æquatio esset $x^3-xyy+aa\sqrt{\overline{ax-yy}}-b3=o$, produceretur $3x^2\dot{x}-\dot{x}yy-2x\dot{y}y+aa\dot{\overline{\sqrt{ax-yy}}}=o$. Ubi si fluxionem $\dot{\overline{\sqrt{ax-yy}}}$ tollere velis, pone $\sqrt{\overline{ax-yy}}=z$, & erit $ax-yy=z^2$ &

& (per hanc Propositionem) $a\dot{x} - 2\dot{y}y = 2\dot{z}z$ seu $\frac{a\dot{x} - 2\dot{y}y}{2z} = \dot{z}$, hoc est $\frac{a\dot{x} - 2\dot{y}y}{2\sqrt{ax - yy}} = \dot{\overline{\sqrt{ax - yy}}}$. Et

inde $3x^2\dot{x} - \dot{x}yy - 2x\dot{y}y + \frac{a^3\dot{x} - 2aa\dot{y}y}{2\sqrt{ax - yy}} = 0$

Et per operationem repetitam pergitur ad fluxiones secundas, tertias & sequentes. Sit æquatio $zy^3 - z^4 + a^4 = 0$, & fiet per operationem primam $\dot{z}y^3 + 3z\dot{y}y^2 - 4\dot{z}z^3 = 0$, per secundam $\ddot{z}y^3 + 6\dot{z}\dot{y}y^2 + 3z\ddot{y}y^2 + 6z\dot{y}^2y - 4\ddot{z}z^3 - 12\dot{z}^2z^2 = 0$, per tertiam $\dddot{z}y^3 + 9\ddot{z}\dot{y}y^2 + 9\dot{z}\ddot{y}y^2 + 18\dot{z}\dot{y}^2y + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3 - 4\dddot{z}z^3 - 36\ddot{z}\dot{z}z^2 - 24\dot{z}^3z = 0$.

Ubi vero sic pergitur ad fluxiones secundas, tertias & sequentes, convenit quantitatem aliquam ut uniformiter fluentem considerare, & pro ejus fluxione prima unitatem scribere, pro secunda vero & sequentibus nihil. Sit æquatio $zy^3 - z^4 + a^4 = 0$, ut supra; & fluat z uniformiter, sitq; ejus fluxio unitas, & fiet per operationem primam $y^3 + 3z\dot{y}y^2 - 4z^3 = 0$, per secundam $6\dot{y}y^2 + 3z\ddot{y}y^2 + 6z\dot{y}^2y - 12z^2 = 0$, per tertiam $9\ddot{y}y^2 + 18\dot{y}^2y + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3 - 24z = 0$.

In

In hujus autem generis æquationibus concipiendum est quod fluxiones in singulis terminis sint ejusdem ordinis, id est vel omnes primi ordinis $\dot{y}$, $\dot{z}$, vel omnes secundi $\ddot{y}$, $\dot{y}^2$, $\dot{y}\dot{z}$, $\dot{z}^2$, vel omnes tertii $\dddot{y}$, $\ddot{y}\dot{y}$, $\ddot{y}\dot{z}$, $\dot{y}^3$, $\dot{y}^2\dot{z}$, $\dot{y}\dot{z}^2$ $\dot{z}^3$ &c. Et ubi res aliter se habet complendus est ordo per subintellectas fluxiones quantitatis uniformiter fluentis. Sic æquatio novissima complendo ordinem tertium fit $9z\dot{y}\ddot{y}^2 + 18\dot{z}\dot{y}^2\dot{y} + 3z\dddot{y}y^2 + 18z\ddot{y}\dot{y}y + 6z\dot{y}^3 - 24z\dot{z}^3 = 0$.

PROP. II. PROB. II.

Invenire Curvas quæ quadrari possunt.

Sit ABC figura invenienda, BC Ordinatim applicata rectangula, & AB abscissa. Producatur CB ad E ut sit BE = 1, & compleatur parallelogrammum ABED: & arearum ABC, ABED fluxiones erunt ut BC & BE. Assumatur igitur æquatio quævis qua relatio arearum definiatur, & inde dabitur relatio ordinatarum BC & BE per Prop. I. Q. E. I. *Fig.* 4.

Hujus rei exempla habentur in Propositionibus duabus sequentibus.

PROP.

PROP. III. THEOR. I.

Si pro abſciſſa AB & area AE ſeu AB×1 promiſcue ſcribatur z, & ſi pro $e + fz^{\eta} + gz^{2\eta} + hz^{3\eta} +$ &c. ſcribatur R: ſit autem area Curvæ $z^{\theta}R^{\lambda}$ erit. ordinatim applicata BC =

$$\theta e \begin{matrix}+\theta\\+\lambda\eta\end{matrix} fz^{\eta} \begin{matrix}+\theta\\+2\lambda\eta\end{matrix} gz^{2\eta} \begin{matrix}+\theta\\+3\lambda\eta\end{matrix} hz^{3\eta} + \text{\&c. in } z^{\theta-1} R^{\lambda-1}.$$

Demonſtratio.

Nam ſi ſit $z^{\theta}R^{\lambda} = v$, erit per Prop. 1, $\theta\dot{z}z^{\theta-1}R^{\lambda} + \lambda z^{\theta}\dot{R}R^{\lambda-1} = \dot{v}$. Pro R^{λ} in primo æquationis termino & z^{θ} in ſecundo ſcribe $RR^{\lambda-1}$ & $zz^{\theta-1}$, & fiet $\theta\dot{z}R + \lambda z\dot{R}$ in $z^{\theta-1}R^{\lambda-1} = \dot{v}$. Erat autem $R = e + fz^{\eta} + gz^{2\eta} + hz^{3\eta}$ &c. & inde per Prop. 1. fit $\dot{R} = \eta f\dot{z}z^{\eta-1} + 2\eta g\dot{z}z^{2\eta-1} + 3\eta h\dot{z}z^{3\eta-1} +$ &c. quibus ſubſtitutis & ſcripta BE ſeu 1 pro $\dot{z}$, fiet

$$\theta e \begin{matrix}+\theta\\+\lambda\eta\end{matrix} fz^{\eta} \begin{matrix}+\theta\\+2\lambda\eta\end{matrix} gz^{2\eta} \begin{matrix}+\theta\\+3\lambda\eta\end{matrix} hz^{3\eta} + \text{\&c. in } z^{\theta-1} R^{\lambda-1} = \dot{v} = BC.$$

Q. E. D.

PROP.

PROP. IV. THEOR. II.

Si Curvæ abſciſſa AB ſit z, & ſi pro $e + fz^n + gz^{2n} + \&c.$ ſcribatur R, & pro $k + lz^n + mz^{2n} + \&c.$ ſcribatur S; ſit autem area Curvæ $z^\theta R^\lambda S^\mu$: erit ordinatim applicata BC =

$$\left.\begin{array}{llllll} \theta ek \; \substack{+\theta\\+\lambda n}\, fkz^n & \substack{+\theta\\+2\lambda n}\, gkz^{2n} & \cdot * \cdot\cdot\cdot\cdot\cdot & * \cdot\cdot\cdot\cdot\cdot & & \\ \quad \substack{+\theta\\+\mu n}\, elz^n & \substack{+\theta\\+\lambda n\\+\mu n}\, flz^{2n} & \substack{+\theta\\+2\lambda n\\+\mu n}\, glz^{3n} & * \cdot\cdot\cdot\cdot\cdot & & \\ & \substack{+\theta\\+2\mu n}\, emz^{2n} & \substack{+\theta\\+\lambda n\\+2\mu n}\, fmz^{3n} & \substack{+\theta\\+2\lambda n\\+2\mu n}\, gmz^{4n} & & \end{array}\right\} \text{in } z^{\theta-1}R^{\lambda-1}S^{\mu-1}$$

Demonſtratur ad modum Propoſitionis ſuperioris.

PROP. V. THEOR. III.

Si Curvæ abſciſſa AB ſit z, & pro $e + fz^n + gz^{2n} + hz^{3n} + \&c.$ ſcribatur R: ſit autem ordinatim applicata $z^{\theta-1}R^{\lambda-1}$ in $a + bz^n + cz^{2n} + dz^{3n} + \&c.$ & ponatur $\frac{\theta}{n} = r$. $r + \lambda = s$. $s + \lambda = t$. $t + \lambda = v$. &c. erit area

$$z^\theta R^\lambda \text{ in } \frac{\frac{1}{n}a}{re} + \frac{\frac{1}{n}b - sfA}{\overline{r+1},e}z^n + \frac{\frac{1}{n}c \substack{-s\\-1} fB - tgA}{\overline{r+2},e}z^{2n} + \frac{\frac{1}{n}d \substack{-s\\-2} fC \substack{-t\\-1} gB - vhA}{\overline{r+3},e}z^{3n}$$

$$+ \frac{\substack{-s\\-3} fD \substack{-t\\-2} gC \substack{-v\\-1} hB}{\overline{r+4},e}z^{4n} + \&c.$$ Ubi A, B, C, D, &c.

denotant totas coefficientes datas terminorum singulorum in serie cum signis suis + & —, nempe A primi termini coefficientem $\frac{\frac{1}{n}a}{re}$, B secundi coefficientem $\frac{\frac{1}{n}b - sfA}{\overline{r+1}, e}$, C tertii coefficientem $\frac{\frac{1}{n}c - \frac{s}{1}fB - tgA}{\overline{r+2}, e}$, & sic deinceps.

Demonstratio.

Sunto juxta Propositionem tertiam,

Curvarum Ordinatæ		& earundem areæ.
1. $\theta eA + \frac{\theta}{+\lambda n} fAz^n + \frac{\theta}{+2\lambda n} gAz^{2n} + \frac{\theta}{+3\lambda n} hAz^{3n}$ &c.	$z^{\theta-1}R^{\lambda-1}$.	$Az^\theta R^\lambda$.
2 .... $\overline{\theta+n}, eBz^n + \frac{\overline{\theta+n}}{+\lambda n} fBz^{2n} + \frac{\overline{\theta+n}}{+2\lambda n} gBz^{3n}$ &c.		$Bz^{\theta+n}R^\lambda$.
3 .... $\overline{\theta+2n}, eCz^{2n} + \frac{\overline{\theta+2n}}{+\lambda n} fCz^{3n}$ &c.		$Cz^{\theta+2n}R^\lambda$.
4 .... $\overline{\theta+3n}, eDz^{3n}$ &c.		$Dz^{\theta+3n}R^\lambda$.

Et si summa ordinatarum ponatur æqualis ordinatæ $a + bz^n + cz^{2n} + dz^{3n} +$ &c. in $z^{\theta-1}R^{\lambda-1}$, summa arearum $z^\theta R^\lambda$ in $A + Bz^n + Cz^{2n} + Dz^{3n} +$ &c. æqualis erit areæ Curvæ cujus ista est ordinata. Æquentur igitur Ordinatarum termini correspondentes, & fiet $a = \theta eA$, $b = \frac{\theta}{+\lambda n} fA + \frac{\theta}{+n} eB$, $c = \frac{\theta}{+2\lambda n} gA + \frac{\overline{\theta+n}}{+\lambda n} fB + \overline{\theta+2n}, eC$ &c. & inde $\frac{a}{\theta e} = A$. $\frac{b - \overline{\theta+\lambda n}, fA}{\overline{\theta+n}, e} = B$. $\frac{c - \overline{\theta+2\lambda n}, gA - \overline{\theta+n+\lambda n}, fB}{\overline{\theta+2n}, e} = C$. Et sic deinceps in infinitum.

nitum. Pone jam $\frac{\theta}{\eta} = r$. $r + \lambda = s$. $s + \lambda = t$ &c. & in area $z^{\theta}R^{\lambda} \times A + Bz^{\eta} + Cz^{2\eta} + Dz^{3\eta}$ &c. ſcribe ipſorum A, B, C, &c. valores inventos & prodibit ſeries propoſita. Q. E. D.

Et notandum eſt quod Ordinata omnis duobus modis iu ſeriem reſolvitur. Nam index η vel affirmativus eſt poteſt vel negativus. Proponatur Ordinata $\frac{3k - lzz}{zz\sqrt{kz - lz^3 + mz^4}}$. Hæc vel ſic ſcribi poteſt $z^{-\frac{5}{2}} \times \overline{3k - lzz} \times \overline{k - lzz + mz^3}|^{-\frac{1}{2}}$, vel ſic $z \times \overline{-l + 3kz^{-2}} \times \overline{m - lz^{-1} + kz^{-3}}|^{-\frac{1}{2}}$. In caſu priore eſt $a = 3k$. $b = 0$. $c = -l$. $e = k$. $f = 0$. $g = -l$. $h = m$. $\lambda = -\frac{1}{2}$. $\eta = 1$. $\theta - 1 = -\frac{5}{2}$. $\theta = -\frac{3}{2} = r$. $s = -1$. $t = -\frac{1}{2}$. $v = 0$. In poſteriore eſt $a = -l$. $b = 0$. $c = 3k$. $e = m$. $f = -l$. $g = 0$. $h = 1$. $\lambda = -\frac{1}{3}$. $\eta = -1$. $\theta - 1 = 1$. $\theta = 2$. $r = -2$. $s = -1\frac{1}{2}$. $t = -1$. $v = -\frac{1}{2}$. Tentandus eſt caſus uterque. Et ſi ſerierum alterutra ob terminos tandem deficientes abrumpitur ac terminatur, habebitur area Curvæ in terminis finitis. Sic in exempli hujus priore caſu ſcribendo in ſerie valores ipſorum a, b, c, e, f, g, h, λ, θ, r, s, t, v, termini omnes poſt primum evaneſcunt in infinitum & area Curvæ prodit $-2\sqrt{\frac{k - lzz + mz^3}{z^3}}$. Et hæc area ob ſignum negativum adjacet abſciſſæ ultra ordinatam productæ. Nam area omnis affirmativa adjacet tam abſciſſæ quam ordinatæ, negativa vero cadit ad contrarias partes ordinatæ & adjacet abſciſſæ productæ, manente ſcilicet ſigno Ordinatæ. Hoc modo ſeries alterutra & nonnunquam utraque ſemper terminatur & finita evadit ſi Curva geometrice quadrari poteſt. At ſi Curva talem quadraturam non admittit, ſeries utraq; continuabitur in infinitum, & earum

rum altera converget & aream dabit approximando, præterquam ubi r (propter aream infinitam) vel nihil est vel numerus integer & negativus, vel ubi $\frac{z}{e}$ æqualis est unitati. Si $\frac{z}{e}$ minor est unitate, converget series in qua index η affirmativus est: sin $\frac{z}{e}$ unitate major est, converget series altera. In uno casu area adjacet abscissæ ad usq; ordinatam ductæ, in altero adjacet abscissæ ultra ordinatam productæ.

Nota insuper quod si Ordinata contentum est sub factore rationali Q & factore surdo irreducibili R^{π}, & factoris surdi latus R non dividit factorem rationalem Q; erit $\lambda - 1 = \pi$ & $R^{\lambda-1} = R^{\pi}$. Sin factoris surdi latus R dividit factorem rationalem semel, erit $\lambda - 1 = \pi + 1$ & $R^{\lambda-1} = R^{\pi+1}$: si dividit bis, erit $\lambda - 1 = \pi + 2$ & $R^{\lambda-1} = R^{\pi+2}$: si ter, erit $\lambda - 1 = \pi + 3$, & $R^{\lambda-1} = R^{\pi+3}$: & sic deinceps.

Si Ordinata est fractio rationalis irreducibilis cum Denominatore ex duobus vel pluribus terminis composito: resolvendus est denominator in divisores suos omnes primos. Et si divisor sit aliquis cui nullus alius est æqualis, Curva quadrari nequit: Sin duo vel plures sint divisores æquales, rejiciendus est eorum unus, & si adhuc alii duo vel plures sint sibi mutuo æquales & prioribus inæquales, rejiciendus est etiam eorum unus, & sic in aliis omnibus æqualibus si adhuc plures sint: deinde divisor qui relinquitur vel contentum sub divisoribus omnibus qui relinquuntur, si plures sunt, ponendum est pro R, & ejus quadrati reciprocum R^{-2} pro $R^{\lambda-1}$, præterquam ubi contentum illud est quadratum vel cubus vel quadrato quadratum, &c. quo casu ejus latus ponen-

ponendum eſt pro R & poteſtatis index 2 vel 3 vel 4 negative ſumptus pro λ. & Ordinata ad denominatorem R^2 vel R^3 vel R^4 vel R^5 &c. reducenda.

Ut ſi ordinata ſit $\frac{z^5+z^4-8z^3}{z^5+z^4-5z^3-z^2+8z-4}$; quoniam hæc fractio irreducibilis eſt & denominatoris diviſores ſunt pares, nempe $z-1$, $z-1$, $z-1$ & $z+2$, $z+2$, rejicio magnitudinis utriuſque diviſorem unum & reliquorum $z-1$, $z-1$, $z+2$ contentum z^3-3z+2 pono pro R & ejus quadrati reciprocum $\frac{1}{R^2}$ ſeu R^{-2} pro $R^{\lambda-1}$. Dein Ordinatam ad denominatorem R^2 ſeu $R^{1-\lambda}$ reduco, & fit $\frac{z^6-9z^4+8z^3}{\overline{z^3-3z+2}|\text{ quad.}}$, id eſt $z^3 \times \overline{8-9z+}z^3 \times \overline{2-3z+z^3}|^{-2}$ Et inde eſt $a=8$. $b=-9$. $c=0$. $d=-1$, &c. $e=2$. $f=-3$. $g=0$. $h=1$. $\lambda-1=-2$. $\lambda=-1$. $\eta=1$. $\theta-1=3$. $\theta=4=r$. $s=3$. $t=2$. $v=1$. Et his in ſerie ſcriptis prodit area $\frac{z^4}{z^3-3z+2}$, terminis omnibus in tota ſerie poſt primum evaneſcentibus.

Si deniq; Ordinata eſt fractio irreducibilis & ejus denominator contentum eſt ſub factore rationali Q & factore ſurdo irreducibili R^π, inveniendi ſunt lateris R diviſores omnes primi, & rejiciendus eſt diviſor unus magnitudinis cujuſq; & per diviſores qui reſtant, ſiqui ſint, multiplicandus eſt factor rationalis Q: & ſi factum æquale eſt lateri R vel lateris illius poteſtati alicui cujus index eſt numerus integer, eſto index ille m, & erit $\lambda-1=-\pi-m$, & $R^{\lambda-1}=R^{-\pi-m}$. Ut ſi Ordinata ſit $\frac{3q^5-q^4x+9q^3xx-qqx^3-6qx^4}{\overline{qq-xx}\sqrt{\text{cub.}\,q^3+qqx-qxx-x^3}}$,

quoniam

quoniam factoris surdi latus R seu $q3+qqx-qxx-x3$ divisores habet $q+x$, $q+x$, $q-x$ qui duarum sunt magnitudinum, rejicio divisorem unum magnitudinis utriusq; & per divisorem $q+x$ qui relinquitur multiplico factorem rationalem $qq-xx$. Et quoniam factum $q3+qqx-qxx-x3$ æquale est lateri R, pono $m=1$. & inde, cum π sit $\frac{1}{3}$, sit $\lambda-1=-\frac{4}{3}$. Ordinatam igitur reduco ad denominatorem $R^{\frac{-4}{3}}$ & sit $Z^{0}\times\overline{3q^{6}+2q^{5}x+8q^{4}xx+8q^{3}x^{3}-7qqx^{4}-6qx^{5}}\times\overline{q^{3}+qqx-qxx-x^{3}}|^{-\frac{4}{3}}$. Unde est $a=3q^{6}$. $b=2q^{5}$ &c. $e=q3$. $f=qq$ &c. $\theta-1=0$. $\theta=1=\eta$. $\lambda=-\frac{1}{3}$. $r=1$. $s=\frac{2}{3}$. $t=\frac{1}{3}$. $v=0$. Et his in serie scriptis prodit area $\frac{3qqx+3x^{3}}{\sqrt{\text{cub.}}\ \overline{a3+aax-axx-x^{3}}}$, terminis omnibus in serie tota post tertium evanescentibus.

$\sqrt{\text{cub.}}\ q^3+qqx-qxx-x^3$

PROP. VI. THEOR. IV.

Si Curvæ abscissa AB sit z, & scribantur R pro $e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}+$ &c. & S pro $k+lz^{\eta}+mz^{2\eta}+nz^{3\eta}$ &c. sit autem ordinatim applicata $z^{\theta-1}R^{\lambda-1}S^{\mu-1}$ in $a+bz^{\eta}+cz^{2\eta}+dz^{3\eta}$ &c. & si terminorum, e, f, g, h, &c. & k, l, m, n. &c. rectangula sint.

ek	fk	gk	hk	&c.
el	fl	gl	hl	&c.
em	fm	gm	hm	&c.
en	fn	gn	hn	&c.

Et

Et si rectangulorum illorum coefficientes numerales sint respective

$$\tfrac{1}{n}\theta = r. \quad r + \lambda = s. \quad s + \lambda = t. \quad t + \lambda = v. \ \&c.$$

$$r + \mu = s'. \quad s + \mu = t'. \quad t + \mu = v'. \quad v + \mu = w'. \ \&c.$$

$$s' + \mu = t''. \quad t' + \mu = v''. \quad v' + \mu = w''. \quad w' + \mu = x''. \ \&c.$$

$$t'' + \mu = v'''. \quad v'' + \mu = w'''. \quad w'' + \mu = x'''. \quad x'' + \mu = y'''. \ \&c.$$

area Curvæ erit hæc

$$z^{\theta}R^{\lambda}S^{\mu} \text{ in } \frac{\frac{1}{n}a}{rek} + \frac{\frac{1}{n}b \begin{smallmatrix} -sfk \\ -s'el \end{smallmatrix} A}{\overline{r+1}, ek} z^{n} + \frac{\frac{1}{n}c \begin{smallmatrix} -\overline{s+1}, fk \\ -\overline{s'+1}, el \end{smallmatrix} B \begin{smallmatrix} -tgk \\ -t'fl \\ -t''em \end{smallmatrix} A}{\overline{r+2}, ek} z^{2n}$$

$$+ \frac{\frac{1}{n}d \begin{smallmatrix} -\overline{s+2}, fk \\ -\overline{s'+2}, el \end{smallmatrix} C \begin{smallmatrix} -\overline{t+1}, gk \\ -\overline{t'+1}, fl \\ -\overline{t''+1}, em \end{smallmatrix} B \begin{smallmatrix} -v\,hk \\ -v'gl \\ -v''fm \\ -v'''en \end{smallmatrix} A}{\overline{r+3}, ek} z^{3n} + \&c.$$

Ubi A denotat termini primi coefficientem datam $\frac{\frac{1}{n}a}{rek}$ cum signo suo + vel —, B coefficientem datam secundi, C coefficientem datam tertii, & sic deinceps. Terminorum vero, a, b, c, &c. e, f, g, &c. k, l, m, &c. unus vel plures deesse possunt. Demonstratur Propositio ad modum præcedentis, & quæ ibi notantur hic obtinent. Pergit autem series talium Propositionum in infinitum, & Progressio seriei manifesta est.

PROP.

PROP. VII. THEOR. V.

Si pro $e + fz^{\eta} + gz^{2\eta} +$ &c. ſcribatur R ut ſupra, & in Curvæ alicujus Ordinata $z^{\theta \pm \eta\sigma} R^{\lambda \pm \tau}$ maneant quantitates datæ θ, η, λ, e, f, g, &c. & pro σ ac τ ſcribantur ſucceſſive numeri quicunq; integri : & ſi detur area unius ex Curvis quæ per Ordinatas innumeras ſic prodeuntes deſignantur ſi Ordinatæ ſunt duorum nominum in vinculo radicis, vel ſi dentur areæ duarum ex Curvis ſi Ordinatæ ſunt trium nominum in vinculo radicis, vel areæ trium ex Curvis ſi Ordinatæ ſunt quatuor nominum in vinculo radicis, & ſic deinceps in infinitum : dico quod dabuntur areæ curvarum omnium. Pro nominibus hic habeo terminos omnes in vinculo radicis tam deficientes quam plenos quorum indices dignitatum ſunt in progreſſione arithmetica. Sic ordinata $\sqrt{a^4 - ax^3 + x^4}$ ob terminos duos inter a^4 & $-ax^3$ deficientes pro quinquinomio haberi debet. At $\sqrt{a^4 + x^4}$ binomium eſt & $\sqrt{a^4 + x^4 - \frac{x^8}{a^4}}$ trinonium, cum progreſſio jam per majores differentias procedat. Propoſitio vero ſic demonſtratur.

CAS. I.

Sunto Curvarum duarum Ordinatæ $pz^{\theta-1} R^{\lambda-1}$ & $qz^{\theta+\eta-1} R^{\lambda-1}$, & areæ pA & qB, exiſtente R quantitate trium nominum $e + fz^{\eta} + gz^{2\eta}$. Et cum per

Prop.

Prop. III. ſit $z^{\theta}R^{\lambda}$ area curvæ cujus Ordinata eſt $\theta e \begin{smallmatrix}+\theta\\ +\lambda\eta\end{smallmatrix} fz^{\eta} \begin{smallmatrix}+\theta\\ +2\lambda\eta\end{smallmatrix} gz^{2\eta}$ in $z^{\theta-1}R^{\lambda-1}$, ſubduc Ordinatas & areas priores de area & Ordinata poſteriori, & manebit $\begin{smallmatrix}\theta e\\ -p\end{smallmatrix} \begin{smallmatrix}+\theta\\ +\lambda\eta\end{smallmatrix} fz^{\eta} \begin{smallmatrix}+\theta\\ +2\lambda\eta\end{smallmatrix} gz^{2\eta}$ in $z^{\theta-1}R^{\lambda-1}$ Ordinata nova Curvæ, & $z^{\theta-1}$
$-qz^{\eta}$

$z^{\theta}R^{\lambda} - pA - qB$ ejuſdem area. Pone $\theta e = p$ & $\theta f + \lambda\eta f = q$ & Ordinata evadet $\begin{smallmatrix}\theta\\ +2\lambda\eta\end{smallmatrix} gz^{2\eta}$ in $z^{\theta-1}R^{\lambda-1}$, & area $z^{\theta}R^{\lambda} - \theta eA - \theta fB - \lambda\eta fB$. Divide utramq; per $\theta g + 2\lambda\eta g$, & aream prodeuntem dic C, & aſſumpta utcunq; r, erit rC area Curvæ cujus Ordinata eſt $rz^{\theta+2\eta-1}R^{\lambda-1}$. Et qua ratione ex areis pA & qB aream rC Ordinatæ $rz^{\theta+2\eta-1}R^{\lambda-1}$ congruentem invenimus, licebit ex areis qB & rC aream quartam puta sD, ordinatæ $sz^{\theta+3\eta-1}R^{\lambda-1}$ congruentem invenire, & ſic deinceps in infinitum. Et par eſt ratio progreſſionis ab areis B & A in partem contrariam pergentis. Si terminorum θ, $\theta+\lambda\eta$, & $\theta+2\lambda\eta$ aliquis deficit & ſeriem abrumpit, aſſumatur area pA in principio progreſſionis unius & area qB in principio alterius, & ex his duabus areis dabuntur areæ omnes in progreſſione utraque. Et contra, ex aliis duabus areis aſſumptis fit regreſſus per analyſin ad areas A & B, adeo ut ex duabus datis cæteræ omnes dentur. Q. E. O. Hic eſt caſus Curvarum ubi ipſius z index θ augetur vel diminuitur perpetua additione vel ſubductione quantitatis η. Caſus alter eſt Curvarum ubi index λ augetur vel diminuitur unitatibus.

CAS. II.

Ordinatæ $pz^{\theta-1}R^{\lambda}$ & $qz^{\theta+n-1}R^{\lambda}$, quibus areæ pA & qB jam reſpondeant, ſi in R ſeu $e+fz^{n}+gz^{2n}$ ducantur ac deinde ad R viciſſim applicentur, evadunt $pe+pfz^{n}+pgz^{2n}\times z^{\theta-1}R^{\lambda-1}$ & $qez^{n}+qfz^{2n}+qgz^{3n}\times z^{\theta-1}R^{\lambda-1}$. Et per Prop. III. eſt $az^{\theta}R^{\lambda}$ area Curvæ cujus Ordinata eſt $\theta ae\begin{smallmatrix}+\theta\\+\lambda n\end{smallmatrix}afz^{n},\begin{smallmatrix}+\theta\\+2\lambda n\end{smallmatrix}agz^{2n}$ in $z^{\theta-1}R^{\lambda-1}$, & $bz^{\theta+n}R^{\lambda}$ area Curvæ cujus ordinata eſt $\begin{smallmatrix}\theta\\+n\end{smallmatrix}bez^{n}\begin{smallmatrix}+\theta\\+n\\+\lambda n\end{smallmatrix}bfz^{2n}\begin{smallmatrix}+\theta\\+n\\+2\lambda n\end{smallmatrix}bgz^{3n}$ in $z^{\theta-1}R^{\lambda-1}$. Et harum quatuor arearum ſumma eſt $pA+qB+az^{\theta}R^{\lambda}+bz^{\theta+n}R^{\lambda}$ & ſumma reſpondentium ordinatarum

$$\begin{array}{llll}
\theta ae & \begin{smallmatrix}+\theta\\+\lambda n\end{smallmatrix}afz^{n} & \begin{smallmatrix}+\theta\\+2\lambda n\end{smallmatrix}agz^{2n} & \begin{smallmatrix}+\theta\\+n\\+2\lambda n\end{smallmatrix}bgz^{3n} \text{ in } z^{\theta-1}R^{\lambda-1}. \\
+pe & \begin{smallmatrix}+\theta\\+n\end{smallmatrix}be & \begin{smallmatrix}+\theta\\+n\\+\lambda n\end{smallmatrix}bf & +qg \\
 & +pf & +pg & \\
 & +qe & +qf &
\end{array}$$

Si terminus primus tertius & quartus ponantur ſeorſim æquales nihilo, per primum fiet $\theta ae+pe=0$ ſeu $-\theta a=p$, per quartum $-\theta b-nb-2\lambda nb=q$, & per tertium (eliminando p & q) $\frac{2ag}{f}=b$. Unde ſecundus fit $\frac{\lambda naff-4\lambda nage}{f}$, adeoq; ſumma quatuor Ordinatarum eſt $\frac{\lambda naff-4\lambda nage}{f}z^{\theta+n-1}R^{\lambda-1}$, & ſumma totidem reſpondentium arearum eſt $az^{\theta}R^{\lambda}+\frac{2ag}{f}z^{\theta+n}R_{\lambda}-\theta aA-\frac{2\theta+2n+4\lambda n}{f}agB$.

Divi-

Dividantur hæ summæ per $\frac{\lambda\eta aff - 4\lambda\eta age}{f}$, & si Quotum posterius dicatur D, erit D area curvæ cujus ordinata est Quotum prius $z^{\theta+\eta-1}R^{\lambda-1}$. Et eadem ratione ponendo omnes Ordinatæ terminos præter primum æquales nihilo potest area Curvæ inveniri cujus Ordinata est $z^{\theta-1}R^{\lambda-1}$. Dicatur area ista C, & qua ratione ex areis A & B inventæ sunt areæ C ac D, ex his areis C ac D inveniri possunt aliæ duæ E & F ordinatis $z^{\theta-1}R^{\lambda-2}$ & $z^{\theta+\eta-1}R^{\lambda-2}$ congruentes, & sic deinceps in infinitum. Et per analysin contrariam regredi licet ab areis E & F ad areas C ac D, & inde ad areas A & B, aliasq; quæ in progressione sequuntur. Igitur si index λ perpetua unitatum additione vel subductione augeatur vel minuatur, & ex areis quæ Ordinatis sic prodeuntibus respondent duæ simplicissimæ habentur; dantur aliæ omnes in infinitum. Q. E. O.

CAS. III.

Et per casus hosce duos conjunctos, si tam index θ perpetua additione vel subductione ipsius η, quam index λ perpetua additione vel subductione unitatis, utcunq; augeatur vel minuatur, dabuntur areæ singulis prodeuntibus Ordinatis respondentes. Q. E. O.

CAS. IV.

Et ſimili augmento ſi ordinata conſtat ex quatuor nominibus in vinculo radicali & dantur tres arearum, vel ſi conſtat ex quinq; nominibus & dantur quatuor arearum, & ſic deinceps: dabuntur areæ omnes quæ addendo vel ſubducendo numerum η indici θ vel unitatem indici λ generari poſſunt. Et par eſt ratio Curvarum ubi ordinatæ ex binomiis conflantur, & area una earum quæ non ſunt geometrice quadrabiles datur. Q. E. O.

PROP. VIII. THEOR. VI.

Si pro $e + fz^{\eta} + gz^{2\eta} + \&c.$ & $k + lz^{\eta} + mz^{2\eta} + \&c.$ ſcribantur R & S ut ſupra, & in Curvæ alicujus Ordinata $z^{\theta \pm \eta\sigma} R^{\lambda \pm \tau} S^{\mu \pm \upsilon}$ maneant quantitates datæ θ,

$\pm n\sigma$

η, λ, μ, e, f, g, k, l, m, &c. & pro σ, τ, & υ, ſcribantur ſucceſſive numeri quicunq; integri: & ſi dentur areæ duarum ex curvis quæ per ordinatas ſic prodeuntes deſignantur ſi quantitates R & S ſunt binomia, vel ſi dentur areæ trium ex curvis ſi R & S conjunctim ex quinq; nominibus conſtant, vel areæ quatuor ex curvis ſi R & S conjunctim ex ſex nominibus conſtant, & ſic deinceps in infinitum: dico quod dabuntur areæ curvarum omnium.

Demonſtratur ad modum Propoſitionis ſuperioris.

PROP.

PROP. IX. THEOR. VII.

Æquantur Curvarum areæ inter ſe quarum Ordinatæ ſunt reciproce ut fluxiones Abſciſſarum.

Nam contenta ſub Ordinatis & fluxionibus Abſciſſarum erunt æqualia, & fluxiones arearum ſunt ut hæc contenta.

COROL. I.

Si aſſumatur relatio quævis inter Abſciſſas duarum Curvarum, & inde per Prop. 1. quæratur relatio fluxionum Abſciſſarum, & ponantur Ordinatæ reciproce proportionales fluxionibus, inveniri poſſunt innumeræ Curvæ quarum areæ ſibi mutuo æquales erunt.

COROL. II.

Sic enim Curva omnis cujus hæc eſt Ordinata $z^{\theta-1}$ in $\overline{e+fz^{\eta}+gz^{2\eta}+\&c.}|^{\lambda}$ aſſumendo quantitatem quamvis pro ν & ponendo $\frac{\eta}{\nu}=s$ & $z^{s}=x$, migrat in aliam ſibi æqualem cujus ordinata eſt $\frac{\nu}{\eta}x^{\frac{\nu\theta-\eta}{\eta}}$ in $\overline{e+fx^{\nu}+gx^{2\nu}+\&c.}|^{\lambda}$.

CO-

COROL. III.

Et Curva omnis cujus Ordinata est $z^{\theta-1}$ in $\overline{a+bz^{\eta}+cz^{2\eta}+\&c.} \times \overline{e+fz^{\eta}+gz^{2\eta}\&c.}|^{\lambda}$, assumendo quantitatem quamvis pro ν & ponendo $\frac{\eta}{\nu}=s$ & $z^{s}=x$, migrat in aliam sibi æqualem cujus ordinata est $\frac{\nu}{\eta}x^{\frac{\nu\theta-\eta}{\eta}}$ in $\overline{a+bx^{\nu}+cx^{2\nu}+\&c.}\times\overline{e+fx^{\nu}+gx^{2\nu}+\&c.}|^{\lambda}$.

COROL. IV.

Et Curva omnis cujus Ordinata est $z^{\theta-1}$ in $\overline{a+bz^{\eta}+cz^{2\eta}+\&c.} \times \overline{e+fz^{\eta}+gz^{2\eta}+\&c.}|^{\lambda}$ $\times\overline{k+lz^{\eta}+mz^{2\eta}+\&c.}|^{\mu}$, assumendo quantitatem quamvis pro ν & ponendo $\frac{\eta}{\nu}=s$ & $z^{s}=x$, migrat in aliam sibi æqualem cujus ordinata est $\frac{\nu}{\eta}x^{\frac{\nu\theta-\eta}{\eta}}$ in $\overline{a+bx^{\nu}+cx^{2\nu}+\&c.}\times\overline{e+fx^{\nu}+gx^{2\nu}+\&c.}|^{\lambda}\times\overline{k+lx^{\nu}+mx^{2\nu}+\&c.}|^{\mu}$

COROL. V.

Et Curva omnis cujus Ordinata est $z^{\theta-1}$ in $\overline{e+fz^{\eta}+gz^{2\eta}+\&c.}|^{\lambda}$ ponendo $\frac{1}{z}=x$ migrat in aliam sibi æqualem cujus ordinata est $\frac{1}{x^{\theta+1}}\times\overline{e+fx^{-\eta}+gx^{-2\eta}+\&c.}|^{\lambda}$ id est $\frac{1}{x^{\theta+1+\eta\lambda}}\times\overline{f+ex^{\eta}}|^{\lambda}$ si duo sunt

$2\eta\lambda$

nomina in vinculo radicis vel $\frac{1}{x^{\theta+1+\eta\lambda}}\times\overline{g+fx^{\eta}+ex^{2\eta}}|^{\lambda}$ si tria sunt nomina ; & sic deinceps.

CO-

COROL. VI.

Et Curva omnis cujus Ordinata est $z^{\theta-1}$ in $\overline{e+fz^{\eta}+gz^{2\eta}+\&c.}|^{\lambda}\times\overline{k+lz^{\eta}+mz^{2\eta}+\&c.}|^{\mu}$ ponendo $\frac{1}{z}=x$ migrat in aliam sibi æqualem cujus ordinata est $\frac{1}{x^{\theta+1}}\times\overline{e+fx^{-\eta}+gx^{-2\eta}+\&c.}|^{\lambda}$ $\times\overline{k+lx^{-\eta}+mx^{-2\eta}+\&c.}|^{\mu}$ id est $\frac{1}{x^{\theta+1+\eta\lambda+\eta\mu}}\times\overline{f+ex^{\eta}}|^{\lambda}$ $\times\overline{l+kx^{\eta}}|^{\mu}$ si bina sunt nomina in vinculis radicum, vel $\frac{1}{x^{\theta+1+2\eta\lambda+\eta\mu}}\times\overline{g+fx^{\eta}+ex^{2\eta}}|^{\lambda}\times\overline{l+kx^{\eta}}|^{\mu}$ si tria sunt nomina in vinculo radicis prioris ac duo in vinculo posterioris: & sic in aliis. Et nota quod areæ duæ æquales in novissimis hisce duobus Corollariis jacent ad contrarias partes ordinatarum. Si area in alterutra curva adjacet abscissæ, area huic æqualis in altera curva adjacet abscissæ productæ.

COROL. VII.

Si relatio inter Curvæ alicujus Ordinatam y & Abscissam z definiatur per æquationem quamvis a fectam hujus formæ, y^{α} in $e+fy^{\eta}z^{\delta}+gy^{2\eta}z^{2\delta}+hy^{3\eta}z^{3\delta}$ $+\&c.=z^{\beta}$ in $k+ly^{\eta}z^{\delta}+my^{2\eta}z^{2\delta}+\&c.$ hæc figura assumendo $s=\frac{\eta-\delta}{\eta}$, $x=\frac{1}{s}z^{s}$ & $\lambda=\frac{\eta-\delta}{\alpha\delta+\beta\eta}$, migrat in aliam sibi æqualem cujus Abscissa x, ex data

Ordinata

Ordinata v, determinatur per æquationem non affectam $\frac{1}{s}v^{\alpha\lambda} \times \overline{e + fv^{n} + gv^{2n} + hv^{3n} + \&c.}|^{\lambda} \times \overline{k + lv^{n} + mv^{2n} + \&c.}|^{-\lambda} = x$.

COROL. VIII.

Si relatio inter Curvæ alicujus Ordinatam y & Abscissam z definitur per æquationem quamvis affectam hujus formæ, y^{α} in $\overline{e + fy^{n}z^{\delta} + gy^{2n}z^{2\delta} + \&c.} = z^{\beta}$ in $\overline{k + ly^{n}z^{\delta} + my^{2n}z^{2\delta} + \&c.} + z^{\gamma}$ in $\overline{p + qy^{n}z^{\delta} + ry^{2n}z^{2\delta} + \&c.}$ hæc figura assumendo $s = \frac{n-\delta}{n}$, $x = \frac{1}{s}z^{s}$, $\mu = \frac{\alpha\delta + \beta n}{n-\delta}$ & $\nu = \frac{\alpha\delta + \gamma n}{n-\delta}$, migrat in aliam sibi æqualem cujus Abscissa x ex data Ordinata v determinatur per æquationem minus affectam v^{α} in $e + fv^{n} + gv^{2n} + \&c. = s^{\mu}x^{\mu}$ in $k + lv^{n} + mv^{2n} + \&c. + s^{\nu}x^{\nu}$ in $p + qv^{n} + rv^{2n} + \&c.$

COROL. IX.

Curva omnis cujus Ordinata est $\pi z^{\theta-1}$ in $\overline{\nu e + \nu{+}n\, fz^{n} + \nu{+}2n\, gz^{2n} + \&c.} \times \overline{e + fz^{n} + gz^{2n}\ \&c.}|^{\lambda-1} \times \overline{a + b\,\overline{ez^{\nu} + fz^{\nu+n} + gz^{\nu+2n} + \&c.}|^{\tau}}|^{\omega}$, si sit $\theta = \nu\lambda$ & assumantur $x = \overline{ez^{\nu} + fz^{\nu+n} + gz^{\nu+2n} + \&c.}|^{\pi}$, $\sigma = \frac{\tau}{\pi}$ & $\vartheta = \frac{\lambda-\pi}{\pi}$, migrat in aliam sibi æqualem cujus ordinata est $x^{\vartheta} \times \overline{a + bx^{\sigma}}|^{\omega}$. Et nota quod ordinata prior in

$z^{\nu+2n}$

in hoc Corollario evadit ſimplicior ponendo $\lambda = 1$, vel ponendo $\tau = 1$ & efficiendo ut radix dignitatis extrahi poſſit cujus index eſt ω, vel etiam ponendo $\omega = -1$ & $\lambda = 1 = \tau = \sigma = \pi$, ut alios caſus prætaeream.

COROL. X.

Pro $ez^{\nu} + fz^{\nu+\eta} + gz^{\nu+2\eta} +$ &c. $\nu e z^{\nu-1} + \overline{\nu+\eta}\, fz^{\nu+\eta-1} + \overline{\nu+2\eta}\, gz^{\nu+2\eta-1} +$ &c. $k + lz^{\eta} + mz^{2\eta} +$ &c. & $\eta l z^{\eta-1} + 2\eta m z^{2\eta-1} +$ &c. ſcribantur R, $\dot{r}$, S & $\dot{s}$ reſpective, & Curva omnis cujus ordinata eſt $\pi S\dot{r} + \varphi R\dot{s}$ in $R^{\lambda-1} S^{\mu-1} \times \overline{aS^{\upsilon} + bR^{\tau}}|^{\omega}$, ſi fit $\frac{\mu - \upsilon\omega}{\lambda} = \frac{\upsilon}{\tau} = \frac{\varphi}{\pi}$, $\frac{\tau}{\pi} = \sigma$, $\frac{\lambda - \pi}{\pi} = \vartheta$, & $R^{\pi} S^{\varphi} = x$, migrat in aliam ſibi æqualem cujus ordinata eſt $x^{\vartheta} \times \overline{a + bx^{\sigma}}|^{\omega}$. Et nota quod Ordinata prior evadit ſimplicior, ponendo unitates pro τ, υ, & λ vel μ, & faciendo ut radix dignitatis extrahi poſſit cujus index eſt ω, vel ponendo $\omega = -1$ vel $\mu = 0$.

PROP. X. PROB. III.

Invenire figuras ſimpliciſſimas cum quibus Curva quævis geometrice comparari poteſt, cujus ordinatim applicata y per æquationem non affectam ex data abſciſſa z determinatur.

C A S. I.

Sit Ordinata $az^{\theta-1}$, & area erit $\frac{1}{\theta}az^{\theta}$, ut ex Prop. V. ponendo $b=o=c=d=f=g=h$ & $e=1$, facile colligitur.

C A S. II.

Sit Ordinata $az^{\theta-1} \times \overline{e + fz^{n} + gz^{2n}}|^{\lambda-1} + \&c.$ & si curva cum figuris rectilineis geometrice comparari potest, quadrabitur per Prop. V. ponendo $b=o=c=d$. Sin minus convertetur in aliam curvam sibi æqualem cujus Ordinata est $\frac{a}{n}x^{\frac{\theta-n}{n}} \times \overline{e + fx + gx^{2} \&c.}|^{\lambda-1}$ per Corol. 2. Prop. IX. Deinde si de dignitatum indicibus $\frac{\theta-n}{n}$ & $\lambda-1$ per Prop. VII. rejiciantur unitates donec dignitates illæ fiant quam minimæ, devenietur ad figuras simplicissimas quæ hac ratione colligi possunt. Dein harum unaquæq; per Corol. 5. Prop. IX. dat aliam quæ nonnunquam simplicior est. Et ex his per Prop. III. & Corol. 9 & 10, Prop. IX. inter se collatis, figuræ adhuc simpliciores quandoq; prodeunt. Deniq; ex figuris simplicissimis assumptis facto regressu computabitur area quæsita.

CAS. III.

Sit Ordinata $z^{\theta-1} \times \overline{a + bz^{\eta} + cz^{2\eta} + \&c.} \times \overline{e + fz^{\eta} + gz^{2\eta} + \&c.}|^{\lambda-1}$, & hæc figura ſi quadrari poteſt, quadrabitur per Prop. V. Sin minus, diſtinguenda eſt ordinata in partes $z^{\theta-1} \times a \times \overline{e + fz^{\eta} + gz^{2\eta} + \&c.}|^{\lambda-1}$, $z^{\theta-1} \times bz^{\eta} \times \overline{e + fz^{\eta} + gz^{2\eta} + \&c.}|^{\lambda-1}$, &c. & per Caſ. 2. inveniendæ ſunt figuræ ſimpliciſſimæ cum quibus figuræ partibus illis reſpondentes comparari poſſunt. Nam areæ figurarum partibus illis reſpondentium ſub ſignis ſuis + & — conjunctæ component aream totam quæſitam.

CAS. IV.

Sit Ordinata $z^{\theta-1} \times \overline{a + bz^{\eta} + cz^{2\eta} + \&c.} \times \overline{e + fz^{\eta} + gz^{2\eta} + \&c.}|^{\lambda-1} \times \overline{k + lz^{\eta} + mz^{2\eta} + \&c.}|^{\mu-1}$: & ſi Curva quadrari poreſt, quadrabitur per Prop. VI. Sin minus, convertetur in ſimpliciorem per Corol. 4. Prop. IX. ac deinde comparabitur cum figuris ſimpliciſſimis per Prop. VIII. & Corol. 6, 9 & 10. Prop. IX. ut fit in Caſu 2 & 3.

CAS. V.

Si Ordinata ex variis partibus conſtat, partes ſingulæ pro ordinatis curvarum totidem habendæ ſunt, & curvæ illæ quotquot quadrari poſſunt, ſigillatim quadrandæ ſunt, earumq; ordinatæ de ordinata tota demendæ. Dein Curva quam ordinatæ pars reſidua deſignat ſeorſim (ut in Caſu 2, 3 & 4,) cum figuris ſimpliciſſimis comparanda eſt cum quibus comparari poteſt. Et ſumma arearum omnium pro area Curvæ propoſitæ habenda eſt.

COROL. I.

Hinc etiam Curva omnis cujus Ordinata eſt radix quadratica affecta æquationis ſuæ, cum figuris ſimpliciſſimis ſeu rectilineis ſeu curvilineis comparari poteſt. Nam radix illa ex duabus partibus ſemper conſtat quæ ſeorſim ſpectatæ non ſunt æquanum radices affectæ. Proponatur æquatio $aayy + zzyy = 2a^3y + {}^2z^3y - z^4$, & extracta radix erit $y = \frac{a^3 + z^3 + a\sqrt{a^4 + {}^2az^3 - z^4}}{aa + zz}$ cujus pars rationalis $\frac{a3 + z3}{aa + zz}$ & pars irrationalis $\frac{a\sqrt{a^4 + 2az^3 - z^4}}{aa + zz}$ ſunt ordinatæ curvarum quæ per hanc Propoſitionem vel quadrari poſſunt vel cum figuris ſimpliciſſimis comparari cum quibus collationem geometricam admittunt.

COROL. II.

Et curva omnis cujus Ordinata per æquationem quamvis affectam definitur quæ per Corol. 7. Prop. IX. in æquationem non affectam migrat, vel quadratur

dratur per hanc Propositionem si quadrari potest vel comparatur cum figuris simplicissimis cum quibus compari potest. Et hac ratione Curva omnis quadratur cujus æquatio est trium terminorum. Nam æquatio illa si affecta sit transmutatur in non affectam per Corol. 7. Prop. IX. ac deinde per Corol. 2 & 5. Prop. IX. in simplicissimam migrando, dat vel quadraturam figuræ si quadrari potest, vel curvam simplicissimam quacum comparatur.

COROL. III.

Et Curva omnis cujus Ordinata per æquationem quamvis affectam definitur quæ per Corol. 8. Prop. IX. in æquationem quadraticam affectam migrat; vel quadratur per hanc Propositionem & hujus Corol. 1. si quadrari potest, vel comparatur cum figuris simplicissimis cum quibus collationem geometricam admittit.

SCHOLIUM.

Ubi quadrandæ sunt figuræ; ad Regulas hasce generales semper recurrere nimis molestum esset: præstat Figuras quæ simpliciores sunt & magis usui esse possunt semel quadrare & quadraturas in Tabulam referre, deinde Tabulam consulere quoties ejusmodi Curvam aliquam quadrare oportet. Hujus autem generis sunt Tabulæ duæ sequentes, in quibus z denotat Abscissam, y Ordinatam rectangulam

gulam & t Aream Curvæ quadrandæ, & d, e, f, g, g. h, n sunt quantitates datæ cum signis suis $+$ & $-$.

TABULA

Curvarum simpliciorum quæ quadrari possunt.

Curvarum formæ.	Curvarum areæ.
Forma prima.	
$dz^{n-1} = y.$	$\frac{d}{n} z^n = t.$
Forma secunda.	
$\frac{dz^{n-1}}{ee + 2efz^n + ffzz^n} = y.$	$\frac{dz^n}{nee + nefz^n} = t$, vel $\frac{-d}{nef + nffz^n} = t.$
Forma tertia.	
1. $dz^{n}_{-1}\sqrt{e + fz^n} = y.$	$\frac{2d}{3nf} R^3 = t$, existente $R = \sqrt{e + fz^n}$
2. $dz^{2n}_{-1}\sqrt{e + fz^n} = y.$	$\frac{-4e + 6fz^n}{15nff} dR^3 = t.$
3. $dz^{3n}_{-1}\sqrt{e + fz^n} = y.$	$\frac{16ee - 24efz^n + 30ffz^{2n}}{105nf^3} dR^3 = t.$
4. $dz^{4n}_{-1}\sqrt{e + fz^n} = y.$	$\frac{-96e^3 + 144eefz^n - 180effz^{2n} + 210f^3z^{3n}}{945nf^4} dR^3 = t.$
Forma quarta.	
1. $\frac{dz^{n-1}}{\sqrt{e + fz^n}} = y.$	$\frac{2d}{nf} R = t.$
2. $\frac{dz^{2n-1}}{\sqrt{e + fz^n}} = y.$	$\frac{-4e + 2fz^n}{3nff} dR = t.$

dz^{3n-1}

3. $\frac{dz^{3\eta-1}}{\sqrt{e+fz^{\eta}}} = y.$ $\frac{16ee-8efz^{\eta}+6ffz^{2\eta}}{15\eta f^{3}} dR = t.$

4. $\frac{dz^{4\eta-1}}{\sqrt{e+fz^{\eta}}} = y.$ $\frac{-96e^{3}+48eefz^{\eta}-36effz^{2\eta}+30f^{3}z^{3\eta}}{105\eta f^{4}} dR = t.$

TABULA

Curvarum simpliciorum quæ cum Ellipsi & Hyperbola compari possunt.

ra

Sit jam aGD vel PGD vel GDS Sectio Conica cujus area ad Quadraturam Curvæ pro- *Fig.* 5,6,7,8.
positæ requiritur, sitq; ejus centrum A, Axis Ka, Vertex a, Semiaxis conjugatus AP, datum Abscissæ principium A vel a vel α, Abscissa AB vel aB vel αB = x, Ordinata rectangula BD = v, & Area ABDP vel aBDG vel αBDG = s, existente αG Ordinata ad punctum α. Jungantur KD, AD, aD. Ducatur Tangens DT occurrens Abscissæ AB in T, & compleatur parallelogrammum ABDO. Et siquando ad quadraturam Curvæ propositæ requiruntur areæ duarum Sectionem Conicarum, dicatur posterioris Abscissa ξ, Ordinata Υ, & Area σ. Sit autem $\div$ differentia duarum quantitatum ubi incertum est utrum posterior de priori an prior de posteriori subduci debeat.

Curva-

	Curvarum Formæ.	Sectionis Conicæ. Abſciſſa.	Sectionis Conicæ. Ordinata.	Curvarum Areæ.
	Forma prima.			
Fig. 5.	1. $\frac{dz^{n-1}}{e+fz^{n}} = y.$	$z^{n} = x.$	$\frac{d}{e+fx} = v.$	$\frac{1}{n}s = t = \frac{\alpha GDB}{n}$
	2. $\frac{dz^{2n-1}}{e+fz^{n}} = y.$	$z^{n} = x.$	$\frac{d}{e+fx} = v.$	$\frac{d}{nf} z^{n} - \frac{e}{nf} s = t.$
	3. $\frac{dz^{3n-1}}{e+fz^{n}} = y.$	$z^{n} = x.$	$\frac{d}{e+fx} = v.$	$\frac{d}{2nf} z^{2n} - \frac{de}{nff} z^{n} + \frac{ee}{nff} s = t.$
	Forma ſecunda.			
Fig. 6, 7.	1. $\frac{dz^{\frac{1}{2}n-1}}{e+fz^{n}} = y.$	$\sqrt{\frac{d}{e+fz^{n}}} = x.$	$\sqrt{\frac{d}{f} - \frac{e}{f}xx} = v.$	$\frac{2xv \div 4s}{n} = t = \frac{4}{n} ADGa.$
	2. $\frac{dz^{\frac{3}{2}n-1}}{e+fz^{n}} = y.$	$\sqrt{\frac{d}{e+fz^{n}}} = x.$	$\sqrt{\frac{d}{f} - \frac{e}{f}xx} = v.$	$\frac{2de}{nf} z^{\frac{n}{2}} + \frac{4es - 2exv}{nf} = t.$
	3. $\frac{dz^{\frac{5}{2}n-1}}{e+fz^{n}} = y.$	$\sqrt{\frac{d}{e+fz^{n}}} = x.$	$\sqrt{\frac{d}{f} - \frac{e}{f}xx} = v.$	$\frac{2de}{3nf} z^{\frac{3n}{2}} - \frac{2dee}{nff} z^{\frac{n}{2}} + \frac{2eexv - 4ees}{nff} = t.$

Forma

Forma tertia.

*Fig.*6,7,8. 1. $\frac{d}{z}\sqrt{e+fz^{\eta}}=y.\ \frac{1}{z^{\eta}}=xx.\ \sqrt{f+exx}=v.\ \frac{4de}{\eta f}$ in $\frac{v^{3}}{2ex}-s=t=\frac{4de}{\eta f}$ in aGDT, vel in APDB÷TDB.

Vel sic, $\frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{8dee}{\eta ff}$ in $s-\frac{1}{2}xv-\frac{fv}{4e}+\frac{ffv}{4eex}=t=\frac{8dee}{\eta ff}$ in aGDA $+\frac{ffv}{4eex}$.

2. $\frac{d}{z^{\eta+1}}\sqrt{e+fz^{\eta}}=y.\ \frac{1}{z^{\eta}}=xx.\ \sqrt{f+exx}=v.\ \frac{-2d}{\eta}s=t=\frac{2d}{\eta}$APDB, seu $\frac{2d}{\eta}$aGDB.

Vel sic, $\frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{4de}{\eta f}$ in $s-\frac{1}{2}xv-\frac{fv}{2e}=t=\frac{4de}{\eta f}\times$ aGDK.

3. $\frac{d}{z^{2\eta+1}}\sqrt{e+fz^{\eta}}=y.\ \frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{-d}{\eta}s=t=\frac{d}{\eta}\times-$aGDB vel BDPK.

4. $\frac{d}{z^{3\eta+1}}\sqrt{e+fz^{\eta}}=y.\ \frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{3dfs-2dv^{3}}{6\eta e}=t.$

Forma quarta.

Fig. 6. 1. $\frac{d}{z\sqrt{e+fz^{\eta}}}=y.\ \frac{1}{z^{\eta}}=xx.\ \sqrt{f+exx}=v.\ \frac{4d}{\eta f}$ in $\frac{1}{2}xv\div s=t=\frac{4d}{\eta f}$ in PAD vel in aGDA.

Vel sic, $\frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{8de}{\eta ff}$ in $s-\frac{1}{2}xv-\frac{fv}{4e}=t=\frac{8de}{\eta ff}$ in aGDA.

2. $\frac{d}{z^{\eta+1}\sqrt{e+fz^{\eta}}}=y.\ \frac{1}{z^{\eta}}=xx.\ \sqrt{f+exx}=v.\ \frac{2d}{\eta e}$ in $s-xv=t=\frac{2d}{\eta e}$ in POD, vel in AODGa.

Vel sic, $\frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{4d}{\eta f}$ in $\frac{1}{2}xv\div s=t=\frac{4d}{\eta f}$ in aDGa.

3. $z^{2\eta}_{+1}$

3. $\frac{d}{z^{2\eta}_{+1}\sqrt{e+fz^{\eta}}}=y.\ \frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{d}{\eta e}$ in $3s\div 2xv=t=\frac{d}{\eta e}$ in $3aDGa\div\triangle aDB$.

4. $\frac{d}{z^{3\eta}_{+1}\sqrt{e+fz^{\eta}}}=y.\ \frac{1}{z^{\eta}}=x.\ \sqrt{fx+exx}=v.\ \frac{10dfxv-15dfs-2dexxv}{6\eta ee}=t.$

Forma quinta.

1. $\frac{dz^{\eta-1}}{e+fz^{\eta}+gz^{2\eta}}=y.\ \sqrt{\frac{d}{e+fz^{\eta}+gz^{2\eta}}}=x.\ \sqrt{\frac{d}{g}+\frac{ff-4eg}{4gg}xx}=v.\ \frac{xv-2s}{\eta}=t.$

Vel sic, $\sqrt{\frac{dz^{2\eta}}{e+fz^{\eta}+gz^{2\eta}}}=x.\ \sqrt{\frac{d}{e}+\frac{ff-4eg}{4ee}xx}=v.\ \frac{2s-xv}{\eta}=t.$

2. $\frac{dz^{2\eta-1}}{e+fz^{\eta}+gz^{2\eta}}=y.\begin{cases}\sqrt{\frac{d}{e+fz^{\eta}+gz^{2\eta}}}=x.\ \sqrt{\frac{d}{g}+\frac{ff-4eg}{4gg}xx}=v.\\ fz^{\eta}+gz^{2\eta}=\xi.\ \frac{1}{e+\xi}=\tau.\end{cases}\frac{d\sigma+2fs-fxv}{2\eta g}=t.$

Forma sexta, ubi scribitur p pro $\sqrt{ff-4eg}$.

1. $\frac{dz^{\frac{1}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}}=y.\begin{cases}\sqrt{\frac{2dg}{f-p+2gz^{\eta}}}=x.\ \sqrt{d+\frac{-f+p}{2g}xx}=v.\\ \sqrt{\frac{2dg}{f+p+2gz^{\eta}}}=\xi.\ \sqrt{d+\frac{-f-p}{2g}\xi\xi}=\tau.\end{cases}\frac{2xv-4s-2\xi\tau+4\sigma}{\eta p}=t.$

2. $\frac{dz^{\frac{3}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}}=y.\begin{cases}\sqrt{\frac{2dez^{\eta}}{fz^{\eta}-pz^{\eta}+2e}}=x.\ \sqrt{d+\frac{-f+p}{2e}xx}=v.\\ \sqrt{\frac{2dez^{\eta}}{fz^{\eta}+pz^{\eta}+2e}}=\xi.\ \sqrt{d+\frac{-f-p}{2e}\xi\xi}=\tau.\end{cases}\frac{4s-2xv-4\sigma+2\xi\tau}{\eta p}=t.$

Forma

Forma septima.

$-2dfgxv+4degv$

1. $\frac{d}{z}\sqrt{e+fz^{n}+gz^{2n}}=y.\left\{\begin{matrix} z^{n}=x.\ \sqrt{e+fx+gxx}=v. \\ \frac{1}{z^{n}}=\xi.\ \sqrt{g+f\xi+e\xi\xi}=\gamma. \end{matrix}\right\}\frac{4dee\xi\gamma+2def\gamma-2dffv-8dee\sigma+4dfgs}{4neg-nff}=t.$

Fig. 6,7. 2. $dz^{n}_{-1}\sqrt{e+fz^{n}+gz^{2n}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{d}{n}s=t=\frac{d}{n}$ in αGDB.

3. $dz^{2n}_{-1}\sqrt{e+fz^{n}+gz^{2n}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{d}{3ng}v^{3}-\frac{df}{2ng}s=t.$

4. $dz^{3n}_{-1}\sqrt{e+fz^{n}+gz^{2n}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{6dgx-5df}{24ngg}v^{3}+\frac{5dff-4deg}{16ngg}s=t.$

Forma octava.

Fig. 6. 1. $\frac{dz^{n-1}}{\sqrt{e+fz^{n}+gz^{2n}}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{8dgs-4dgxv-2dfv}{4neg-nff}=t=\frac{8dg}{4neg-nff}$ in αGDB $\pm\Delta$DBA.

2. $\frac{dz^{2n-1}}{\sqrt{e+fz^{n}+gz^{2n}}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{-4dfs+2dfxv+4dev}{4neg-nff}=t.$

3. $\frac{dz^{3n-1}}{\sqrt{e+fz^{n}+gz^{2n}}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{\begin{smallmatrix}3dff\\-4deg\end{smallmatrix}s\ \begin{smallmatrix}-2dff\\+4deg\end{smallmatrix}xv-2defv}{4negg-nffg}=t.$

$-15df3 \quad +10df3 \quad \begin{smallmatrix}+10deff\\-16deeg\end{smallmatrix}$ 4. $\frac{dz^{4n-1}}{\sqrt{e+fz^{n}+gz^{2n}}}=y.\ z^{n}=x.\ \sqrt{e+fx+gxx}=v.\ \frac{\begin{smallmatrix}36defg\\-15deff\end{smallmatrix}s\ \begin{smallmatrix}+8degg\\-2dffg\end{smallmatrix}xxv\ \begin{smallmatrix}-28defg\\+10deff\end{smallmatrix}xv\ \begin{smallmatrix}-16deeg\\+10deff\end{smallmatrix}v}{24neg3-6nffg}=t.$

Forma

Forma nona.

$$1.\ \frac{dz_{-1}^{\eta}\sqrt{e+fz^{\eta}}}{g+hz^{\eta}}=y.\ \sqrt{\frac{d}{g+hz^{\eta}}}=x.\ \sqrt{\frac{df}{h}+\frac{eh-fg}{h}xx}=v.\ \frac{\substack{4fg\\-4eh}s\substack{-2fg\\+2eh}xv+\frac{2dfv}{x}}{\eta fh}=t.$$

$$2.\ \frac{dz_{-1}^{2\eta}\sqrt{e+fz^{\eta}}}{g+hz^{\eta}}=y.\sqrt{\frac{d}{g+hz^{\eta}}}=x.\sqrt{\frac{df}{h}+\frac{eh-fg}{h}xx}=v.\frac{\substack{4egh\\-4fgg}s\substack{-2egh\\+2fgg}xv+\frac{2}{3}dh\frac{v^3}{x^3}-2dfg\frac{v}{x}}{\eta fhh}=t.$$

Forma decima.

Fig. 6,7. $$1.\ \frac{dz^{\eta-1}}{g+hz^{\eta}\ \sqrt{e+fz^{\eta}}}=y.\ \sqrt{\frac{d}{g+hz^{\eta}}}=x.\ \sqrt{\frac{df}{h}+\frac{eh-fg}{h}xx}=v.\ \frac{2xv-4s}{\eta f}=t=\frac{4}{\eta f}ADGa.$$

$$2.\ \frac{dz^{2\eta-1}}{g+hz^{\eta}\ \sqrt{e+fz^{\eta}}}=y.\ \sqrt{\frac{d}{g+hz^{\eta}}}=x.\ \sqrt{\frac{df}{h}+\frac{eh-fg}{h}xx}=v.\ \frac{4gs-2gxv+\frac{2dv}{x}}{\eta fh}=t.$$

Forma undecima.

$$1.\ dz^{-1}\sqrt{\frac{e+fz^{\eta}}{g+hz^{\eta}}}=y.\left\{\begin{array}{l}\sqrt{g+hz^{\eta}}=x.\ \sqrt{\frac{eh-fg}{h}+\frac{f}{h}xx}=v.\\ \sqrt{h+gz^{-\eta}}=\xi.\ \sqrt{\frac{fg-eh}{g}+\frac{e}{g}\xi\xi}=\Upsilon.\end{array}\right\}\frac{dxv^3z^{-\eta}-4dfs-4de\sigma}{\eta fg-\eta eh}=t.$$

$$2.\ dz_{-1}^{\eta}\sqrt{\frac{e+fz^{\eta}}{g+gz^{\eta}}}=y.\ \sqrt{g+hz^{\eta}}=x.\ \sqrt{\frac{eh-fg}{h}+\frac{f}{h}xx}=v.\ \frac{2d}{\eta h}s=t.$$

$$3.\ dz_{-1}^{2\eta}\sqrt{\frac{e+fz^{\eta}}{g+hz^{\eta}}}=y\ \sqrt{g+hz^{\eta}}=x.\ \sqrt{\frac{eh-fg}{h}+\frac{f}{h}xx}=v.\ \frac{dhxv^3\substack{-3dfg\\-deh}s}{2\eta fhh}=t.$$

In Tabulis hisce, series Curvarum cujusq; formæ utrinq; in infinitum continuari potest. Scilicet in Tabula prima, in numeratoribus arearum formæ tertiæ & quartæ, numeri coefficientes initialium terminorum (2, —4, 16, —96, 868, &c.) generantur multiplicando numeros —2, —4, —6, —8, —10, &c. in se continuo, & subsequentium terminorum coefficientes ex initialibus derivantur multiplicando ipsos gradatim, in Forma quidem tertia, per $-\frac{3}{2}$, $-\frac{5}{4}$, $-\frac{7}{6}$, $-\frac{9}{8}$, $-\frac{11}{10}$ &c. in quarta vero per $-\frac{1}{2}$, $-\frac{3}{4}$, $-\frac{5}{6}$, $-\frac{7}{8}$, $-\frac{9}{10}$, &c. Et Denominatorum coefficientes 3, 15, 105, &c. prodeunt multiplicando numeros 1, 3, 5, 7, 9, &c. in se continuo.

In secunda vero Tabula, series Curvarum formæ primæ, secundæ, quintæ, sextæ, nonæ & decimæ ope solius divisionis, & formæ reliquæ ope Propositionis tertiæ & quartæ, utrinq; producuntur in infinitum.

Quinetiam hæ series mutando signum numeri n variari solent. Sic enim, e. g. Curva $\frac{d}{z}\sqrt{e+fz^n}=y$, evadit $\frac{d}{z^{\frac{1}{2}n+1}}\sqrt{f+ez^n}=y$

PROP. IX. THEOR. VIII.

Sit ADIC Curva quævis Abscissam habens AB $=z$ & Ordinatam BD $=y$, & sit AEKC Curva alia cujus Ordinata BE æqualis est prioris areæ *Fig. 9.*
ABC

ADB ad unitatem applicatæ, & AFLC Curva tertia cujus Ordinata BF æqualis est secundæ areæ AEB ad unitatem applicatæ, & AGMC Curva quarta cujus Ordinata BG æqualis est tertiæ areæ AFB ad unitatem applicatæ, & AHNC Curva quinta cujus Ordinata BH æqualis est quartæ areæ AGB ad unitatem applicatæ, & sic deinceps in infinitum. Et sunto A, B, C, D, E, &c. Areæ Curvarum Ordinatas habentium y, zy, z^2y, z^3y, z^4y, & Abscissam communem z.

Detur Abscissa quævis $AC=t$, sitq; $BC=t-z=x$, & sunto P, Q, R, S, T areæ Curvarum Ordinatas habentium x, xy, xxy, x^3y, x^4y & Abscissam communem x.

Terminenter autem hæ areæ omnes ad Abscissam totam datam AC, nec non ad Ordinatam positione datam & infinite productam CI: & erit arearum sub initio positarum prima $ADIC=A=P$, secunda $AEKC=tA-B=Q$. Tertia $AFLC=\frac{ttA-2tB+C}{2}=\frac{1}{2}R$. Quarta $AGMC=\frac{t^3A-3ttB+3tC-D}{6}=\frac{1}{6}S$. Quinta $AHNC=\frac{t^4A-4t^3B+6ttC-4tD+E}{24}=\frac{1}{24}T$.

CO-

COROL.

Unde ſi Curvæ quarum Ordinatæ ſunt y, zy, z^2y, z^3y, &c. vel y, xy, x^2y, x^3y, &c. quadrari poſſunt, quadrabuntur etiam Curvæ ADIC, AEKC, AFLC, AGMC, &c. & habebuntur Ordinatæ BE, BF, BG, BH areis Curvarum proportionales.

SCHOLIUM.

Quantitatum fluentium fluxiones eſſe primas, ſecundas, tertias, quartas, aliaſq; diximus ſupra. Hæ fluxiones ſunt ut termini ſerierum infinitarum convergentium. Ut ſi z^n ſit quantitas fluens & fluendo evadat $\overline{z+o}|^n$, deinde reſolvatur in ſeriem convergentem $z^n + nozn^{n-1} + \frac{nn-n}{2}ooz^{n-2} + \frac{n^3-3nn+2n}{6}o^3z^{n-3}$ + &c. terminus primus hujus ſeriei z^n erit quantitas illa fluens, ſecundus noz^{n-1} erit ejus incrementum primum ſeu differentia prima cui naſcenti proportionalis eſt ejus fluxio prima, tertius $\frac{nn-n}{2}ooz^{n-2}$ erit ejus incrementum ſecundum ſeu differentia ſecunda cui naſcenti proportionalis eſt ejus fluxio ſecunda, quartus $\frac{n^3-3nn+2n}{6}o^3z^{n-3}$ erit ejus incrementum tertium ſeu differentia tertia cui naſcenti fluxio tertia proportionalis eſt, & ſic deinceps in infinitum.

Exponi

Exponi autem poſſunt hæ fluxiones per Curvarum Ordinatas BD, BE, BF, BG, BH, &c. Ut ſi Ordinata BE $(=\frac{ADB}{1})$ ſit quantitas fluens, erit ejus fluxio prima ut ordinata BD. Si BF $(=\frac{AEB}{1})$ ſit quantitas fluens, erit ejus fluxio prima ut Ordinata BE & fluxio ſecunda ut Ordinata BD. Si BH $(=\frac{AGB}{1})$ ſit quantitas fluens, erunt ejus fluxiones, prima, ſecunda, tertia & quarta, ut Ordinatæ BG, BF, BE, BD reſpective.

Et hinc in æquationibus quæ quantitates tantum duas incognitas involvunt, quarum una eſt quantitas uniformiter fluens & altera eſt fluxio quælibet quantitatis alterius fluentis, inveniri poteſt fluens illa altera per quadraturam Curvarum. Exponatur enim fluxio ejus per Ordinatam BD, & ſi hæc ſit fluxio prima, quæratur area $ADB = BE \times 1$, ſi fluxio ſecunda, quæratur area $AEB = BF \times 1$, ſi fluxio tertia, quæratur area $AFB = BG \times 1$, &c. & area inventa erit exponens fluentis quæſitæ.

Sed & in æquationibus quæ fluentem & ejus fluxionem primam ſine altera fluente, vel duas ejuſdem fluentis fluxiones, primam & ſecundam, vel ſecundam & tertiam, vel tertiam & quartam, &c. ſine alterutra fluente involvunt: inveniri poſſunt fluentes per quadraturam Curvarum. Sit æquatio $aa\dot{v} = av + vv$, exiſtente $v = BE$, $\dot{v} = BD$, $z = AB$ & $\dot{z} = 1$, & æquatio illa complendo dimenſiones fluxionum, evadet $aa\dot{v} = av\dot{z} + vv\dot{z}$, ſeu $\frac{aa\dot{v}}{av + vv} = \dot{z}$. Jam fluat v uniformiter & ſit

ſit ejus fluxio $\dot{v}=1$ & erit $\frac{aa}{av+vv}=\dot{z}$, & quadrando Curvam cujus Ordinata eſt $\frac{aa}{av+vv}$ & Abſciſſa v, habebitur fluens z. Adhæc ſit æquatio $aa\dot{v}=a\dot{v}+\dot{v}\dot{v}$ exiſtente $v=BF$, $\dot{v}=BE$, $\ddot{v}=BD$ & $z=AB$ & per relationem inter $\ddot{v}$ & $\dot{v}$ ſeu BD & BE invenietur relatio inter A B & B E ut in exemplo ſuperiore. Deinde per hanc relationem invenietur relatio inter AB & BF quadrando Curvam AEB.

Æquationes quæ tres incognitas quantitates involvunt aliquando reduci poſſunt ad æquationes quæ duas tantum involvunt, & in his caſibus fluentes invenientur ex fluxionibus ut ſupra. Sit æquatio $a-bx^m=cxy^n\dot{y}+d\dot{y}^{2n}\dot{y}\dot{y}$. Ponatur $y^n\dot{y}=\dot{v}$ & erit $a-bx^m cx\dot{v}+d\dot{v}\dot{v}$. Hæc æquatio quadrando Curvam cujus Abſciſſa eſt x & Ordinata $\dot{v}$ dat aream v, & æquatio altera $y^n\dot{y}=\dot{v}$ regrediendo ad fluentes dat $\frac{1}{n+1}y^{n+1}=v$. Unde habetur fluens y.

Quinetiam in æquationibus quæ tres incognitas involvunt & ad æquationes quæ duas tantum involvunt reduci non poſſunt, fluentes quandoq; prodeunt per quadraturam Curvarum. Sit æquatio $\overline{ax^m+bx^n}|^p=rex^{r-1}y^s+sex^r\dot{y}y^{s-1}-f\dot{y}y^t$, exiſtente $\dot{x}=1$. Et pars poſterior $rex^{r-1}y^s+sex^r\dot{y}y^{s-1}-f\dot{y}y^t$, x=1.
regrediendo ad fluentes, fit $ex^r y^s-\frac{f}{t+1}y^{t+1}$, quæ proinde eſt ut area Curvæ cujus Abſciſſa eſt x & Ordinata $\overline{ax^m+bx^n}|^p$, & inde datur fluens y.

Sit æquatio $\dot{x} \times \overline{ax^m + bx^n}|^p = \frac{\dot{y}y^{n-1}}{\sqrt{e + fy^n}}$. Et fluens cujus fluxio est $\dot{x} \times \overline{ax^m + bx^n}|^p$ erit ut area Curvæ cujus Abſciſſa eſt x & Ordinata eſt $\overline{ax^m + bx^n}|^p$. Item fluens cujus fluxio eſt $\frac{d\dot{y}y^{n-1}}{\sqrt{e + fy^n}}$ erit ut area Curvæ cujus Abſciſſa eſt y & Ordinata $\frac{d\dot{y}y^{n-1}}{\sqrt{e + fy^n}}$, id eſt (per Caſum 1. Formæ quartæ Tab. I.) ut area $\frac{2d}{nf}\sqrt{e + fy^n}$. Pone ergo $\frac{2d}{nf}\sqrt{e + fy^n}$ æqualem areæ Curvæ cujus Abſciſſa eſt x & Ordinata $\overline{ax^m + bx^n}|^p$ & habebitur fluens y.

Et nota quod fluens omnis quæ ex fluxione prima colligitur augeri poteſt vel minui quantitate quavis non fluente. Quæ ex fluxione ſecunda colligitur augeri poteſt vel minui quantitate quavis cujus fluxio ſecunda nulla eſt. Quæ ex fluxione tertia colligitur augeri poteſt vel minui quantitate quavis cujus fluxio tertia nulla eſt. Et ſic deinceps in infinitum.

Poſtquam vero fluentes ex fluxionibus collectæ ſunt, ſi de veritate Concluſionis dubitatur, fluxiones fluentium inventarum viciſſim colligendæ ſunt & cum fluxionibus ſub initio propoſitis comparandæ. Nam ſi prodeunt æquales Concluſio recte ſe habet:

bet: sin minus, corrigendæ sunt fluentes sic, ut earum fluxiones fluxionibus sub initio propositis æquentur. Nam & Fluens pro lubitu assumi potest & assumptio corrigi ponendo fluxionem fluentis assumptæ æqualem fluxioni propositæ, & terminos homologos inter se comparando.

Et his principiis via ad majora sternitur.

FINIS.

ERRATA.

BOOK I. *Of Opticks.*

PArt 1. p.3. l.20. *Properties which*, ib.p.5. l.5. *and that C*, p.6. l.9. *DE*, p.21. l.23. *are two Rays*, p.27. l.6. *in the Margin put Fig.*14 & 15, p.30.l.7. *MN*, l.9. *M*, p. 44. l.15. *as was proposed*, p.52. l. 17. *a paper Circle*, p.57. l.ult. *emerging*, p.60. l.25. *contain with the*, p.64. l.18. *and* 14*th*, p.65. l.13. *at the*, p.66. l.3.*Semicircular*, p.67. l.25. *Center*, l.31. $4\frac{7}{8}$ *Inches*, p.68. l.8. *to* 16, l.9. *or* $5\frac{1}{3}$, p.71. l.1. *bisect*, p.72.l.13. *falls*, l.20. *being*. Part II. p.86. l.5. *lelopipede*, p.89. l.9. *made by*, p.93. l.18. *to* $77\frac{1}{3}$, l.28,29, *by the third Axiom of the first Part of this Book, the Laws*, p.105. l.5. *see represented*, p. 144. l. 24, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{16}$, $\frac{1}{9}$. p. 118, 119. for *Lib.*1. *Lib.*2. write *Part* 1. *Part* 2. p.122. l.9. *indico*, p. 130. l.19. *to the Angle*, p.132. l.6. *by the brightness*, p.135. l.14. *For if in the*, l.16. *first Part you*, p.136. l.26. *first Part*, l.27. *lights*, p.137. l.20. *green, accordingly as*, p. 138. l. 21. *Prop.*6. *Part.*2. p.139. l.5. *on which*, p.142. l.17. *XY which have been*, p.143. l.7. *purple*, l.16. *several Lights*, l.24. *of white.*

BOOK. II.

P.5. l. 5. *nicely the*, p. 7. l.9. *y*, *z denote*, l.28. *them divers*, p. 10. l. 24. 1000 *to* 1024, p.11. l.11. *oliquities*, *I.* p.17. l.4. $14\frac{1}{3}$ *to* 9, p. 25. l. 11. $10\frac{1}{5}$, p.31. l. 12. *more compounded*, p.55. l.3. *sizes reflect*, l.24. *and therefore their Colours arise*, p.65. l.5. *corpuscles can*, p.71. l.17. *given breadth*, p.84. l. 4. *are to those*, p. 96. l. 24. *Observation of this Part of this Book*, p.103. l.17. *was to the thickness*, p. 105. l. 19. *of this white Ring*, p.107. l.20. *become equal to the third of those.*

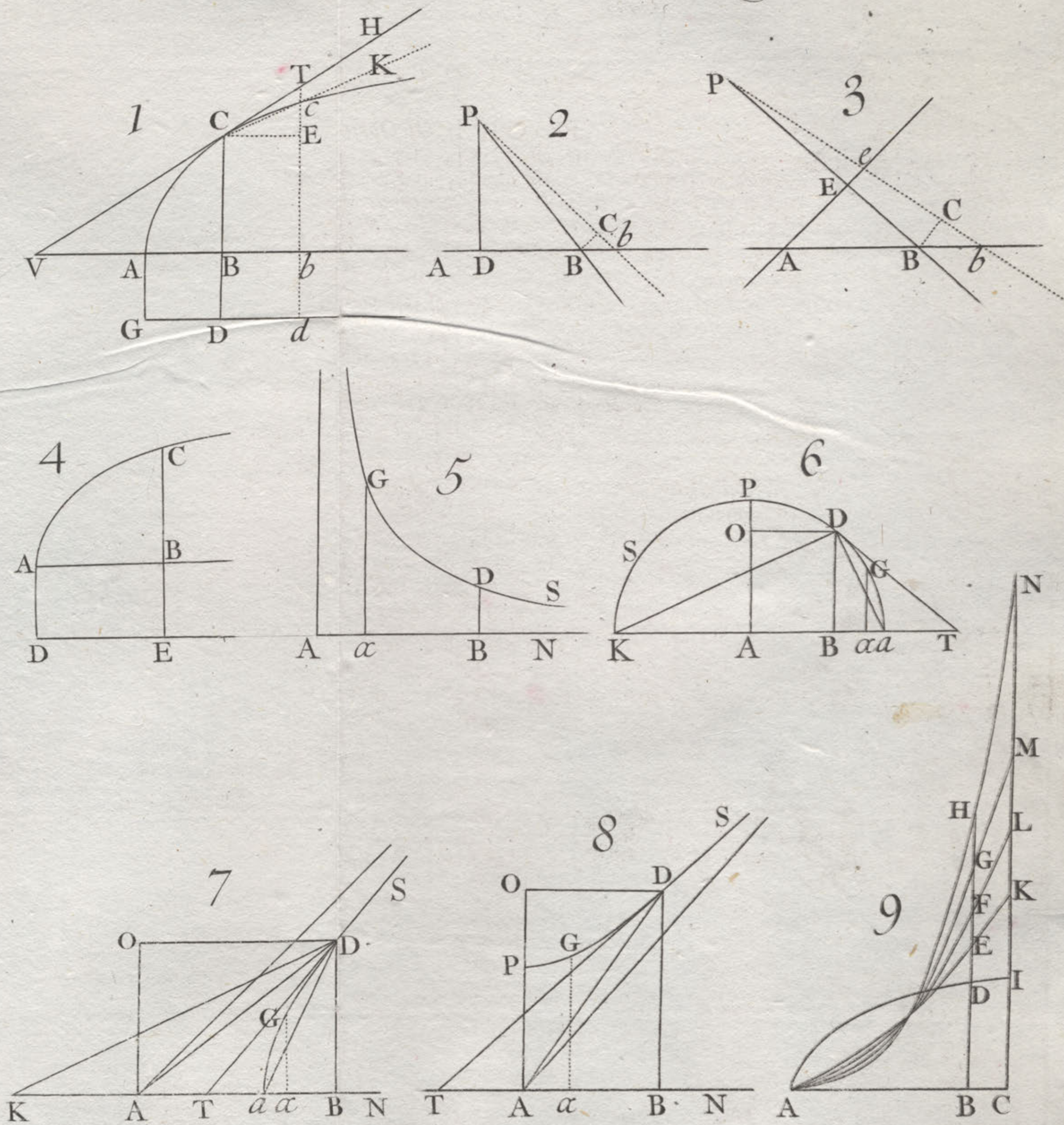
Enumeratio Linearum.

p.143. l.20. *datas signis suis*, p. 144. l.27. *respiciunt*, p. 146. l.5. *sunt Asymptoto*, p. 154. l.13. $cx+d$ *dat Ordinatam* $y =$, l.14. *quæ generatur.*

Quadratura Curvarum.

p.168. l.24. *recta AB*, p.176. l.ult. $\begin{matrix}+\theta\\+\lambda\eta\end{matrix}\,fz^{\eta}$, p.183. l.13. *a*, *b*, *c*, &c. *e*, *f*, *g*, &c. *k*, *l*, *m*, &c. p. 185. l.4. *in* $z^{\theta-1}$, p.188. l.14. $z^{\theta\pm\eta\sigma}$, p. 190. l. 19. *vel* $\frac{1}{x^{\theta-1+2\eta\lambda}}$. p. 192. l. 18. $gz^{\nu+2\eta}$. p.193. l.11. $\overline{aS\upsilon+bR\tau}/\omega$.

Quadr. Tab. I.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NEWTON'S *OPTICKS* (1704)

Annotations and corrections in Sir Isaac Newton 's *Opticks, or, A treatise of the reflexions, refractions, inflexions and colours of light : also two treatises of the species and magnitude of curvilinear figures* (London : Printed for Sam. Smith, and Benj. Walford ..., 1704), in the copy in Rare Books and Special Collections, McGill University Library.

Page numbers in brackets [] mean that the only correction on the page is the correction that listed in the *Errata* (p.212 of the second pagination).

Underlined page numbers indicate that other corrections in addition to those in the *Errata* appear on the page.

Page numbers neither underlined nor in brackets [] contain additional corrections and / or annotations.

A) Book I:

pp. 2, [3], [5], [6], 7, 8, 13, 21, [27], 28, 29, 30, 35, 37, 42, 44, 45(?), 50, [52], [57], [60], [64], 65, 66, [67], [68], [71], [72], 73, 80, [86], [89], 91, 93, 94, 102, [105], [118], [119], 122, 125, 126, [130], [132], [135], [136], [137], [138], 139, 140, [142], [143], 144.

The corrections called for in the *Errata* on page 144 have not been made; 144 is an error page 114.

B) Book II:

pp. 4, 9, 10, 12, 17, 25, 36, 47, 50, 66, 73, 74, 75, 83, 95, 96, 97, 99, 107, 115, 117, 125, 127, 131, 134, 135, 143, [144], 145, [146], 154, 155, Tab[le] III.

The corrections called for in the *Errata* were not made on the following pages: 5, 7, 11, 17, 25, 31, 55, 65, 71, 84, 103, 105, 107 in Book II.

C) Tractates:

pp. [168], [176], 180, 182, [183], [185], [188], [190 but correction incorrectly made in line 18 not 19], [192], 193, 199, 203, 205, 209 and on *Errata* leaf — Book. II. p. 10. l. 24. 10000 to 1024

D) On the verso of rear fly leaf.

6 October 2008

L.1. p.80. l.15. square square root. Lib 11. p.31 Fig 6 in margine. L.1, p.6.
DE